Probabilistic forward modeling of the PSF

Mohammadjavad Vakili[1], David W. Hogg[1,2,3]

ABSTRACT

1. Introduction

This paper is structured as follows. In section 2, we discuss our proposed data-driven model for the PSF estimation. In section 3, we discuss the optimization method we use in order to infer the model parameters, and also the training and validation criteria for truncating the optimization. In section 4, we demonstrate application of our model in esimating the PSF for the data from *Deep Lens Survey*. In section 5 we perform systematic tests, relavant to weak lensing cosmology, to asses the quality of the results obtained from our model. Finally, we discuss and conclude in 6.

2. Model

We want to use N observed stars for modeling the PSF. Each star postage-stamp has M pixels, and can be represented by an M-dimensional vector $\mathbf{y}_i$:

$$\mathbf{y}_i = \begin{bmatrix} y_{i1} \\ . \\ . \\ . \\ y_{iM} \end{bmatrix}, \tag{1}$$

Therefore, all observations can be compactly represented by $N \times M$ matrix $\mathbf{y}$, such that each row i is the observed star $\mathbf{y}_i$. The central pixel of the postage stamp is chosen such that

[1]Center for Cosmology and Particle Physics, New York University

[2]Center for Data Science, New York University

[3]Max-Planck-Institut für Astronomie

the estimated centroid of the star (x_i, y_i) is in the central pixel. Let us denote the vector connecting the the center of the postage-stamp of star i to its centroid by $(\delta x_i, \delta y_i)$, and the flux of star by f_i. An uncertainty σ_{ij} is associated with j-th pixel of the i-th star. If the only contribution to per-pixel uncertainty is uncorrelated Gaussian noise, we will have $\sigma_{ij} = \sigma_i \delta_{ij}$. However, one can use a more general form of uncertainty to account for the effect of outlier pixels such as those hit by Cosmic rays, or overlapping stars.

A common procedure in the PSF estimation involves the follwoing steps: (i) estimattion of the mean-PSF along each pixel $j \in \{1, ..., M\}$ by iteritavely rejecting the bad pixels, (ii) replacing the rejected pixel of each star by the estimated mean-PSF along that pixel, (iii) normalizing the flux of each star, (iv) applying of a sub-pixel shift to the stars such that the centroid of each star lies on center of the postage-stamp of that star, (v) subtracting the mean from each column of $\mathbf{Y}$, and applying PCA to $\mathbf{Y}$.

We want to model the per-pixel intensity of stars y_{ij} with a linear combination of Q components a_{iq} where $q = \{1, ..., Q\}$, and $Q \ll M$.

$$y_{ij} = \sum_{q=1}^{Q} f_i K_i \odot \big(a_{iq} w_{qj}\big) + \sigma_{ij}, \tag{2}$$

where f_i is the flux of star i, $K_i = K_i(\delta x_i, y_i)$ is a linear operator that shifts the center of the linear combination $\sum_{q=1}^{Q} a_{iq} w_{qj}$ to the centroid of star i, w_{qj} is the j-th component of the q-th basis function, and a_{iq} is projection of star i onto the q-th basis function. The shift operation is done by interpolation, such as bilinear interpolation, or cubic spline interpolation.

Let us consider the case of bilinear interpolation. The model for each star i is M dimensional vector $\mathbf{z}_i$ shifted by the operator K_i, and multiplied by flux value f_i. One can rewrite the vector $\mathbf{z}_i$ as a $p \times p$ postage-stamp, where $p^2 = M$. Applying a bilinear shift operator $K(\delta x_i, \delta y_j)$ results in another postage-stamp whose elements are given by

$$\begin{aligned} \big(K_i \odot z_i\big)_{rs} &= \delta x_i \delta y_i \, z_{i,r,s} \\ &+ (1-\delta x_i)\delta y_i \, z_{i,r+1,s} \\ &+ \delta x_i (1-\delta y_i) \, z_{i,r,s+1} \\ &+ (1-\delta x_i)(1-\delta y_i) \, z_{i,r+1,s+1} \, , \end{aligned} \tag{3}$$

where appropriate boundary conditions need to be used.

In order to estimate the basis functions $\{w\}$, the amplitudes $\{a\}$, and the flux values

$\{f\}$, we need to optimize the chi-squared function

$$\chi^2 = -\log p(\mathbf{Y}|\{w, a, f\}) \tag{4}$$

$$= \text{const} + \sum_{i,j=1}^{N,M} \frac{\left[y_{ij} - \sum_{q=1}^{Q} f_i K_i \odot \left(a_{iq} w_{qj}\right)\right]^2}{\sigma_{ij}^2}, \tag{5}$$

3. Optimization

4. Application to Deep Lens Survey

5. Systematic tests

6. Discussion