Números primos y factorizaciones en primos

Funciones para encontrar la lista de los primeros k primos y para factorizar números. Incluye factores de fermat.

```cpp
#include <algorithm>
#include <cassert>
#include <cmath>
#include <iostream>
#include <numeric>
#include <vector>

using ll = long long;

// Represents p^a
struct prime_to_power
{
    prime_to_power(ll prime, ll power) : p(prime), a(power) {}
    ll p;
    ll a;

    explicit operator ll() const { return std::pow(p, a); }
};

class Factorization
{
public:
    using value_type = prime_to_power;
    explicit operator ll() const
    {
        ll t = 1;

        for (auto& pa : m_prime_factors)
            t *= ll(pa);

        return t;
    }

    // returns the power of prime p
    ll operator[](ll p) const
    {
        auto& PF = m_prime_factors;
        auto it = std::partition_point(
          PF.begin(), PF.end(), [p](const prime_to_power& p_a) {
                return p_a.p < p;
```

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        });

        if (it == PF.end() || it->p != p)
            return 0;

        return it->a;
    }

    ll& operator[](ll p)
    {
        auto& PF = m_prime_factors;
        auto it = std::partition_point(
          PF.begin(), PF.end(), [p](const prime_to_power& p_a) {
              return p_a.p < p;
          });

        if (it == PF.end())
        {
            PF.emplace_back(p, 0);
            return PF.back().a;
        }

        // if it exists, everything is fine
        if (it->p == p)
            return it->a;

        // Has to insert into correct position
        ++it;
        it = PF.insert(it, prime_to_power(p, 0));
        return it->a;
    }

    void emplace_back(ll p, ll a)
    {
        assert(m_prime_factors.empty() || p > m_prime_factors.back().p);
        m_prime_factors.emplace_back(p, a);
    }

    auto begin() const { return m_prime_factors.begin(); }
    auto end() const { return m_prime_factors.end(); }

    ll size() const { return m_prime_factors.size(); }

private:
    std::vector<prime_to_power> m_prime_factors;
```

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};

std::ostream& operator<<(std::ostream& os, const Factorization& F)
{
    ll i = 0;

    for (auto f : F)
    {
        os << f.p;

        if (f.a != 1)
            os << "^" << f.a;

        if (i + 1 != F.size())
            os << " * ";

        ++i;
    }

    return os;
}

// returns the biggest integer t such that t*t <= n
ll integral_sqrt(ll n)
{
    ll t = std::round(std::sqrt(n));
    if (t * t > n)
        return t - 1;

    return t;
}

bool is_square(ll N)
{
    ll t = std::round(std::sqrt(N));
    return t * t == N;
}

ll FermatFactor(ll N)
{
    assert(N % 2 == 1);
    ll a = std::ceil(std::sqrt(N));
    ll b2 = a * a - N;

    while (!is_square(b2))
```

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    {
        ++a;
        b2 = a * a - N;
    }

    return a - integral_sqrt(b2);
}


class PrimeFactorizer
{
public:
    explicit PrimeFactorizer(ll primes_up_to = 1000000)
    {
        eratosthenes_sieve(primes_up_to);
    }

    ll size() const { return primes.back() + 1; }

    bool is_prime(ll p) const
    {
        if (p < primes.back())
        {
            return std::binary_search(primes.begin(), primes.end(), p);
        }

        if (p <= primes.back() * primes.back())
        {
            return bf_is_prime(p);
        }

        ll a = FermatFactor(p);
        return a == 1;
    }

    auto begin() const { return primes.begin(); }
    auto end() const { return primes.end(); }

    const auto& Primes() const { return primes; }

    /// Make sure sqrt(n) <
    /// primes.back()*primes.back(), otherwise
    /// this could spit out a wrong factorization.
    Factorization prime_factorization(ll n) const
    {
        Factorization F;
```

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        if (n <= 1)
            return F;

        for (auto p : primes)
        {
            ll a = 0;
            auto qr = std::div(n, p);

            while (qr.rem == 0)
            {
                n = qr.quot;
                qr = std::div(n, p);
                ++a;
            }

            if (a != 0)
                F.emplace_back(p, a);

            if (p * p > n)
                break;
        }

        if (n > 1)
            ++F[n];

        return F;
    }

private:
    void eratosthenes_sieve(ll n)
    {
        std::vector<bool> primecharfunc = {false, false, true};
        primecharfunc.resize(n + 1, true);
        //      for (ll i = 4; i < primecharfunc.size(); i += 2)
        //          primecharfunc[i] = false;

        primes.reserve((1.1 * n) / log(n) + 10); // can remove this line

        ll p = 3;
        for (; p * p <= n; p += 2)
        {
            if (primecharfunc[p])
            {
                primes.emplace_back(p);
```

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                for (ll i = 3 * p; i <= n; i += 2 * p)
                    primecharfunc[i] = false;
            }
        }

        for (; p < n; p += 2)
        {
            if (primecharfunc[p])
                primes.emplace_back(p);
        }
    }

    // private because n has to be odd, and maybe
    // is already a factor in something.
    void fermat_factorization(ll n, Factorization& F)
    {
        assert(n % 2 == 1);
        assert(n > 5);
        ll a = FermatFactor(n);

        ll b = n / a;

        assert(a * b == n);

        if (a == 1)
        {
            ++F[b];
        }
        else
        {
            fermat_factorization(a, F);
            fermat_factorization(b, F);
        }
    }

private:
    std::vector<ll> primes = {2};
    bool bf_is_prime(ll n) const
    {
        for (auto p : primes)
        {
            if (p * p > n)
                break;

            if (n % p == 0)
```

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                return false;
        }

        return true;
    }
};

class EulerPhi
{
public:
    EulerPhi(const PrimeFactorizer& P) : m_phi(P.size())
    {
        m_phi[0] = 0;
        m_phi[1] = 1;
        using iter = std::vector<ll>::const_iterator;
        using pll = std::pair<ll, iter>;
        std::vector<pll> frontier;
        frontier.emplace_back(1, P.begin());

        while (!frontier.empty())
        {
            auto t = frontier.back();
            frontier.pop_back();
            ll a = t.first;
            auto prime = t.second;

            for (auto it = prime; it != P.end() && (*it) * a < m_phi.size();
                 ++it)
            {
                ll p = *it;
                ll multiplier = p - 1;
                if (a % p == 0)
                    multiplier = p;
                m_phi[p * a] = multiplier * m_phi[a];
                frontier.emplace_back(p * a, it);
            }
        }
    }

    ll operator()(ll k) { return m_phi[k]; }

private:
    std::vector<ll> m_phi;
};
```

```cpp
int main()
{
    PrimeFactorizer P;

    std::cout << "Primes: ";
    for (auto it = P.Primes().begin(); it != P.Primes().begin() + 100; ++it)
        std::cout << *it << ' ';
    std::cout << std::endl;

    for (ll n = 2; n <= 30; ++n)
    {
        std::cout << n << " = " << P.prime_factorization(n) << std::endl;
    }

    return 0;
}
```

Output:

```
Primes: 2 3 5 7 11 13 17 19 23 29 31 37 41 43 47 53 59 61 67 71 73 79 83 89 97 101 103 1
2 = 2
3 = 3
4 = 2^2
5 = 5
6 = 2 * 3
7 = 7
8 = 2^3
9 = 3^2
10 = 2 * 5
11 = 11
12 = 2^2 * 3
13 = 13
14 = 2 * 7
15 = 3 * 5
16 = 2^4
17 = 17
18 = 2 * 3^2
19 = 19
20 = 2^2 * 5
21 = 3 * 7
22 = 2 * 11
23 = 23
24 = 2^3 * 3
25 = 5^2
26 = 2 * 13
27 = 3^3
```

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28 = 2^2 * 7
29 = 29
30 = 2 * 3 * 5
```