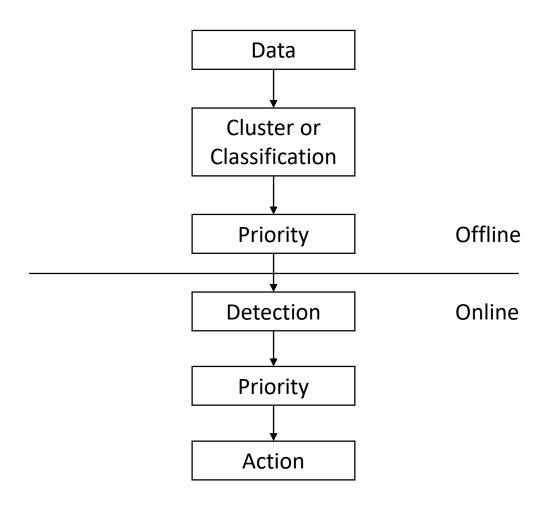
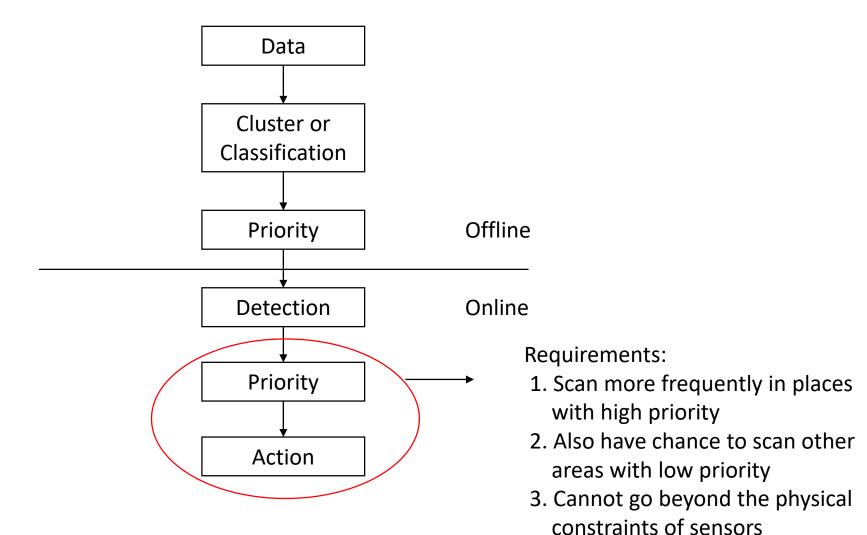
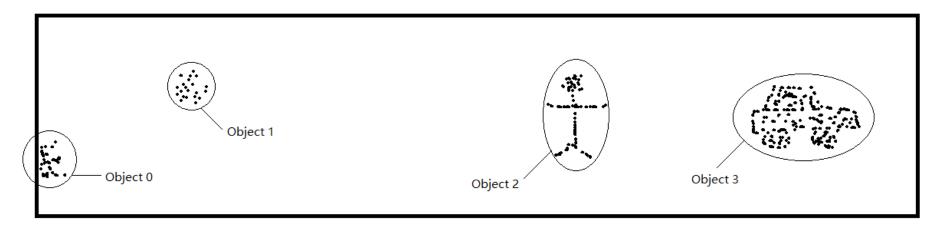
# Pipeline for Active Perception



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# LiDAR Data and Classification



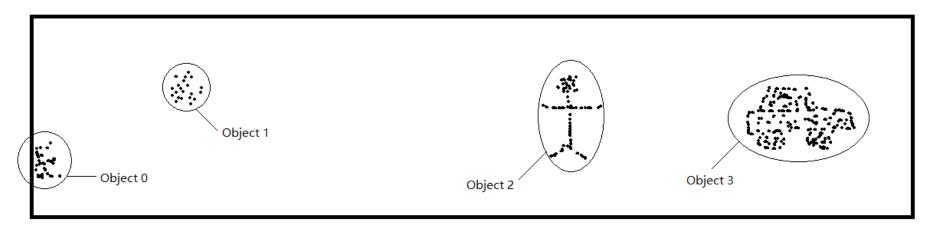
0 degree 360 degree

Object i at time t has following properties :

$$\{r_t^i, \theta_t^i, \Delta \overrightarrow{V_t^i}\}$$

Both these properties decide the priority  $p_t^i$  of Object i:

$$p_t^i = f_p(r_t^i, \theta_t^i, \Delta \overrightarrow{V_t^i})$$



0 degree

Let  $s_t$  denote the state at time t, and let  $s_t^i$  denote the distribution of  $p_t^i$ , so  $b(s_t)$  will be :

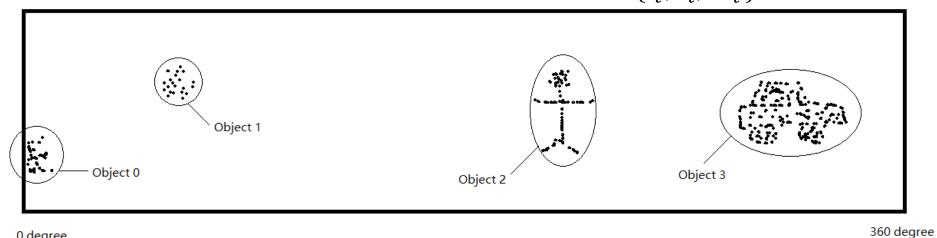
$$b(s_t) = P\{s_t^i | i = 0,1,2 \dots\}$$

Assume  $s_t^i \sim N(\mu_t^i, \sigma_t^i)$ , and  $\mu_t^i, \sigma_t^i$  fulfill :

$$\mu_t^i = \theta_t^i \qquad \qquad \text{Properties of Object $i$ at time $t$}$$
 
$$\{r_t^i, \theta_t^i, \Delta \overrightarrow{V_t^i}\}$$
 
$$\sigma_t^i \propto \frac{1}{r_t^i}$$

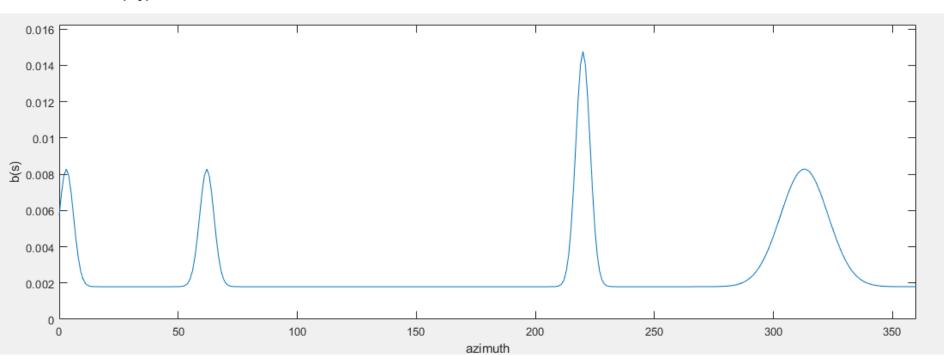


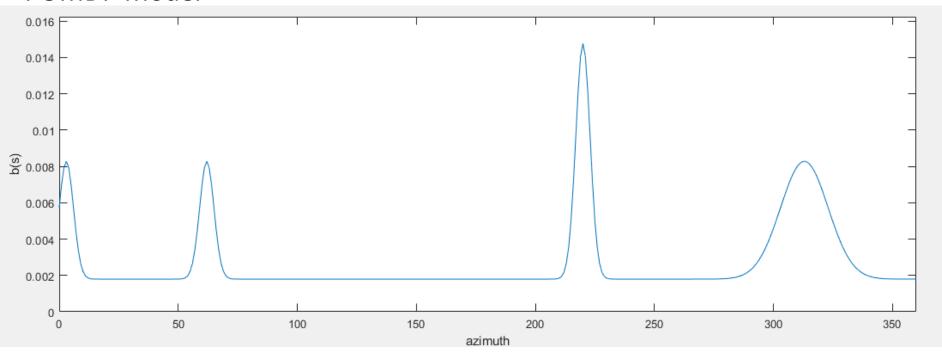
# Properties of Object i at time t $\{r_t^i, \theta_t^i, \Delta V_t^i\}$



# Then $b(s_t)$ will be like :

0 degree





Mathematical representation for  $b(s_t)$ :

$$b(s_t) = \frac{\sum \pi_t^i N(\mu_t^i, \sigma_t^i) + U(0, 2\pi)}{N_t}$$

Properties of Object i at time t

$$\{r_t^i,\theta_t^i,\Delta\overrightarrow{V_t^i}\}$$

$$\pi_t^i \propto \frac{1}{r_t^i}$$

$$N_t = \int (\sum \pi_t^i N(\mu_t^i, \sigma_t^i) + U(0, 2\pi)) ds_t$$
 - Normalization Factor

Mathematical representation for POMDP model:

$$b(s_t') = P(s_t'|s_t, a_t)b(s_t)$$

$$b(s_{t+1}) = \beta P(O_t|s_t', a_t)b(s_t')$$

First, what is  $b(s_t)$ ?

Mathematical representation for POMDP model:

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First, what is  $b(s_t)$ ?

 $b(s_t)'$  means the *belief* of  $s_t$  after action  $a_t$  but before  $O_t$ 

So what makes the change of  $b(s_t)$  from  $b(s_t)$ ?

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Action  $a_t$  and time lapse!

So  $b(s_t)$  should be like:

$$b(s_t') = \frac{\sum \pi_t^{i'} N(\mu_t^{i'}, \sigma_t^{i'}) + U(0, 2\pi)}{N_t'}$$

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$$\pi_t^{i'} = f_{\pi}(\pi_t^{i}, r_t^{i}, \theta_t^{i}, \Delta \overrightarrow{V_t^{i}}, a_t)$$

$$\mu_t^{i'} = f_{\mu}(\mu_t^{i}, r_t^{i}, \theta_t^{i}, \Delta \overrightarrow{V_t^{i}}, a_t)$$

$$\sigma_t^{i'} = f_{\sigma}(\sigma_t^{i}, r_t^{i}, \theta_t^{i}, \Delta \overrightarrow{V_t^{i}}, a_t)$$

Properties of Object i at time t  $\{r_t^i, \theta_t^i, \Delta \overrightarrow{V_t^i}\}$ 

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Then, how to calculate  $b(s_{t+1})$  or how to get  $P(O_t|s_t',a_t)$ ?

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 $\mathcal{O}_t$  denotes the state that we can observe because of action  $a_t$ 

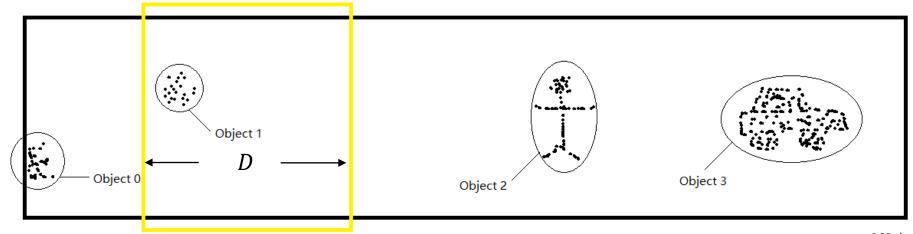
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360 degree

Mathematical representation for POMDP model:

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$$b(s_{t+1}) = \beta P(O_t|s_t', a_t)b(s_t')$$

So  $p(O_t|s_t', a_t)$  should be like that:

$$P(O_t|s_t', a_t) = \sum_{\substack{\mu_{o,t}^j \in [a_t - \frac{D}{2}, a_t + \frac{D}{2}]}} \pi_{o,t}^j N(\mu_{o,t}^j, \sigma_{o,t}^j)$$

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So how to calculate  $b(s_{t+1})$ ?

KL-divergence!

Recall:

$$b(s_t') = \frac{\sum \pi_t^{i'} N(\mu_t^{i'}, \sigma_t^{i'}) + U(0, 2\pi)}{N_t'}$$

$$P(O_t|s_t',a_t) = \sum_{\substack{\mu_{o,t}^j \in [a_t - \frac{D}{2}, a_t + \frac{D}{2}]}} \pi_{o,t}^j N(\mu_{o,t}^j, \sigma_{o,t}^j)$$

Calculate  $b(s_{t+1})$  (  $p(O_t|s_t',a_t)$ ,  $b(s_t')$  ):

for gaussian distribution  $N(\mu_{o,t}^j, \sigma_{o,t}^j)$  in  $P(O_t|s_t', a_t)$ :

find gaussian distribution  $N(\mu_t^{i\prime}, \sigma_t^{i\prime})$  which fulfills :

1. 
$$\mu_t^{i'} \in [a_t - \frac{D}{2}, a_t + \frac{D}{2}]$$

2.  $KL(N(\mu_{o.t}^j, \sigma_{o.t}^j), N(\mu_t^{i\prime}, \sigma_t^{i\prime}))$  is the smallest

$$N(\mu_{t+1}^i, \sigma_{t+1}^i) = \beta N(\mu_t^{i'}, \sigma_t^{i'}) N(\mu_{o,t}^j, \sigma_{o,t}^j)$$

for gaussian distribution  $N(\mu_t^{i'}, \sigma_t^{i'})$  in  $b(s_t')$  that has not been calculated:

$$N(\mu_{t+1}^i, \sigma_{t+1}^i) = N\left({\mu_t^i}', {\sigma_t^i}'\right)$$
 // TODO: This equation is ugly and needs change 
$$b(s_{t+1}) = \frac{\sum \pi_{t+1}^i N(\mu_{t+1}^i, \sigma_{t+1}^i) + U\left(0, 2\pi\right)}{N_{t+1}}$$
 return  $b(s_{t+1})$ 

Mathematical representation for POMDP model:

$$b(s_t') = P(s_t'|s_t, a_t)b(s_t)$$

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So next step is to choose action  $a_t$  based on present  $b(s_t)$ .

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So next step is to choose action  $a_t$  based on present  $b(s_t)$ .

Recall the requirements for active perception:

- 1. Scan more frequently in places with high priority
- 2. Also have chance to scan other areas with low priority
- 3. Cannot go beyond the physical constraints of sensors

Traditional way for POMDP model to choose action is based on the reward function:

$$R(a_t) = f(a_t)$$

$$a_t = \operatorname*{argmax}_{a_t} R(a_t)$$

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This is useful but it cannot fulfill the second requirement. So we need to sample  $a_t$  from the distribution of  $a_t$  instead of directly choose a  $a_t$ .

Also, to fulfill the third requirement, the  $a_t$  should be like :

$$a_t = \{a_t^p, a_t^g\}$$

Requirements for active perception:

- 1. Scan more frequently in places with high priority
- 2. Also have chance to scan other areas with low priority
- 3. Cannot go beyond the physical constraints of sensors

To fulfill the third requirement,  $a_t$  should be like :

$$a_t = \{a_t^p, a_t^g\}$$

 $a_t^p$  denotes the present angle of sensor and  $a_t^g$  denotes the present goal of sensor.

 $a_t^p$  should increase or decrease toward  $a_t^g$  and both of them should fulfill the requirements below :

1. The difference between  $a_{t-1}^g$  and  $a_t^g$  should be small, which is:

$$\left| a_t^g - a_{t-1}^g \right| < \varepsilon_g$$

- 2. The difference between  $a_t^p$  and  $a_{t-1}^p$  should be in the physical constraint of sensor.
- 3. If  $a_t^p$  and  $a_t^g$  is very close, then  $a_{t+1}^g$  should not be close to  $a_t^g$  any more.

 $a_t^p$  and  $a_t^g$  should fulfill the requirements below :

- 1. The difference between  $a_{t-1}^g$  and  $a_t^g$  should be small, which is:  $|a_t^g a_{t-1}^g| < \varepsilon_a$
- 2. The difference between  $a_t^p$  and  $a_{t-1}^p$  should be in the physical constraint of sensor.
- 3. If  $a_t^p$  and  $a_t^g$  is very close, then  $a_{t+1}^g$  should not be close to  $a_t^g$  any more.

So the distribution of  $a_t^g$  should be like:

$$R(a_t^g) = \frac{|a_t^g - a_t^p|^m}{|a_t^g - a_{t-1}^g|^n} \int_{a_t^g - \frac{D}{2}}^{a_t^g + \frac{D}{2}} b(s_t) ds_t$$

$$P(a_t^g|s_t) = \frac{R(a_t^g)}{\int R(a_t^g) da_t^g}$$

Sample  $a_t^g$  from its distribution and choose  $a_t^p$  which fulfills the requirements.