Internet Advertising and the Generalized Second-Price Auction: Selling Billions of Dollars Worth of Keywords

By Benjamin Edelman et. al.

February 17, 2014

Outline

Outline

1. GSP and VCG
   - Formal definitions
   - Locally envy free
   - A special strategy profile

VCG (Vickrey-Clarke-Groves)

- N advertising slots for a given keyword
- K bidders competing for the slots
- Probability that an ad gets clicked α_i
- value per click to the advertiser - s_k
- $\forall i, j \; if \; i < j \Rightarrow \alpha_i > \alpha_j$
- Let $b^{(j)}$be the bid of the jth highest bidder and $g(j)$be its identity
- Slots are allocated according to the bid rank of the advertiser
- Payment per click $P^{V,i}$is the "negative externality" of the advertiser
- Payoff - $\alpha_i s_i - P^{V,i}$

VCG (Vickrey-Clarke-Groves)

- N advertising slots for a given keyword
- K bidders competing for the slots
- Probability that an ad gets clicked α_i
- value per click to the advertiser - s_k
- $\forall i, j \; if \; i < j \Rightarrow \alpha_i > \alpha_j$
- Let $b^{(j)}$be the bid of the jth highest bidder and $g(j)$be its identity
- Slots are allocated according to the bid rank of the advertiser
- Payment per click $P^{V,i}$is the “negative externality” of the advertiser
- Payoff - $\alpha_i s_i - P^{V,i}$

VCG (Vickrey-Clarke-Groves)

- N advertising slots for a given keyword
- K bidders competing for the slots
- Probability that an ad gets clicked α_i
- value per click to the advertiser - s_k
- $\forall i, j \; if \; i < j \Rightarrow \alpha_i > \alpha_j$
- Let $b^{(j)}$be the bid of the jth highest bidder and $g(j)$be its identity
- Slots are allocated according to the bid rank of the advertiser
- Payment per click $P^{V,i}$is the "negative externality" of the advertiser
- Payoff - $\alpha_i s_i - P^{V,i}$

VCG Example

- $< \alpha_1, \alpha_2 > = < 200, 100 >$
- $< s_1, s_2, s_3 > = < 10, 4, 2 >$
- $< p_{V,1}, p_{V,2}, p_{V,3} > = < 600, 200, 0 >$
- $< u_1, u_2, u_3 > = < 1400, 200 >$

VCG Example

- $< \alpha_1, \alpha_2 > = < 200, 100 >$
- $< s_1, s_2, s_3 > = < 10, 4, 2 >$
- $< p_{V,1}, p_{V,2}, p_{V,3} > = < 600, 200, 0 >$
- $< u_1, u_2, u_3 > = < 1400, 200 >$

VCG Example

- $< \alpha_1, \alpha_2 >=< 200, 100 >$
- $< s_1, s_2, s_3 >=< 10, 4, 2 >$
- $< p_{V,1}, p_{V,2}, p_{V,3} >=< 600, 200, 0 >$
- $< u_1, u_2, u_3 >=< 1400, 200 >$

VCG Example

- $<\alpha_1, \alpha_2> = <200, 100>$
- $<s_1, s_2, s_3> = <10, 4, 2>$
- $<p_{V,1}, p_{V,2}, p_{V,3}> = <600, 200, 0>$
- $<u_1, u_2, u_3> = <1400, 200>$

VCG Example

- $< \alpha_1, \alpha_2 >=< 200, 100 >$
- $< s_1, s_2, s_3 >=< 10, 4, 2 >$
- $< p_{V,1}, p_{V,2}, p_{V,3} >=< 600, 200, 0 >$
- $< u_1, u_2, u_3 >=< 1400, 200 >$

GSP

- Payment per click - $\alpha_i b^{(i+1)}$, everything else same as VCG
- Payoff on click - $\alpha_i(s_i - b^{(i+1)})$

GSP

- Payment per click - $\alpha_i b^{(i+1)}$, everything else same as VCG
- Payoff on click - $\alpha_i(s_i - b^{(i+1)})$

GSP

- Payment per click - $\alpha_i b^{(i+1)}$, everything else same as VCG
- Payoff on click - $\alpha_i(s_i - b^{(i+1)})$

GSP vs VCG

Remark1

If all advertisers were to bid the same amounts under the two mechanisms, then each advertiser's payment would be at least as large under GSP as under VCG.

Proof.

For $i = min\{K, N\}, p^{(i)} = p^{V,i} = \alpha_i b^{(i+1)}$
For any $i < min\{K, N\}, p^{V,i} - p^{V,i+1} = (\alpha_i - \alpha_{i+1}) b^{(i+1)} \leq \alpha_i b^{(i+1)} - \alpha_{i+1} b^{(i+2)} = p^{(i)} - p^{(i+1)}$ $\square$

GSP vs VCG

Remark1

If all advertisers were to bid the same amounts under the two mechanisms, then each advertiser's payment would be at least as large under GSP as under VCG.

Proof.

For $i = min\{K, N\}, p^{(i)} = p^{V,i} = \alpha_i b^{(i+1)}$
For any $i < min\{K, N\}, p^{V,i} - p^{V,i+1} = (\alpha_i - \alpha_{i+1}) b^{(i+1)} \leq \alpha_i b^{(i+1)} - \alpha_{i+1} b^{(i+2)} = p^{(i)} - p^{(i+1)}$ □

GSP vs VCG

Remark1

If all advertisers were to bid the same amounts under the two mechanisms, then each advertiser's payment would be at least as large under GSP as under VCG.

Proof.

For $i = min\{K, N\}, p^{(i)} = p^{V,i} = \alpha_i b^{(i+1)}$
For any $i < min\{K, N\}, p^{V,i} - p^{V,i+1} = (\alpha_i - \alpha_{i+1})b^{(i+1)} \leq \alpha_i b^{(i+1)} - \alpha_{i+1} b^{(i+2)} = p^{(i)} - p^{(i+1)}$ □

GSP vs VCG

Remark1

If all advertisers were to bid the same amounts under the two mechanisms, then each advertiser's payment would be at least as large under GSP as under VCG.

Proof.

For $i = min\{K, N\}, p^{(i)} = p^{V,i} = \alpha_i b^{(i+1)}$

For any $i < min\{K, N\}, p^{V,i} - p^{V,i+1} = (\alpha_i - \alpha_{i+1}) b^{(i+1)} \leq \alpha_i b^{(i+1)} - \alpha_{i+1} b^{(i+2)} = p^{(i)} - p^{(i+1)}$ □

GSP vs VCG

Remark2

Truth-telling is a dominant strategy under VCG.

Remark3

Truth-telling is not a dominant strategy under GSP.

GSP vs VCG

Remark2

Truth-telling is a dominant strategy under VCG.

Remark3

Truth-telling is not a dominant strategy under GSP.

Outline

1. GSP and VCG
   - Formal definitions
   - Locally envy free
   - A special strategy profile

Locally Envy Free

- In addition to equilibrium strategies what additional gaming can the advertisers indulge in, to increase the payoff?
- Advertisers might be envious of the advertisers who won, exactly one slot above them
- They might actually bid higher to get one slot above them if that leads to higher profits.
- "Current" payoff >= payoff after moving up by one slot$\Longrightarrow \alpha_i s_{g(i)} - p^{(i)} \geq \alpha_{i-1} s_{g(i)} - p^{(i-1)}$

Locally Envy Free

- In addition to equilibrium strategies what additional gaming can the advertisers indulge in, to increase the payoff?
- Advertisers might be envious of the advertisers who won, exactly one slot above them
- They might actually bid higher to get one slot above them if that leads to higher profits.
- "Current" payoff >= payoff after moving up by one slot $\Longrightarrow \alpha_i s_{g(i)} - p^{(i)} \geq \alpha_{i-1} s_{g(i)} - p^{(i-1)}$

Locally Envy Free

- In addition to equilibrium strategies what additional gaming can the advertisers indulge in, to increase the payoff?
- Advertisers might be envious of the advertisers who won, exactly one slot above them
- They might actually bid higher to get one slot above them if that leads to higher profits.
- "Current" payoff >= payoff after moving up by one slot $\Longrightarrow \alpha_i s_{g(i)} - p^{(i)} \geq \alpha_{i-1} s_{g(i)} - p^{(i-1)}$

Outline

1. GSP and VCG
   - Formal definitions
   - Locally envy free
   - A special strategy profile

A special strategy profile

- Consider a strategy profile for the GSP auction
- $j < k \implies s_j > s_k$, Advertisers ordered according to there valuations
- strategy profile $B^*$is such that for each advertiser i $b^{(i)} = p^{V,i-1}/\alpha_{i-1}$

Theorem

$B^*$is locally envy free equilibrium of the GSP auction and the outcome of this equilibrium is same as that of the dominant strategy equilibrium of VCG. In any other locally envy free equilibrium of GSP the total payments to the seller is at least as high as in B^*

A special strategy profile

- Consider a strategy profile for the GSP auction
- $j < k \Longrightarrow s_j > s_k$, Advertisers ordered according to there valuations
- strategy profile $B^*$is such that for each advertiser i $b^{(i)} = p^{V,i-1}/\alpha_{i-1}$

Theorem

$B^*$is locally envy free equilibrium of the GSP auction and the outcome of this equilibrium is same as that of the dominant strategy equilibrium of VCG. In any other locally envy free equilibrium of GSP the total payments to the seller is at least as high as in B^*

A special strategy profile

- Consider a strategy profile for the GSP auction
- $j < k \implies s_j > s_k$, Advertisers ordered according to there valuations
- strategy profile $B^*$is such that for each advertiser i $b^{(i)} = p^{V,i-1}/\alpha_{i-1}$

Theorem

$B^*$is locally envy free equilibrium of the GSP auction and the outcome of this equilibrium is same as that of the dominant strategy equilibrium of VCG. In any other locally envy free equilibrium of GSP the total payments to the seller is at least as high as in B^*

Proof

Part 1

$B^*$is locally envy free equilibrium of the GSP auction

Proof

Part 1

B^* is locally envy free equilibrium of the GSP auction

Proof

Part 1

B^* is locally envy free equilibrium of the GSP auction

Proof

Part 2

The outcome of this equilibrium is same as that of the dominant strategy equilibrium of VCG.

Proof

Part 2

The outcome of this equilibrium is same as that of the dominant strategy equilibrium of VCG.

Proof

Part 2

The outcome of this equilibrium is same as that of the dominant strategy equilibrium of VCG.

Payment comparison with VCG

Part 3

In any other locally envy free equilibrium of GSP the total payments to the seller is at least as high as in B^*

- In any dominant strategy equilibrium of VCG auction, the total payments made by the advertisers will be lower than in any locally envy free equilibrium in GSP.

Payment comparison with VCG

Part 3

In any other locally envy free equilibrium of GSP the total payments to the seller is at least as high as in B^*

- In any dominant strategy equilibrium of VCG auction, the total payments made by the advertisers will be lower than in any locally envy free equilibrium in GSP.

Payment comparison with VCG

Part 3

In any other locally envy free equilibrium of GSP the total payments to the seller is at least as high as in B^*

- In any dominant strategy equilibrium of VCG auction, the total payments made by the advertisers will be lower than in any locally envy free equilibrium in GSP.