Modelling of Inverted Pendulum System(IPS)

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Let $x_1 = x$, $x_2 = \dot{x}$, $\theta_1 = \theta$ and $\theta_2 = \dot{\theta}$, the nonlinear model for the inverted pendulum is as the following

$$\dot{x}_1 = x_2 + w_1 \tag{1}$$
$$\dot{x}_2 = u + w_2 \tag{2}$$
$$\dot{\theta}_1 = \theta_2 + w_3 \tag{3}$$
$$\dot{\theta}_2 = -k\theta_2 - ncos\theta_1 u + tsin\theta_1 + w_4 \tag{4}$$

where x is the position of the cart, θ is the angle of the pendulum, u is the acceleration of the cart as control input. $t = \frac{3g}{2l}$, $n = \frac{3}{2l}$, $k = \frac{3B_\theta}{ml^2}$. w_1, w_2, w_3 and w_4 is the noise in each dimension (normal distribution with zero mean). The following is the linearisation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & \frac{3g}{2l} & -\frac{3B_\theta}{ml^2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -\frac{3}{2l} \end{bmatrix} u + \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix}$$

The value of parameters are as the following: m = 0.1314; l = 0.68; g = 9.81; B_θ= 0.06. Moreover, tc=0.02s is use for creating a discrete time model based on the equations above. The safety set of the system is $[-0.4\ 0.4] \times [-3\ 3] \times [-\frac{\pi}{18}\ \frac{\pi}{18}] \times [-\frac{\pi}{3}\ \frac{\pi}{3}]$. The number of grids on each dimension is 15, 20, 20, 20. The input space is $[-1\ 1]$ and the number of grids for input space is 30. The covariance of noise on each dimension is 2.5×10^{-5}, 1.0×10^{-6}, 1.0×10^{-6} and 1.0×10^{-4} accordingly. The time horizon for the safety controller should be 20 seconds.