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Life expectancy, retirement and endogenous growth

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Abstract

In this paper I address the links between life expectancy, retirement age and economic growth. I build a finite horizon *OLG* model with exogenous retirement in which human capital accumulation drives endogenous growth. The return on individual investment in human capital depends positively on the remaining active years. Postponing retirement age raises the return and investment in human capital, and the proportion of working individuals, thus increasing the sustainable growth rate. Increments in life expectancy do *not* increase the growth rate by themselves, but reduce it: optimal investment in human capital is not affected and the proportion of retirees becomes larger. Therefore, increases in life expectancy lead to higher growth rates *only if* they are accompanied by simultaneous increments in the working period.

JEL classifications: D91; E17; O41

Keywords: Life expectancy; Retirement; Growth

1. Introduction

'Life expectancy alone is one of the strongest explanatory variables of growth in GDP', World Health Organization.

In this paper I address the issue of how life expectancy and retirement age are linked to economic growth. To this end I build a finite horizon *OLG* model with

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exogenous retirement in which human capital accumulation drives endogenous growth and which I solve numerically.

The issue of the effects of the reduction in mortality rates and the resulting increases in life expectancy upon economic growth rates has recently been addressed in the literature through both empirical and theoretical studies.

Concerning empirical studies, the hypothesis that the reduction in mortality rates has caused higher levels of investment in human capital and, therefore, augmented growth rates is partially supported for various economies. Thus, Kalemli-Ozcan et al. (2000) show an increase in life expectancy at birth along with an increase in average numbers of years of schooling in England. They claim that after averaging across lower income countries, there is apparently a connection between increases in life expectancy at birth and increases in gross secondary school enrollment that is positively connected with higher growth rates observed.[1] Rodríguez and Sachs (1999) also show a positive relationship between life expectancy and growth for the case of Venezuela. Preliminary data from Latin American and Caribbean countries show that *GDP* growth is statistically associated with life expectancy: for instance, estimates based on data from Mexico suggest that for any additional year of life expectancy there will be an additional 1% increase in *GDP* 15 years later. (World Health Organization, 1999, Box 1.2, p. 9). Barro and Sala-i-Martin (1995) estimate for a sample of 97 countries that a 13-year increment in life-expectancy would increase the per capita growth rate by 1.4% per year, although they also find some exceptions.[2] Malmberg (1994) gives another exception: higher growth was achieved when middle-aged persons were numerous, whereas increases in dependent age groups led to lower per capita growth in Sweden.

A common result in most theoretical studies which include some sort of *human capital* is that an increase in life expectancy lengthens the period needed to recover investment in human capital, which translates into higher returns on individual education or human capital investment. This augmented return will give rise to higher levels of investment in human capital which in turn will raise growth rates. Ehrlich and Lui (1991) show in a three overlapping generations model how improvements in longevity lower fertility, thus raising educational investment and long term growth. In a similar setup Meltzer (1995) obtains that mortality reductions may favor economic growth by increasing educational investment. Kalemli-Ozcan et al. (2000) show in an overlapping generations model à la Blanchard–Yaari that if mortality drops, life expectancy increases, so that an augmented life horizon to enjoy the return on human capital investment gives rise to higher schooling (human capital investment), although growth is not affected as the growth rate in their model is identically equal to zero. In a similar setup, Hu (1995) simulates that the projected population aging in the US is likely to increase the growth rate of output by approximately 0.4%. In this case, the interpretation is that population aging increases saving, and thereby capital accumulation. In a slightly modified model,

[1] See other references cited there: Ram and Schultz (1979), Preston (1980) and Meltzer (1995).

[2] Countries in which a higher life expectancy has not resulted in a higher growth: Ghana, Mozambique, Uganda, Zaire, Haiti, Guyana, Uruguay and Venezuela, even though all of them have exhibited a higher schooling level.

Hu (1999) reaches the same result: demographic changes leading to population aging can have a positive growth effect on the economy because they induce an increase in human capital investment. De la Croix and Licandro (1999) also take the Blanchard–Yaari model as the starting point. They build an economy in which there is *no* physical capital (an *AH* economy), and where individuals accumulate human capital as a function of the optimal length of the schooling period that they choose. The effect of lower mortality rates upon growth turns out to be ambiguous as three effects follow: (i) individuals die later on average, thus the depreciation rate of aggregate human capital decreases: (ii) agents tend to study more because the expected flow of future wages has risen, and the human capital per capita increases; but (iii) the economy consists of more old agents who completed their schooling a long time ago. The first two effects have positive influence on the growth rate, but the third a negative one.[3]

A similar result can be found in Reinhart (1999), but this time through a different channel as only *physical capital* is assumed. In a Blanchard–Yaari type model he shows that longer life expectancy generates higher growth: if the mortality rate goes down, individuals discount the future less so that savings (and physical investment) increase, thus raising the growth rate. Futagami and Nakajima (2001), however, obtain the opposite result. In a finite horizon *OLG* model (again, without human capital), they show that higher life expectancy increases growth: even though higher life expectancy causes a lower savings rate (for a given growth rate), in equilibrium the growth rate must be higher because the growth rate has a strong positive effect on the savings rate. Uhlig and Yanagawa (1996) pose a different issue, but one can use their *OLG* model with only physical capital to answer the question that I am dealing with here.[4] By simulating their model it can be shown, nevertheless, that an increase in the number of periods of life (and/or the number of active periods) might result either in higher or in lower growth rates depending on the parameterization of the model.[5]

The model that I use here draws on Nerlove et al. (1993) and Echevarría and Iza (2000): a finite horizon *OLG* model in which individuals are allowed to invest in human capital, and into which the economy's productivity or knowledge enters as an externality in the stock of human capital with which individuals are endowed at birth and in the individual production of human capital. The private return on individual investment in human capital at any age before retirement depends positively on the remaining active years. Consequently, postponing the retirement age raises the return and investment in human capital; additionally, the proportion of working individuals in the society is enlarged. Therefore, the sustainable growth

[3] There is an additional negative effect: if individuals spend more time at school, they enter the labor market later. Therefore, the share of active population becomes smaller.

[4] They show that under rather mild conditions, higher capital income taxes (and lower labor income tax rates) may lead to faster growth rates in *OLG* economies with endogenous growth. However, greater life expectancy (higher number of periods of life) implies that the result of a positive capital income tax rate to maximize growth may no longer hold.

[5] In particular it depends on the value assigned to the parameter in the aggregate production function representing an *AK* technology.

rate becomes higher. Increments in life expectancy at birth do *not* increase the growth rate by themselves, however, but reduce it: the optimal investment in human capital is not affected and the proportion of retirees becomes larger. This is in line with a commonly accepted view: if life expectancy increases, the ratio of retirees (dissavers) to workers (savers) goes up, so the aggregate saving rate falls and so do accumulation in physical capital and growth. In short, increases in life expectancy at birth cause higher growth rates *only if* they are accompanied by simultaneous increments in the working period.

In the light of this simple model, I believe that the theoretical reasoning often used in the literature to explain how a higher life expectancy gives rise to a higher level of economic growth through human capital arguments must be rephrased. When one considers *infinite horizon* models à la Blanchard–Yaari, for instance, there is usually no room for retirement age. Therefore any increase in life expectancy represents an enlarged working period, thus increasing the return on human capital investment. But when *finite horizon* economies are considered instead, the existence of a retirement age becomes essential to characterize the return on human capital investment. Increments in life expectancy by themselves do not necessarily cause changes in human capital investment. The growth rate might even turn out to be lowered as explained above. The argument might perhaps be right for models in which there is no retirement (so that the last active period coincides with the last period alive) and for less developed economies in which retirement takes place late in individuals' lives (because they keep on working as long as they are physically capable). In this case, improvements in food and health conditions lead to higher life expectancies *and* longer working periods.

The paper is organized as follows. Section 2 sets up and solves the model: the individual problem (Section 2.1), the aggregates of individual choices (Section 2.2), the production side (Section 2.3), and the characterization of the steady state growth rate (Section 2.4). Section 3 sets out and discusses the values assigned to the parameters in the model. Section 4 shows the results for the benchmark model (Section 4.1), and presents some sensitivity analyses (Section 4.2). Section 5 concludes. A mathematical appendix is included at the end of the paper (Appendix A).

2. The model

2.1. The individual problem

In this subsection I, first, briefly describe the economy and, second, I set up and solve the individual problem.

Following the lines of Nerlove et al. (1993) and Echevarría and Iza (2000), I assume a discrete time, finite horizon, overlapping generations model of endogenous growth. Individuals in this economy make two choices: optimal consumption and optimal investment in human capital. Current utility depends only on consumption. I assume that there is no leisure choice, so that labor supply is inelastic and normalized to one unit (until retirement age is reached). Human capital (which allows individuals to obtain higher productivity and labor income in the future) is home produced by making use of two inputs: income not consumed nor saved as financial wealth (the human capital investment) and the state of knowledge in the economy. Individuals take the latter as given, although it is the result of equilibrium aggregate individual production of human capital. Concerning production, I assume that there is a unique good in the economy which is produced out of physical and human capital, and which can be consumed, accumulated as physical capital or used to produce human capital.

For simplicity, the analysis is conducted only for steady state paths, thus leaving aside transition paths. Along steady state paths it turns out that aggregate variables, when measured in efficiency units (i.e. relative to aggregate human capital), remain constant and when measured in per capita terms, grow at an endogenous, constant rate. This greatly simplifies the job because the stock of physical capital per efficient unit of labor is held constant and so are factor prices.

Let us assume an individual born at time t with a finite lifetime horizon T who maximizes the present discounted value of future utilities. Current utility depends on current consumption, denoting the consumption that the individual born at time t makes at age a as $C_{t,a}$.[6]

I assume that individuals enter the labor market at age 1, and from that moment onwards they supply inelastically one unit of labor per period until they retire at age T' (exogenously given). During this first (working) phase, individuals invest in both physical and human capital. Expenditure at age a consists of expenditure on consumption $C_{t,a}$, plus expenditure on human capital, $H_{t,a}$. In the second part, after retirement, individuals live only on accumulated savings (that is, physical capital income) until they die. Given the absence of mandatory social security, individuals must necessarily accumulate enough physical capital while active to guarantee consumption for the retirement period.

As for revenues, I first assume that individuals are born with a non-negative *human* capital or innate productivity level that depends on the state of knowledge in the society at their birth date, $A_t^b > 0$. This represents the services that a physically healthy person offers in his first year of active life (Romer, 1990, p. S79). Individuals obtain labor income $E_{t,a}$ while active, that is, for $1 < a < T'$. Individual knowledge at birth is assumed to be positively related to the state of knowledge in the economy at that time. In particular, I assume that

$$A_t^b = mB_t, \quad m \geqslant 0, \tag{1}$$

where B_t denotes the level of knowledge in the society at time t.

Labor income at age a is given by the product of wage per efficiency unit w times the human capital accumulated until that period or the number of effective units of labor $A_{t,a}$, that is:[7]

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[6] When two subscripts are shown at the bottom of individual variables, the former will denote birth date, while the latter will stand for age.

[7] I drop the time subscript for w because I will focus on steady state equilibria and, as we shall see later on, these are characterized by constant factor prices.

$$E_{t,a}=wA_{t,a}, \quad \text{for } 1\leqslant a\leqslant T'. \tag{2}$$

The (home) production technology of human capital is the same as that used by Nerlove et al. (1993). Namely, we have that

$$h_{t,a+1}=GH_{t,a}^{\gamma}B_{t+a}^{1-\gamma}, \quad \text{for } 1\leqslant a\leqslant T', \tag{3}$$

where $h_{t,a+1}$ is the flow of human capital which is incorporated into the stock of the individual at the age of $a+1$, $H_{t,a}$ is the expenditure on human capital made at the age of a, B_{t+a} is the state of knowledge of the economy at the beginning of time $t+a$, and G is a positive constant playing the role of a scaling factor. Knowledge in the society, therefore, gives rise to two sources of externality: stock of human capital at birth and human capital production.[8] Notice first that an individual's human capital investment will increase his/her own productivity level according to the state of knowledge of the economy which he takes as given (when $\gamma<1$). But, as we will see later, the state of knowledge of the economy will itself depend on the aggregate production of human capital.[9] Note also the timing of human capital production: there is a one period lag between investment at age a ($H_{t,a}$) and production at age $a+1(h_{t,a+1})$. Finally, individuals will invest in human capital only until retirement age. Assuming that human capital depreciates at a constant rate δ_H, the number of effective labor units of an individual at age a (whenever he/she remains active) is given by

$$A_{t,a}=\begin{cases} A_t^b, & \text{for } a=1 \\ A_t^b(1-\delta_H)^{a-1}+\sum_{j=2}^{a}h_{t,j}(1-\delta_H)^{a-j}, & \text{for } 2\leqslant a\leqslant T' \end{cases}. \tag{4}$$

Along these lines, Stokey and Rebelo (1995) claim that the largest source of depreciation of aggregate human capital comes from the fact that lifetimes are finite. Therefore, overlapping generations models allow a more satisfactory treatment of this issue than infinite horizon representative agent models. This, in turn, raises a new problem: how human capital is transmitted from one generation to the next. In this model current generations learn from previous generations: they take advantage of the accumulated knowledge in society while they are employed.

I assume that there are no human capital bequests. In other words, individuals do not directly inherit their knowledge from their parents, although the economy as a whole does transfer knowledge from one period to the next through B. I am here

[8] I will discuss the law of motion for B_t later. The production of human capital depends on the state of knowledge in the period in which the individual makes the investment H, one period before the production h incorporates into his human capital stock. But, as mentioned above, I will focus on steady states in which social knowledge grows at a constant rate.

[9] See Lucas (1990b) and Einarsson and Marquis (1996) for empirical evidence in favor of externalities in human capital production.

considering a concept of human capital in the spirit of Romer (1990), which is closely related to the knowledge that an individual accumulates in his/her lifetime and that dies with him/her. However, the state of knowledge that may exist outside the individual (a scientific law, a mechanical principle, the wheel, etc.) increases over time as long as there are individuals who allocate resources to becoming skilled and to generating knowledge. Thus, B represents the level of knowledge accumulated in the past by training and research institutions and it increases in every period with the contributions of researchers. As in Romer (1990), human capital (associated with the labor force) gets paid, while knowledge does not. A remarkable point is that the growth of B is due to external effects: it is a non-rival and non-excludable good (a public good, in short) from which individuals get profit without having to pay for it.

Finally, I assume the existence of perfect capital markets that allow individuals to lend and borrow at the same interest rate r, which remains constant because of the steady state assumption. Formally, assuming that current utility from consumption is isoelastic (the intertemporal elasticity of substitution being given by $1/\sigma$, $\sigma>0$) and that the subjective discount factor is $0<\beta<1$, the problem that an individual faces at the beginning of his/her active life can be expressed as

$$\max_{\{C_{t,a}\}_{a=1}^{T},\{H_{t,a}\}_{a=1}^{T'}} \sum_{a=1}^{T} \beta^{a} \frac{C_{t,a}^{1-\sigma}}{1-\sigma} \tag{5}$$

subject to

$$\sum_{a=1}^{T} \frac{C_{t,a}}{(1+r)^{a}} = \sum_{a=1}^{T'} \frac{E_{t,a}}{(1+r)^{a}} - \sum_{a=1}^{T'} \frac{H_{t,a}}{(1+r)^{a}}, \tag{6}$$

in addition to Eqs. (1)–(4), where Eq. (6) represents the intertemporal budget constraint.

Substituting Eqs. (1)–(4) into Eq. (6), the problem can be reduced to maximizing Eq. (5) with respect to $\{C_{t,a}\}_{a=1}^{T}$ and $\{H_{t,a}\}_{a=1}^{T'}$ subject to a single constraint. Given the absence of leisure from the utility function, the problem can be solved in two stages. In the first stage, individuals choose the path of optimal plans of investment in human capital ($H_{t,a}^{*}$, for $1\leqslant a\leqslant T'$) that maximizes the present discounted value of the difference between their labor income and their expenditure on human capital and which will be denoted by RHS_t^{*}. In the second stage, for the previous optimal investment in human capital, individuals choose the sequence of optimal consumption levels ($C_{t,a}^{*}$, for $1\leqslant a\leqslant T$) that maximize the present discounted sum of future utilities. Thus, upon solving the first stage, one has that (Appendix A)[10]

$$H_{t,a}^{*} = \left\{ \frac{\gamma G w}{\rho-\Delta} \left[1-(\Delta/\rho)^{T'-a}\right] \right\}^{1/(1-\gamma)} B_{t+a}, \quad \text{for } 1\leqslant a\leqslant T', \tag{7a}$$

[10] In what follows, a '*' symbol over a choice variable will denote 'optimal value'.

and where $\Delta\equiv(1-\delta_H)$ and $\rho\equiv 1+r$.

Notice that Eq. (7a) can be rewritten as

$$\rho=Gw\gamma\left[\frac{B_{t+a}}{H^*_{t,a}}\right]^{1-\gamma}\left[\frac{1-(\Delta/\rho)^{T'-a}}{1-\Delta/\rho}\right],\quad \text{for } 1\leqslant a\leqslant T', \tag{7b}$$

which has an intuitive interpretation: along the optimal investment path the return on investment in physical capital must necessarily be equal to the *private* return on investment in human capital.

Note that, firstly, as long as the human capital depreciation rate δ_H is less than one (or $\Delta>0$), the investment in human capital which an individual makes when he/she is a periods old will increase his/her future labor income until he/she retires (i.e. from the period in which he/she turns $a+1$ until the period in which he/she turns T').

Secondly, and more importantly, the return on human capital investment depends positively on the years remaining until retirement age, i.e. $T'-a$. Younger workers have stronger incentives than older ones to invest in human capital. In particular, given the time structure of the production technology of human capital, it will turn out that optimal investment in human capital in the last period of active life must be zero, that is to say, $H^*_{t,T'}\equiv 0$. This will help us to understand the result concerning the effects of changing T': a (say) reduction in T' will imply a lower private return on human capital, a lower investment and, consequently, a smaller sustainable growth rate of knowledge in the economy.

Thirdly, if the economy is growing at a positive rate, so that B_t is increasing over time, investment in human capital becomes more and more productive: this effect works in the opposite direction than the one just mentioned above, making $H_{t,a}$ increase with age.[11]

After the second stage is solved, the optimal consumption is given by:

$$C^*_{t,a}=\frac{(\rho\beta)^{a/\sigma}RHS^*_t}{Q},\quad \text{for } 1\leqslant a\leqslant T, \tag{8}$$

where by definition

$$Q\equiv\left[(\beta\rho^{1-\sigma})^{1/\sigma}-(\beta\rho^{1-\sigma})^{(T+1)/\sigma}\right]\left[1-(\beta\rho^{1-\sigma})^{1/\sigma}\right]^{-1}.$$

(Appendix A.)

From Eqs. (3), (7a) and (7b) I obtain the production and the stock of human capital as, respectively,

$$h^*_{t,a+1}=\left\{\frac{\gamma w}{\rho-\Delta}[1-(\Delta/\rho)^{T'-a}]\right\}^{\gamma/(1-\gamma)}G^{1/(1-\gamma)}B_{t+a},\quad \text{for } 1\leqslant a\leqslant T', \tag{9}$$

[11] When the externality $1-\gamma$ in Eq. (3) is high enough, an inverted-U shape pattern for $H_{t,a}$ can be obtained, so that human capital expenditure increases in the early periods of life.

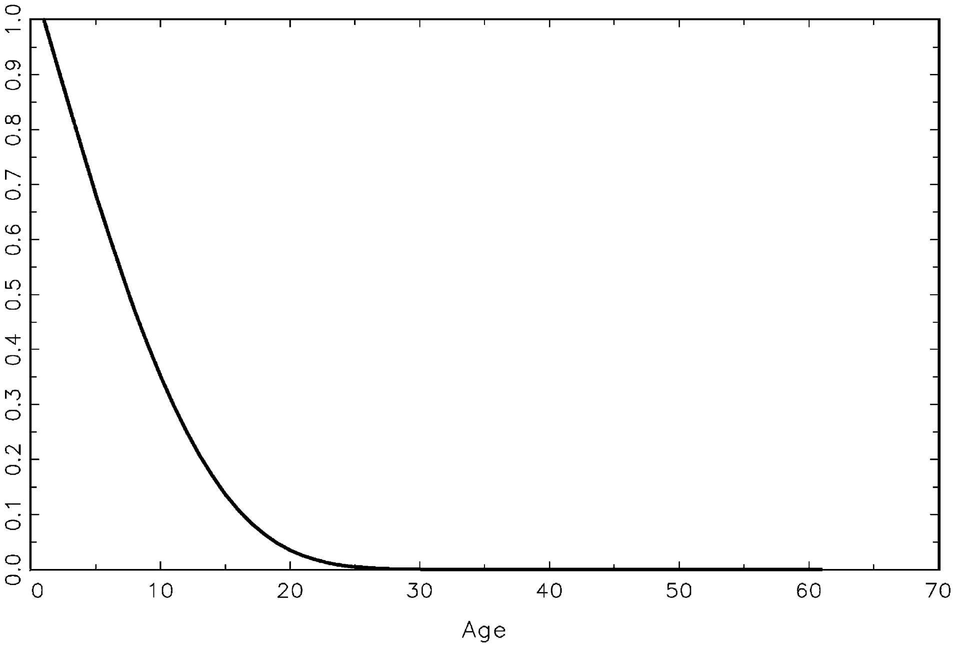

Fig. 1. Human capital investment.

and

$$A^*_{t,a}=\begin{cases} A^b_t, & \text{for } a=1 \\ A^b_t\Delta^{a-1}+G^{1/(1-\gamma)}\sum_{j=2}^{a}\Delta^{a-j}\left\{\dfrac{w\gamma[1-(\Delta/\rho)^{(T'+1-j)}]}{\rho-\Delta}\right\}^{\gamma/(1-\gamma)}B_{t+j-1}, & \text{for } 2\leqslant a\leqslant T' \end{cases}. \tag{10}$$

Once I have obtained the optimal paths for consumption, and investment, production and stock of human capital, I solve for the optimal stock of physical capital (or financial wealth) $W^*_{t,a}$ held at the beginning of the period in which the individual is a periods old. Thus, the optimal path for financial wealth is given by

$$W^*_{t,a+1}=\begin{cases} W^*_{t,a}(1+r)-C^*_{t,a}-H^*_{t,a}+E^*_{t,a}, & \text{for } 1\leqslant a\leqslant T', \\ W^*_{t,a}(1+r)-C^*_{t,a}, & \text{for } T'+1\leqslant a\leqslant T, \end{cases} \tag{11}$$

with $W^*_{t,T+1}\equiv W^*_{t,1}\equiv 0$, that is, the individual's terminal and initial wealths are zero.

The pattern followed by human capital expenditure is shown in Fig. 1; the

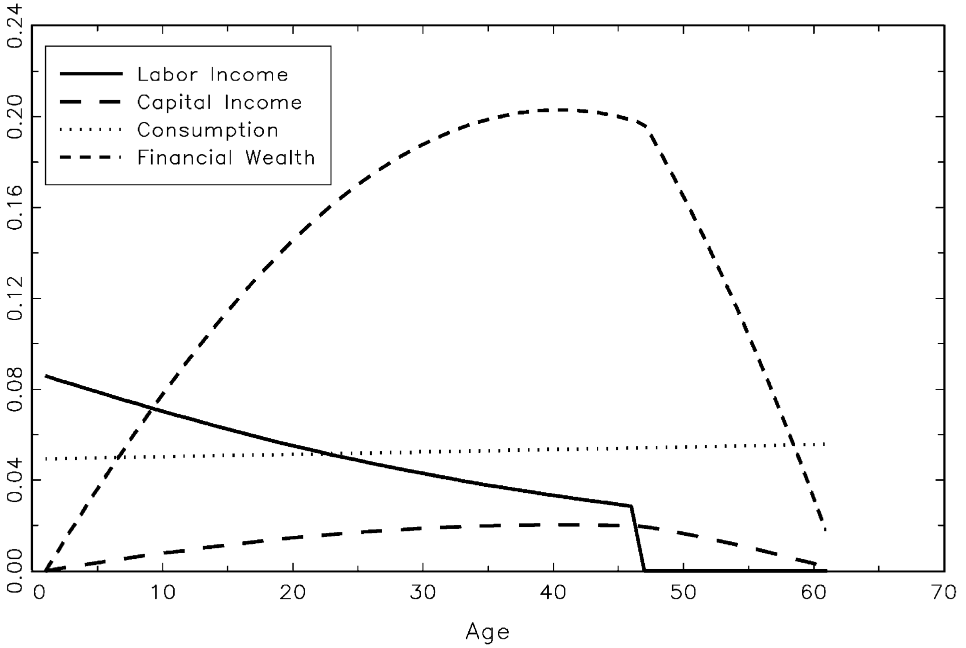

Fig. 2. Labor income, capital income, consumption, financial wealth.

patterns followed by consumption, labor income, capital income and financial wealth are shown in Fig. 2. In both cases parameter values used are those discussed in Section 3.

Given these parameter values, human capital investment strictly decreases with respect to age, concentrating mainly during the first years of life: e.g. at $a=10$ individuals in this economy have already made over 82% of their lifetime education expenditure.

As one would expect, consumption follows a smooth pattern with a (constant) growth rate given by $\beta\rho$. Capital income (and financial wealth) start from zero and become negative in the early years, attaining a maximum near retirement age and falling from that moment onwards until death when both become zero. Given the pattern followed by education expenditure and a positive depreciation rate δ_H it is not surprising that labor income declines over time.[12]

2.2. The aggregates

Having solved the individual problem, in this subsection I obtain the (per capita) aggregates of the individuals' optimal decisions across the population. All we need

[12] Under alternative parameter values that allow for lower γ's, labor income can display an inverted-U shape, declining near retirement (see previous footnote) (See Hansen (1993) for the US economy and Park (1997) for the South Korean).

is to specify the age distribution. It is straightforward to show that for the case of a stable population (i.e. one with a constant rate of growth and a time invariant age structure) growing at a rate n and whose individuals have a certain and finite lifetime horizon T, age distribution is characterized by

$$f_a = \frac{n(1+n)^{T-a}}{(1+n)^T - 1}, \quad \text{for } 1 \leqslant a \leqslant T, \tag{12}$$

where f_a stands for the proportion of individuals a periods old.

A caveat is in order before we go any further. The population growth rate n is endogenously determined by the joint distribution of birth and mortality rates across ages. (See, e.g. how the *characteristic function* is defined in Schoen (1988).) In this model, however, for the sake of simplicity, it is treated as a given parameter. Therefore, whenever I assume that there is a higher life expectancy at birth T (as a result of a drop in the mortality rate), a change in the birth rate distribution is implicitly assumed so that the population growth rate is kept constant.

Thus, from Eqs. (7a), (7b), (8)–(10) and Eq. (12), summing up for all ages, in per capita terms I obtain the aggregates of consumption (C_t), investment (H_t), production (h_t), and stock of human capital (A_t), respectively, where I have assumed that the economy is in its t-th period (Appendix A).

2.3. The production sector

In this subsection I close the model by specifying the aggregate production sector of the economy. For better tractability, the most convenient way to do this is by modeling the production sector by means of a representative firm whose production and factor demands represent aggregate production and aggregate factor demands, respectively.[13] Thus, I assume that all firms maximize profits in perfectly competitive product and factor markets and that, in addition to their using the same technology, this is characterized by exhibiting constant returns to scale. In this way, the existence of such a representative firm is guaranteed (Sargent, 1987, Ch. 1).

The most commonly used functional form for production technology in the related literature is that of a Cobb–Douglas production function. This implies that the elasticity of substitution between factors is one.[14] As claimed by Stokey and Rebelo (1995), the assumption is not so restrictive as it might seem because factor substitution elasticity is not a crucial parameter in explaining growth.[15]

[13] I am certainly aware of the problems that the aggregation of economic agents (firms or individuals) poses, and also aware that the 'representative agent' paradigm offers an easy way out. However, the focus here is on the aggregation of individuals, leaving aside the aggregation of firms.

[14] See, e.g. Davies and Whalley (1991), Pecorino (1993), Nerlove et al. (1993), Jones et al. (1993), Razin and Yuen (1996), Mendoza et al. (1997), Meijdam and Verbon (1997), Kalemli-Ozcan et al. (2000), Hu (1995, 1999), Milesi-Ferretti and Roubini (1998), Roeger and De Fiore (1999), Devereux and Love (1994) and Uhlig and Yanagawa (1996).

[15] 'Elasticities may be important for welfare conclusions, however, since they are critical in determining the size of the distortion in input ratios resulting from asymmetric taxation of factor incomes' (Stokey and Rebelo, 1995).

Thus, I assume that production in period t is

$$Y_t = F(P_tK_t)^{\theta}(P_tA_t)^{1-\theta}, \quad 0<\theta<1, \tag{13}$$

where Y_t, P_tK_t, and P_tA_t denote, respectively, the aggregates of output, stock of physical capital and stock of human capital, P_t is the total population, and F a scaling factor that allows us to redefine the units of the aggregate commodity produced in this economy.

Assuming that the firm maximizes instantaneous profits with respect to physical and human capital in perfectly competitive markets for output and factors, the problem that the firm faces can be formally expressed as

$$\max_{P_tK_t,P_tA_t} F(P_tK_t)^{\theta}(P_tA_t)^{1-\theta} - wP_tA_t - (r+\delta_K)P_tK_t, \tag{14}$$

so that factor prices w and r are given by

$$r = F\theta k^{\theta-1} - \delta_K, \quad \text{and} \quad w = F(1-\theta)k^{\theta}, \tag{15}$$

where I define $k \equiv K_t/A_t$, i.e. the stock of physical capital per efficient unit of labor (which, as claimed before, remains constant along steady state paths), $\delta_K > 0$ denoting the constant depreciation rate of physical capital. In words, Eq. (15) states that production factor (gross) payments are given by their respective marginal productivity levels.

2.4. Steady state and equilibrium in the goods market

In this subsection I characterize the equilibrium in the goods market in the steady state. Before specifying the behavior of the economy along its steady state path, I describe the dynamics of the economy's state of knowledge B_t. Assuming, first, that in every period it depreciates at the same rate δ_H as individual human capital stock and, second, that it is enlarged by the (flow of aggregate) production of human capital (h_t), we have that

$$B_t = (1-\delta_H)B_{t-1} + h_t. \tag{16a}$$

Five remarks concerning Eq. (16a) are in order. First, this way of specifying the dynamics of B_t differs from the one Nerlove et al. (1993) consider. I do not assume that the flow which is incorporated into B_t is the expenditure on or the investment in human capital (H_t), but the *production* of human capital (h_t) in period t as a result of the investment made in period $t-1$ (which, in turn, will depend on the state of knowledge in that period, B_{t-1}].[16] That is, the gross increment in the economy's productivity at time t $[B_t - (1-\delta_H)B_{t-1}]$ will depend on the economy's

[16] I run a sensitivity analysis below in which I assume the specification in Nerlove et al. (1993).

state of knowledge at time $t-1$. This is in line with the endogenous growth literature (Romer, 1990).

Second, we should bear in mind the public good characteristic exhibited by B_t, which fairly extends to h_t. This is why the latter is not multiplied by P_t in Eq. (16a).

Third, the engine of growth in this economy is best seen if we take a close look at the equation of motion for B_t, substitute h_t by its definition (Appendix A), and rewrite it properly, thus obtaining

$$B_t - B_{t-1} = -\delta_H B_{t-1} + \sum_{a=1}^{T'} f_a h_{t-a+1,a}. \tag{16b}$$

In words, the change in the aggregate knowledge in society in any period with respect to the immediately previous one is the result of two opposite sign forces, a negative one (the depreciation of the existing knowledge at a rate δ_H) and a positive one (the production of human capital *by workers*, as only those individuals between 1 and T' are taken into account).[17] Moreover, if we focus on the second term on the right-hand-side of Eq. (16b), we notice that aggregate knowledge is the result of adding human capital production of workers born on different dates who, therefore, have met different sequences for the stock of knowledge of society. Consequently, aggregate knowledge and how it is transmitted from generation to generation depend on the age distribution of population (or, more precisely, of workers). In other words, this model fairly falls within the *vintage human capital growth* literature.[18]

Fourth, alternatively a law of motion for aggregate stock of human capital (instead of knowledge B_t) can be specified by noting that A_t depends on the (net of depreciation) human capital of workers who were in the labor force in the previous period and their current production, but also on the human capital brought by the newly incorporated young workers and the human capital gone with the last generation of retiree workers. A_t and B_t are related to each other. Note that along steady state paths, they *must* both grow at the same rate, i.e. they are proportional (Appendix A). There are at least two reasons in favor of the alternative which I have followed here: in addition to its easier analytical tractability, it better captures how human knowledge is transmitted from generation to generation and accumulates over time in spite of the finite lives of individuals.

And, five, the way in which social knowledge accumulates is close to Arrow (1962) *learning-by-doing* model. In this economy, individual human capital fully depreciates at death, so that knowledge accumulated by society B_t is transmitted from generation to generation as a pure externality. No resources are allocated to the production of knowledge which is simply a by-product of individual human capital production.[19]

[17] Compare, for instance, Eq. (16b) with equation 28 in Boucekkine et al. (2002) p. 362.

[18] See, e.g. Boucekkine et al. (2002) and references therein.

[19] I owe this interpretation of Eq. (16a) to an anonymous referee.

In the steady state k and y (output per efficient unit of labor) remain constant, as do w and r; furthermore, B_t, A_t, H_t, h_t and C_t (per capita aggregates) will grow at a constant rate g. Let us see why. The economy is in equilibrium when the exogenously given path for B_t coincides with the real path given by the law of motion Eq. (16a). In particular, if consumers take B_t as growing at a rate g, then the economy is in equilibrium if the optimal decisions of consumers (as represented by the law of motion) give rise to the same growing path for B_t. If B_t grows over time at a rate g, then H_t, h_t and A_t also grow at the same rate g (see the corresponding expressions in the Appendix A). Consequently, the earnings of individuals will depend on their age but also on the period in which they were born and, therefore, RHS_t^* will also depend on t. Therefore, I can drop the time subscripts from k, y, r and w, but not from B_t, A_t, H_t, h_t and C_t.

To see why this model allows for growth in the steady state of variables in per capita term (although *not* in efficiency units), at the same rate g as B_t, just rewrite the right-hand-side RHS_t^* of the budget constraint in Eq. (6) using Eqs. (1)–(4), (7a) and (7b). This yields

$$wmB_t\left[\frac{1-(\Delta/\rho)T'}{\rho-\Delta}\right]+wGB_t\sum_{a=2}^{T'}\sum_{j=2}^{a}\frac{\Delta^{a-j}}{\rho^a}\left\{\frac{\gamma Gw}{\rho-\Delta}[1-(\Delta/\rho)^{T'-j+1}]\right\}^{\gamma/(1-\gamma)}$$
$$\times(1+g)^{j-1}-B_t\sum_{a=1}^{T'}\frac{1}{\rho^a}\left\{\frac{\gamma Gw}{\rho-\Delta}[1-(\Delta/\rho)^{T'-a}]\right\}^{1/(1-\gamma)}(1+g)^a. \tag{17}$$

Therefore, the lifetime resources available to an individual born at time t are proportional to B_t: thus, if B_t grows at a rate g, then per capita aggregates will also grow at the same rate g. In Nerlove et al. (1993) the (physical capital) endowment that individuals receive is constant, so that per capita variables remain constant along the steady state paths. In Echevarría and Iza (2000) the (human capital) endowment that individuals receive at birth is constant so that there is no growth of per capita variables either. In this case, however, a human capital birth endowment which is proportional to society's knowledge allows for non-negative growth rates along steady state paths.

Thus, from Eq. (16a) and the definition of g as the growth rate of B_t, so that $B_t = B_{t-1}(1+g)$, it also holds that in the steady state $B_t = (1-\delta_H)B_t(1+g)^{-1} + h_t$ must be true. Upon substitution of h_t (per capita human capital production, see Appendix A), and after some rewriting, the growth rate g is implicitly given by

$$1=\frac{1-\delta_H}{1+g}+\frac{n(1+n)^{T-1}m}{(1+n)^T-1}$$
$$+\frac{n(1+n)^T G^{1/(1-\gamma)}}{[(1+n)^T-1](1+g)}\left(\frac{\gamma w}{\rho-\Delta}\right)^{\gamma/(1-\gamma)}\sum_{a=2}^{T'}\frac{[1-(\Delta/\rho)^{T'-a+1}]^{\gamma/(1-\gamma)}}{(1+n)^a}. \tag{18a}$$

Finally, if population grows at a constant rate n and society's knowledge grows at g, aggregate gross investment in physical capital at time t along the steady state path should be $(g+n+gn+\delta_K)P_tK_t$, so that equilibrium in the goods market along the steady state path implies that

$$y=c+i_H+(g+n+gn+\delta_K)k, \tag{19}$$

where

$$y\equiv Y_t/(P_tA_t),\ c\equiv C_t/A_t,\ \text{and}\ i_H\equiv H_t/A_t.$$

To sum up, the steady state is characterized by four equations: the two in Eq. (15) plus Eq. (18a) and Eq. (19) which give us the stationary values for k, r, w and g.[20] The expression for Eq. (19) becomes quite messy once y, c and i_H are replaced by the corresponding expressions Fk^θ, C_t/A_t and H_t/A_t, respectively. It is easy to check, nevertheless, that Eq. (19) depends on the equilibrium values for k, r, w and g, so that the number of equations coincides with the number of unknown variables. (See Appendix A.)

3. Parameterization of the model

In this section I specify and discuss the values that I assign to the parameters in the model to compute the steady state equilibrium. Two goals are pursued simultaneously in choosing the values for the parameters: convergence of the numerical algorithm (an equilibrium solution), and also a sensible equilibrium. One way to accomplish the latter consists in making the equilibrium display some (the more, the better, of course) key aspects or stylized facts of a real world economy. In particular, I have chosen the US case, by far the most often used in calibration exercises, for which more references and data are available than for any other economy. Additionally, uniqueness of equilibrium is also desired. The high non-linearity of the equations, however, makes it difficult to prove uniqueness, Therefore, one has to rely on the fact that when solving the model I tried different starting the values so that either the algorithm converged to the same equilibrium or did not converge at all. Parameter values are summarized in Table 2 at the end of the section.

(i) *Preference parameters*. I assume that the rate of time preference, as represented by the discount factor β, is 0.975. A similar value has been used in other references.[21] See, for instance, Lucas (1990a), Davies and Whalley (1991), Pecorino (1993), Meijdam and Verbon (1997) and Kalemli-Ozcan et al. (2000) who assume 0.97. Concerning σ (the inverse of the elasticity of intertemporal substitution) I assume $\sigma=2$ as in Lucas (1990a).

[20] Because the equations involved are highly non-linear, I solve the model numerically. In particular, I use the GAUSS subroutine for non-linear equations *NLSYS*. The program files are available from the author upon request.

[21] When comparing our value of β to other values used in continuous time models, I used the following approximation $\beta\approx e^{-\varepsilon}$ with ε being the corresponding rate of time preference in a continuous time setup.

Table 1
Demographic projections

Year	n	Mortality rate	LEB	Mean age	Median age
2000	0.0091	0.0087	77.1	36.5	35.8
2005	0.0085	0.0086	77.8	37.2	36.7
2010	0.0081	0.0086	78.5	37.9	37.4
2015	0.0081	0.0086	79.2	38.6	37.6
2020	0.0078	0.0087	79.9	39.2	38.1
2025	0.0078	0.0090	80.6	39.7	38.5
2030	0.0077	0.0093	81.3	40.2	38.9
2040	0.0069	0.0098	82.6	40.6	39.0
2050	0.0067	0.0098	83.9	40.7	38.8

LEB, Life Expectancy at Birth. Projections for net growth rates n have been obtained from US Census Bureau, *National Population Projections, II. Detailed Files, Components of Change,* (*NP-D3*) *Annual Demographic Components of Change for the Resident Population by Race and Hispanic Origin: Lowest, Middle, Highest Series and Zero International Migration Series, 1999–2100*. Projections for life expectancy at birth have been obtained from the projections of *middle series* life tables in US Census Bureau, *National Population Projections, II. Detailed Files,* (*NP-D5*) *Component Assumptions of the Resident Population by Age, Sex, Race and Hispanic Origin: Lowest, Middle and Highest Series, 1999–2100*. Projections for mean and median age have been obtained from US Census Bureau, *National Population Projections, II. Detailed Files, Total Population by Age, Sex, Race, Hispanic Origin, and Nativity:* (*NP-Dl-A*)*, Annual Projections of the Resident Population by Age, Sex, Race, and Hispanic Origin: Lowest, Middle, Highest Series and Zero International Migration Series, 1999–2100*. In all cases, projections refer to the middle series hypothesis, as four hypotheses are usually considered depending on the assumptions on fertility, life expectancy and net migration.

(ii) *Production function*. Having assumed a Cobb–Douglas production function, I characterize this by the capital income share, θ. I assume that it is 0.36, the same value assumed by, among others, Prescott (1986), King and Rebelo (1990), Jones et al. (1993), Devereux and Love (1994, 1995). Given the complexity of the model and the problems in finding the roots of the equations, I introduce a scaling factor $F=0.123$ into the production function. In this way, just by redefining units, I am able to make the algorithm converge to reasonable values.

(iii) *Lifetime horizon, retirement age and rate of population growth*. I assume in what I will refer to as the 'benchmark model' that T' and T are 46 and 61, respectively. Assuming that individuals enter the labor market at, for instance, 16, these values could be reasonably interpreted as if individuals started working at 16, retired at 62, and died at 77. Population growth rate is set at $n=0.0096$.

Table 2
Parameter values: benchmark case

$F=0.123$	$\sigma=2$
$\delta_K=0.1$	$T=61$
$\theta=0.36$	$\beta=0.975$
$G=1.13$	$T'=46$
$\gamma=0.95$	$n=0.0096$
$\delta_H=0.025$	$m=1.997057$

Why make these choices? Regarding T', Gendell and Siegel (1992) estimate the time trends in retirement age among US workers by sex from 1950 to 2005. They report estimates of 62.3 and 62.0 for men and women, respectively, in 1995–2000, and 61.7 and 61.2 in 2000–2005. Gendell (1998) reports estimates of 62.2 among male workers and 62.7 among female workers for 1990–1995. Projecting their time trends to 2000–2005, one obtains 61.5 and 61.6 for men and women. As for T, my choice allows us to replicate a sensible life expectancy at birth of 77. According to the *2000 Statistical Abstract of the United States*, in 1998 life expectancy (at birth) was 73.9 years for men and 79.4 for women.[22] The value of the rate of population growth n is assigned because, according to the *1999 Statistical Abstract of the United States*, it is the value observed in 1997.

Given the values for T and n, I obtain a median age of 42, and a mean age of 44, substantially higher than those currently observed (35.8 and 36.5, respectively for 2001; see Table 1). Given the simplicity of the demographic structure in this model (characterized by Eq. (12)), it is quite hard to reproduce observed moments of the age distribution. Projections for some demographic variables for the next half century are shown in Table 1. Patterns are clear: reductions in growth and mortality rates go together with increments in life expectancy at birth and in mean and median ages.

(iv) *Human capital*. Concerning parameters in Eq. (3), I assume that $G=1.13$ and $\gamma=0.95$. Parameter m in Eq. (1) is set at 1.997057. I assume that human capital depreciates at a rate of $\delta_H=0.025$ which is fairly close to other values assumed in the literature. As pointed out before, however, the main source of depreciation of aggregate human capital is that lifetimes are finite (aspect captured in finite horizon *OLG* models).

These parameter values are chosen for three reasons. First, redefining units by changing G facilitates convergence of the algorithm implemented to solve the equilibrium numerically. Second, by assigning m, γ and δ_H proper values, I have been able to replicate the observed average rate of growth for per capita *GDP* in the last 20 years of 0.02 (i.e. g). The same growth rate is obtained in Devereux and Love (1994) and King and Rebelo (1990). This is slightly higher than, for instance, the 1.5% obtained in Lucas (1990a) and Kim (1992).[23] And, third, a high value of γ is needed to replicate observed patterns for education expenditure: the higher the γ, the more it concentrates in the beginning of individuals' lives, as the externality of B_t in the individual human capital production loses importance.

(v) *Depreciation rate of physical capital*. I assume that $\delta_K=0.1$. The same value can be found in some previous studies: Prescott (1986), Jones et al. (1993), Mendoza et al. (1997), Devereux and Love (1994, 1995), and King and Rebelo (1990).

[22] See Table 116, p. 84, in *Vital Statistics, 2000 Statistical Abstract of the United States*, United States Department of Commerce, Bureau of the Census.

[23] Data for the growth rate of the per capita *GDP* are readily available in the *2000 Statistical Abstract of the United States*, US Census Bureau, for instance.

4. Results

4.1. The benchmark case

Some caution is needed in interpreting the results obtained before going any further. These should be useful *only* for illustrating how changes in lifetime horizon and/or in the length of the working period affect the growth rate in *this model*. In order to go beyond that and predict the effects in an *actual* economy such as the US economy, a more detailed calibration exercise might perhaps be needed and additional ingredients should be included to enable the model to reproduce a larger set of stylized facts.

The model does reproduce the observed per capita growth rate ($g=0.02$), and reasonable values for the interest rate ($r=3\%$), for the capital–output ratio ($K/Y=2.8$), for the ratio of consumption (including education expenditure) to output (0.64, for which the mean observed value for the 1960–2000 period is 0.66), for the education expenditure output ratio (1.6%, for which the mean observed value for 1990, 1995, 1998 and 1999 is 1.5%), and for the education consumption ratio (2.4%, the mean observed in the same years being 2.3%). However, the physical capital investment to output ratio obtained (0.36) more than doubles the mean observed value dining the same period (0.16). In any case, when comparing these values obtained in the model with those observed one should be aware of at least three facts. First, the model does not include the government sector nor the outside sector (with public spending and net imports representing 20 and 1.4% of *GDP*, respectively, in the same period). Second, human capital is home produced: a second sector and a price for education services would surely improve the results. And, third, investment in physical capital is treated here as a residual: it is the value needed to keep the capital per worker (in efficiency units) constant along steady state paths, but not the result of maximizing decision by firms.[24]

After solving the model for the parameter values explained in the previous section, I solve the model for a grid of values for lifetime horizon and retirement age (i.e. T and T'). The results are summarized in Table 3.

Reading along the rows of the Table, the pattern is clear. For any retirement age, an increase in life expectancy always results in lower growth rates: for every additional year, the growth rate would be reduced by 0.04% on average. The interpretation is obvious: first, an increase in lifetime horizon in this model does *not* affect (directly) the optimal choice of human capital investment (only indirectly through changes in w and r). In any period before retirement, that depends on the remaining active years, regardless of the length of the retirement phase. And, second, the share of passive population (retirees) becomes higher as the time span of the retirement regime is enlarged: as long as $T'<T$, workers do not die in this economy.

Reading the Table by columns, a neat result shows up: for any given life expectancy, higher retirement ages always give rise to higher growth rates. For

[24] See *Statistical Abstract of the United States. 2001 Edition,* www.census.gov/statab/www, Table 640 p. 417 and Table 648, p. 423.

Table 3
Life expectancy at birth, retirement age and per capita growth

T'	T									$\overline{\Delta g}$
	57	58	59	60	61	62	63	64	65	
42	2.161	2.106	2.054	2.006	1.960	1.916	1.875	1.837	1.801	−0.045
43	2.168	2.114	2.063	2.015	1.970	1.928	1.889	1.851	1.817	−0.044
44	2.174	2.122	2.072	2.025	1.982	1.941	1.902	1.866	1.833	−0.043
45	2.182	2.130	2.081	2.036	1.993	1.953	1.916	1.881	1.849	−0.042
46	2.189	2.138	2.091	2.047	2.000	1.966	1.930	1.897	1.865	−0.041
47	2.197	2.147	2.101	2.058	2.017	1.980	1.945	1.912	1.881	−0.040
48	2.205	2.156	2.111	2.069	2.030	1.993	1.959	1.927	1.898	−0.038
49	2.213	2.165	2.121	2.080	2.042	2.006	1.973	1.942	1.914	−0.037
50	2.221	2.175	2.132	2.091	2.054	2.020	1.987	1.957	1.929	−0.037
$\overline{\Delta g}$	0.008	0.009	0.010	0.011	0.012	0.013	0.014	0.015	0.016	−0.029

Case I: Benchmark case. *Note:* Life expectancy at birth and retirement age are given, respectively, by $T+16$ and $T'+16$. Thus, e.g. $T=61$ and $T'=46$ would stand for a life expectancy at birth of 77 and a retirement age of 62. Per capita growth rates are expressed in percentage points. $\overline{\Delta g}$ denotes average increment of growth rate per additional year.

every additional year, the growth rate would go up by 0.01% on average. Again, the intuition behind the result is straightforward. When retirement is postponed, two effects follow in this model: first, investment in human capital increases; and, second, the active (employed) population is enlarged. There is an additional effect that works in the opposite direction, however: higher retirement also implies an older labor force with workers who invested in human capital when the state of knowledge was lower than in more recent periods, thus reducing the average human capital in the economy.[25]

Finally, when changes in both life expectancy and retirement age are simultaneously considered, the effects of changes in the former slightly outweigh those of changes in the latter. Thus, reading the Table along the diagonals, increments in lifetime horizons together with increments in working periods would reduce growth rates.

4.2. Sensitivity analysis

I also perform four sensitivity analyses (cases II through V) concerning the specification for the dynamics of aggregate knowledge, the depreciation rates of both types of capital, the elasticity of intertemporal substitution and the size of the externality in human capital production $1-\gamma$. Results are shown in turn.

[25] This is in contrast, for instance, to a result in the above mentioned Futagami and Nakajima (2001) when a *pay-as-you-go* social security system is allowed for: any increase in retirement age gives rise to higher present discounted value of wage income, thus increasing consumption, *reducing* savings rate and growth.

Table 4
Life expectancy at birth, retirement age, and per capita growth

T'	T 57	58	59	60	61	62	63	64	65	$\overline{\Delta g}$
42	1.801	1.777	1.754	1.732	1.711	1.689	1.669	1.649	1.629	−0.022
43	1.877	1.854	1.831	1.808	1.787	1.765	1.745	1.724	1.705	−0.022
44	1.951	1.928	1.905	1.882	1.860	1.839	1.818	1.797	1.777	−0.022
45	2.023	1.999	1.976	1.953	1.931	1.910	1.889	1.868	1.848	−0.022
46	2.092	2.068	2.043	2.022	2.000	1.978	1.957	1.937	1.916	−0.022
47	2.160	2.135	2.112	2.089	2.067	2.045	2.024	2.003	1.983	−0.022
48	2.224	2.200	2.177	2.154	2.131	2.109	2.088	2.067	2.047	−0.022
49	2.287	2.263	2.239	2.216	2.193	2.171	2.150	2.129	2.109	−0.022
50	2.348	2.324	2.300	2.277	2.254	2.232	2.210	2.189	2.169	−0.022
$\overline{\Delta g}$	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.068	0.046

Case II: $B_t=(1-\delta_H)B_{t-1}+H_t$. *Note:* Table is built under the assumption that the dynamics of B_t are given by Eq. (17)′ rather than Eq. (17). Parameter values $m=1.807$, $G=12.769$, $\gamma=0.48$, $\delta_H=0.02$ and $F=0.02$ ensure that, for $T=61$ and $T'=46$, per capita growth rate $g=2\%$ and interest rate $r=0.03$.

4.2.1. Case II

First, I assume different dynamics for knowledge in society. Following Nerlove et al. (1993), I assume that the flow which is incorporated into B_t is the *expenditure* on or the investment in human capital (H_t), not the *production* of human capital h_t. Thus, we have that

$$B_t=(1-\delta_H)B_{t-1}+H_t. \tag{16c}$$

In this case, the equation for the growth rate must be rewritten. Thus, from Eq. (16c) and the definition of g as the growth rate of B_t, so that $B_t=B_{t-1}(1+g)$, in the steady state it must be true that $B_t=(1-\delta_H)B_t(1+g)^{-1}+H_t$. Substitution of H_t (per capita human capital production, see Appendix A), after some rewriting, yields the growth rate g implicitly given by

$$1=\frac{1-\delta_H}{1+g}+\frac{n(1+n)^T}{[(1+n)^T-1]}\left[\frac{G\gamma w}{\rho-\Delta}\right]^{1/(1-\gamma)}\sum_{a=1}^{T'}\frac{[1-(\Delta/\rho)^{T'-a}]^{1/(1-\gamma)}}{(1+n)^a}. \tag{18b}$$

In order to obtain a 2% growth rate for $T=61$ and $T'=46$ as in the benchmark case, m, G, δ_H, γ and F are adjusted to 1.807, 12.769, 0.02, 0.48 and 0.02, respectively. The results are shown in Table 4.

Following the rows, a similar pattern to that of the benchmark case shows up again. Increments in life expectancy alone always lead to lower growth rates: 0.02% per additional year on average (lower reductions than in the benchmark case). When increments in retirement age are considered in turn, I obtain higher growth rates: 0.07% per additional year on average. If increments in both T and T' are allowed

Table 5
Life expectancy at birth, retirement age and per capita growth (Case III: $\delta_H=0$)

T'	T									$\overline{\Delta g}$
	57	58	59	60	61	62	63	64	65	
42	4.741	4.677	4.616	4.557	4.501	4.447	4.394	4.344	4.296	−0.056
43	4.741	4.678	4.617	4.558	4.503	4.448	4.397	4.347	4.299	−0.055
44	4.742	4.679	4.618	4.560	4.504	4.450	4.399	4.349	4.302	−0.055
45	4.743	4.680	4.620	4.562	4.506	4.452	4.401	4.352	4.305	−0.055
46	4.744	4.681	4.621	4.563	4.508	4.455	4.404	4.355	4.308	−0.055
47	4.745	4.682	4.623	4.565	4.510	4.457	4.407	4.358	4.312	−0.054
48	4.746	4.684	4.624	4.567	4.512	4.460	4.410	4.362	4.316	−0.054
49	4.747	4.685	4.626	4.569	4.515	4.463	4.413	4.366	4.321	−0.053
50	4.749	4.687	4.628	4.571	4.517	4.466	4.417	4.370	4.325	−0.053
$\overline{\Delta g}$	0.001	0.001	0.002	0.002	0.002	0.002	0.003	0.003	0.004	−0.052

Note: The remaining parameters remain unchanged relative to the benchmark case.

for, a different pattern is obtained: the growth rate becomes *higher* by 0.05% on average. Therefore, the positive effect of increments in T' more than offsets the negative effects of increments in T.

4.2.2. Case III

Secondly, I also try a different depreciation rate for human capital. In particular, I assume the extreme case that $\delta_H=0$. The results are shown in Table 5.

The main, obvious consequence is a generalized increment in growth rates. Concerning the effects of changes in T and T', the results qualitatively parallel those obtained in the benchmark case. Increments in life expectancy alone give rise to lower growth rates: 0.05% per additional year on average (higher reductions than in the benchmark case). Increments in retirement age make growth rates go up: 0.02% per additional year on average (slightly higher increases than in the benchmark case). And, finally, if increments in both T and T' are allowed for, the growth rate becomes lower (as in the benchmark case) by 0.05% on average. Therefore, the positive effect of increments in T' fails to offset the negative effects of increments in T as in the benchmark case.

4.2.3. Case IV

Thirdly, I also run the experiment of trying two different values for σ (the inverse of the intertemporal elasticity of substitution): 0.5 and 1 (the logarithmic case). The results concerning the growth rates are exactly the same as those obtained under the benchmark case specification; therefore, they are not shown again.[26]

[26] This is an example of how differently infinite horizon, representative agent models and finite horizon, overlapping generations models may behave: in the former type of setup the intertemporal elasticity of substitution has been shown to be a *critical* parameter in explaining economic growth (see, e.g. Stokey and Rebelo, 1995).

Table 6
Life expectancy at birth, retirement age and per capita growth (Case VI: $\gamma = 0.75$)

T'	T									$\overline{\Delta g}$
	57	*58*	59	60	61	62	63	64	65	
42	4.903	4.854	4.807	4.761	4.716	4.673	4.631	4.591	4.551	−0.044
43	4.958	4.909	4.861	4.815	4.771	4.728	4.686	4.646	4.607	−0.044
44	5.010	4.961	4.914	4.868	4.824	4.781	4.740	4.700	4.660	−0.044
45	5.060	5.012	4.965	4.919	4.875	4.832	4.791	4.751	4.712	−0.044
46	5.109	5.061	5.014	4.969	4.925	4.882	4.841	4.801	4.762	−0.043
47	5.157	5.108	5.062	5.016	4.973	4.930	4.889	4.849	4.811	−0.043
48	5.202	5.154	5.108	5.063	5.019	4.977	4.936	4.896	4.858	−0.043
49	5.247	5.199	5.152	5.107	5.064	5.021	4.981	4.941	4.903	−0.043
50	5.289	5.242	5.195	5.150	5.107	5.065	5.024	4.985	4.947	−0.043
$\overline{\Delta g}$	0.048	0.049	0.049	0.049	0.049	0.049	0.049	0.049	0.050	0.006

Note: The rest of parameters remain unchanged relative to the benchmark case.

4.2.4. Case V

Finally, I check the effects of a change in the size of the externality $1-\gamma$ on the production of human capital. In particular, I assume that $\gamma = 0.75$, so that the externality is enlarged compared to the benchmark case. The results are shown in Table 6.

As a first result, a higher (positive) externality would imply in general (i.e. for all pairs T, T') increased growth rates compared to those in the benchmark case, as one would expect. As for the effects of changes in T and/or T' upon growth rates, the qualitative results obtained above for the benchmark case are maintained. Higher values for life expectancy and for retirement age give rise to lower (0.04% on average per additional year) and higher (0.05% on average per additional year) rates of per capita growth, respectively. And when both T and T' are increased simultaneously, the growth rate goes up (0.01% on average per additional year).

5. Conclusion and final remarks

In this paper I address the issue of how life expectancy and retirement age are linked to economic growth. I start from empirical evidence and theoretical literature that shows a positive correlation between life expectancy at birth and per capita growth rate. I build a finite horizon *OLG* model in which human capital accumulation drives endogenous growth. The return on individual investment in human capital depends positively on the remaining active years. Consequently, postponing retirement age raises the return and investment in human capital; additionally, the proportion of working individuals in the society is enlarged. Although the working population also becomes older on average (which tends to reduce the average human capital in the economy), the sustainable growth rate becomes higher. Increments in life expectancy at birth do *not* increase growth rate by themselves, however, but reduce it: the optimal investment in human capital is not affected and the proportion of retirees becomes larger. In short, increases in life expectancy at birth cause higher growth rates *only* if they are accompanied by simultaneous increments in working period.

I believe that the theoretical reasoning often used in literature to explain how a higher life expectancy gives rise to a higher level of economic growth through human capital arguments must be rephrased. When one takes *infinite horizon* models à la Blanchard–Yaari, there is no retirement age. Therefore any increase in life expectancy represents an enlarged working period, thus increasing the return on human capital investment. But when *finite horizon* economies are considered instead, the retirement age becomes essential to characterize the return to human capital investment. Increments in life expectancy itself need not cause changes in human capital investment. Even the growth rate might turn out to be lowered, as explained above. The argument might perhaps be right for underdeveloped economies in which retirement takes place late in individuals' lives (because they keep on working as long as they are physically capable). In this case, improvements in food and health conditions lead to higher life expectancies *and* longer working periods.[27]

Given the crucial role played by the retirement age, I strongly believe that a promising line for further research would be one which would endogenize the retirement age (e.g. Boucekkine et al., 2002) as there is empirical evidence that workers choose their retirement to some extent in response to, for instance, social security incentives. (See Coile and Gruber (2000) for the US case and Gruber et al. (1999) for several countries' economies.)

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Appendix A

The individual problem

The intertemporal budget constraint is given by Eq. (6).

Substituting Eqs. (1)–(4) into Eq. (6), recalling that I have defined $\Delta \equiv (1-\delta_H)$ and $\rho \equiv 1+r$, and rearranging terms, the right-hand-side can be rewritten as

$$\sum_{a=1}^{T'} \frac{wA_{t,a}}{\rho^a} - \sum_{a=1}^{T'} \frac{H_{t,a}}{\rho^a} = \frac{wA_t^b}{\rho} + \frac{wA_t^b}{\rho}\left[\frac{(\Delta/\rho)-(\Delta/\rho)^{T'}}{1-(\Delta/\rho)}\right] + Gw\sum_{a=2}^{T'} \frac{1}{\rho^a} \sum_{j=2}^{a} \Delta^{a-j} H_{t,j-1}^{\gamma} B_{t+j-1}^{1-\gamma} - \sum_{a=1}^{T'} \frac{H_{t,a}}{\rho^a} \equiv \mathrm{RHS}_t(H_{t,1}, H_{t,2}, \ldots, H_{t,T'}). \tag{A1}$$

[27] See, for instance Sen (1999) and Mayer (2001), and *Health and Developement in the 20th Century* in World Health Organization (1999) Ch. 1, pp. 1–12, on health and development.

Maximizing $\mathrm{RHS}_t(\cdot)$ with respect to $H_{t,1}$, $H_{t,2}$,..., $H_{t,T'-1}$ and $H_{t,T'}$, subject to $H_{t,a} \geqslant 0$, $1 \leqslant a \leqslant T'$, one obtains Eq. (7a).

Substituting $H^*_{t,a}$ in Eq. (3), human capital production $h^*_{t,a+1}$ is given by Eq. (9).

Substituting Eq. (9) in Eq. (4), human capital stock $A^*_{t,a}$ is obtained as Eq. (10).

Substituting $H^*_{t,a}$ in RHS_t in Eq. (A1), rearranging terms, and taking into account that $A^b_t = mB_t$, and that along steady state paths B_t grows at a rate g so that $B_{t+j-1} = B_t(1+g)^{j-1}$, and $B_{t+a} = B_t(1+g)^a$, one obtains that $\mathrm{RHS}^*_t = \mathrm{RHS}_t(H^*_{t,1}, H^*_{t,2}, H^*_{t,3}, \ldots, H^*_{t,T'})$ is given by,

$$\mathrm{RHS}^*_t = E_1 B_t, \tag{A2}$$

where

$$E_1 \equiv wm\left[\frac{1-(\Delta/\rho)^{T'}}{\rho-\Delta}\right] + wG\sum_{a=2}^{T'}\sum_{j=2}^{a}\frac{\Delta^{a-j}}{\rho^a}\left\{\frac{\gamma Gw}{\rho-\Delta}\left[1-(\Delta/\rho)^{T'-j+1}\right]\right\}^{\gamma/(1-\gamma)}(1+g)^{j-1}$$
$$-\sum_{a=1}^{T'}\frac{1}{\rho^a}\left\{\frac{\gamma Gw}{\rho-\Delta}\left[1-(\Delta/\rho)^{T'-a}\right]\right\}^{1/(1-\gamma)}(1+g)^a.$$

In the second stage of the optimization problem, I obtain the optimal programs for consumption from the necessary (and sufficient) first order conditions of the problem:

$$\max_{\{C_{t,a}\}_{a=1}^{T}} \sum_{a=1}^{T} \beta^a \frac{C_{t,a}^{1-\sigma}}{1-\sigma}$$

subject to

$$\sum_{a=1}^{T} \frac{C_{t,a}}{(1+r)^a} = \mathrm{RHS}^*_t.$$

Thus, optimal consumption is given by Eq. (8).

The aggregates

From Eqs. (7a), (7b), (8)–(10) and the age distribution characterized by Eq. (12), assuming that the economy is in the steady state (where all variables in per capita terms, B_t included, grow at a constant rate g), one obtains the aggregates of consumption (C_t), investment in human capital (H_t), production of human capital (h_t), and stock of human capital (A_t) as follows.

(1) Aggregate consumption

$$C_t = \sum_{a=1}^{T} f_a C^*_{t-a+1,a} = \sum_{a=1}^{T} \frac{n(1+n)^{T-a}}{(1+n)^T - 1} \frac{(\rho\beta)^{a/\sigma}\mathrm{RHS}^*_{t+1-a}}{Q},$$

and taking into account that RHS^*_t grows at g

$$C_t = \sum_{a=1}^{T} \frac{n(1+n)^{T-a}}{(1+n)^T - 1} \frac{(\rho\beta)^{a/\sigma} \mathrm{RHS}_t^*(1+g)^{1-a}}{Q} = E_2 \mathrm{RHS}_t^*, \tag{A3}$$

where

$$E_2 \equiv \frac{n(1+n)^{T-1}}{Q[(1+n)^T - 1]} \left\{ \frac{(\beta\rho)^{1/\sigma} - \dfrac{(\beta\rho)^{(T+1)/\sigma}}{[(1+n)(1+g)]^T}}{1 - \dfrac{(\beta\rho)^{1/\sigma}}{[(1+n)(1+g)]}} \right\}.$$

(2) Aggregate human capital investment

$$H_t = \sum_{a=1}^{T'} f_a H^*_{t-a+1,a} = \sum_{a=1}^{T'} \frac{n(1+n)^{T-a}}{(1+n)^T - 1} \left\{ \frac{\gamma G w}{\rho - \Delta} [1 - (\Delta/\rho)^{T'-a}] \right\}^{1/(1-\gamma)} B_t = E_3 B_t, \tag{A4}$$

where

$$E_3 \equiv \frac{n S_H}{[(1+n)^T - 1]} \left[\frac{\gamma G w}{\rho - \Delta} \right]^{1/(1-\gamma)},$$

and where

$$S_H \equiv \sum_{a=1}^{T'} (1+n)^{T-a} [1 - (\Delta/\rho)^{T'-a}]^{1/(1-\gamma)}.$$

(3) Aggregate human capital production

$$\begin{aligned} h_t &= \sum_{a=1}^{T'} f_a h^*_{t-a+1,a} = \frac{n(1+n)^{T-1}}{(1+n)^T - 1} A_t^b + \sum_{a=2}^{T'} \frac{n(1+n)^{T-a}}{(1+n)^T - 1} h^*_{t-a+1,a} \\ &= \frac{n(1+n)^{T-1}}{(1+n)^T - 1} A_t^b + \sum_{a=2}^{T'} \frac{n(1+n)^{T-a} G^{1/(1-\gamma)}}{(1+n)^T - 1} \left\{ \frac{\gamma w}{\rho - \Delta} [1 - (\Delta/\rho)^{T'-a+1}] \right\}^{\gamma/(1-\gamma)} B_{t-1}, \end{aligned}$$

and, taking into account that $A_t^b = m B_t$ and that $B_{t-1} = B_t (1+g)^{-1}$,

$$h_t = E_4 B_t, \tag{A5}$$

where

$$E_4 \equiv \frac{n(1+n)^{T-1} m}{(1+n)^T - 1} + \frac{n(1+n)^T G^{1/(1-\gamma)}}{(1+n)^T - 1} \left[\frac{\gamma w}{\rho - \Delta} \right]^{\gamma/(1-\gamma)} \frac{S_h}{1+g},$$

where by definition,

$$S_h \equiv \sum_{a=2}^{T'} (1+n)^{-a}[1-(\Delta/\rho)^{T'-a+1}]^{\gamma/(1-\gamma)}.$$

(4) Aggregate human capital

$$\begin{aligned} A_t &= \sum_{a=1}^{T'} f_a A^*_{t-a+1,a} = \sum_{a=1}^{T'} \frac{n(1+n)^{T-a}}{(1+n)^T-1} A^b_{t+1-a}\Delta^{a-1} + \sum_{a=2}^{T'} \frac{n(1+n)^{T-a}}{(1+n)^T-1} \sum_{j=2}^{a} h^*_{t+j-a,a}\Delta^{a-j} \\ &= \sum_{a=1}^{T'} \frac{n(1+n)^{T-a}}{(1+n)^T-1} A^b_{t+1-a}\Delta^{a-1} + \sum_{a=2}^{T'} \frac{n(1+n)^{T-a}}{(1+n)^T-1} \sum_{j=2}^{a} \Delta^{a-j} G^{1/(1-\gamma)} \left[\frac{\gamma w}{\rho-\Delta}\right]^{\gamma/(1-\gamma)} \\ &\quad \times [1-(\Delta/\rho)^{T'-j+1}]^{\gamma/(1-\gamma)} B_{t+j-a-1}. \end{aligned}$$

Recalling that $A^b_{t+1-a} = mB_{t+1-a}$, and that $B_{t+1-a} = B_t(1+g)^{1-a}$ and $B_{t+j-a-1} = B_t(1+g)^{j-a-1}$, A_t can be finally expressed as

$$A_t = E_5 B_t, \tag{A6}$$

where

$$E_5 \equiv \frac{nm}{(1+n)^T-1} \sum_{a=1}^{T'} (1+n)^{T-a}(1+g)^{1-a}\Delta^{a-1} + \frac{n\left[\frac{\gamma w}{\rho-\Delta}\right]^{1/(1-\gamma)} G^{1/(1-\gamma)}}{(1+n)^T-1}$$
$$\times \sum_{a=2}^{T'} (1+n)^{T-a} \sum_{j=2}^{a} \Delta^{a-j}[1-(\Delta/\rho)^{T'-j+1}]^{\gamma/(1-\gamma)}(1+g)^{j-a-1}.$$

(5) The equilibrium

The equilibrium values for k, r, w and γ are characterized by Eqs. (15), (18a) and (19). In order to check that the 4 equations involve only the four previous variables, Eq. (19) must be rewritten properly:

i. the left-hand-side of Eq. (19) [See Eq. (13)] can be expressed as Fk^{θ}; and,
ii. the right-hand-side of Eq. (19) [See Eqs. (A2), (A3), (A4) and (A6)] can be rewritten as $(E_1E_2)/E_5 + (E_3/E_5) + (g+n+gn+\delta_K)k$, and where E_1, E_2, E_3 and E_5 depend only on r, w and γ.

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