Models and Tools for Cyber-Physical Systems

Digital Written Exam, June the 9th 2021, 10:00-16:00

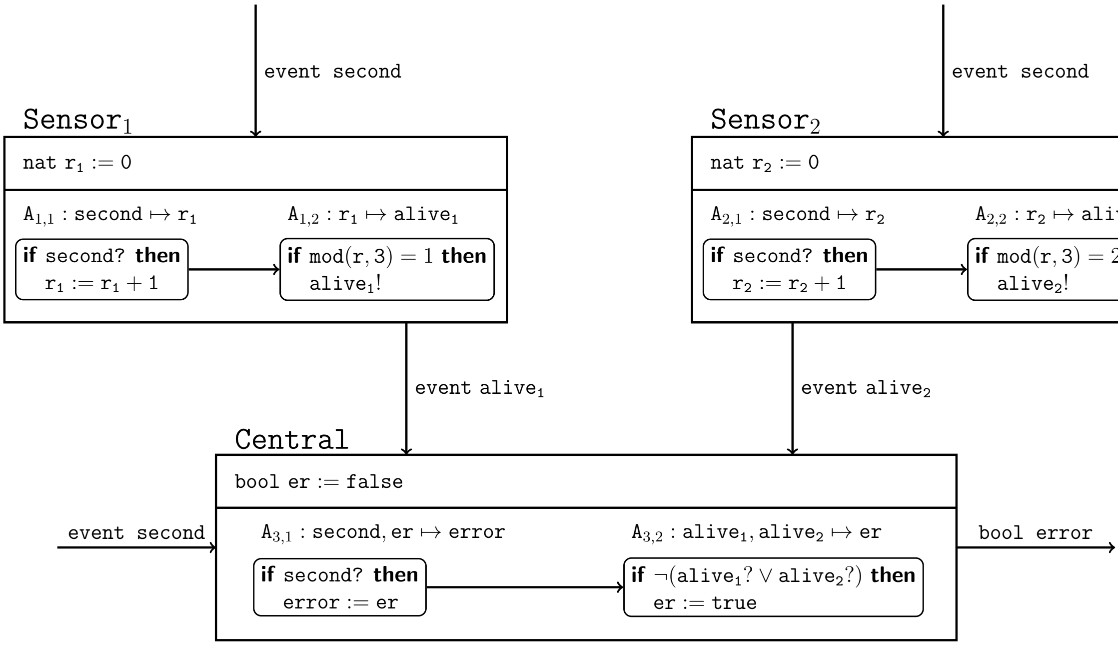
Please read the following before solving the exercises.

- This exam contains 7 exercises. Each exercise is compulsory. The solution has to be composed in English. If you believe that the assignment wording is ambiguous or erroneous, then write down what additional assumption you are using and outline your reasons.
- In some exercises you are asked to extend incomplete UPPAAL and MATLAB files. Solutions have to be uploaded to Digital Exam in the form of UPPAAL, MATLAB, pdf or text files.
- Solutions submitted as digital pictures should be of sufficient quality (high resolution, enough light, not blurry, etc.).
- Allowed aids is the course material (lecture slides and videos, exercise sheets, the book of Alur, UPPAAL/MATLAB tutorials, ect.), the software tools UPPAAL and MATLAB, and your own notes. Anything else is rendered illegal, including, in particular, Googling or asking other persons for help.
- In case of emergencies: Students can contact the instructors during the exam by approaching the study secretary, as outlined in the guidelines for online exams. Keep an eye on your student mail for potential announcements during the exam.

Last but not least, good luck!

Exercise 1: Synchronous Model – Wireless Sensor Network 10 Points

Consider the block diagram for a wireless sensor network given below. The network consists of two sensors $\texttt{Sensor}_1$ and $\texttt{Sensor}_2$ and a central unit `Central`. At every second, the component `Central` monitors the well functioning of the sensors by checking if an event $\texttt{alive}_1$ or $\texttt{alive}_2$ is present. If no event is present, it sets the state variable `er` to `true`.



(a) Consider the synchronous parallel product $\texttt{Sensor}_1 \| \texttt{Sensor}_2 \| \texttt{Central}$. Give the resulting state variables S, output variables O, input variables I, and the precedence relation $\prec$ among tasks (e.g. $A_{1,1} \prec A_{1,2}$).

(b) Can the output variable `error` be set to `true`? If yes, provide a corresponding execution.

.... Solution ....

(a)
- $S = \{\texttt{er}, \texttt{r}_1, \texttt{r}_2, \}$
- $O = \{\texttt{error}, \texttt{alive}_1, \texttt{alive}_2\}$
- $I = \{\texttt{second}\}$
- $\prec = \{(A_{1,1}, A_{1,2}), (A_{2,1}, A_{2,2}), (A_{3,1}, A_{3,2}), (A_{1,2}, A_{3,2}), (A_{2,2}, A_{3,2})\}$

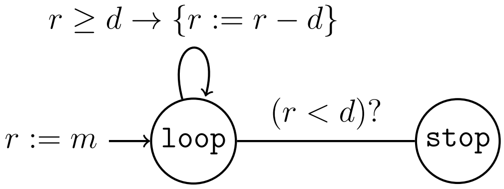
(b) Lets assume ordering on state variables $(\texttt{er}, \texttt{r}_1, \texttt{r}_2)$, and for outputs , $\texttt{error}, \texttt{alive}_1, \texttt{alive}_2$ then an execution

$$(false, 0, 0) \xrightarrow{\top / false, \top, \bot} (false, 1, 1) \xrightarrow{\top / false, \bot, \top} (false, 2, 2) \xrightarrow{\top / false, \bot, \bot}$$
$$(true, 3, 3) \xrightarrow{\top / true, \top, \bot} (true, 4, 4) \to \ldots$$

Exercise 2: Symbolic Transition System

10 Points

The following state machine finds the remainder $\texttt{REM}(m, d)$ resulting from the division of positive integers $m = 290$ and $d = 9$.



(a) Describe the underlying transition system symbolically giving, state variables, initialization formula, and transition formula φ.

(b) Given the region $A : (100 \leq r \leq 290)$, compute the image using the transition formula φ. Describe the required steps.

.... Solution ....

(a) The transition system `REM(m,n)` has state variable r and $mode$ of the enumerated type $\{loop, stop\}$. The initialization is given by the formula

$$(mode = loop) \wedge (r = m)$$

The transition formula φ is given as:

$$\begin{aligned} & [(mode = loop) \wedge (r \geq d) \wedge (r' = r - d) \wedge (mode' = loop)] \\ \vee \; & [(mode = loop) \wedge (r < d) \wedge (r' = r) \wedge (mode' = stop)] \end{aligned}$$

(b)
- Conjuction of A and φ, note $A \equiv (100 \leq r \wedge r \leq 290)$

$$(100 \leq r \leq 290) \wedge [(mode = loop) \wedge (r \geq 9) \wedge (r' = r - 9) \wedge (mode' = loop)]$$

- Existententially quantify $mode$

$$(100 \leq r \leq 290) \wedge (r \geq 9) \wedge (r' = r - 9) \wedge (mode' = loop)$$

- Existententially quantify r

$$(100 \leq r' + 9) \wedge (r' + 9 \leq 290) \wedge (mode' = loop)]$$

- Renaming

$$(91 \leq r \leq 281) \wedge (mode' = loop)$$

Exercise 3: Asynchronous Model - Shared Registers UPPAAL 10 Points

The following process increases a shared atomic register `n` by using a local register `r` and read-write operations.

```
global int k:=10; int n:=0;
process P_i
local int j:=0;
local int r:=0;
while ( j < k ) {
  r := n; r := r + 1; n := r;
  j := j + 1;
}
```

(a) Consider the product $(P_1 \| P_2)$, what is the minimal final value of global variable `n`? *Hint:* use the UPPAAL model `ex3.xml` with a suitable query to find the value.

(b) Explain how the minimal value for `n` is obtained.

(c) Consider the product $(P_1 \| P_2 \| P_3)$ what is the minimal final value of global variable `n`?

.... Solution ....

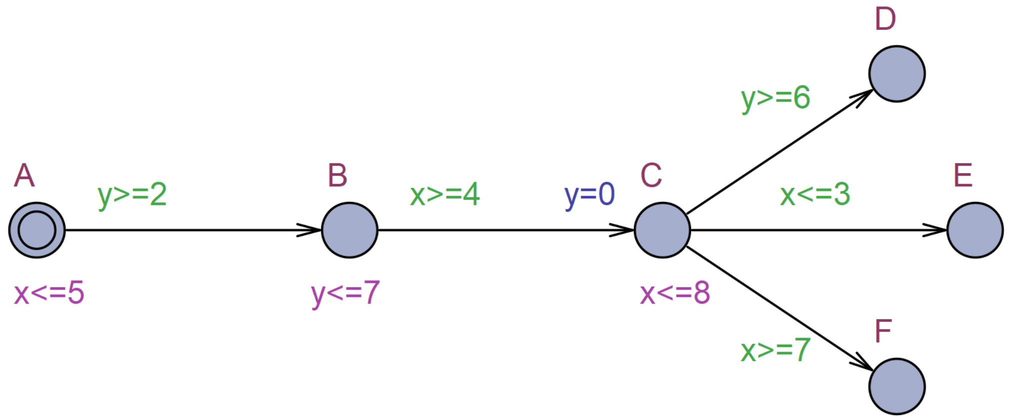
(a) $n = 2$

(b) One process e.g. P_2 is able to store in r the value 0, then P_1 executes the loop 9 times, then P_2 inteferes and sets $n = 1$. P_1 reads n, sets $r_1 = 1$ and increments to $r_1 = 2$. Then P_2 executes to completion. Finally P_1 sets $n = r_1 = 2$ and exits the loop.

(c) $n = 2$. For three processes and $k = 10$ the state space it too big and UPPAAL might not be able to explore it. To help your intuition you can set the value of $k = 5$ and observe that similar executions as for $(P_1 \| P_2)$ occur.

Exercise 4: Exploration of Timed Automata 10 Points

Consider the timed automaton $\mathcal{A}$ below with two clocks x and y.



(a) Which of the three locations D, E and F are reachable from the initial state $(\mathtt{A}, \mathtt{x} = 0, \mathtt{y} = 0)$?

(b) For each of these three goal locations that is reachable, provide a timed transition sequence that leads to the location from the initial state.

(c) For each of the three goal locations that is reachable, what is the fastest time of reaching that location. Provide a witness timed transition sequence.

(d) Describe using difference constraints the reachable zones upon entry and after delay for the locations A, B and C.

(e) For each of the three goal locations that is NOT reachable, suggest a weakening of the guard leading to the location, so that the location becomes reachable.
NOTE: `x<=7` is weaker than `x<=5` and `x>=2` is weaker than `x>=4`.

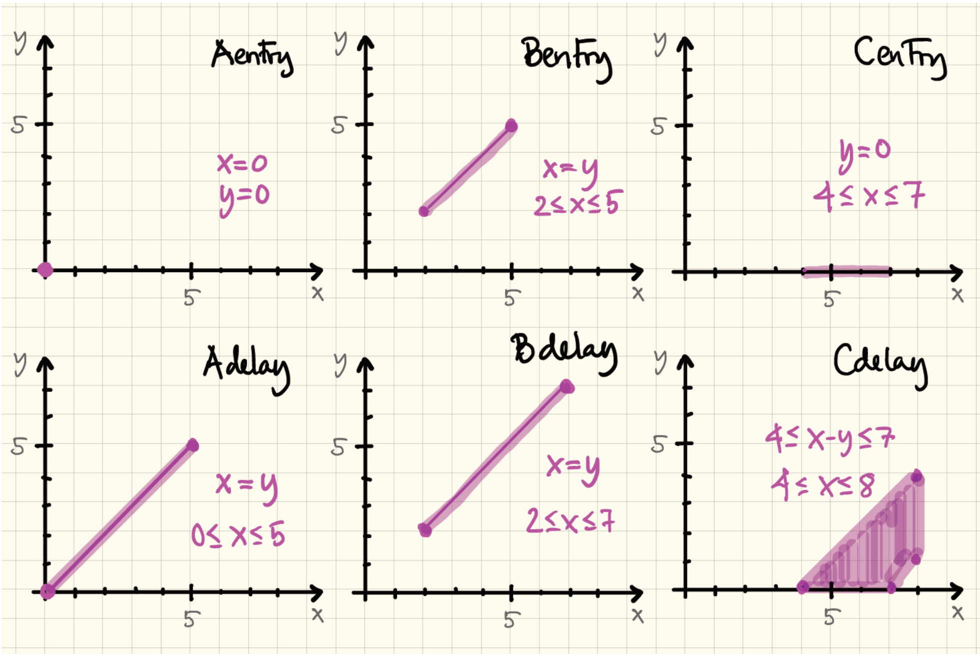
Solution

(a) F

(b)
$$
\begin{array}{ll}
(\mathtt{A}, x = 0, y = 0) & \xrightarrow{2} \\
(\mathtt{A}, x = 2, y = 2) & \rightarrow \\
(\mathtt{B}, x = 2, y = 2) & \xrightarrow{2} \\
(\mathtt{B}, x = 4, y = 4) & \rightarrow \\
(\mathtt{C}, x = 4, y = 0) & \xrightarrow{3} \\
(\mathtt{C}, x = 7, y = 3) & \rightarrow \\
(\mathtt{F}, x = 7, y = 3) & \rightarrow
\end{array}
$$

(c) F is reachable in 7 time units. The above timed transition sequence is a witness.

(d) See figure below



(e) The guard to D should be weakened to `y>=4`. The guard to E should be weakend to `x<=4`.

Exercise 5: The Druzba Mutual Exclusion Problem 15 Points

The problem is based on a true story of one of the lectures of this course experienced during the conference CONCUR in 2002 in Brno. During this – otherwise extremely nice conference – accommodation was arranged in the local Druzba hostel. Rooms being nice, there was the unexpected surprise of sharing the shower with the neighbor (causing some screaming in at least one occasion), see Fig. 1 below.

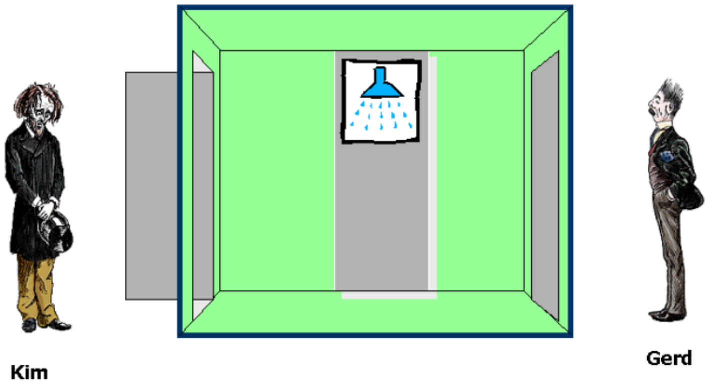


Figure 1: Sharing a Shower in Druzba

During the conference a lot of possible solutions for how to obtain mutual exclusion in the shower were discussed. Your job is to help find a good solution.

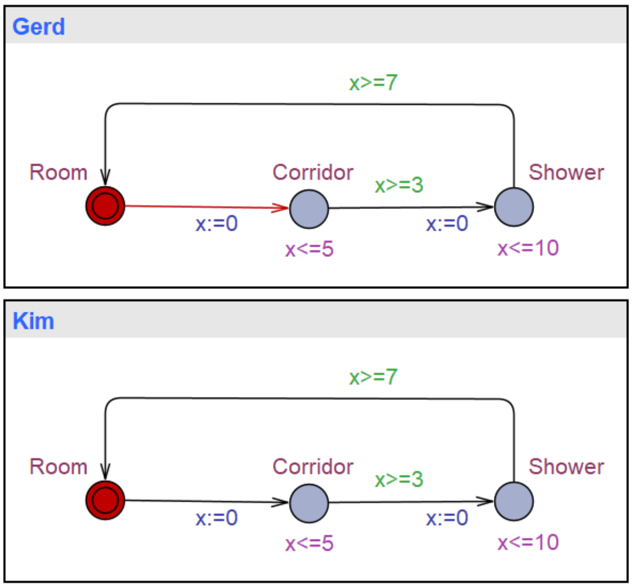


Figure 2: First model of the Druzba Mutex problem

In Fig. 2 you see an initial solution in UPPAAL to the problem. You can find the complete UPPAAL model in Digital Exam in the file `Druzba.xml`. Here the two users of the shower (Gerd and Kim) may at any moment in time make a go for the shower. This first requires waiting between 3 and 5 minutes in the `Corridor`. The actual use of the `Shower` will take between 7 and 10 minutes.

(a) Formulate as a UPPAAL property ϕ_M (in TCTL) the desired mutual exclusion on the `Shower` location.

(b) Check in UPPAAL whether the initial solution satisfies the mutual exclusion property ϕ_M. If not use UPPAAL to generate a violating trace. Please provide the corresponding Message Sequence Chart (MSC) found in the simulator of UPPAAL.

(c) Formulate as a UPPAAL property ϕ_G (in TCTL) the desired liveness property that whenever Gerd enters the `Corridor` he will eventually get to the `Shower`. Formulate a similar liveness property ϕ_K for Kim.

(d) Check in UPPAAL whether the initial solution satisfies the above liveness properties ϕ_G and ϕ_K and report the answer.

(e) Please upload to Digital Examn your extension of the initial solution with the properties ϕ_M, ϕ_G and ϕ_K in a file with name `DruzbaSol1.xml`.

Now assume that the bathroom has a Light which can be checked and switched on before entering the bathroom – and switched back off when leaving the bathroom.

(a) Extend the initial model with a Boolean variable `L` to represent whether the Light is on or off.

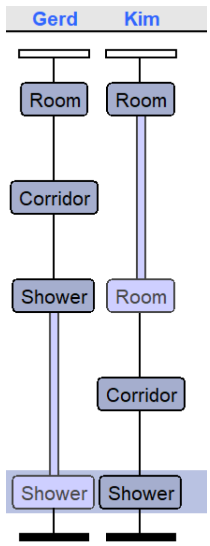
(b) As an improved solution, use `L` to "check and switch on" upon entering the `Cooridor`. Check in UPPAAL whether the properties ϕ_M, ϕ_G and ϕ_K are satisfied for the improved solution.

(c) Please upload to Digital Examn your proposal for the improved solution in a file with name `DruzbaSol2.xml`

. . . . Solution . . . .

(a) $\phi_M =$ `A[] not (Kim.Shower and Gerd.Shower)`

(b) The property does not hold. The following is a MSC witness.

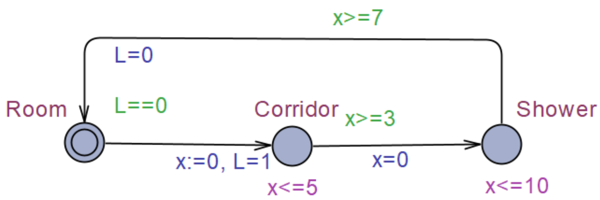


(c) $\phi_G =$ `Gerd.Corridor --> Gerd.Shower` and $\phi_K =$ `Kim.Corridor --> Kim.Shower`

(d) Both ϕ_G and ϕ_K holds for the initial model.

Extension of the model with a Boolean variable `L`:

(a) The extension results in the following model:;



(b) All properties ϕ_M, ϕ_G and ϕ_K holds.

Exercise 6: Continuous System 10 Points

In this exercise, you may (but are not obliged to) justify your answers using MATLAB. Any use of MATLAB must be however documented by crisp snippets of MATLAB's command window featuring the relevant inputs and outputs.

Let the following matrices be given:

$$A = \begin{pmatrix} -3 & 2 \\ -2 & 2 \end{pmatrix} \qquad B = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

(a) Prove that $x = 0$ is not a stable equilibrium of $\frac{d}{dt}x(t) = Ax(t)$.

(b) Compute a gain matrix K for which the feedback loop matrix $A - BK$ has eigenvalues -2 and -3.

(c) Is there an initial condition $x(0) \neq 0$ for which $\frac{d}{dt}x(t) = Ax(t)$ admits a solution that converges towards zero? If yes, provide such an initial condition, i.e., $x(0) \neq 0$ and $\lim_{t\to\infty} x(t) = 0$. If not, argue why such an initial condition does not exist.

.... Solution ....

(a) Since our system is $\frac{d}{dt}x(t) = Ax(t)$ and A is a 2×2 matrix, variable x is a vector $x(t) = \binom{x_1(t)}{x_2(t)}$.

With MATLAB: Input

Listing 1: Matlab input

```
A = [ [-3 2]; [-2 2] ];
eig(A)
```

yields output

Listing 2: Matlab output

```
ans =
-2
 1
```

Hence, $+1$ is an eigenvalue of A and the discussion from the course implies that $x = 0$ is not a stable equilibrium.

By hand: As discussed in the course, it suffices to prove that A has an eigenvalue with positive real part. The eigenvalues are the roots of the characteristic polynomial $p(\lambda) = |A - \lambda I|$, where $|\cdot|$ denotes the determinant. With this, we obtain

$$p(\lambda) = |A - \lambda I| = \begin{vmatrix} -3-\lambda & 2 \\ -2 & 2-\lambda \end{vmatrix} = (-3-\lambda)(2-\lambda) - (-2)2 = \lambda^2 + \lambda - 2$$

High school math then shows that $\lambda^2 + \lambda - 2 = (\lambda - 1)(\lambda + 2)$, implying that the eigenvalues of A are 1 and -2.

(b) *With MATLAB:* Input

Listing 3: Matlab input

```
A = [ [-3 2]; [-2 2] ];
B = [ 1; 0 ];
K = place(A,B,[-2,-3])
lambdas = eig(A - B*K)    % double check, not required
```

yields output

Listing 4: Matlab output

```
K = [4   -8]
lambdas =
-3
-2
```

By hand:

$$A - BK = A - \begin{pmatrix} 1 \\ 0 \end{pmatrix} (k_1, k_2) = \begin{pmatrix} -3 - k_1 & 2 - k_2 \\ -2 & 2 \end{pmatrix}$$

we observe that

$$\begin{aligned} |(A - BK) - \lambda I| &= \begin{vmatrix} -3 - k_1 - \lambda & 2 - k_2 \\ -2 & 2 - \lambda \end{vmatrix} \\ &= (-3 - k_1 - \lambda)(2 - \lambda) - (-2)(2 - k_2) \\ &= \lambda^2 + \lambda(-2 + 3 + k_1) - 2 - 2k_1 - 2k_2 \end{aligned}$$

Since

$$(\lambda - (-2))(\lambda - (-3)) = \lambda^2 + 5\lambda + 6,$$

matching coefficients yields $k_1 = 4$ and $k_2 = -8$.

(c) From the course, we know that the eigenvectors underlying eigenvalues with negative real parts yield solutions converging to zero (instead, eigenvectors underlying eigenvalues with positive real part yield diverging solutions). From (a) we know that -2 is an eigenvalue of A. Because of this, solving the linear system of equations $Ax = -2x$ implies that $x = (2a, a)^T$ is, for any $a \neq 0$, an eigenvector for eigenvalue -2 (this can be computed by hand or for instance via `[V,D] = eig(A)` in MATLAB). Consequently, any $x(0) = (2a, a)^T$ with $a \neq 0$ constitutes an initial condition converging to zero.

(Add-on, not required.) Note that this does not contradict the fact that $x = 0$ is an unstable equilibrium. Indeed, for unstability, it suffices to have *at least one* eigenvalue with positive real part. This is here the case since $\lambda = 1$ is an eigenvalue of A. The corresponding eigenvectors solve the equation $Ax = x$ and are given by $x = (a, 2a)^T$ for any $a \neq 0$, while the corresponding diverging solutions are $x(t) = (e^t a, e^t 2a)$. On the other hand, the converging solutions are given by $x(t) = (e^{-2t} 2a, e^{-2t} a)$. One can cross-check that this is indeed the case by differentiating the expression:

$$\begin{aligned} \frac{d}{dt} x_1(t) &= -2e^{-2t} 2a = -3e^{-2t} 2a + 2e^{-2t} a = -3x_1(t) + 2x_2(t) \\ \frac{d}{dt} x_2(t) &= -2e^{-2t} a = -2e^{-2t} 2a + 2e^{-2t} a = -2x_1(t) + 2x_2(t) \end{aligned}$$

A similar check can be done by differentiating $x(t) = (e^t a, e^t 2a)$.

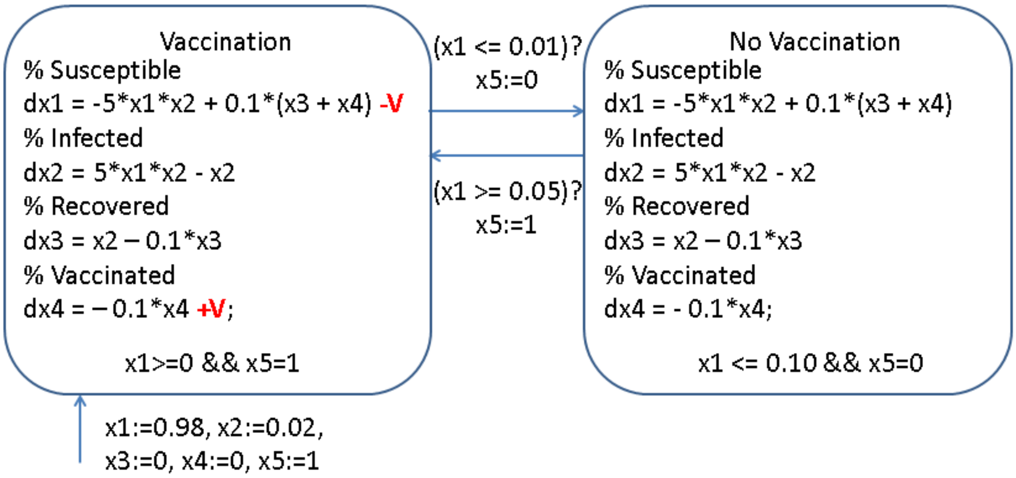


Figure 3: Hybrid model of a vaccination program.

Exercise 7: Hybrid Vaccination Model 10 Points

Consider the hybrid system of Figure 3 and the following incomplete MATLAB script below which can be found in Digital Exam:

Listing 5: Incomplete MATLAB script `vaccination.m`

```
function vaccination()
    T = 5.0;
    % Initial percentages of susceptible, infected, recovered, vaccinated and the initial mode
    x0 = [0.98, 0.02, 0.00, 0.00, 1]';
    dt = T / 100;
    I = 0:dt:T;

    figure
    hold on
    % ... add code ...
    [I,x]=ode45(@(t,x)drift(t,x,V),I,x0);
    % ... add code ...
end

function dx = drift(t,x,V)
    dx = zeros(5,1);

    % ... add code ...

    if(x(5) == 1)
        dx(1) = -5*x(1)*x(2) - V + 0.1*x(3) + 0.1*x(4);
        dx(2) = 5*x(1)*x(2) - x(2);
        dx(3) = x(2) - 0.1*x(3);
        dx(4) = V - 0.1*x(4);
```

```
    else
        dx(1) = -5*x(1)*x(2) + 0.1*x(3) + 0.1*x(4);
        dx(2) = 5*x(1)*x(2) - x(2);
        dx(3) = x(2) - 0.1*x(3);
        dx(4) = - 0.1*x(4);
    end
end
```

Extend the MATLAB script `vaccination.m` so that it

- Computes for each vaccination rate $V \in \{0.1, 0.2, \ldots, 1.0\}$ an execution of the hybrid system on the time interval $[0; 5]$ and;
- Plots the infection forecasts x2 of all ten executions in a common figure.

Note: Solutions defining new MATLAB functions or making use of MATLAB commands other than `plot` or `ode45` will be not considered.

.... Solution ....

Listing 6: Complete MATLAB solution

```
function vaccination()
    T = 5.0;
    % Initial percentages of susceptible, infected, recovered,
       vaccinated and the initial mode
    x0 = [0.98, 0.02, 0.00, 0.00, 1]';
    dt = T / 100;
    I = 0:dt:T;

    figure
    hold on
    % added code
    for i = 1 : 10
        V = i*0.1;
        [I,x]=ode45(@(t,x)drift(t,x,V),I,x0);
        plot(I(:),x(:,2));
    end
end

function dx = drift(t,x,V)
    dx = zeros(5,1);

    % added code
    if(x(5) == 1 && x(1) <= 0.01 )
        x(5) = 0;
    elseif(x(5) == 0 && x(1) >= 0.05)
        x(5) = 1;
    end

    if(x(5) == 1)
        dx(1) = -5*x(1)*x(2) - V + 0.1*x(3) + 0.1*x(4);
```

```
        dx(2) = 5*x(1)*x(2) - x(2);
        dx(3) = x(2) - 0.1*x(3);
        dx(4) = V - 0.1*x(4);
    else
        dx(1) = -5*x(1)*x(2) + 0.1*x(3) + 0.1*x(4);
        dx(2) = 5*x(1)*x(2) - x(2);
        dx(3) = x(2) - 0.1*x(3);
        dx(4) = - 0.1*x(4);
    end
end
```