NEUR 2110 STATISTICAL NEUROSCIENCE – HOMEWORK 2

Exercise 1 (4 Points): Spectrograms based on periodogram (rectangular window), Hann taper and multitaper.

(a) Load the seizure data from the file MG29S1.mat (Truccolo et al., Nature Neurosci, 2011). The file contains the variable "LFP" and the corresponding sample times in the variable "t". The LFP was recorded from a microelectrode array intracortically implanted in the middle temporal gyrus of a person with pharmacologically intractable seizures. The LFP was downsampled to 1 kHz and has units in microvolts. The seizure starts at time zero and ends at about 66.5 seconds. Plot the LFP. Based on the visual inspection of the time domain signals, what are the main rhythms occurring during the seizure? Some of those rhythms may occur transiently for tens of seconds and then disappear, or they might gradually slow down in frequency.

(b) Compute the spectrogram based on the periodogram (rectangular window), Hann taper, and the multitaper approaches. Adapt the code provided in the warm-up script. Use a time window W = 0.5 s and bandwidth R = 8 Hz. For the step size use half the size of the time window. Plot the corresponding spectrograms in dB as a 2D images (e.g. using imagesc.m), showing time in seconds and frequency in Hz. For guidance, you can refer to the spectrogram of the same data shown in the handout for Lecture 4. When converting to dB, do not normalize by the max power. How do the three approaches/spectrograms compare? How many different transient rhythms can be detected? What are their main frequency bands?

Exercise 2 (1 Point): The Fourier transform of the Gaussian probability density function

$$g_\sigma(t) = \frac{1}{(2\pi\sigma^2)^{1/2}} \exp(-\frac{t^2}{2\sigma^2})$$

is also a Gaussian

$$G_\sigma(f) = \exp(-2\pi^2 f^2 \sigma^2).$$

Note that (a) since a zero mean Gaussian is an even function about zero, its Fourier transform is also a real valued function; (b) since it is a density, its integral corresponds to 1 and is invariant as the variance approaches zero. What happens to the Fourier transform of the Gaussian as the variance approaches zero? Plot the Gaussian and the corresponding Fourier transform for $\sigma = 4, 1, 0.1, 0.001$, over the range [-10, 10]. Apply this result to the interpretation of the case of the Fourier transform of a white noise autocovariance function (δ-function) and its corresponding power spectrum. The above is based on the classic approach to the Dirac δ-function. Modern approaches use generalized functions / distribution theory.

Exercise 3 (4 points): Rectangular window, zero padding and frequency resolution.

(a) Compute the discrete Fourier transform of the following cosine signal:

$$x(t) = \cos(2\pi f t),$$

for $f = 50$ Hz, sampled during 1 second at every $\Delta = 0.001$ s.

Plot the power spectrum using the periodogram (i.e. the one-sided power spectrum based on the DFT) in dB. Use three different zero padding conditions: 1, 2 and 6 second zero padding.

(b) The power spectrum does not look like a line spectrum, i.e. a δ-function at 50 Hz. Why?

(c) Why does the shape of the power spectrum change with different zero padding conditions?

(d) Consider the following signal

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t),$$

for $f_1 = 57$ Hz and $f_2 = 60$ Hz. What is the maximum frequency resolution for a Fourier transform applied to a 200 ms time window? Would you be able to discriminate between the two frequencies? Would zero padding help the discrimination? What should one do in order to discriminate the two frequencies?

Exercise 4 (1 Point): Multitaper single trial confidence intervals. Generate a single (4-second long) realization of a *f* = 40 Hz sinusoid plus Gaussian white noise (zero mean, unit variance)

$$X(t) = \cos(2\pi ft) + \varepsilon(t).$$

Use a sampling rate of 1000 Hz. Compute the multitaper power spectral function estimate using a bandwidth of 4 Hz. Plot also the corresponding 95% confidence intervals based on the χ^2 approximation. Based on Lecture 4, how does the performance of the χ^2 approximation depends on the number of degrees of freedom?

Exercise 5 (Optional): Theoretical and sample power spectrum for an AR(6) (Babadi and Brown, 2014). Consider the AR(6) process

$$X_t = a_1 X_{t-1} + a_2 X_{t-2} + \ldots + a_6 X_{t-6} + \varepsilon_t,$$

where $t \geq 0$, $X_0 \sim N(0,1)$, $a = [3.9515\ \text{-}7.8885\ 9.7340\ \text{-}7.7435\ 3.8078\ \text{-}0.9472]$ and $\{\varepsilon_t\}$ is a (zero mean, unit variance) Gaussian white noise process.

(a) Sample a 1-second long realization of the process (N = 2^10).

(b) Compute and plot the (one-sided) power spectrum based on the periodogram. Plot the sample power spectrum and the theoretical power spectrum (dB, frequency in Hz) corresponding to this AR(6) process. For the computation of the theoretical power spectrum, use the script provided in the warm-up code. Contrast and discuss the result in terms of bias and variance of the periodogram estimator (Lecture 2).

(c) Plot the (one-sided) power spectrum using the multitaper method. Use 4 tapers, and a bandwidth of 5 Hz. Plot the power spectrum based on the periodogram, multitaper and theoretical spectra. (Use the function multitaperSpectrum.m)

(**Note**: although your plots should look qualitatively similar to the figures in Babadi and Brown (2014), there will be an offset difference due to using seconds as the time unit in the simulation of the process.)

Exercise 6 (Optional): Computing the DPSS sequences or tapers (Babadi and Brown, 2014). Construct the Dirichlet matrix

$$(\Phi_R)_{m,n} := \frac{\sin(\pi R \Delta(m-n))}{\pi(m-n)},$$

where m=1,…,M, n=1:N, (use M=N=2^10), and bandwidth R = 2 α/(NΔ), α=4, $\Delta = 1$.

(a) Compute the multitapers (i.e. discrete prolate spheroidal sequences, dpss) as the eigenvectors of the matrix.

(b) Plot the first 7 tapers ordered accordingly to corresponding decreasing eigenvalues, i.e. leading/higher eigenvalues first.

(c) Plot the power spectrum (based on DFT only) of the first, second and seven tapers. How does the energy concentration change across the different tapers? How different energy concentrations affect the bias of the power spectrum estimator?