Optical communication

Sheet (1)

Example 2.1

A silica optical fiber with a core diameter large enough to be considered by ray theory analysis has a core refractive index of 1.50 and a cladding refractive index of 1.47.

Determine: (a) the critical angle at the core–cladding interface; (b) the *NA* for the fiber; (c) the acceptance angle in air for the fiber.

Solution: (a) The critical angle ϕ_c at the core–cladding interface is given by Eq. (2.2) where:

$$\phi_c = \sin^{-1}\frac{n_2}{n_1} = \sin^{-1}\frac{1.47}{1.50}$$
$$= 78.5°$$

(b) From Eq. (2.8) the *NA* is:

$$NA = (n_1^2 - n_2^2)^{\frac{1}{2}} = (1.50^2 - 1.47^2)^{\frac{1}{2}}$$
$$= (2.25 - 2.16)^{\frac{1}{2}}$$
$$= 0.30$$

(c) Considering Eq. (2.8) the acceptance angle in air θ_a is given by:

$$\theta_a = \sin^{-1} NA = \sin^{-1} 0.30$$
$$= 17.4°$$

Example 2.2

A typical relative refractive index difference for an optical fiber designed for long-distance transmission is 1%. Estimate the *NA* and the solid acceptance angle in air for the fiber when the core index is 1.46. Further, calculate the critical angle at the core–cladding interface within the fiber. It may be assumed that the concepts of geometric optics hold for the fiber.

Solution: Using Eq. (2.10) with $\Delta = 0.01$ gives the *NA* as:

$$\begin{aligned} NA = n_1(2\Delta)^{\frac{1}{2}} &= 1.46(0.02)^{\frac{1}{2}} \\ &= 0.21 \end{aligned}$$

For small angles the solid acceptance angle in air ζ is given by:

$$\zeta \simeq \pi\theta_a^2 = \pi \sin^2 \theta_a$$

Hence from Eq. (2.8):

$$\begin{aligned} \zeta \simeq \pi(NA)^2 &= \pi \times 0.04 \\ &= 0.13 \text{ rad} \end{aligned}$$

Using Eq. (2.9) for the relative refractive index difference Δ gives:

$$\Delta \simeq \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1}$$

Hence

$$\begin{aligned} \frac{n_2}{n_1} = 1 - \Delta &= 1 - 0.01 \\ &= 0.99 \end{aligned}$$

From Eq. (2.2) the critical angle at the core–cladding interface is:

$$\begin{aligned} \phi_c = \sin^{-1}\frac{n_2}{n_1} &= \sin^{-1} 0.99 \\ &= 81.9^\circ \end{aligned}$$

Example 2.3

An optical fiber in air has an NA of 0.4. Compare the acceptance angle for meridional rays with that for skew rays which change direction by 100° at each reflection.

Solution: The acceptance angle for meridional rays is given by Eq. (2.8) with $n_0 = 1$ as:

$$\theta_a = \sin^{-1} NA = \sin^{-1} 0.4$$
$$= 23.6°$$

The skew rays change direction by 100° at each reflection, therefore $\gamma = 50°$. Hence using Eq. (2.17) the acceptance angle for skew rays is:

$$\theta_{as} = \sin^{-1}\left(\frac{NA}{\cos \gamma}\right) = \sin^{-1}\left(\frac{0.4}{\cos 50°}\right)$$
$$= 38.5°$$

Example 2.4

A multimode step index fiber with a core diameter of 80 μm and a relative index difference of 1.5% is operating at a wavelength of 0.85 μm. If the core refractive index is 1.48, estimate: (a) the normalized frequency for the fiber; (b) the number of guided modes.

Solution: (a) The normalized frequency may be obtained from Eq. (2.70) where:

$$V \simeq \frac{2\pi}{\lambda} a n_1 (2\Delta)^{\frac{1}{2}} = \frac{2\pi \times 40 \times 10^{-6} \times 1.48}{0.85 \times 10^{-6}} (2 \times 0.015)^{\frac{1}{2}} = 75.8$$

(b) The total number of guided modes is given by Eq. (2.74) as:

$$M_s \simeq \frac{V^2}{2} = \frac{5745.6}{2}$$
$$= 2873$$

Hence this fiber has a V number of approximately 76, giving nearly 3000 guided modes.

Example 2.5

A graded index fiber has a core with a parabolic refractive index profile which has a diameter of 50 µm. The fiber has a numerical aperture of 0.2. Estimate the total number of guided modes propagating in the fiber when it is operating at a wavelength of 1 µm.

Solution: Using Eq. (2.69), the normalized frequency for the fiber is:

$$V = \frac{2\pi}{\lambda} a(NA) = \frac{2\pi \times 25 \times 10^{-6} \times 0.2}{1 \times 10^{-6}}$$

$$= 31.4$$

The mode volume may be obtained from Eq. (2.95) where for a parabolic profile:

$$M_g \simeq \frac{V^2}{4} = \frac{986}{4} = 247$$

Hence the fiber supports approximately 247 guided modes.

Example 2.6

Estimate the maximum core diameter for an optical fiber with the same relative refractive index difference (1.5%) and core refractive index (1.48) as the fiber given in Example 2.4 in order that it may be suitable for single-mode operation. It may be assumed that the fiber is operating at the same wavelength (0.85 μm). Further, estimate the new maximum core diameter for single-mode operation when the relative refractive index difference is reduced by a factor of 10.

Solution: Considering the relationship given in Eq. (2.96), the maximum V value for a fiber which gives single-mode operation is 2.4. Hence, from Eq. (2.70) the core radius a is:

$$a = \frac{V\lambda}{2\pi n_1 (2\Delta)^{\frac{1}{2}}} = \frac{2.4 \times 0.85 \times 10^{-6}}{2\pi \times 1.48 \times (0.03)^{\frac{1}{2}}}$$

$$= 1.3\ \mu\text{m}$$

Therefore the maximum core diameter for single-mode operation is approximately 2.6 μm.

Reducing the relative refractive index difference by a factor of 10 and again using Eq. (2.70) gives:

$$a = \frac{2.4 \times 0.85 \times 10^{-6}}{2\pi \times 1.48 \times (0.003)^{\frac{1}{2}}} = 4.0\ \mu\text{m}$$

Hence the maximum core diameter for single-mode operation is now approximately 8 μm.

Example 2.7

A graded index fiber with a parabolic refractive index profile core has a refractive index at the core axis of 1.5 and a relative index difference of 1%. Estimate the maximum possible core diameter which allows single-mode operation at a wavelength of 1.3 μm.

Solution: Using Eq. (2.97) the maximum value of normalized frequency for single-mode operation is:

$$V = 2.4(1 + 2/\alpha)^{\frac{1}{2}} = 2.4(1 + 2/2)^{\frac{1}{2}}$$
$$= 2.4\sqrt{2}$$

The maximum core radius may be obtained from Eq. (2.70) where:

$$a = \frac{V\lambda}{2\pi n_1(2\Delta)^{\frac{1}{2}}} = \frac{2.4\sqrt{2} \times 1.3 \times 10^{-6}}{2\pi \times 1.5 \times (0.02)^{\frac{1}{2}}}$$
$$= 3.3\ \mu\text{m}$$

Hence the maximum core diameter which allows single-mode operation is approximately 6.6 μm.

Example 2.8

Determine the cutoff wavelength for a step index fiber to exhibit single-mode operation when the core refractive index and radius are 1.46 and 4.5 μm, respectively, with the relative index difference being 0.25%.

Solution: Using Eq. (2.98) with $V_c = 2.405$ gives:

$$\lambda_c = \frac{2\pi a n_1(2\Delta)^{\frac{1}{2}}}{2.405} = \frac{2\pi 4.5 \times 1.46(0.005)^{\frac{1}{2}}}{2.405}\ \mu\text{m}$$
$$= 1.214\ \mu\text{m}$$
$$= 1214\ \text{nm}$$

Hence the fiber is single-moded to a wavelength of 1214 nm.

Assignment (1)

Using simple ray theory, describe the mechanism for the transmission of light within an optical fiber. Briefly discuss with the aid of a suitable diagram what is meant by the acceptance angle for an optical fiber. Show how this is related to the fiber numerical aperture and the refractive indices for the fiber core and cladding.

An optical fiber has a numerical aperture of 0.20 and a cladding refractive index of 1.59. Determine:

(a) the acceptance angle for the fiber in water which has a refractive index of 1.33;

(b) the critical angle at the core–cladding interface.

Comment on any assumptions made about the fiber.

The velocity of light in the core of a step index fiber is 2.01×10^8 m s^{-1}, and the critical angle at the core–cladding interface is 80°. Determine the numerical aperture and the acceptance angle for the fiber in air, assuming it has a core diameter suitable for consideration by ray analysis. The velocity of light in a vacuum is 2.998×10^3 m s^{-1}.

Define the relative refractive index difference for an optical fiber and show how it may be related to the numerical aperture.

A step index fiber with a large core diameter compared with the wavelength of the transmitted light has an acceptance angle in air of 22° and a relative refractive index difference of 3%. Estimate the numerical aperture and the critical angle at the core–cladding interface for the fiber.

A step index fiber has a solid acceptance angle in air of 0.115 radians and a relative refractive index difference of 0.9%. Estimate the speed of light in the fiber core.

Briefly indicate with the aid of suitable diagrams the difference between meridional and skew ray paths in step index fibers.

Define the normalized frequency for an optical fiber and explain its use in the determination of the number of guided modes propagating within a step index fiber.

A step index fiber in air has a numerical aperture of 0.16, a core refractive index of 1.45 and a core diameter of 60 μm. Determine the normalized frequency for the fiber when light at a wavelength of 0.9 μm is transmitted. Further, estimate the number of guided modes propagating in the fiber.

Describe with the aid of simple ray diagrams:

(a) the multimode step index fiber;

(b) the single-mode step index fiber.

Compare the advantages and disadvantages of these two types of fiber for use as an optical channel.

Explain what is meant by a graded index optical fiber, giving an expression for the possible refractive index profile. Using simple ray theory concepts, discuss the transmission of light through the fiber. Indicate the major advantage of this type of fiber with regard to multimode propagation.

The relative refractive index difference between the core axis and the cladding of a graded index fiber is 0.7% when the refractive index at the core axis is 1.45. Estimate values for the numerical aperture of the fiber when:

(a) the index profile is not taken into account; and
(b) the index profile is assumed to be triangular.
Comment on the results.

A multimode graded index fiber has an acceptance angle in air of 8°. Estimate the relative refractive index difference between the core axis and the cladding when the refractive index at the core axis is 1.52.

A graded index fiber with a core axis refractive index of 1.5 has a characteristic index profile (α) of 1.90, a relative refractive index difference of 1.3% and a core diameter of 40 μm. Estimate the number of guided modes propagating in the fiber when the transmitted light has a wavelength of 1.55 μm, and determine the cutoff value of the normalized frequency for single-mode transmission in the fiber.

A single-mode step index fiber has a core diameter of 7 μm and a core refractive index of 1.49. Estimate the shortest wavelength of light which allows single-mode operation when the relative refractive index difference for the fiber is 1%.