Exam Model: 4629

Zagazig University,Faculty of Engineering **Final Exam**
Academic Year: 2016/2017
Specialization: Computer & Systems Eng.
Course Name: Robotics
Course Name: CSE629/CSE514/CSE513
Examiner: Dr. **Mohammed Nour**

Date: 04/02/2017
Exam Time: 3 hours
No. of Pages: 9
No. of Questions: 5
Full Mark: [70]

▷ Please **answer all questions**. Use **3** decimal digits approximation.

▷ Use your answer sheet as a draft for solutions. **Attach last exam page** to it.

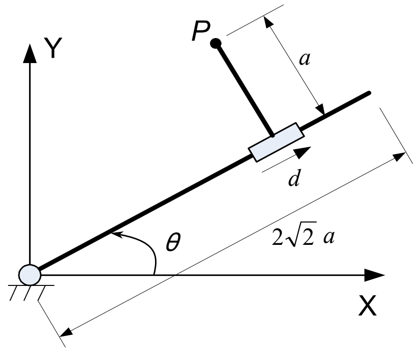
▷ Mark your answers for all questions in the table provided in the last page.

Question 1. [16 Marks] *(2 × 8)*

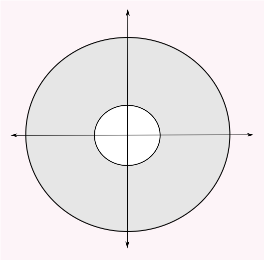
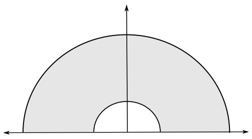
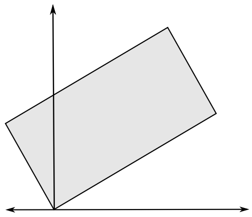
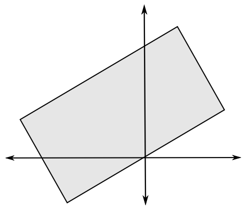
1. Which of the following terms refers to the rotational motion of a robot arm?
   a) swivel b) axle c) roll d) yaw
2. What is the name for the space inside which a robot unit operates?
   a) environment b) spatial base c) danger zone d) work envelop
3. Which of the following terms **is not** one of the five basic parts of a robot?
   a) peripheral tools b) end effectors c) controller d) sensor
4. The number of moveable joints in the base, the arm, and the end effectors of the robot determines ...... ?
   a) degrees of freedom b) operational limits c) flexibility d) cost
5. For a robot unit to be considered a functional industrial robot, typically, how many degrees of freedom would the robot have?
   a) three b) four c) six d) eight
6. . Which of the basic parts of a robot unit would include the computer circuitry that could be programmed to determine what the robot would do?
   a) controller b) arm c) end effector d) drive
7. End effectors can be classified into two categories which are...
   a) elbows and wrists b) grippers and end of arm tooling
   c) grippers and wrists d) end of arm tooling and elbows
8. The amount of weight that a robot can lift is called...
   a) tonnage b) payload c) dead lift d) horsepower

Question 2. [8 Marks] *(2 × 4)*

A 2-DOF planar manipulator has a rotational joint and a prismatic joint. The two links are perpendicular to each other and their dimensions are as indicated. P is the tip (end-effector) of the manipulator.



9. This manipulator workspace is sketched as:

a) 
b) 
c) 
d) 

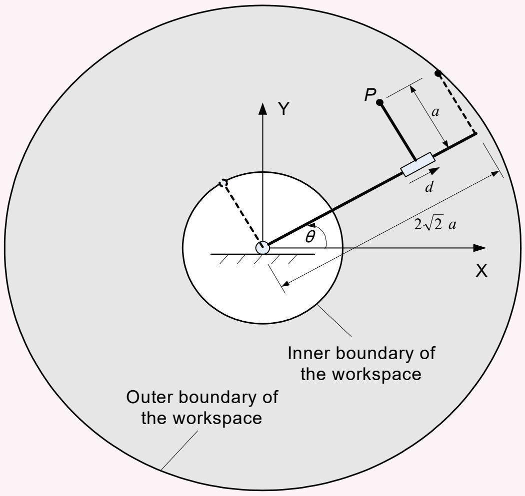
10. The dimensions of the workspace can be mathematically expressed as:

a) $x^2 + y^2 \leq 8\,a^2$ b) $x + y \leq a\cos\theta$

c) $a^2 \leq x^2 + y^2 \leq (3a)^2$ d) $x \leq 2\sqrt{2}\,a$ and $y \leq a$

Solution

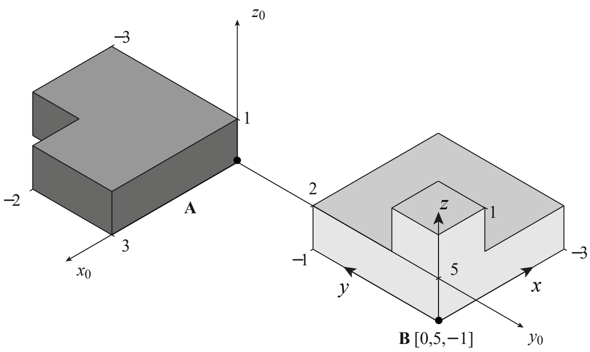
The Cartesian workspace sketch is shown in the figure in shadowed area.

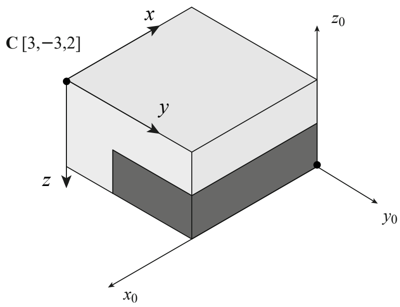


It can be mathematically expressed as: $l^2 \leq x^2 + y^2 \leq (3l)^2$.

Question 3. [10 Marks] $(2 + 2 + 1 + 1 + 2 + 2)$

Consider the pose of the objects A and B in space, as shown on the left. The goal is to displace object B into a new pose **C** on A, so that both objects are connected as shown on the right:





11. The homogeneous transformation matrix $\mathbf{H}_B^0$ to represent **B** w.r.t. frame 0 is:

a) $\begin{bmatrix}1 & 0 & 0 & 3\\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 2\\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1\\ 0 & 0 & 0 & 1\end{bmatrix}$ b) $\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 5\\ 0 & 0 & 1 & -1\\ 0 & 0 & 0 & 1\end{bmatrix}$ c) $\begin{bmatrix}1 & 0 & 0 & 3\\ 0 & -1 & 0 & 2\\ 0 & 0 & -1 & 1\\ 0 & 0 & 0 & 1\end{bmatrix}$ d) $\begin{bmatrix}1 & 0 & 0 & -3\\ 0 & -1 & 0 & 8\\ 0 & 0 & -1 & 3\\ 0 & 0 & 0 & 1\end{bmatrix}$

12. The homogeneous transformation matrix $\mathbf{H}_C^0$ to represent $\mathbf{C}$ w.r.t. frame 0 is:

a) $\begin{bmatrix}1 & 0 & 0 & 3\\ 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 2\\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 1\\ 0 & 0 & 0 & 1\end{bmatrix}$ b) $\begin{bmatrix}-1 & 0 & 0 & 0\\ 0 & -1 & 0 & 5\\ 0 & 0 & 1 & -1\\ 0 & 0 & 0 & 1\end{bmatrix}$ c) $\begin{bmatrix}1 & 0 & 0 & 3\\ 0 & -1 & 0 & 2\\ 0 & 0 & -1 & 1\\ 0 & 0 & 0 & 1\end{bmatrix}$ d) $\begin{bmatrix}1 & 0 & 0 & -3\\ 0 & -1 & 0 & 8\\ 0 & 0 & -1 & 3\\ 0 & 0 & 0 & 1\end{bmatrix}$

13. A sequence of transformations that reaches the desired pose is:

a) $\mathbf{C} = T(3,2,1)\,R_x\left(\frac{\pi}{2}\right)$ b) $\mathbf{C} = T(-3,0,-1)\,R_x\left(\frac{\pi}{2}\right)$

c) $\mathbf{C} = T(3,2,1)\,R_x(\pi)$ d) $\mathbf{C} = T(0,-5,1)\,R_x(\pi)$

Consider the following homogeneous transformation matrix $\mathbf{F}$ and rotation matrix $\mathbf{R}$:

$$\mathbf{F} = \begin{bmatrix}x_1 & 0 & -1 & 5\\ x_2 & 0 & 0 & 3\\ x_3 & -1 & 0 & 2\\ 0 & 0 & 0 & 1\end{bmatrix} \qquad \mathbf{R} = \begin{bmatrix}x_1 & 0 & -1\\ x_2 & 0 & 0\\ x_3 & -1 & 0\end{bmatrix}$$

14. We can split the transformation matrix $\mathbf{F}$ into:

a) $\begin{bmatrix}0 & -1 & 5\\ 0 & 0 & 3\\ -1 & 0 & 2\end{bmatrix}$ and $\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}$ b) $\begin{bmatrix}x_1 & 0 & -1\\ x_2 & 0 & 0\\ x_3 & -1 & 0\end{bmatrix}$ and $\begin{bmatrix}5\\ 3\\ 2\end{bmatrix}$

c) translation followed by a rotation d) both b) and c)

15. In the rotation matrix $\mathbf{R}$:

a) $Rank(\mathbf{R}) = 4$ b) $\mathbf{R}\mathbf{R}^T = \mathbf{I}$

c) dot product of any two columns is one d) cross product of any two rows is zero

16. In the rotation matrix $\mathbf{R}$, values of x_1, x_2 and x_3 are calculated as:

a) $1,0,0$ b) $0,1,0$ c) $0,0,1$ d) $1,1,1$

Solution

We can select an arbitrary sequence of displacements, where object B is first rotated for 180° about axis x_0. It is easy to realize, that we shall reach the final pose $\mathbf{C}$ using translations only.

The object is first lifted for at least 1 unit in z_0 direction, in order not to collide with object A. Afterwards we slide over object A for 3 units in the x_0 direction. After displacing the object for two units in y_0 direction, the objects A and B are connected. As we are dealing with a transformations in a reference frame, the individual transformations are written in reverse order:

$$\mathbf{C} = T(3,2,1)\,R_x(\pi)$$

Since $\mathbf{R}$ is a rotation matrix, then:

$$\mathbf{R}\mathbf{R}^T = \mathbf{I}$$

$$\begin{bmatrix} x_1 & 0 & -1 \\ x_2 & 0 & 0 \\ x_3 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

from which we get:

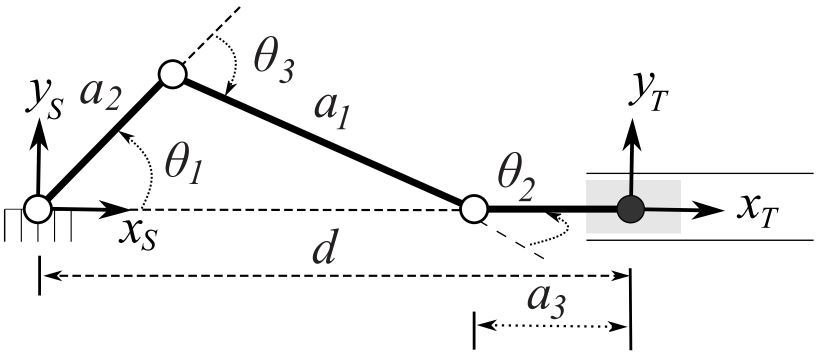
$$\begin{aligned} x_1^2 + 1 = 1 \quad &\Rightarrow \quad x_1 = 0 \\ x_2^2 = 1 \quad &\Rightarrow \quad x_2 = 1 \\ x_3^2 + 1 = 1 \quad &\Rightarrow \quad x_3 = 0 \end{aligned}$$

Note that $x_2 = -1$ is rejected because for a rotation matrix $|R| = 1$ as $\mathbf{R} \in SO(3)$ With the above solution can verfy that:

$$x_1\, x_2 = 0, \quad x_1\, x_3 = 0, \quad x_3\, x_2 = 0$$

Question 4. [12 Marks] *(2 + 2 + 4 + 4)*

Consider the slider-crank mechanism shown below with θ_1 as **actuated** joint and $\{x_S, y_S\}$ is the base frame and $\{x_T, y_T\}$ is the tool frame:



17. This mechanism has ········· degrees of freedom.

a) one b) two c) three d) four

18. Gruebler formula **can not** be used because:

a) the mechanism moves in one dimension (in the x–axis direction) only.
b) it is used with 2D parallel mechanisms.
c) it is used with either 3D closed chains manipulators.
d) it is applicable for inverse kinematics.

19. the manipulator forward kinematics (i.e. given θ_1 find $[x_T \quad y_T]^T$)

a) $a_3 + \dfrac{a_2\, \sin(\theta_1 + \theta_2)}{\sin \theta_1}$ b) $\dfrac{a_2\, \sin(\theta_1 + \theta_2)}{\sin \theta_1}$ c) $a_2\, \sin(\theta_1 + \theta_2)$ d) $d \sin(\theta_1)$

20. the manipulator inverse kinematics (i.e. given $[x_T \quad y_T]^T$ find θ_1)

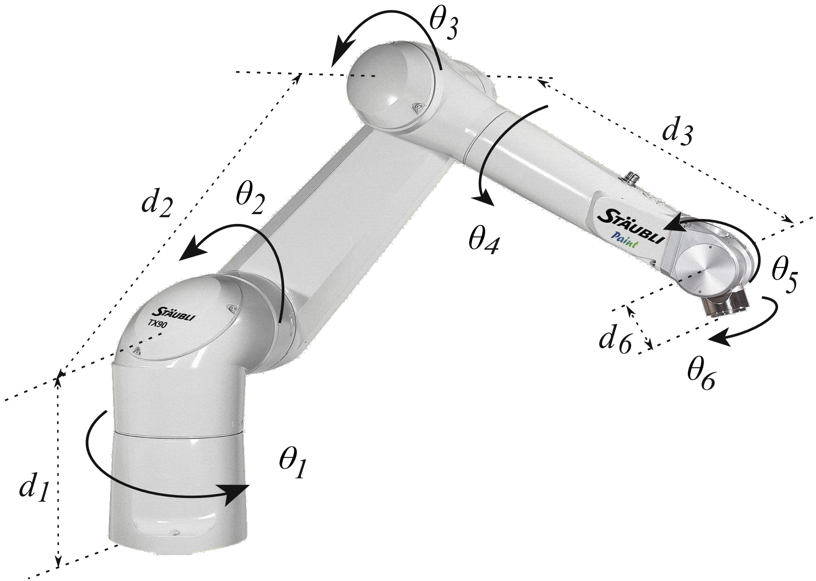
a) $\cos\left(a^2 + a_2^2 - a_1^2\right)$ b) $a_2^2 + a^2 - 2\, a_2\, a \cos \theta_1$ c) $2\, a_2\, a \cos \theta_1$ d) $\arccos\left[\dfrac{a^2 + a_2^2 - a_1^2}{2 a_2\, a}\right]$

Solution

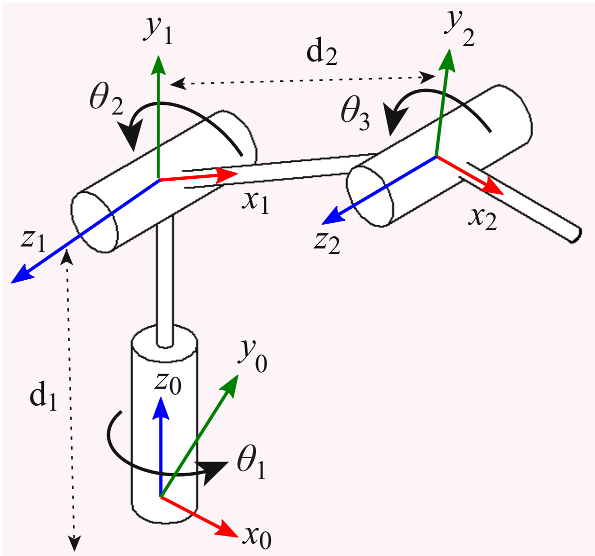
- This mechanism has **one** degree of freedom as it has only one *independent* joint variable (θ_1) and all other joints depend on it.
- Gruebler formula **can not** be used because the mechanism moves in one dimension (in the x–axis direction) only While Gruebler formula is used with either 2D or 3D.
- **Manipulator Forward Kinematics**
  - Since the mechanism is restricted to move in the x–axis direction only, then:
  $$y_T = y_S = 0$$
  - in figure, let a be the length of the triangle side formed by the three joints. We get:
  $$x_T = d = a + a_3$$
  - applying sine rule to the triangle:
  $$\frac{\sin\theta_1}{a_1} = \frac{\sin\theta_2}{a_2} = \frac{\sin(\pi - \theta_3)}{a} \qquad = \frac{\sin[\pi - (\theta_1 + \theta_2)]}{a} = \frac{\sin(\theta_1 + \theta_2)}{a}$$
  $$\theta_2 = \arcsin\left[\frac{a_2}{a_1}\sin\theta_1\right] \quad \Rightarrow \quad a = \frac{a_2\,\sin(\theta_1 + \theta_2)}{\sin\theta_1}$$
  $$x_T = d = a + a_3 = a_3 + \frac{a_2\,\sin(\theta_1 + \theta_2)}{\sin\theta_1}$$
- **Manipulator Inverse Kinematics**
  - at any time:
  $$x_T = d = a + a_3 \quad \Rightarrow \quad a = x_T - a_3$$
  - using cosine rule:
  $$a_1^2 = a_2^2 + a^2 - 2\,a_2\,a\cos\theta_1 \quad \Rightarrow \quad \theta_1 = \arccos\left[\frac{a^2 + a_2^2 - a_1^2}{2a_2\,a}\right]$$

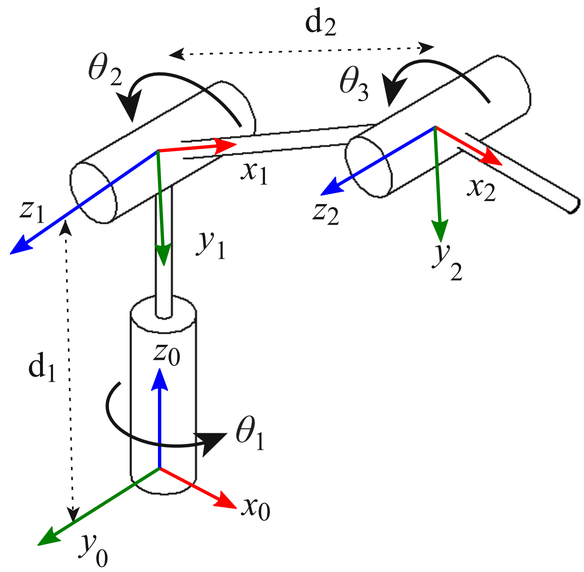
Question 5. [24 Marks] *(3 × 8)*

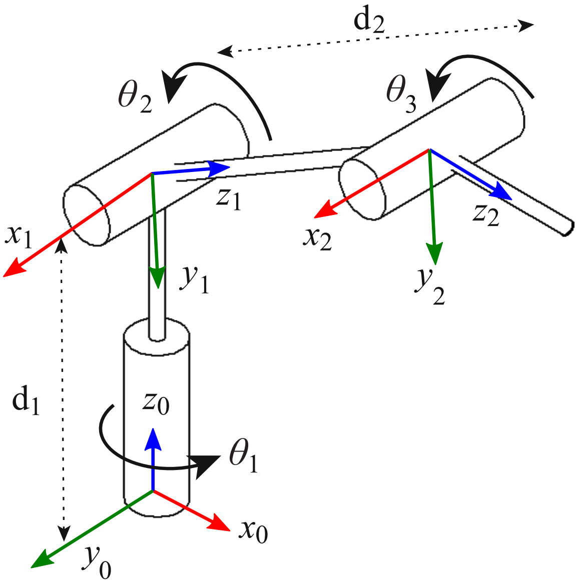
The Stäubli robot is an anthropomorphic robot with a spherical wrist as shown with its dimensions:

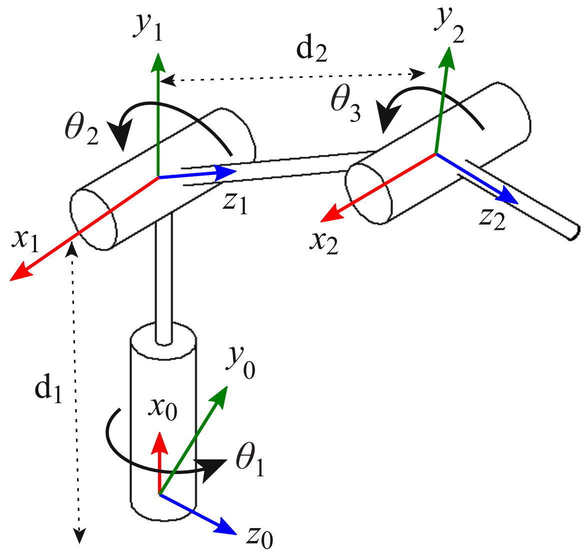


21. According to the DH conventions, we can assign frames to the first three joints as:

a)


b)


c)


d)


22. The DH parameters of the **spherical** wrist joints:

a)

Link	a_i	α_i	d_i	θ_i
4	0	-90	0	θ_4
5	0	90	0	θ_5
6	0	0	d_6	θ_6

b)

Link	a_i	α_i	d_i	θ_i
4	0	90	d_4	θ_4
5	0	90	0	θ_5
6	0	0	d_6	θ_6

c)

Link	a_i	α_i	d_i	θ_i
4	0	0	d_4	θ_4
5	0	0	0	θ_5
6	d_2	90	d_6	θ_6

d)

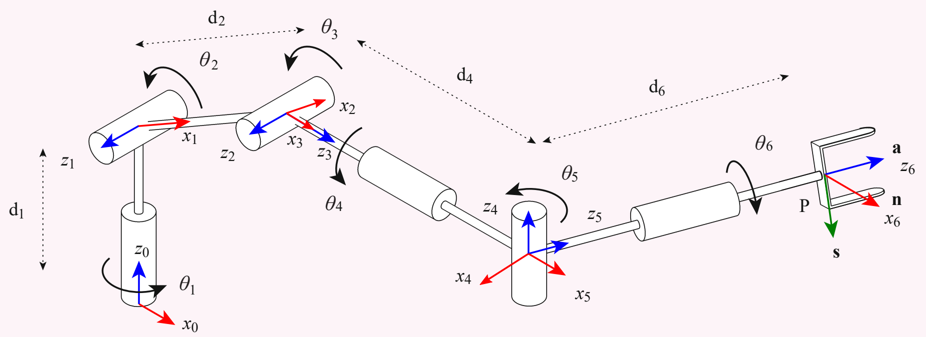
Link	a_i	α_i	d_i	θ_i
4	0	90	d_4	θ_4
5	0	-90	0	θ_5
6	0	0	d_6	θ_6

23. the homogeneous transformation matrices A_3 is found as:

a) $\begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & 1 & 0 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ b) $\begin{bmatrix} c_3 & -s_3 & 0 & d_3\,c_3 \\ s_3 & c_3 & 0 & d_3\,s_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ c) $\begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ d) $\begin{bmatrix} s_3 & 0 & c_3 & 0 \\ c_3 & 0 & -s_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Solution

According to the DH conventions, we can assign frames to all joints as:



The DH parameters of the robot joints:

Link	a_i	α_i	d_i	θ_i
1	0	90	d_1	θ_1
2	d_2	0	0	θ_2
3	0	-90	0	θ_3
4	0	90	d_4	θ_4
5	0	-90	0	θ_5
6	0	0	d_6	θ_6

The matrices describing the relative poses of the neighboring coordinate frames:

$$A_1 = \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ s_1 & 0 & -c_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_2 = \begin{bmatrix} c_2 & -s2 & 0 & d_2\,c_2 \\ s_2 & c_2 & 0 & d_2\,s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_3 = \begin{bmatrix} c_3 & 0 & s_3 & 0 \\ s_3 & 0 & -c_3 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4 = \begin{bmatrix} c_4 & 0 & s_4 & 0 \\ s_4 & 0 & -c_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_5 = \begin{bmatrix} c_5 & 0 & s_5 & 0 \\ s_5 & 0 & -c_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} A_6 = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ s_6 & c_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The geometric model of the robot arm is represented by the product of first three matrices:

$$H_3^0 = A_1\,A_2\,A_3 = \begin{bmatrix} c_1c_{23} & -s_1 & -c_1s_{23} & d_2c_1c_2 \\ s_1c_{23} & c_1 & -s_1s_{23} & d_2s_1c_2 \\ s_{23} & 0 & c_{23} & d_1+d_2s_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Even more complex is the geometric model of robot wrist, represented by the product of the last three matrices:

$$H_6^3 = A_4\,A_5\,A_6 = \begin{bmatrix} c_4c_5c_6 - s_4s_6 & -c_4c_5s_6 - s4c_6 & -c_4s_5 & -d_6c_4s_5 \\ s_4c_5c_6 + c_4s_6 & -s_4c_5s_6 + c_4c_6 & -s_4s_5 & -d_6s_4s_5 \\ s_5c_6 & -s_5s_6 & c_5 & d_4+d_6c_5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Supplementary Material

Note: you *may* need some or none of these identities:

$$R_Z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad R_Y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}, \quad R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$A_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i\sin\theta_i & \sin\alpha_i\sin\theta_i & a_i\cos\theta_i \\ \sin\theta_i & \cos\alpha_i\cos\theta_i & -\sin\alpha_i\cos\theta_i & a_i\sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos(a \pm b) = \cos(a)\cos(b) \mp \sin(a)\sin(b)$$

$$\sin(a \pm b) = \sin(a)\cos(b) \pm \cos(a)\sin(b)$$

$$\sin(\theta) = -\sin(-\theta) = -\cos(\theta + 90^\circ) = \cos(\theta - 90^\circ)$$

$$\cos(\theta) = \cos(-\theta) = \sin(\theta + 90^\circ) = -\sin(\theta - 90^\circ)$$

$$\cos^2(\theta) + \sin^2(\theta) = 1$$

$$\textbf{if}\ \cos(\theta) = b,\ \textbf{then}\ \theta = \operatorname{atan2}\left(\pm\sqrt{1-b^2}, b\right)$$

$$\textbf{if}\ \sin(\theta) = b,\ \textbf{then}\ \theta = \operatorname{atan2}\left(b, \pm\sqrt{1-b^2}\right)$$

$$\textbf{if}\ a\cos(\theta) + b\sin(\theta) = c,\ \textbf{then}\ \theta = \operatorname{atan2}(b,a) + \operatorname{atan2}\left(\pm\sqrt{a^2+b^2-c^2}, c\right)$$

$$\textbf{if}\ a\cos(\theta) - b\sin(\theta) = 0,\ \textbf{then}\ \theta = \operatorname{atan2}(a,b) + \operatorname{atan2}(-a,-b)$$

For a triangle: $A^2 = B^2 + C^2 - 2BC\cos(a), \quad \frac{\sin(a)}{A} = \frac{\sin(b)}{B} = \frac{\sin(c)}{C}$

اسم الطالب :

التاريخ : / /

رقم الجلوس

رقم النموذج

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