Diameter Maximum distance between 2 nodes in network

Radius Half the diameter

Spanning tree A subgraph which is a tree and reaches all nodes. Has $N-1$ edges.

Network Complexities

Star network Central server, everything else connected to it. $O(n)$ messages.

Chain network Nodes connected to chain, need to go through $n+i$ nodes to reach master. $O(n^2)$ messages if every sends value, $O(n)$ if aggregated.

Tree network $O(|E|)$ where $|E|$ is number of edges.

Global Message Broadcast

- Flooding for Broadcast

  - Flood type, Unique ID, Data.
  - Send to all neighbours, if seen before discard otherwise forward and add to *seen list*.
  - Each node needs to store flood IDs.
  - Message complexity is $O(|E|)$, at worst $O(n^2)$.
  - To reduce complexity slightly, don't send to where you received from.
  - Time complexity is *diameter of G*.

Computing Tree from a network

- BFS tree search

  - Every node has a parent pointer
  - Zero or more child pointers
  - Flood at every node, parent is the one who contacted it first.
  - If many, choose one with smallest id
  - Child informs parent of selection, parent creates child pointer.
  - Message complexity is $O(|E|)$, at worst $O(n^2)$.
  - Time complexity is *diameter of G*.

Tree Based Broadcast

- Send message to all nodes using tree.
- Receive from parent, send to children.
- Message complexity is $O(n-1)$.

Aggregating Sum of Values Using BFS (Convergecast)

- Start from leaves.
- Every node waits for values from children, sum, and send up.
- Without the tree

  - Every node waits for $O(|E|)$ messages.
  - $O(n|E|)$ messages in total.
  - Good fault tolerance.

- With the tree

  - Any node can use broadcast.
  - Bad fault tolerance: need to rebuild.
  - Shortest path: from any node q, follow parent pointers to root.

BFS Trees for Routing

- Create a BFS tree at every node.
- Store parent pointers to other nodes' BFS trees.
- $O(n|E|)$ message complexity for routing table.

Bit Complexity

- Sometimes we calculate the amount of bits exchanged to assess complexity.
- Each node needs $\Theta(\log n)$ bits to store ID

Minimum Spanning Tree Spanning tree with lowest sum of edge weights $w(e)$. Using it for broadcast has the smallest possible cost. Useful in point to point routing.

Cut Optimality

- Every edge of the MST partitions the graph into 2 disjoint sets.
- No other edge can have smaller weight than the MST edge.
- Every non-MST edge when added to MST creates a cycle.

Prim's Algorithm

- Initialise $P = x$ and $Q = E$.
- While $P \neq V$

  - Select edge (u, v) in the cut $(P, V \setminus P)$ with smallest weight
  - Add v to P

- If we search for minimum each time it's $O(mn)$
- If we use heaps it's $O(m \log n)$ or $O(m + n \log n)$

Distributed Prim's Algorithm

- In every round, find the minimum edge
- Use a convergecast every round for n rounds
- Complexity?
- Does not use distributed computation.
- Tree spreads from one point, rest of network is idle.

Kruskal's Algorithm

- Each node is its own tree
- Sort all edges by weight.
- For each tree

  - Find the least weight boundary edge.
  - Add it to the set of edges: merges two trees into one

- To know which edge is boundary:

  - Maintain ID for each tree.
  - Check that endpoint has different tree ID.
  - Update tree ID of all nodes when merging (smaller tree). The cost of updating IDs is $O(n \log n)$.

GHS Distributed Algorithm

- In level 0 each node its own tree.
- Each tree has a leader (leader id = tree id).
- At each level k:

  - All leaders do a convergecast to find minimum boundary edge.
  - It then broadcasts this in the tree so the node knows.
  - The node informs the node on "the other side" which informs the leader.

- Possibly merging more than 2 trees at the same time.
- We get tree of trees: no cycles.
- Complexity

  - $O(n \log n)$ time
  - $O(n \log n + |E|)$ messages

- Weights need to be unique: use IDs to resolve ties

Independent Set A subset of vertices in the network such that no two vertices are connected by an edge of the network.

Maximum Independent Set

- Largest such set, can be used for interference-free transmission in Wi-Fi.
- NP-hard to compute this set.

Maximal Independent Set

- No more nodes can be added to it while keeping it an IS.
- Local:
    - Start with Q={v}
    - Repeat while Q non-empty:
        * Choose a node p in Q
        * Put p in IS
        * Remove all neighbours from Q
- Distributed:
    - Select root
    - Remove neighbours of root from possibility
    - Select IS in neighbours of neighbours etc.
- It could be pretty bad compared to the optimal Maximum IS.

UTC Universal Coordinated Time. Kept within 0.9s of Greenwich

Piezoelectric effect Squeeze a quartz crystal: generates electric field. Apply electric field: crystal bends.

Quartz crystal clock Resonates like a tuning fork. Accurate to parts per million

Skew Time difference between 2 clocks.

Drift Difference in rate between 2 clocks.

Detecting a clock skew

- It is 5s behind
    - Advance by 5s to correct.
- It is 5s ahead
    - Pushing back is bad: could be received before sent.
- Monotonicity: time is always increasing
    - If behind, decrease clock rate.
    - If ahead, increase clock rate.

How Clocks Synchronise

- Get time from server
- Delays in message transmission
- Delays due to processing time
- Server's time may be inaccurate

Christian's Algorithm

- Request sent at T_0, reply received at T_1
- Assume delays are symmetric, T_{server} is time from reply
- $T_{new} = T_{server} + (T_1 - T_0)/2$
- If minimum message transit time T_{min} is known
    - Range: $T_1 - T_0 - 2T_{min}$, accurate within $(T_1 - T_0 - 2T_{min})/2$

Berkeley Algorithm

- Assume no machine has perfect time
- Takes average of participants
- Sync everyone to average
- Master-slaves pattern
    - Master polls each machine for time
    - Computes average
    - Send each clock the offset to adjust time
- Fault tolerance
    - Ignore slaves with large skews
    - If master fails, elect new one

Network Time Protocol

- Enable clients to synchronise to UTC
- Reliable: Redundant servers and paths
- Scalable: Enable many clients to sync frequently
- Security: Authenticate sources
- Servers in layers
    - Layer 1: Directly connected to atomic clock
    - Layer 2: Few μs off layer 1
- Uses multiple rounds of messages, large number of servers and MST for inter-server sync

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Logical Clocks

- Determine what happened before what without clocks.
- Use a counter at each process.
- Increment after each event.
- Can also increment when there are no events.
- Each event has an associated time

Happened Before

- $a \rightarrow b$ means a before b.
- a is send of message m and b is receive.
- Transitive property
- Events without a "happened before" relation are concurrent.
- Preserves causal relations.
- Implies partial order
    - Ordering between pairs of events.
    - No ordering between concurrent events.

Lamport Clocks

- A logical clock
- Sent with every message
- On receiving a message, set own clock to $max(own, message) + 1$
- For any event e, write $c(e)$ for logical time
- If $a \rightarrow b$ then $c(a) < c(b)$
- If $a \rightarrow b$ then no Lamport clock exists with $c(a) == c(b)$
- If $e_1 || e_2$ then there exists a Lamport clock such that $c(a) == c(b)$
- If we order all events by their Lamport clock then we get partial order satisfying causal relations
- Total order from Lamport clocks
    - If event e occurs in process j at time $c(e)$
    - Give it time $(c(e), j)$
    - Order events by (c, id)

Vector Clocks

- If $a \rightarrow b$ then $c(a) < c(b)$.
- Also if $c(a) < c(b)$ then $a \rightarrow b$.
- Each process i maintains a vector V_i.
- V_i has n elements
    - keeps clock $V_i[j]$ for every other process j
    - On every local event: $V_i[i] = V_i[i] + 1$
    - On sending a message at i
        * Adds 1 to $V_i[i]$
        * Sends entire V_i
    - On receiving a message at j
        * Take max element by element
        * $V_j[k] = max(V_j[k], V_i[k])$ for all k
        * Adds 1 to $V_j[j]$ (local event)
- $V == V'$ iff $V[i] == V'[i]$ for all i
- $V < V'$ iff $V[i] < V'[i]$ for all i
- $V \leq V'$ iff $V[i] \leq V'[i]$ for all i
- $a \rightarrow b$ if $V(a) < V(b)$
- Two events are concurrent if neither $<$ nor $>$ is true
- Drawbacks
    - Entire vector sent with message

- All vector elements (n) have to be checked
- $\Omega(n)$ per message communication complexity, increases with time

Distributed Snapshots

- Take a snapshot of the system
- Global state: state of all processes and comm. channels
- Consistent cuts: set of states of all processes is a consistent cut if: for any states s, t in the cut $s||t$.
- If $a \to b$, then b cannot be before cut and a after cut

Distributed Snapshot Algorithm

- Ask each process to record state.
- The set of states must be a consistent cut.
- Assumptions
  - Communication channels are FIFO
  - Processes communicate only with neighbours
  - We assume for now that everyone is a neighbour
  - Processes do not fail

Chandy and Lamport Algorithm

- Send Rule at i
  - Process i records state
  - On every outgoing channel where a marker has not been sent i sends a marker on the channel before sending any other message.
- Receive Rule at i on channel C
  - i has not received a marker before
    * Record state of i
    * Record state of C as empty
    * Follow Send Rule
  - Otherwise
    * Record state of C as set of messages received on C since recording i's state and before receiving marker on C.
- Algorithm stops when all processes have received marker on all channels.
- $O(l)$ message complexity: l is number of links, plus the messages sent by normal execution of processes
- $O(d)$ time complexity: d is diameter

Snapshot Properties

- If s_1 (in p_1) $\to s_2$ (in p_2)
  - Then s_2 before cut $\implies s_1$ before cut
  - Proof by contradiction: s_1 after cut
    * p_1 recorded its state before s_1
    * Message m from p_1 to p_2: this causes $s_1 \to s_2$ to be true
    * p_1 must have recorded state before sending m
    * p_1 must have sent marker to p_2 before sending m
    * p_2 must have received marker before m and before s_2
    * s_2 must be after cut: contradiction

Application of Snapshots

- Detection of stable predicates
- A property that once it becomes true, stays true
- Examples
  - Deadlocked: every process in some subset is waiting for another
  - Terminated: once ended, computation remains stopped
  - Loss of token: in mutex, process with token can access a resource. If token gets lost, it stays lost.
  - Garbage: If no-one has a reference to a file, that file can be deleted

- So if such a property was true before the snapshot, it is true in snapshot, and can be detected by checking the snapshot

Non-stable Predicates

- Predicate may have happened but state has changed, e.g. "Was this file opened at some time?
- Two types
  - Possibly B: B could have happened
  - Definitely B: B definitely happened
- Collecting global states
  - Each process notes its every state & vector timestamp
  - Sends it to server
  - We only need to save state changes affecting the predicate
  - The server looks at these and tries to figure out if predicate B was possibly or definitely true

Possible States

- Server checks for possible states: consistent cuts for B
- Create a lattice where a downward path from initial state to final state is a valid execution
- **Possibly** B occurs on at least downward path
  - Do BFS search from start
  - If there is one state with B true then possibly b is true
- **Definitely** B occurs on all downward paths
  - Do BFS search from start
  - Do not visit nodes with B true
  - If BFS reaches final state and B is not true there then definitely B is false, otherwise it is true
- Complexity for both is $O(k^N)$ where k is max number of events at a single process

Mutual Exclusion Multiple processes should not use same resource at once. Restricts access to critical section to at most one process at a time.

Critical Section Part of code that uses the restricted resource

Mutex Properties

- **Safety**: Two processes should not use critical section simultaneously.
- **Liveness**: Every live request for CS is eventually granted.
- **Fairness**: Requests must be granted in the order they are made.

Mutex Algorithms Assumptions

- Only one resource
- All channels are FIFO

Central Server Algorithm

- There is a coordinator that holds a *token* for the resource.
- Other processes send token requests to coordinator.
- Server puts incoming requests into a queue.
- Sends token to first process in queue.
- Process returns token when done.
- Advantages: simple and constant complexity per message
- Disadvantages:
  - Central point of failure
  - Central bottleneck
  - Does not preserve order in async systems
  - Coordinator must be elected

Token Ring Algorithm

- Processes arranged in a ring
- Token is continuously passed in one direction
- If process does not need to enter CS, it passes token
- Otherwise it holds token, executes CS and then passes
- Disadvantages
    - Not in order
    - Long delay in getting token
    - One failure breaks ring
    - Passes token even without requests

Lamport's Mutex Algorithm

- Every node i has a queue of requests (sorted by timestamps)
- Process i sends CS request
    - $REQUEST(timestamp, i)$ to all processes
    - Enters $(timestamp, i)$ in own queue
- Process j receives $REQUEST(timestamp, i)$
    - Send timestamped $REPLY$ to i
    - Enters $(timestamp, i)$ in queue
- Process i enters CS if
    - $(timestamp, i)$ at head of own queue
    - Received $REPLY$ from all processes
- To release CS: send $RELEASE$ to all
- On receiving $RELEASE$ at j remove $(timestamp, i)$ from queue
- Complexity: $3(n-1)$ messages per CS request

Ricart and Agrawala's Algorithm

- Node j does not send $REPLY$ if j has request with timestamp lower than i request
- Node j delays the $REPLY$ until after own $RELEASE$
- Process i sends CS request
    - $REQUEST(timestamp, i)$ to all processes
- Process j receives $REQUEST(timestamp, i)$
    - If j has no own outstanding requests earlier than $timestamp$ or is not executing CS
        * Send $REPLY$ to i
        * Enters $(timestamp, i)$ to own queue
    - Else keep $(timestamp, i)$ pending
- Process i enters CS if it has received $REPLY$ from all.
- To release CS: send $REPLY$ to pending processes.
- Complexity: $2(n-1)$ messages per CS request.

Maekawa's Quorum Algorithm

- Instead of getting permission from all, get it from subset
- For each process i we have a voting quorum V_i
    - For all $i, j : V_i \cap V_j \neq \emptyset$
    - For all $i : i \in V_i$
    - Voting sets are same size
    - Each node part of same number of sets
- Arrange nodes in a square grid
- Quorum for node i are all nodes in same row or column
- Any two quorums intersect
- Complexity: $O(\sqrt{n})$ per CS request.

Packets Messages sent in (fixed-size) packets.

Local Area Networks

- Medium: Broadcast
- Message from one computer to all other computers
- Ethernet LAN is a broadcast medium and so is Wireless LAN
- Advantages
    - Sending a common message to everyone is easy
    - Finding destination is easy: destination field
- Disadvantage: Medium access - multiple transmissions at the same time

Medium Access

- Only one transmission at a time
- Protocols
    - TDMA: every node has a periodic slot
    - CSMA: see if anyone else transmitting, defer
    - ACKs are used to confirm transmission
- More complicated for wireless (hidden terminal)

Routing

- Finding a path in the network
- Every node has a routing table
- Equivalent to a BFS tree at every node
- Smaller routing tables by combining addresses

Location-based Routing

- Uses nodes' locations to discover paths
- Greedy algorithm: forward to neighbour closest

Transport Management

- UDP: send packet, hope it gets delivered
    - not FIFO
    - used in streaming
- TCP: send packet, ensure arrival
    - is FIFO
    - waits for ACKs, otherwise resends
    - slows down packet stream when packets not ACKed (assumes routers dropping them)

Overlay Network

- parts of the network sometimes ignored
- nodes that carry but not participate or edges not used
- used in P2P networks where communication only with known nodes

Computation

- Synchronous
    - Operation in rounds
    - In a round, a node performs computation, and then sends messages
    - All messages sent at the end of round x are delivered at start of round $x + 1$
    - Can be implemented with $m + c$ duration
        * if message transmission time bounded by m
        * Computation times are bounded by c
        * Clocks are synchronised
    - Easier to design, starting point for design
- Asynchronous
    - No synchronisation or rounds
    - Nodes compute and send at different speeds
    - No assumptions about speeds
    - Simplifying assumptions can be made e.g.
        * Channels are FIFO
        * Code blocks are atomic (uninterrupted by messages)
        * Either communication or computation bounded

Failures

- Hardware failures
- Out of power failure
- Software failure

- Can be permanent/temporary
- Nodes can fail individually or together (correlated)

Stopping Failure Node stops working, assumptions about what it finished and who knows about failure.

Byzantine Failure Node behaves arbitrarily or as adversary.

Link Failure Can be noise (waves at similar frequencies) or interference (nodes nearby communicating). Channels can become silent & unusable, active & unusable, or active & erroneous message

Security

- Unauthorised access/modification
- Attack on nodes: causing nodes to fail, reading data, or taking control
- Attack on links: blocking communication, reading channel data, corrupting data

Mobility

- Movement makes it harder to design distributed systems
- Communication is difficult: delays, lost messages, edge weights change

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Leader Election

- Agreement is simpler with a master but single point of failure (SPOF)
- When one master fails, another takes over

Failure Detectors

- Detecting crashed processes
- Detecting "working" is easier (they respond), detecting "failed" is harder.
- Unreliable detectors: reply with "suspected (failed)" or "unsuspected".
- **Example**
  - Suppose all messages delivered within D seconds.
  - Then we can require heartbeat every T seconds to failure detector.
  - If a failure detector does not get heartbeat in $T + D$ seconds, it marks process as "suspected" or "failed".
- Synchronous: simple, send a message and wait for $2D + \epsilon$ (no need for detector)
- If T or D too large: long failure timeouts
- If T too small: too much pressure on clients
- If D too small: "working" could get marked as "failed"

Real World

- Both synchronous and asynchronous too rigid
- Have 2 values: $D1$ and $D2$
- Use probabilities: delivery time is a distribution, estimate probability of failure
- a: probability process fails within time T
- b: probability a message not received in $T + D$
- Want to estimate $P(a|b)$ using Bayes Theorem

Distinguished Leader
: Leader must have a property other nodes do not have: node with highest ID.

Aggregation Tree Leader Election

- Node r detects leader failed, initiates election
- Node r creates BFS tree
- Asks for max node ID via aggregation (convergecast)
- If all n nodes start election needs n trees
  - $O(n^2)$ communication and $O(n)$ memory per node

Ring Leader Election (Chang and Roberts)

- Nodes send to right max of received from left and own ID
- When max ID node receives the ID it knows everyone has seen it and declares itself the leader
- If multiple elections at the same time: it sends the ID only if greater than own ID
- Message complexity: $O(n^2)$

Ring Leader Election (Hirschberg and Sinclair)

- $k - neighborhood$ of node p: set of all nodes within distance k.
- Message has a time-to-live (ttl) variable decremented on receiving. When zero, not forwarded.
- Algorithm operates in phases, each phase ttl is doubled
- Node sends messages right and left with ID and ttl
- Node returns message if ID in message greater and ttl is zero
- Otherwise it forwards it and decreases ttl
- If both returned, node is leader of $k - neighborhood$
- When $2^i >= n/2$ only 1 process survives: the leader
- Number of rounds: $O(\log n)$
- Number of messages: $O(n)$ per phase

Bully Algorithm

- Each node knows IDs of all nodes
- Synchronous (round) operation
- Node p initiates election
- p sends message to all nodes with higher ID
- If p does not get any replies, it declares itself a leader
- Higher ID gets message, responds and starts election
- If higher ID gets leader declaration it starts election again
- Message complexity: $O(n^2)$

Multicast

- Send message to multiple nodes with only 1 message
- Happens only within a group (LAN)
- Nodes can accept message and join group (tree at each node)
- IP addresses from 224.0.0.0 to 239.255.255.255
- A message to one of the addresses sent to all nodes subscribed to the group (same network)
- When joining node informs OS which informs network: Internet Group Mgmt protocol (IGMP)
- IP Multicast
  - Sender sends only once
  - Every router forwards only once
  - Uses UDP: no guarantees

Reliable Multicast

- Sending process is in multicast group
- Nodes may fail
- One to one communication between processes
- No network fails, no messages undelivered
- $multicast(g, m)$: message m to group g
- $receive(m)$: OS receives message and gives to multicasting process
- $deliver(m)$: multicast process delivers to application
- Integrity: A working process p in group g delivers m at most once, and m was multicast by some working process.
- Agreement: If a working process delivers m then all other working processes in group g will deliver m

Basic Reliable Multicast

- $send(m, q)$ to each process q in group
- On receive, pretend it was multicast

- Does not implement Agreement: sender could crash mid-send

Reliable Multicast Implementation

- Initialize *Received* =
- Send message to each process in group
- On receive if not in *Received*

  - Add it to *Received*
  - Forward *m* along
  - Deliver to application

- Satisfies Integrity: $send(m, q)$ is reliable
- Satisfies Agreement: forwards before delivers

Multicast Ordering

- We want messages delivered in correct order
- FIFO: if a process performs 2 multicasts, every process sees them in correct order
- Causal: if $multicast(m) \rightarrow multicast(m')$ then deliver m before m' (implies FIFO)
- Total: All working processes deliver messages in same order
- Our Multicast is FIFO assuming it sends to group members in same order and channels are FIFO

Causally-ordered Multicast

- Each process has a Vector clock
- Suppose p sends multicast m
- q receives m and holds until

  - It has delivered any earlier message by p
  - It has delivered any multicast message delivered by p before m

- Checking using vector timestamps

Totally-ordered Multicast

- Using a sequencer process

  - Process p wants to multicast
  - It requests sequence number from sequencer
  - Send multicast with sequence
  - Multicasts are delivered by sequence number
  - SPOF and bottleneck

- Using collective agreement

  - Process p sends basic multicast to the group
  - Each process picks a sequence number
  - Processes run protocol to agree on sequence number
  - Messages delivered according to the agreement

Basic Consensus

- Set of processes each with state undecided
- Termination: Set their decision variable and enter decided state
- Agreement: If processes entered decided state their decisions are equal
- Integrity: If all processes proposed value v then all of them have decision v
- Simple algorithm

  - Use reliable multicast to communicate all values
  - Rule e.g. min or max decides

Byzantine Generals Consensus

- Commander issues attack
- One or more processes may be faulty
- Termination: everyone decides
- Agreement: non-faulty processes agree
- Integrity: Non-faulty commander decides
- No solution with $n < 3f$ processes

Interactive Consensus

- Processes have to agree on a vector of values
- Each process contributes only to part of the vector but all processes have same vector in the end
- Termination: everyone decides
- Agreement: same vector V
- If p_i proposes x then $V_i = x$ for all

Termination Detection

- Computation started by s by sending messages
- Process s starts with weight 1.0
- When any process sends a message it puts part of its weight in the message
- When any process receives a message it adds weight to own weight
- When a process finishes computing it sends current weight to s
- When s has weight = 1.0 it knows the deed is done
- No message is lost (TCP required)

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Dijkstra-Scholten Algorithm

- Maintains a tree of which node initiated computation at which other node
- Each node has active children counter cc
- When node x sends message to y

  - x increments cc
  - If y was idle it becomes active and x is parent
  - If y was active send ack to x

- When x receives ack it decrements cc
- When y finishes computation and has $cc = 0$ it sends ack to parent

Distributed OS

- OSes on different computers are like a single OS
- Process does not know that other resources/processes are at other computers
- One interface to all resources in the network
- Disadvantages

  - What if part of network fails and there are 2 sets of processes now.
  - Say A and B start on some machine but OS moves one of them elsewhere. Inefficient if they communicate a lot.
  - Access to off-site resources has to be through explicit connection (cannot have all machines in same OS).

Distributed Computation in Networked OS

- Distributed algorithms for synchronisation, consistent ordering, mutex, leader election, failure detection etc.

Virtualisation

- Virtual machine runs as an application on a computer
- It emulates hardware of the computer
- When application ran in guest OS, the VM emulates the process of the OS as well as the application
- Good for sandboxing, testing, backup
- Server farms run several VMs on one physical server
- Advantages

  - More flexibility
  - Easier to turn on/off (scale)
  - Easier to backup
  - VMs can be moved from one machine to another

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