Math Assignment

Set 29

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1 Answers

(1) here,
$f(x) = sin^2 x$
We know that,

$$\Rightarrow \cos x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2n!}$$
$$\Rightarrow cosx = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \ldots.$$
$$\Rightarrow cos2x = 1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} - \frac{(2x)^6}{6!} -$$
$$\Rightarrow \cos 2x = \sum_{n=0}^{\infty} \frac{(2x)^{2n+2}}{2(2n+2)!} [Answer]$$

(2)
$\lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x} ....(i)$
we know,
$\Rightarrow \cos x =$
$\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2x)!}$
$\Rightarrow 1 - cosx \quad = \frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - \cdots ...(ii)$

Again,

$$1 + x - e^x = 1 + x - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right)$$
$$= -\left(\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x4}{4!} + \cdots\right) ....(iii)$$

$as,$
$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!} \quad = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + ...|$

$$= \lim_{x \to 0} \frac{\frac{x^2}{2!} - \frac{x^4}{4!} + \frac{x^6}{6!} - ......}{-\left(\frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} - \cdots\right)}$$
$$= \lim_{x \to 0} \frac{\frac{1}{2!} - \frac{x^2}{4!} + \frac{x4}{6!} - ...}{-\left(\frac{1}{2!} + \frac{x}{3!} + \frac{x^2}{4!} + \cdots\right)}$$
$$= \frac{\frac{1}{2!}}{-\frac{1}{2!}}$$
$$= -1$$

$$thus, \quad \lim_{x \to 0} \frac{1 - \cos x}{1 + x - e^x} = -1[Ans]$$

$(3) given,$

$\cosh x \geq 1 + \frac{x^2}{2}$

$we know,$

$\cos hx = \frac{e^x + e^{-x}}{2}$

$= f(x)$

$now,$

$f(x) = \cos hx$

$= \frac{e^x + e^{-x}}{2}$

$\Rightarrow f(0) = \frac{e^0 + e^0}{2} = 1$

$f'(x) = \frac{e^x - e^{-x}}{2}$

$f'(0)) = \frac{1-1}{2} = 0$

$f'(x) = \frac{e^x + e^{-x}}{2}$

$f''(0) = 1$

$f'''(x) = \frac{e^x - e^{-x}}{2}$

$f'''(0) = 0$

$$\Rightarrow \cosh x = f(0) + \frac{f'(0)}{1!}x + \frac{f''(0)}{2!}x + \frac{f''(x)}{3!}x^2 + ... = 1 + 0 + \frac{1}{2!}x^2 + \frac{0}{3!}x^3 + \frac{1}{4}x^4 + \cdots$$

$\Rightarrow \cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots$

Thus:

$$\Rightarrow coshx \geq 1 + \frac{x^2}{2}[Ans]$$

(4)

f(x, y)=x $e^{\sin(x^2y)} \frac{}{(x^2+y^2)^{\frac{3}{2}}}$

$$\Rightarrow \frac{d}{dx}f(x,y) = \frac{\frac{d}{dx}xe^{\sin(x^2y)}\left(x^2+y^2\right)^{\frac{3}{2}} - \frac{d}{dx}(x^2+y^2)^{\frac{3}{2}}xe^{\sin\left(x^2y\right)}}{\frac{d}{dx}\left(x^2+y^2\right)^{\frac{3}{2}}}$$

$$\Rightarrow f_x(x,y) = e^{\sin\left(x^2y\right)}\left(x^5\cos\left(x^2y\right)\frac{dy}{dx} + 2x^4\cos\left(x^2y\right)y + x^3\cos(x^2y)y^2\frac{dy}{dx}\right.$$
$$+2x^2\cos\left(x^2y\right)y^3 - 2x^2 - 3xy\frac{dy}{dx} + y^2$$

$$\overline{}$$
$$\frac{d}{dx}(x^2+y^2)^{\frac{5}{2}}$$

$\Rightarrow f_x(1,0) = e^{-}(1 \cdot 1 \cdot 0 + 21 \cdot 1) \cdot 0 + 1 \cdot 1 \cdot 0 \cdot 0 + 2.1 \cdot 1.0$

$\frac{-2\cdot1-3\cdot1\cdot0\cdot0+0}{(1+0)^{\frac{5}{2}}}$

$\Rightarrow f_x(1,0) = -2[Ans]$

(5)

$given,$

$z = f(x+at) + g(x-at)$

$u = x + at$

$v = x - at$

$thus:$

$z = f(u) + g(v)$

$we\ know, \Rightarrow a = x + a+$
$\Rightarrow \frac{\partial u}{\partial t} = a$
$v = x - at$
$\Rightarrow \frac{\partial v}{\partial t} = -a$

now , if we partial differentiate,

$$\Rightarrow \frac{\partial z}{\partial t} = \frac{\partial u}{\partial t} f'(u) + \frac{\partial v}{\partial t} g'(v)$$

= a f$'(u) + (-a)g'(v)$
$= a\frac{\partial y}{\partial t} f''(u) + (-a)\frac{\partial v}{\partial t} g''(u)$
$= a \cdot a \cdot f''(u) - a(-a)g''(v)$....(i)

$$\Rightarrow \frac{\partial z}{dt^2} = a^2 f''(u) + a^2 g''(v)$$

$z = f(u) + g(v)$
$\Rightarrow \frac{\partial_z}{\partial x} = \frac{\partial u}{\partial x} f'(u) + \frac{\partial u}{\partial x} g'(v)$
$\Rightarrow \frac{\partial_z}{dx} = 1f'(u) + 1g'(v)$
$again:$
$\Rightarrow \frac{\partial^2 z}{dx^2} = \frac{\partial u}{\partial x} f''(u) + \frac{\partial v}{\partial x} g''(v)$
$\Rightarrow \frac{\partial^2 z}{dx^2} = f''(u) + g''(v).....(ii)$

$From$
$(i) \Rightarrow$
$\Rightarrow \frac{\partial^2 z}{dx^2} = af''(u) + ag''(v)$

$$From(ii) \Rightarrow a\frac{\partial^2 z}{\partial x^2} = af''(u) + ag''(v) \Rightarrow = \frac{\partial^2 z}{\partial t^2}$$

$$\Rightarrow \frac{\partial^2 z}{\partial t^2} = a\frac{\partial^2 z}{\partial t^2}[proved]$$

(6) $f(x, y) = \frac{x+y}{y-x}\Big[given\Big]$
$now,\ keeping\ y\ constant, if\ we\ increasing\ x$
$\Rightarrow \frac{\partial f}{\partial x} = \frac{1(y-x)-(-1)(x+y)}{(y-x)^2}$

$\Rightarrow \frac{\partial f}{\partial x} = \frac{y-x+x+y}{(y-x)^2}$

$now,\ at\ (0, 7) point:$
$\Rightarrow \quad \frac{\partial f}{dx} = \frac{2.7}{(7-0)^2} = \frac{2}{7} > 0$
$thus:$
$when\ \ y\ is\ constant\ but\ x\ varies\ at$
$point(0, 7),\ the\ graph\ is\ increasing[Ans]$

$if\ x\ is\ constant\ and\ y$
$varies, then \Rightarrow \frac{\partial f}{\partial y} = \frac{1(y-x)-1(y+x)}{(y-x)^2}$
$= \frac{-2x}{(y-x)^2}$

$at\ point,$

$$(0,7) \Rightarrow \quad \frac{\partial f}{\partial y} = \frac{-2.0}{(7-0)^2} = 0$$

$therefor\ if x\ is\ constant\ and\ y$
$varies\ then\ f(x,y) is\ neither$
$increasing\ nor\ decreasing\ at (0,7) [ans]$

Figure 1: Screenshots of my assignment 2