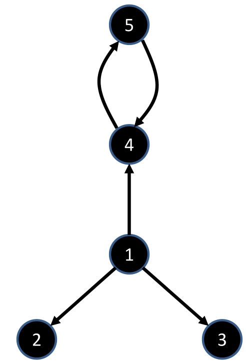
NEUR 2110 STATISTICAL NEUROSCIENCE – HOMEWORK 3

Exercise 1 (5 points): Power, coherence and partial coherence based on multitaper spectra. Consider the vector AR(3) process ("~ 20 Hz β –network")

$$
\begin{aligned}
X_{1,t} &= 0.95\sqrt{2}X_{1,t-1} - 0.9025X_{1,t-2} + \varepsilon_{1,t} \\
X_{2,t} &= 0.5X_{1,t-2} + \varepsilon_{2,t} \\
X_{3,t} &= -0.4X_{1,t-3} + \varepsilon_{3,t} \\
X_{4,t} &= -0.5X_{1,t-2} + 0.25\sqrt{2}X_{4,t-1} + 0.25\sqrt{2}X_{5,t-1} + \varepsilon_{4,t} \\
X_{5,t} &= -0.25\sqrt{2}X_{4,t-1} + 0.25\sqrt{2}X_{5,t-1} + \varepsilon_{5,t}
\end{aligned}
,
$$



where X_t is a 5-dimensional column vector. The noise vector is Gaussian white with diagonal covariance matrix, where the diagonal entries correspond to [0.6, 0.5, 0.3, 0.3, 0.6]. (Note: a diagonal matrix is used here for simplicity. We could explore instantaneous noise correlations with a nondiagonal matrix.)

(a) (1 point) Is the process stationary? Check that the VAR(3) is stable.

(b) (1 point) Using the script provided in the warm-up code, sample 200 trials, each 3-second long with a sampling rate of 200 Hz. The initial condition can be zero mean unit variance Gaussian. Remember to throw way the first 200 samples or so to remove the transient. Plot a single trial showing all of the 5 channels. For sampling, we define the coefficient matrix in Matlab as (M = 5 is the dimension of the AR process of order = 3)

```
A = zeros(M,M,order);
a=sqrt(2);
A(1,1,1) =  0.95*a;
A(1,1,2) = -0.9025;
A(2,1,2) =  0.5;
A(3,1,3) = -0.4;
A(4,1,2) = -0.5;
A(4,4,1) =  0.25*a;
A(4,5,1) =  0.25*a;
A(5,4,1) = -0.25*a;
A(5,5,1) =  0.25*a;
```

Adapt the script provided in the warm-up code to estimate the spectral matrix based on the multitaper method. Use the provided function "multitaperSpectrum.m" and a 5 Hz bandwidth.

(c) (3 points) Compute and plot the power spectrum, spectral coherence and partial spectral coherence based on the estimated spectral matrix. (Your results should replicate the figure shown in the Lecture 5.)

Exercise 2 (5 points): Power, coherence and partial coherence computed from estimated VAR(p) models. Based on the sample from the model generated in exercise (1), adapt the script provided in the warm-up code to estimate the VAR(p) model using the maximum entropy method for the multivariate case. Use the provided function "var_maxent.m". This function expects the data X to be a 3D array in the format of X(M,N,R), where M = 5 is the dimension of the model, N is number of samples per trial, and R is the number of trials. Based on the estimated coefficient matrices and noise covariance:

(a) (2 points) Compute the transfer function and from it the spectral matrix (see script in warm-up code). Note: because of numerical issues, you will need to enforce the diagonal of the spectral matrix to be real. The matlab function “real.m” can be used for that purpose. Compute the corresponding (auto) power spectrum for each of the 5 components.

(b) (3 points) As in exercise 1(d), compute and plot the power spectrum, spectral coherence and partial spectral coherence, but now based on the AR(p) estimated spectral matrix. Your results should replicate the figure shown in Lectures 5-6.