NEUR 2110 STATISTICAL NEUROSCIENCE – HOMEWORK 1 (Lectures 1 & 2)

Exercise 1 (4 points): Load the file Ch2-EEG-4.mat into MATLAB (Kramer & Eden, Chapter 2). You'll find two variables:

EEG = the EEG data, consisting of 1,000 trials, each observed for 1 s; *t* = the time axis, which ranges from 0 s to 1 s. These data have a similar structure to the data studied in chapter 2. To collect these data, a stimulus was presented to the subject at 0.25 s. Analyze these data for the presence of an evoked response. To do so, answer the following questions.

(a) What is the sampling rate (samples/s)? Is the sampling interval the same across the recording interval? Plot all the individual trials of these data using the function "imagesc.m" (similarly to Fig. 2.4, Chapter 2). Explain what you observe in words. From your visual inspection, do you expect to find an ERP in these data?

(b) Compute the ERP for these data, and plot the results. Do you observe an ERP (i.e., times at which the 95% confidence intervals do not include zero)? Include 95% confidence intervals in your ERP plot, and label the axes. Use the nonparametric bootstrap (Chapter 2, pages 34 – 37). Explain in a few sentences the results of your analysis, as you would to a collaborator who collected these data.

(c) Compute the ensemble variance for these data, and plot the results. Show in the same plot the corresponding 95% confidence interval obtained via nonparametric bootstrap.

(d) Is the process mean stationary? Is the process variance stationary? Explain what you see.

Exercise 2 (1 point): Consider the following stochastic process (random walk)

$$X_t = \alpha X_{t-1} + \varepsilon_t ,$$

for $\alpha = 1$. Assume that $t \geq 1$, $X_0 = 0$ and that the sequence $\{\varepsilon_t\}$ is a zero mean Gaussian white noise process with variance σ^2.

(a) Sample *R* = 100 realizations (sample paths) of this process, using $\sigma^2 = 2$, $t = 1, 2, \ldots, T$, $T = 1000$. Compute and plot the sample ensemble mean and variance functions of the process. Is the process mean and variance stationary?

(b) Repeat the above for the case of the random walk with a drift (use $\mu = 0.1$)

$$X_t = \mu + \alpha X_{t-1} + \varepsilon_t .$$

Exercise 3 (5 points): Consider the AR(6) process

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_6 X_{t-6} + \varepsilon_t ,$$

where $t \geq 0$, $X_0 \sim N(0,1)$, $a = [3.9515 \text{ -}7.8885 \ 9.7340 \text{ -}7.7435 \ 3.8078 \text{ -}0.9472]$ and $\{\varepsilon_t\}$ is a Gaussian white noise process (zero mean, unit variance).

(a) Show that the process is stationary by establishing that it is stable based on the analysis of the companion matrix (augmented Markov representation) of the process.

(b) (Optional) Compute and plot the (truncated) impulse response function up to the first 600 time steps.

(c) Simulate the process via direct iteration of the difference equation; Generate a single realization of the process and throw away the first 500 transient samples keeping only the last N = 1024 samples. Plot the sample path.

(d) (Optional) Simulate the process via convolution of the above impulse response with the noise sequence. For comparison purposes, use reflective boundary conditions for iteration of the difference equation and zeros for the convolution. Plot the sample paths from obtained via direct iteration and convolution, using the same input noise sequence.

(e) Compute and plot the biased and unbiased autocovariance and autocorrelation functions. Why does the biased case approach zero? (See Kramer & Eden, Chapter 3.) Plot the approximate 95% confidence interval for the autocorrelation function.

(f) From the simulated sample path, estimate the AR(6) process coefficients using the Burg maximum entropy method (see example in the warm-up code).

(g) (Optional) Try different model orders AR(p), p =1, …, 20. Determine the model order using the Bayesian Information Criterion (BIC). Plot the BIC as a function of the model order.

(h) (Optional) Plot the partial autocorrelation function up to lag 20 and the corresponding approximate 95% confidence interval.

(i) Whiteness test: compute and plot the autocorrelation function of the residual error and the corresponding approximate 95% confidence interval for the estimated AR(6).