

Introduction to GW/BSE in GPAW: Monolayer MoS₂ as an example

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@dpmodeling-abacus, August 6, 2021

Outline

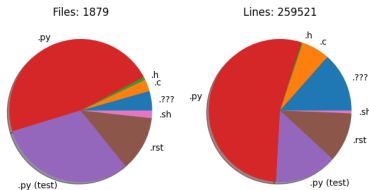
- ① Introduction
 - DFT calculations in GPAW
 - Band-gap problem
- ② GW/BSE in GPAW
 - A glance at theory
 - Workflow in GPAW
- ③ Example: Gap engineering of ml-MoS₂
 - Background
 - Model
 - Calculations

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What is GPAW?

- Open-source DFT code with projector augmented-wave (not Global Pandemic App Watch)
- Developed by Thygesen group in Technical University of Denmark, v1.0.0 released on 2016-03-17
- Basis: plane-wave, LCAO with NAO and real-space grids
- Written in Python and C, based on ASE, LIBXC, NumPy and SciPy
- Available by pip, conda, Linux repos, Docker image ...



<https://wiki.fysik.dtu.dk/gpaw/>

J. Enkovaara et al., J. Phys.: Condens. Matter **22**, 253202 (2010)

Line counts from talk by Jens Jørgen Mortensen in GPAW 2021

Quick example of GPAW with plane-wave

Ground state of silicon with PBE: Si_gs.py

```
from ase.build import bulk
from gpaw import GPAW, FermiDirac
from gpaw.wavefunctions.pw import PW

atoms = bulk('Si', 'diamond', a=5.43)

c = GPAW(mode=PW(400), xc='PBE', eigensolver='dav',
          kpts={'size': (8, 8, 8), 'gamma': True},
          occupations=FermiDirac(0.01), txt='Si_gs.txt')
atoms.calc = c

atoms.get_potential_energy()
c.write ('Si_gs.gpw')
```

Running serially and parallelly

Serial call

```
$ time python Si_gs.py  
real 0m9.746  
...
```

Parallel

```
$ time mpirun -np 4 gpaw python Si_gs.py  
real 0m4.425s  
...
```

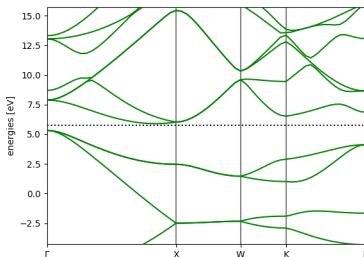
- automatically handle parallel I/O
- use parallelization over \mathbf{k} /spin/band
- support MPI/OpenMP and ScaLAPACK

Band structure

```
from gpaw import GPAW

c = GPAW('Si_gs.gpw', nbands=16,
         fixdensity=True,
         symmetry='off',
         kpts={'path': 'GXWKL',
              'npoints': 60},
         convergence={'bands': 8})
c.get_potential_energy()
c.write('Si_bs.gpw')

bs = c.band_structure()
bs.plot(emax=16.0, emin=-4.0,
       filename="Si_bs.png")
```



Si_bs.png

Compute band structure properties manually

Retrieve eigenvalues on the irreducible \mathbf{k} points

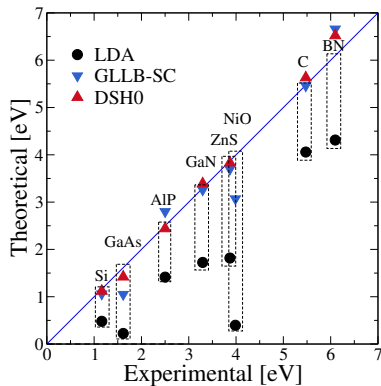
```
c = GPAW('Si_gs.gpw', txt=None)

eigens = np.array([[c.get_eigenvalues(ik, isp)
                    for ik, _ in enumerate(c.get_ibz_k_points())]
                  for isp in range(c.get_number_of_spins())])

ivb = int(np rint(c.get_number_of_electrons()))//2 - 1

vbs = eigens[0, :, ivb]
cbs = eigens[0, :, ivb+1]
indir_gap = min(cbs) - max(vbs)
dir_gap = min(cbs - vbs)
```


Band-gap problem in LDA/GGA



Underestimation stems from the so-called derivative discontinuity

$$E_g^{\text{fund}} = \text{IP} - \text{EA}$$

$$E_g^{\text{KS}} = \epsilon_{\text{CBM}}^{\text{KS}} - \epsilon_{\text{VBM}}^{\text{KS}}$$

$$E_g^{\text{fund}} = E_g^{\text{KS}} + \Delta_{\text{xc}}$$

$\Delta_{\text{xc}} \neq 0$ for exact KS potential

Solutions within DFT framework

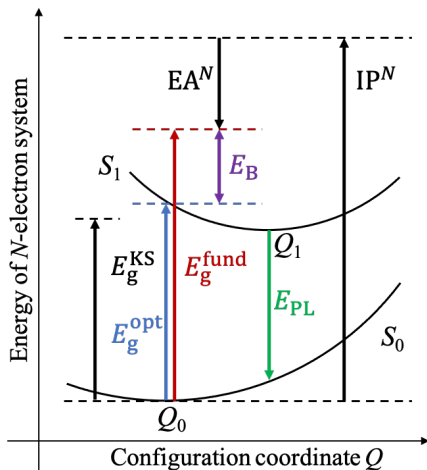
- meta-GGA, e.g. mBJ, GLLB-SC
- Hybrid functional, e.g. HSE06, doubly screened hybrid (DSH)

J. P. Perdew et al., Phys. Rev. Lett. **49**, 1691–1694 (1982)

F. Tran, S. Ehsan, and P. Blaha, Phys. Rev. Materials **2**, 023802 (2018)

Z.-H. Cui et al., J. Phys. Chem. Lett. **9**, 2338–2345 (2018)

Definition of band gaps

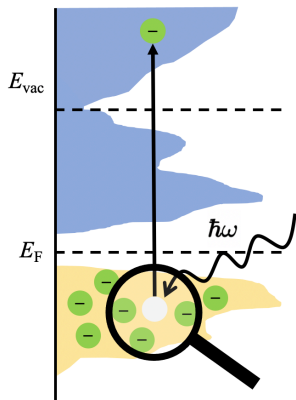


Property	Measurement
IP / EA, E_g^{fund}	PES / IPS
E_g^{opt}	DRS, ellipsometry
E_{PL}	photoluminescence

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Green's function in many-body perturbation theory



Time-ordered Green's function ($1 = \mathbf{r}_1 t_1$)

$$iG(1, 2) = \langle \Psi_N | T \left\{ \hat{\psi}_H(1) \hat{\psi}_H^\dagger(2) \right\} | \Psi_N \rangle$$

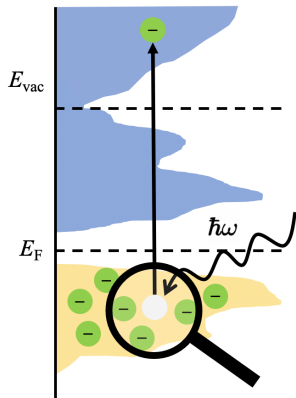
G can be formally solved in frequency domain

$$G(\mathbf{r}, \mathbf{r}', \omega) = \sum_n \frac{\varphi_n(\mathbf{r}, \omega) \varphi_n^*(\mathbf{r}', \omega)}{\omega - \epsilon_n(\omega)}$$

$$\begin{aligned} [h_0(\mathbf{r}) + v_H(\mathbf{r})] \varphi_n(\mathbf{r}, \omega) + \int d\mathbf{r}' \Sigma(\mathbf{r}, \mathbf{r}', \omega) \varphi_n(\mathbf{r}', \omega) \\ = \epsilon_n(\omega) \varphi_n(\mathbf{r}, \omega) \end{aligned}$$

Σ : self-energy

Green's function in many-body perturbation theory



Time-ordered Green's function ($1 = \mathbf{r}_1 t_1$)

$$iG(1, 2) = \langle \Psi_N | T \left\{ \hat{\psi}_H(1) \hat{\psi}_H^\dagger(2) \right\} | \Psi_N \rangle$$

G can be formally solved in frequency domain

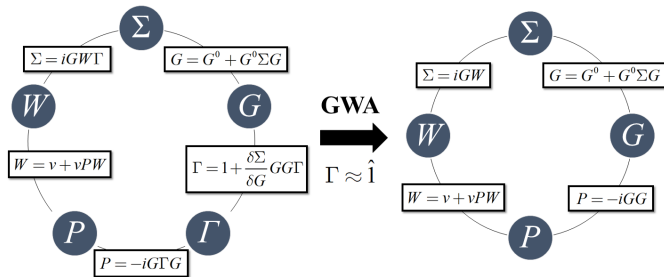
$$G(\mathbf{r}, \mathbf{r}', \omega) = \sum_n \frac{\varphi_n(\mathbf{r}, \omega) \varphi_n^*(\mathbf{r}', \omega)}{\omega - \epsilon_n(\omega)}$$

$$[h_0(\mathbf{r}) + v_H(\mathbf{r})] \varphi_n^{\text{QP}}(\mathbf{r}) + \int d\mathbf{r}' \Sigma(\mathbf{r}, \mathbf{r}', \epsilon_n^{\text{QP}}) \varphi_n^{\text{QP}}(\mathbf{r}') = \epsilon_n^{\text{QP}} \varphi_n^{\text{QP}}(\mathbf{r})$$

$$(\Sigma \text{ small}) \quad \quad \quad = \epsilon_n^{\text{QP}} \varphi_n^{\text{QP}}(\mathbf{r})$$

Σ : self-energy ϵ^{QP} : quasi-particle (QP) energy

Exact solution for Σ : Hedin's equations



Hedin's pentagon with

- v (W): bare (screened) Coulomb interaction
- polarization function P and vertex function Γ

G_0W_0/GW_0 approximation for calculating Σ

$$\Sigma(\mathbf{r}, \mathbf{r}', \omega) = \frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega' e^{i\omega'\delta} G_0(\mathbf{r}, \mathbf{r}', \omega + \omega') W_0(\mathbf{r}', \mathbf{r}, \omega')$$

$$G_0(\mathbf{r}, \mathbf{r}', \omega) = \sum_{n\mathbf{k}} \frac{\psi_{n\mathbf{k}}(\mathbf{r}) \psi_{n\mathbf{k}}^*(\mathbf{r}')}{\omega - \epsilon_{n\mathbf{k}}^{\text{KS}} - i\eta \text{sgn}(\mu - \epsilon_{n\mathbf{k}}^{\text{KS}})}$$

$$W_0(\mathbf{r}, \mathbf{r}', \omega) = \int d\mathbf{r}'' \epsilon_0^{-1}(\mathbf{r}, \mathbf{r}'', \omega) v(\mathbf{r}'', \mathbf{r}')$$

$$\epsilon_0(\omega) = \mathbf{1} - \mathbf{v} \mathbf{P}_0(\omega) = \mathbf{1} - \mathbf{v} \chi_0(\omega)$$

$$\chi_0(\omega) = -\frac{i}{2\pi} \int_{-\infty}^{\infty} d\omega' e^{i\omega'\delta} \mathbf{G}_0(\omega + \omega') \mathbf{G}_0(\omega')$$

$$\mathbf{G}^{-1} = \mathbf{G}_0^{-1} - \Sigma$$

GW Implementation (recp. space and freq. domain)

$$M_{nm}^i(\mathbf{k}, \mathbf{q}) = \sqrt{N_k} \int_V d\mathbf{r} [\chi_i^{\mathbf{q}}(\mathbf{r}) \psi_{m\mathbf{k}-\mathbf{q}}(\mathbf{r})]^* \psi_{n\mathbf{k}}(\mathbf{r})$$

$$P_{ij}(\mathbf{q}, \omega) = \frac{1}{\Omega} \sum_{\mathbf{k}} \sum_{nm} F_{nm}(\mathbf{k}, \mathbf{q}, \omega) M_{nm}^i(\mathbf{k}, \mathbf{q}) [M_{nm}^j(\mathbf{k}, \mathbf{q})]^*$$

$$F_{nm}(\mathbf{k}, \mathbf{q}, \omega) = 2f_{n\mathbf{k}} (1 - f_{m\mathbf{k}-\mathbf{q}}) \left\{ \frac{1}{\omega - \epsilon_{m\mathbf{k}-\mathbf{q}} + \epsilon_{n\mathbf{k}} + i\eta} - \frac{1}{\omega + \epsilon_{m\mathbf{k}-\mathbf{q}} - \epsilon_{n\mathbf{k}} - i\eta} \right\}$$

$$\Sigma_{n\mathbf{k}}^x = -\frac{1}{N_k} \sum_{\mathbf{q}} \sum_i v_i(\mathbf{q}) \sum_m f_{m\mathbf{k}-\mathbf{q}} |M_{mn}^i(\mathbf{k}, \mathbf{q})|^2 \text{ (exact exchange, EXX)}$$

$$\Sigma_{n\mathbf{k}}^c = \frac{i}{2\pi N_k} \int_{-\infty}^{\infty} d\omega' \sum_m \sum_{\mathbf{q}} \frac{X_{nm}(\mathbf{k}, \mathbf{q}, \omega')}{\omega + \omega' - \epsilon_{m\mathbf{k}-\mathbf{q}} + i\eta \text{sgn}(\epsilon_{m\mathbf{k}-\mathbf{q}} - \mu)}$$

$$X_{nm}(\mathbf{k}, \mathbf{q}, \omega) = \sum_{ij} [M_{nm}^i(\mathbf{k}, \mathbf{q})]^* [\epsilon_{ij}(\mathbf{q}, \omega) - 1] M_{nm}^j(\mathbf{k}, \mathbf{q})$$

- Auxiliary basis $\chi_i^{\mathbf{q}}(\mathbf{r})$: plane-wave, NAO, product basis, ...
- \sum_{nm} : core states, high-lying states, basis-set completeness ...
- Frequency integration $\int d\omega$: plasmon-pole model, full frequency, ...

(Static) Bethe-Salpeter equation

Similar to the Casida equations in TD-DFT

$$\begin{pmatrix} R & C \\ -C^* & -R^* \end{pmatrix} \begin{pmatrix} X^m \\ Y^m \end{pmatrix} = \Omega_m \begin{pmatrix} X^m \\ Y^m \end{pmatrix}$$

with

$$R_{ia,jb} = \left(\varepsilon_a^{\text{QP}} - \varepsilon_i^{\text{QP}} \right) \delta_{ij} \delta_{ab} + 2v_{ia,jb} - W_{ij,ab}$$

$$C_{ia,jb} = 2v_{ia,bj} - W_{ib,aj}$$

$$v_{ia,jb} = (ia | jb) = \langle ij | ab \rangle = \int d\mathbf{r} d\mathbf{r}' \psi_i^*(\mathbf{r}) \psi_j^*(\mathbf{r}') v(\mathbf{r}, \mathbf{r}') \psi_a(\mathbf{r}) \psi_b(\mathbf{r}')$$

$$W_{ia,jb} = \int d\mathbf{r} d\mathbf{r}' \psi_i^*(\mathbf{r}) \psi_j^*(\mathbf{r}') W(\mathbf{r}, \mathbf{r}', \omega = 0) \psi_a(\mathbf{r}) \psi_b(\mathbf{r}')$$

Solve BSE to obtain the neutral excitation energy Ω_m and electron-hole eigen-state

$$\Psi_m^{\text{eh}}(\mathbf{r}_e, \mathbf{r}_h) = \sum_{ia} [X_{ia}^m \psi_i(\mathbf{r}_h) \psi_a(\mathbf{r}_e) + Y_{ia}^m \psi_i(\mathbf{r}_e) \psi_a(\mathbf{r}_h)]$$

Why GW/BSE in GPAW

- open-source and well-documented
- powered by Python ecosystem
- good performance due to efficient parallelization and native C code for computation-extensive parts
- various functionalities including vertex correction, Coulomb truncation and interpolation schemes

F. Hüsler, T. Olsen, and K. S. Thygesen, Phys. Rev. B 88, 245309 (2013), F. Hüsler, T. Olsen, and K. S. Thygesen, Phys. Rev. B 87, 235132 (2013), T. Olsen et al., npj Comput. Mater. 5, 1–23 (2019), F. A. Rasmussen et al., Phys. Rev. B 94, 155406 (2016), P. S. Schmidt, C. E. Patrick, and K. S. Thygesen, Phys. Rev. B 96, 205206 (2017), K. S. Thygesen, 2D Mater. 4, 022004 (2017)

Workflow for G_0W_0/GW_0

GW functionality is provided in GPAW since 2014.

The typical workflow follows 3 steps:

Step 1 SCF for converged density

Step 2 non-SCF calculation for occupied and unoccupied states (practically on a smaller \mathbf{k} -point mesh)

Step 3 GW calculations

Workflow for G_0W_0/GW_0

GW functionality is provided in GPAW since 2014.

Step 1: SCF

```
from ase.build import bulk
from gpaw import GPAW, FermiDirac
from gpaw.wavefunctions.pw import PW

atoms = bulk('C', 'diamond', a=3.567)
c = GPAW(mode=PW(400), xc='PBE',
          kpts={'size': (4, 4, 4), 'gamma': True},
          convergence={'energy': 1e-8},
          occupations=FermiDirac(0.001),
          txt='C_gs.txt')
atoms.calc = c
atoms.get_potential_energy()
c.write('C_gs.gpw')
```

Workflow for G_0W_0 / GW_0

Step 2: get all bands (here on the same \mathbf{k} grids as SCF)

```
c.diagonalize_full_hamiltonian()  
c.write('C_gs_full.gpw', mode='all')
```

Note that the full GPAW file can be huge, depending on the PW cut-off and \mathbf{k} -point mesh

Workflow for G_0W_0/GW_0

Step 3: GW

```
from gpaw.response.g0w0 import G0W0

gw = G0W0(calc='C_gs_full.gpw', bands=(1, 7),
          nbands=144, ecut=100,
          filename='C_g0w0-k4-ecut100')
result = gw.calculate()
```

Run serially and the following files are generated:

C_g0w0-k4-ecut100.exx.npy	C_g0w0-k4-ecut100.exx.txt
C_g0w0-k4-ecut100_results.pckl	C_g0w0-k4-ecut100.txt
C_g0w0-k4-ecut100.vxc.npy	C_g0w0-k4-ecut100.w.txt

Glimpse of GW output files

filename.exx.txt: details of EXX calculation

```
...
Distributing spins, k-points and bands (1 x 64 x 4) over 1 process
Number of blocks: 1
Using Wigner-Seitz truncated coulomb interaction.
Inner radius for 4x4x4 Wigner-Seitz cell: 4.119 Ang
Range-separation parameter: 1.214 Ang-1
FFT size for calculating truncated Coulomb: 60x60x60
EXX eigenvalue contributions in eV:
[[[-20.239 -20.239 -20.239 -10.109 -10.109 -10.108]
  [-22.192 -20.633 -20.633 -9.845 -7.755 -9.301]
  ...
  [-22.737 -20.861 -20.861 -8.851 -8.851 -11.025]]]
```

Glimpse of GW output files

filename.w.txt: details of response function (from Chi0)

```
...
Point group included. Time reversal included. Disabled non symmorphic
↪ symmetries. Found 8 allowed symmetries. 18 groups of equivalent kpoints.
↪ 71.875% reduction.
```

```
( 1  0  0) ( 1  0 -1) ( 0  1  0) ( 0  1 -1) ( 0  1 -1) ( 0  1  0)
( 0  1  0) ( 0  1 -1) ( 1  0  0) ( 1  0 -1) ( 0  1  0) ( 0  1 -1)
( 0  0  1) ( 0  0 -1) ( 0  0  1) ( 0  0 -1) (-1  1  0) (-1  1  0)
```

```
( 1  0 -1) ( 1  0  0)
( 1  0  0) ( 1  0 -1)
( 1 -1  0) ( 1 -1  0)
```

```
Integrating response function.
Integral kind: spectral function
Distributing domain [18, 1] over 1 process
Number of blocks: 1
1.3598370552062988s |-----| Time: 1.349s
...
```

repeated for 8 (N_k in IBZ) times

Glimpse of GW output files

filename_results.pkl: binary file (pickle) for saving data

```
>>> import pickle
>>> data = pickle.load(open('C_g0w0-k4-ecut100_results.pkl', 'rb'))
>>> list(data.keys())
['f', 'eps', 'vxc', 'exx', 'sigma', 'dsigma', 'Z', 'qp', 'iqp']
# occupancies at Gamma
>>> data['f'][0, 0]
array([1., 1., 1., 0., 0., 0.])
# KS gap at Gamma
>>> data['eps'][0, 0, 3] - data['eps'][0, 0, 2]
5.5509672399201015
# QP gap at Gamma
>>> data['qp'][0, 0, 3] - data['qp'][0, 0, 2]
7.335502210222952
```

QP band structures

Use **GWBands** object to obtain QP band structure along a particular path by interpolating the results of a regular **k**-mesh

```
from gpaw.response.gw_bands import GWBands
```

```
L = np.array([0.5, 0.5, 0.5])
```

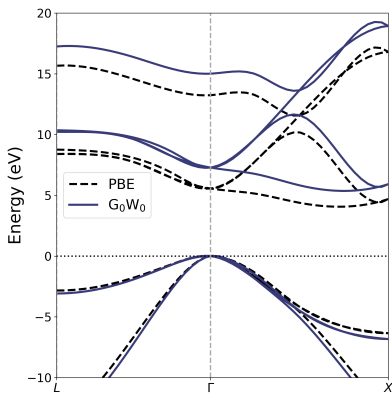
```
X = np.array([0.5, 0.5, 0.0])
```

```
G = np.array([0.0, 0.0, 0.0])
```

```
kpoints = np.array([L, G, X])
```

```
gwbands = GWBands(calc=gwscf,  
                  gw_file=pckl,  
                  kpoints=kpoints,  
                  bandrange=(0,8))
```

```
gw = gwbands.get_gw_bands(nk_Int=50,  
                          interpolate=True)
```



C_band.png

Workflow for BSE based on QP energies

```
import numpy as np
from gpaw.response.bse import BSE

fn = 'Si_g0w0-k8-ecut100_results.pkl'
qp = pickle.load(open(fn, 'rb'))['qp']
# initialize BSE object
bse = BSE('Si_gs.gpw', ecut=20,
          gw_skn=qp[:, :, :], # should conform vb and cb
          valence_bands=[1, 2, 3],
          conduction_bands=[4, 5, 6],
          nbands=48, txt='Si_bse.txt')

# start real work
bse.get_dielectric_function(q_c=[0.0, 0.0, 0.0], eta=0.05,
                           write_eig="eig-Gamma.dat",
                           w_w=np.linspace(0, 10, 10001),
                           filename="dielec-Gamma.csv")
```

Instead of QP energies from GW, one can also use KS energies modified by scissors operator, especially when the \mathbf{k} -mesh is rather dense.

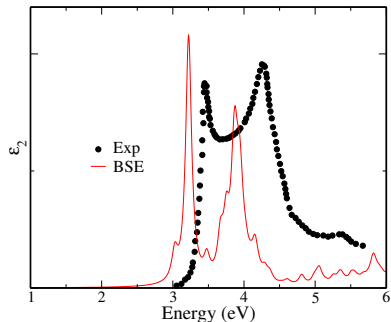
Glimpse of BSE output

eig-Gamma.dat

BSE eigenvalues in eV

0	3.024679	0.0000005562442000
1	3.024683	0.0000013704094051
2	3.024692	0.0000631292840484
3	3.026919	0.0000048426899972
...		
7	3.027818	0.0007571072157299
8	3.033229	0.0000001653798213
9	3.221671	3.5624202893152370
10	3.221758	0.0018907765523448
11	3.221800	1.6440997878927592
...		
163	3.874447	1.2703810427209152
...		

Plot with dielec-Gamma.csv

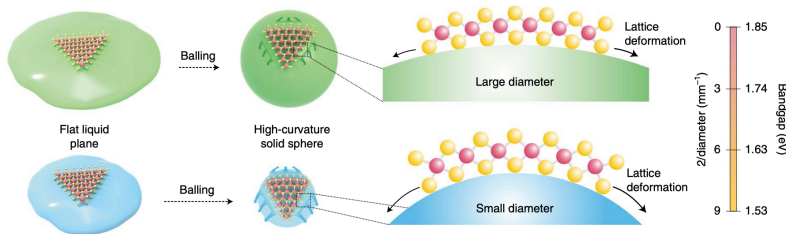


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Background

Efficient gap tuning of mono-layer MoS_2 by sphere diameter engineering

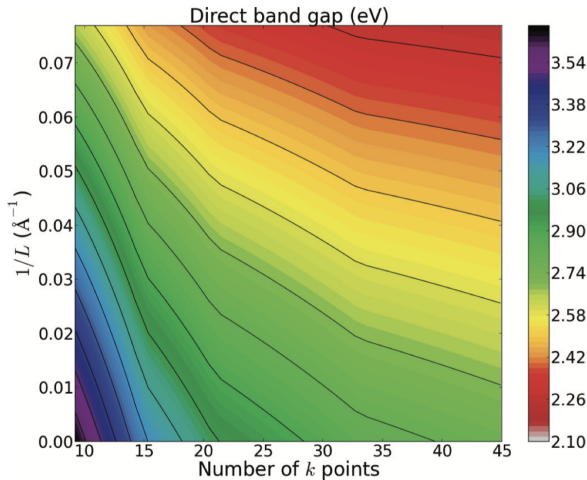


Argument to be verified: biaxial or isotropic strain is more efficient to tune the band gap of monolayer MoS_2 than uniaxial strain.

Modelling of gap engineering by strains

- strain: ml-MoS₂ with scaled lattice constant (internally relaxed)
- band gap: measured by photoluminescence (PL) spectra in experiment
(ignore Stokes shift) → optical absorption spectra
(ignore phonon-assisted excitation) → GW/BSE without moment transfer

Error cancellation in QP energy of 2D system



Solutions in GPAW

The convergence can be improved by

- 2D truncation of Coulomb interaction for L
- analytic correction from averaging $\epsilon(\mathbf{q} \rightarrow 0)$ around Γ for \mathbf{k} -mesh

```
strain = 'bi_0p'  
diagfile = f'MoS2_{strain}_fulldiag.gpw'  
domega0, omega2 = 0.01, 10  
fn = f'MoS2_g0w0_{strain}'  
  
gw = G0W0(calc=diagfile, method="G0W0",  
          filename=fn, bands=(8,18), # vb at index 12  
          ecut=200, ecut_extrapolation=True,  
          truncation='2D', q0_correction=True,  
          domega0=domega0, omega2=omega2)  
gw.calculate()
```

Cut-off of response function and frequency integration should also be checked (against our interested property: first absorption peak)

QP band with SOC considered

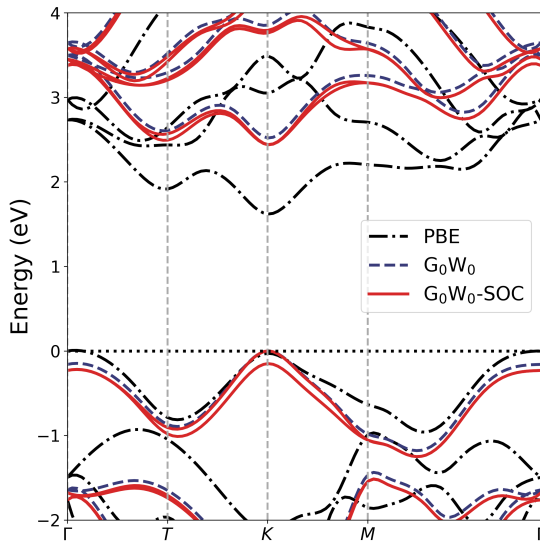
For ML-MoS₂, spin-orbit coupling is essential to get qualitatively correct absorption spectra

```
from gpaw.response.gw_bands import GWBands

K = np.array([1/3, 1/3, 0.0])
M = np.array([0.0, 0.5, 0.0])
G = np.array([0.0, 0.0, 0.0])
kpoints = np.array([G, K/2, K, M, G])

GW = GWBands(calcfile=gsfile,
              gwpckl=f'{filename}.pckl',
              kpoints=kpoints, bandrange=(8,17))
soc = GW.get_gw_bands(SO=True, dft=False,
                      interpolate=True, vac=False)
```

QP band with SOC considered



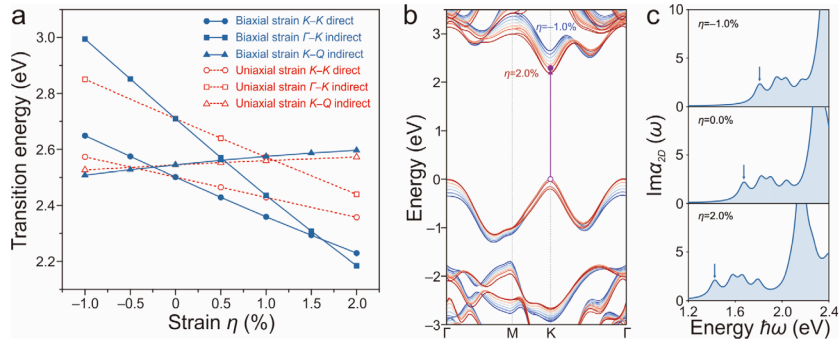
BSE on top of GW-SOC

Modification to the original BSE object

```
bse = BSE(gsfile, txt='Gamma_bse.txt'  
          spinors=True, gw_skn_soc=gw_skn_soc,  
          truncation='2D',  
          valence_bands=[11, 12],  
          conduction_bands=[13, 14],  
          nbands=nbands_bse, ecut=ecut)
```

- shape of `gw_skn_soc` conforms that of eigenvalues in ground-state calculations, which is different from `gw_skn`
- parameter `gw_skn_soc` in fact was not working properly in v1.5.2
- in v21.1.0, `gw_skn_soc` is removed and SOC is handled by `spinors`

Results



Thank you for listening!

References

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