

# Bit Manipulation - 2

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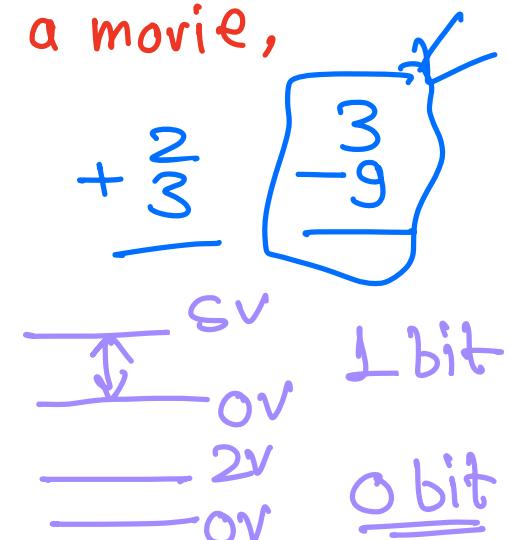
*Reachable in Scaler Lounge* 

"Brain is the hardware. Mindset is the operating system.  
Thoughts are the programming instructions, and  
emotions are the output"

\* ChatGPT, Maps, video call, watching a movie, creating an animation.

AND, OR, XOR, NOT

Computers = fast + dumb



### § Application of left shift operator:

$$(00000100)_2 \rightarrow 2^2 = 4 \quad (1 \ll 2)$$

$$(0010000)_2 \rightarrow 2^4 = 16 \quad (1 \ll 4)$$

④ Set  $i^{th}$  bit in a number

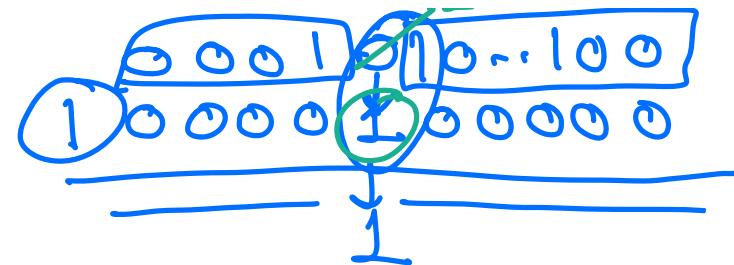
ex:  $N = 45$   
 $i = 4$

$$\begin{array}{r} \text{set } 4 \text{ at } 4 \\ 101101 \\ \Rightarrow 1\textcircled{1}1101 \\ \hline 2^4 \end{array}$$

ex:  $N = 45 \Rightarrow 101\textcircled{0}01$

$i^{th}$  bit

$$N = N | (1 \ll i)$$



$$N=26$$

$$i=4$$

$\begin{array}{r} 00011010 \\ \text{---} \\ 76543210 \end{array}$

$$\text{OR } \begin{array}{r} 00010000 \\ \hline 00011010 \end{array}$$

N  
with 4th bit  
is set

$$N=26$$

$$i=2$$

$\begin{array}{r} 00011010 \\ \text{---} \\ 76543210 \end{array}$

$00000100$

$$N = \underline{\underline{00011110}}$$

\* Application of XOR (^) operator

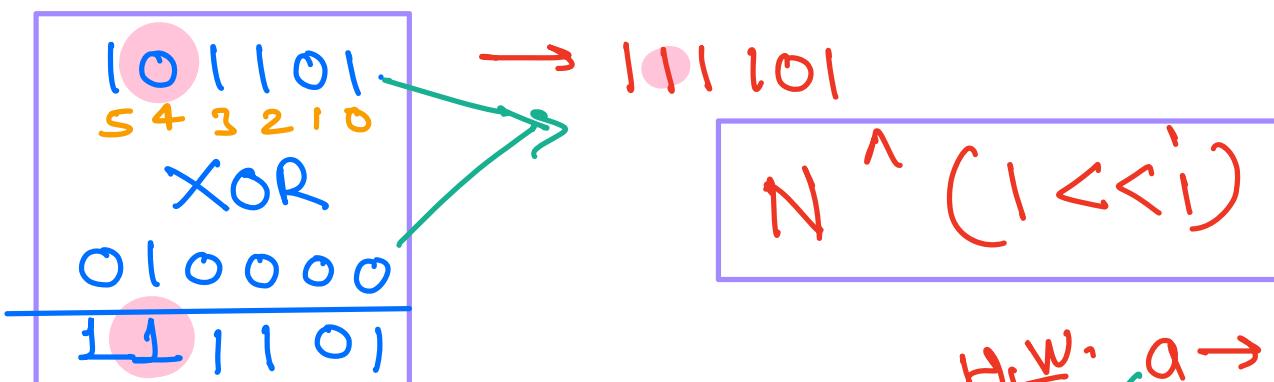
\* Flip (toggle) the  $i^{\text{th}}$  bit

XOR  
Same bit  $\rightarrow 0$   
different bit  $\rightarrow 1$

ex.  $N = 45$   
 $i = 2$

$101\ 101$  →  $101001$   
S 4 3 2 1 0

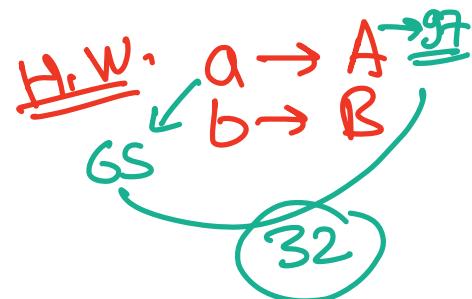
ex.  $N = 45$   
 $i = 4$   
 $i << i$



& operator.



Check if  $i^{\text{th}}$  bit is set or not?



ex.  $N = 45$   
 $i = 2$

$101\ 101$  ⇒ true  
S 4 3 2 1 0

$\begin{array}{r} 101\ 101 \\ S 4 3 2 1 0 \\ \& 000100 \\ \hline 000100 \end{array}$  ( $i << 2$ )

Obsn: If  $i^{\text{th}}$  bit is not set then  
 $(N \wedge (i << i)) = 0$

$$N = 45 \quad i = 4$$

$$\begin{array}{r} 101101 \\ \& 010000 \\ \hline 000000 \end{array} (i << 4)$$

```

if (N & (1 << i) == 0) {
    print ("not set")
}
else
    print ("bit is set")

```

$0 \rightarrow \text{false}$   
 $!0 \rightarrow \text{true}$

$N \& (1 << i)$   $\Rightarrow$  get  $i^{\text{th}}$  bit is set or Not

II Sol<sup>n</sup>: using OR operation

$$N = 45 \quad S = 101101$$

$$i = 1 \quad \text{OR} \quad \begin{array}{r} 000010 \\ \hline 101111 \end{array}$$

$$i = 2 \quad \begin{array}{r} 101101 \\ \text{OR} \quad 000100 \\ \hline 101101 \end{array}$$

gf (  $N = (N | (1 \ll i))$  {

    print(i<sup>th</sup> bit is set)

Q Unset the  $i^{\text{th}}$  bit of the Number  $N$  if it is set.

ex:

$N = 6$        $\begin{smallmatrix} 3 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{smallmatrix}$       ans = 0010  
 $i = 2$

ex:

$N = 6$        $\begin{smallmatrix} 3 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{smallmatrix}$       ans = 0110  
 $i = 3$

idea:

- ① Check if  $i^{\text{th}}$  bit is set
- ② If yes, unset  $i^{\text{th}}$  bit

int

unsetBit (  $N, i$  ) {

$X = N \& (1 \ll i)$

gf ( $X \neq 0$ ) {

$N = N ^ (1 \ll i)$

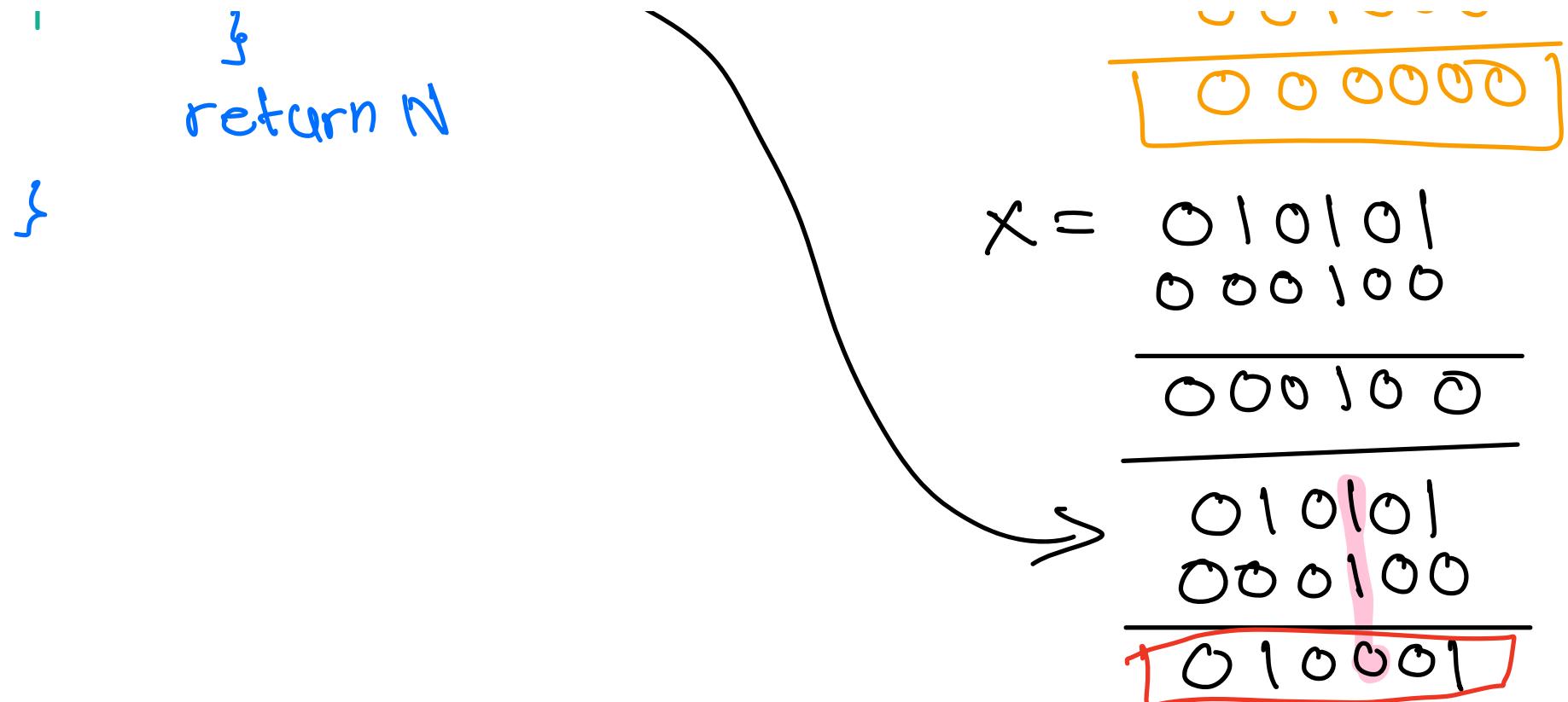
)

$N = 21, i = 3, 2$

$\begin{smallmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{smallmatrix}$

ans = 010101

$X = \begin{smallmatrix} 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{smallmatrix}$



Q: Given an integer N, Count the total number of set bits.

ex:  $N = 12, \dots 0\overset{3}{1}\overset{2}{1}00$  ans = 2       $\frac{S_9}{32} \frac{32}{27}$

$N = 59, \dots \overset{5}{1}\overset{4}{1}\overset{3}{1}\overset{2}{0}\overset{1}{1}0$  ans = 5

idea: ① we know how to check if  $i^{\text{th}}$  bit is set or not.

② If we do the ① for all the bits, we will get  
our answer.

```
bool checkbit (N, i){  
    if (N & (1<<i) != 0){  
        return true;  
    } else  
        return false;  
}  
  
int countbits (int N){  
    int setbits = 0;  
    for(i=0; i<32; i++){  
        if (checkbit(N, i)){  
            setbits++;  
        }  
    }  
    return setbits;  
}
```

int 4 bytes  
↓  
32 bits  
-----  
32 bits



(II)

Right shift

$$\begin{array}{c} \overbrace{10101}^{\text{10101}} \\ \Rightarrow \\ \gg 1 \\ \hline \end{array} \Rightarrow \underline{01010}$$

idea: ① check the first bit (LSB)  
② right shift the number.

$$N = 45$$

$$101101$$

$$\Rightarrow \underline{4}$$

Steps:

(I)

$$\begin{array}{r} 101101 \\ & \& 0000001 \\ \hline 1 & ! = 0 \end{array}$$

Count  
1

(II)

$$\begin{array}{r} 0101101 \\ & \& 0000001 \\ \hline 0 \end{array}$$

right shift by 1

count won't change

(III)

$$\begin{array}{r} 001011 \\ & \& 0000001 \\ \hline : & & 1 & ! = 0 \end{array}$$

count +1

③

if  $N == 0$ , set bit = 0

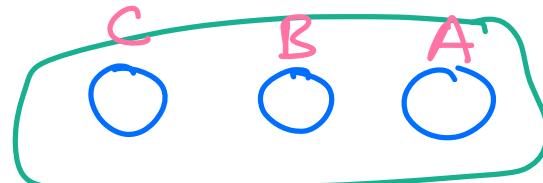
```
int countOfSetBits (N) {  
    int count = 0  
    while (N > 0) {  
        if (N & 1) {  
            count++;  
        }  
        N = N >> 1  
    }  
    return count;  
}
```

§. Negative numbers:

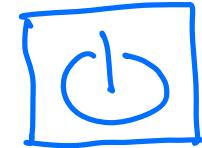
3 cathode ray tubes

0 0 0	0	off
0 0 1	1	off
0 1 0	2	off
0 1 1	3	off
1 0 0	4	-0
1 0 1	5	-1
1 1 0	6	-2
1 1 1	7	-3

How to denote negative numbers.



$1 \rightarrow \text{on}$   
 $0 \rightarrow \text{off}$



$$\underline{2} \times \underline{2} * 2 = 8$$

punch cards

C ✓  
B X  
A ✓  
101



Addition

AND  
XOR

$$-2 + 3 = 1$$

①

$$\begin{array}{r} -2 \\ +3 \\ \hline 110 \\ 011 \\ \hline 001 \end{array}$$

$$-3 + 1 = -2$$

$$\begin{array}{r} -3 \\ +1 \\ \hline 111 \\ 001 \\ \hline 100 \end{array}$$

$\cancel{1} \cancel{0} \cancel{0} \neq 2$

2's complement

- ① 1's complement
- ② +1 to the number

to make subtraction easier

$$\begin{array}{r} 110 \\ +1 \\ \hline 111 \end{array}$$

discarded 2's	1's complement	000	001	010	011	100	101	110	111	→ 0	→ 1	→ 2	→ 3	+11:00
1000	111	000	001	010	011	100	101	110	111					
111	110	000	001	010	011	100	101	110	111					
110	101	000	001	010	011	100	101	110	111					
101	100	000	001	010	011	100	101	110	111					
100	011	000	001	010	011	100	101	110	111					
011	010	000	001	010	011	100	101	110	111					
010	001	000	001	010	011	100	101	110	111					
001	000	000	001	010	011	100	101	110	111					



8 bit number



→ take 2's complement to get the value

$$\begin{array}{r} \underline{\underline{45}} \quad 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1 \Rightarrow +45 \\ -45 \Rightarrow 1\ 1\ 0\ 1\ 0\ 0\ 1\ 0 \\ \hline 1\ 1\ 0\ 1\ 0\ 0\ 1\ 1 \Rightarrow -45 \end{array}$$

2's complement makes subtraction easier.

$$\begin{array}{r} (\underline{\underline{45}} - 45) \Rightarrow 45 + (-45) \\ \hline 1\ 1\ 1\ 1\ 1\ 1\ 1 \\ \Rightarrow \begin{array}{r} 45 \\ + -45 \\ \hline 0 \end{array} \quad \begin{array}{r} 0\ 0\ 1\ 0\ 1\ 1\ 0\ 1 \\ | \ 1\ 0\ 1\ 0\ 0\ 1\ 1 \\ \hline 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0 \end{array} \end{array}$$

$$\begin{array}{r} \text{---} \\ \Rightarrow \quad \begin{array}{r} 00001100 \\ 11110011 \\ \hline 11110100 \end{array} \quad +12 \\ \text{---} \end{array}$$

(2's complement)

$$\underline{\underline{45 - 12}} \Rightarrow 45 + (-12)$$

$$\begin{array}{r} & \text{1} & \text{1} & \text{1} & \text{1} & \text{1} \\ & 0 & 0 & 1 & 0 & 1 \\ + & 1 & 1 & 1 & 0 & 1 \\ \hline & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}$$

45 - 12 = 33

$$\begin{array}{r} 12 \\ -45 \\ \hline 11010011 \end{array}$$

$$\Rightarrow -0010000 + 1$$

$$\Rightarrow -0010000 \Rightarrow -33 \quad \text{can display}$$

\* Building a 3-bit computer.  $\Rightarrow$  8 number

	<u>+ve</u>	<u>MSB &amp; -ve</u>	<u>1's comple</u>	<u>2's comple</u>
000	0	1	0	0
001	1	2	1	1
010	2	3	2	2
011	3	4	3	3
100	4	-0	-1	-4
101	5	-1	-2	-3
110	6	-2	-3	-2
111	7	-3	-0	-1

CAT

PIE

format

$$\begin{array}{r} 00 \\ 11 \\ + 1 \\ \hline 10 \end{array}$$

111

2's complement  $\Rightarrow$  subtraction is trivial

Quiz  $-3$  in 2's complement

$$+3 \Rightarrow \underline{\quad 0 \quad 0 \quad 0 \quad 0} \quad \underline{\quad 0 \quad 0 \quad 1 \quad 1}$$

$$1's \Rightarrow \begin{array}{r} 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \\ + 1 \\ \hline \end{array} \quad 0 \quad 0$$

$$\Rightarrow -3, 2's \Rightarrow \boxed{1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1}$$

Quiz  $-10$  in 2's complement?

$$+10 \Rightarrow \underline{\quad 0 \quad 0 \quad 0 \quad 0} \quad \underline{\quad 1 \quad 0 \quad 1 \quad 0}$$

$$\begin{array}{r} 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 0 \quad 1 \\ + 0 \quad 1 \\ \hline \end{array} \quad \boxed{1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1 \quad 1 \quad 0}$$

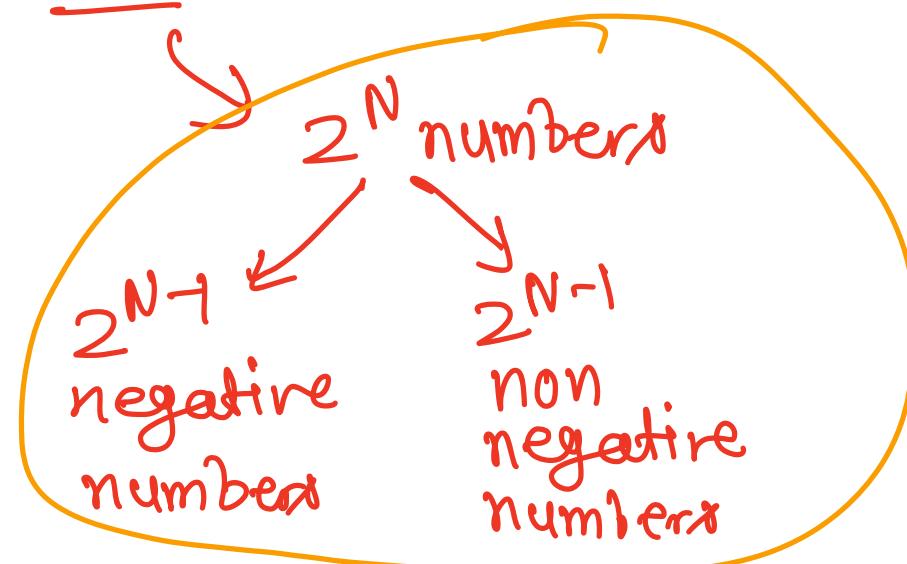
\* Range of N-Bit system: (2's complement)

3 bit system  $[-4, -3, -2, -1, 0, 1, 2, 3]$

$2^3$   $\downarrow$   $-/+ \Rightarrow [-2^2 \leftarrow 0 \rightarrow 2^2 - 1]$

$\Rightarrow [-2^2 \rightarrow 2^2 - 1]$

$N$  bit system:  $\Rightarrow [-2^{N-1}, 2^{N-1} - 1]$

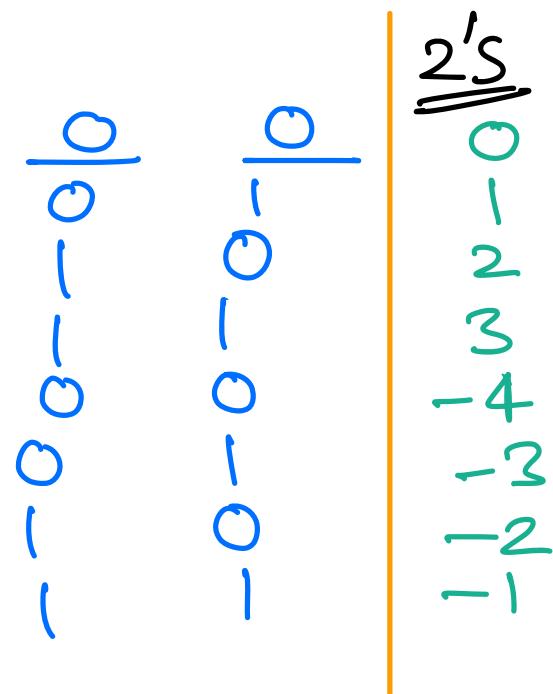


\* Constraints:

int  $\Rightarrow$  32 bits  
  ↓  
  - 31    21    7

$$N \in \{1, 10^6\}$$

A vertical stack of circles representing a linked list. The top four circles are pink and contain blue 'C' characters, with a green arrow pointing to the top-most 'C'. Below them are three light blue circles, each containing a blue 'I' character.



$$\begin{aligned}
 & \left[ -2^{\lceil \frac{30}{3} \rceil}, 2^{\lceil \frac{30}{3} \rceil} - 1 \right] \\
 & \downarrow \\
 & \boxed{2^{10} = 1024 \approx 10^3} \\
 & \left( -\left(2^{10}\right)^3 * 2, \left(2^{10}\right)^3 * 2 - 1 \right) \\
 \Rightarrow & \left( -\left(10^3\right)^3 * 2, \left(10^3\right)^3 * 2 - 1 \right) \\
 \Rightarrow & \underline{\left( -2 \times 10^9, 2 \times 10^9 - 1 \right)} \\
 & \text{int range}
 \end{aligned}$$

$$\Rightarrow [-4, 3]$$

$$\frac{a}{b} = \frac{2}{3} \quad \underline{\underline{a+b=6}}$$

$$a = 010 \Rightarrow 2^3 \Rightarrow \underline{\underline{2+2+2}}$$

$$b = 011$$

$\Rightarrow$

$$\begin{array}{r} 010 \\ 011 \\ \hline 101 \\ \hline 110 \end{array}$$

$\Rightarrow a = 2$

$\Rightarrow b = 3$

$\Rightarrow a+b = \cancel{5} = -2 \quad \text{overflow}$

int  $a = 10^5$   
 int  $b = 10^6$

$\Rightarrow a*b = 10^{11}$

int  $[-2 \times 10^9, 2 \times 10^9 - 1]$   
 $\cancel{X}$  4 bytes

$$\begin{aligned} \text{int } c &= a * b \\ &= 10^{11} \end{aligned}$$

$$\begin{aligned} \text{int } a &= 10^5 \\ \text{int } b &= 10^6 \end{aligned}$$

$$\text{long } c = a * b; \quad \times$$

long 8 bytes  
(64 bits)  
 $[-2^{63}, 2^{63}-1]$   
 $\approx [-9 \times 10^{18}, 9 \times 10^{18}]$   
range of long

(compiler treats int, as int)

$$= \underline{\underline{\text{long}(a)}} * \underline{\underline{b}} \quad \checkmark$$

$$= \text{long}(a \ b) \quad \underbrace{0 \ 0 \ 0 \ 0}_{32 \text{ bits}} \quad \underbrace{-0}_{32}$$

$$= 1L * a * b \quad \checkmark$$

Q: Given N elements, find the sum!

$$1 \leq N \leq 10^5$$

$1 \leq A[i] \leq 10^6$   
long  
~~int~~ sum = 0  
 for (int i=0; i<N; i++) {  
 sum += A[i]  
 }  
 return sum

$[10^6, 10^6 \dots 10^6]$   $\Rightarrow 10^{11}$   
 $\underbrace{10^6}_{10^5}$   
 Out of  
int range  
 $(-2 \times 10^9, 2 \times 10^9)$

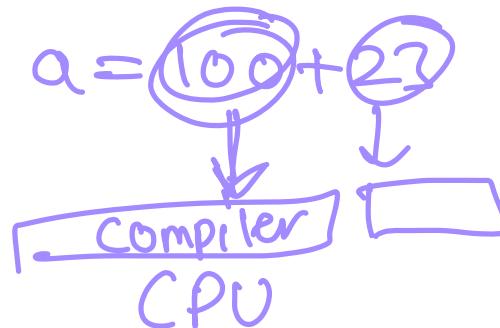
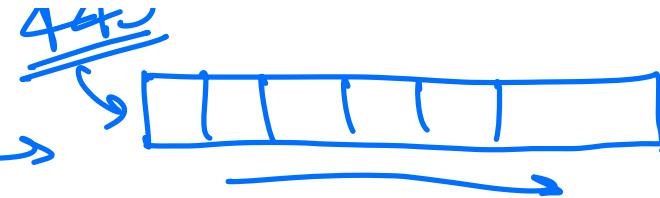
Doubt

11  $\Rightarrow$  is it eleven or three?

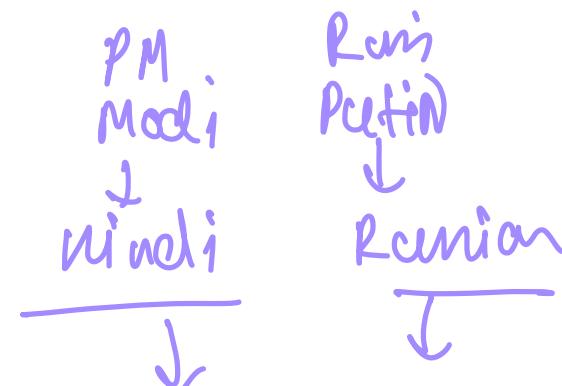
t-Shirt  $\Rightarrow$  there are 10 types of people  
 one who understand binary  
 and one who don't

11  $\Rightarrow$  decimal if you are writing in the code  
 $\Rightarrow$  internally everything is binary

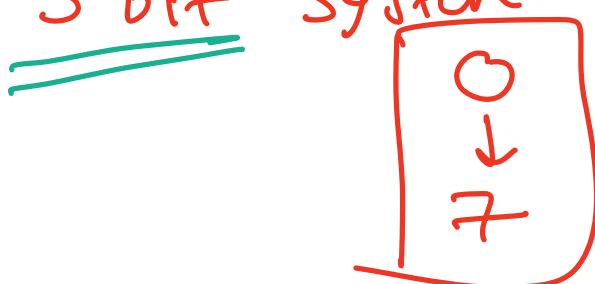
int a = 100;



printf(a) = 123



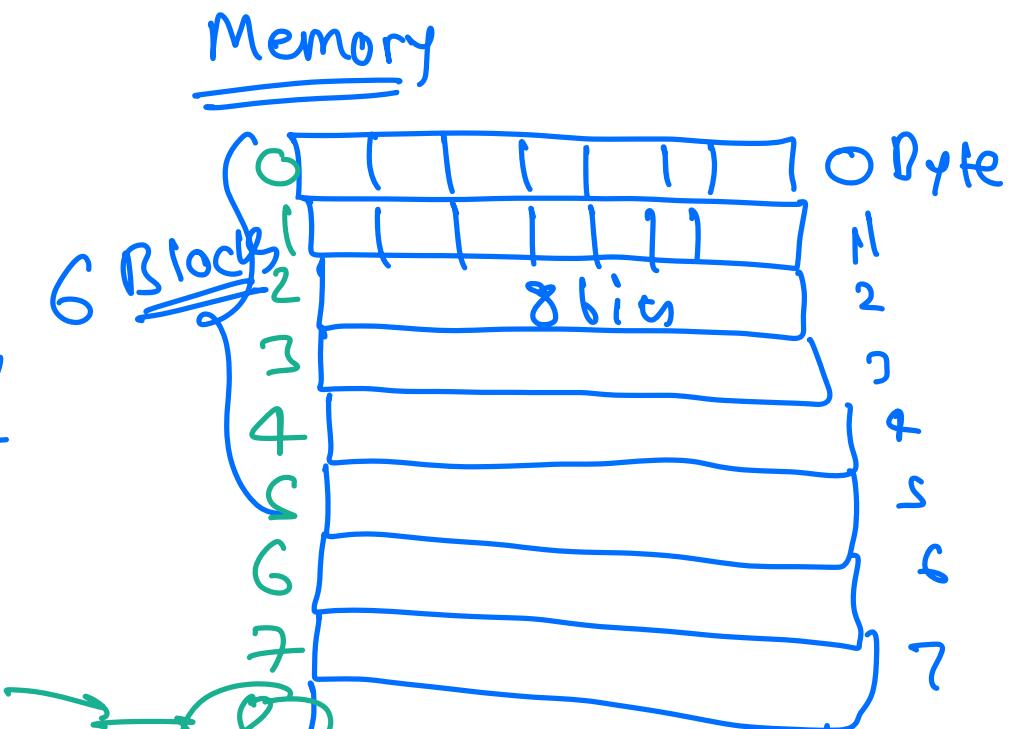
④ 3 bit system

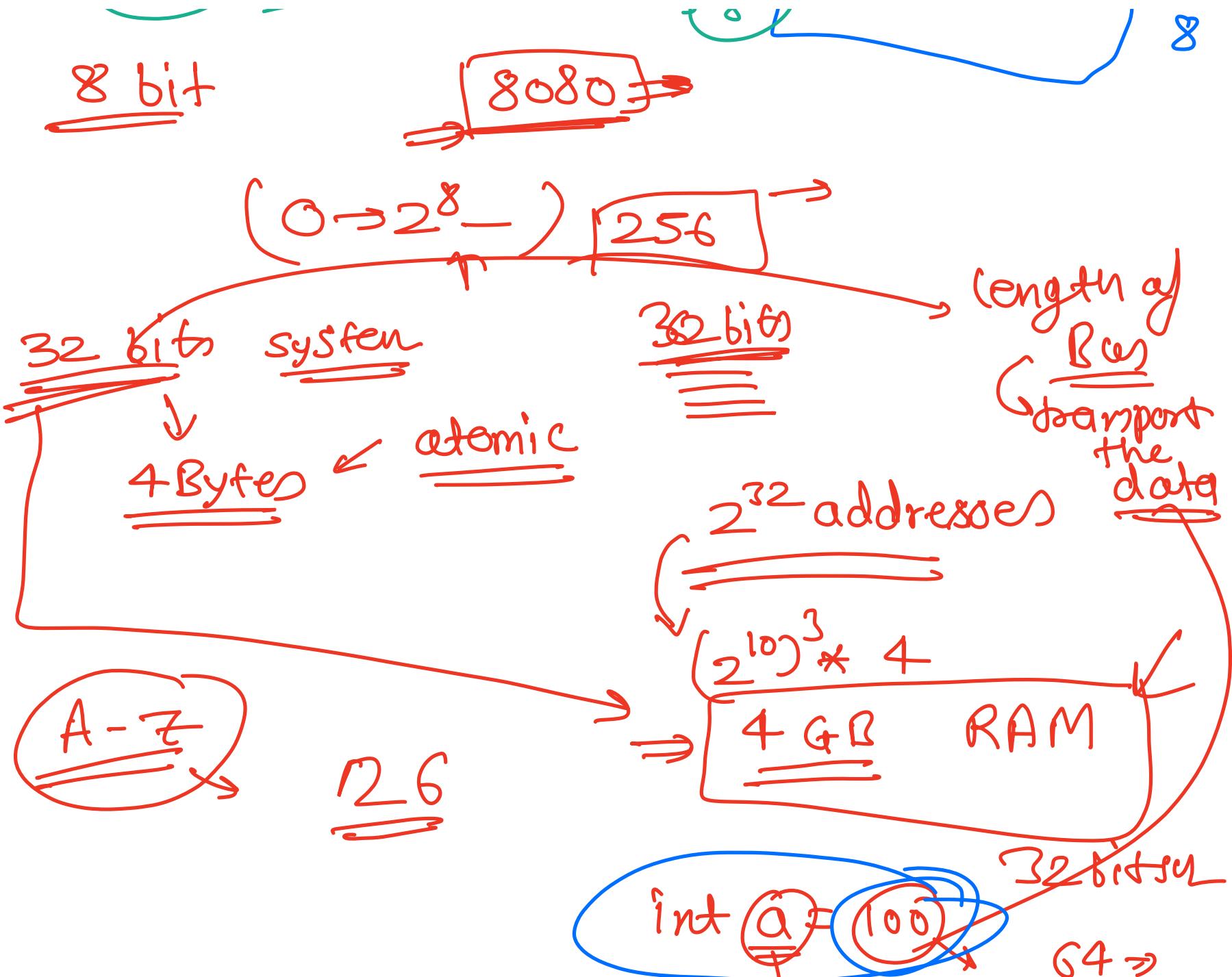


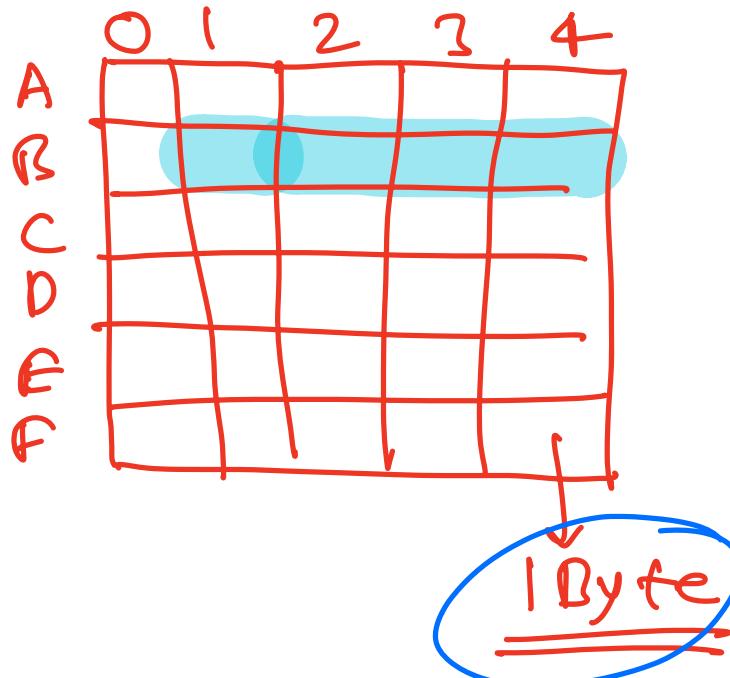
010 → 2<sup>nd</sup> memory block

110 → 6<sup>th</sup> block

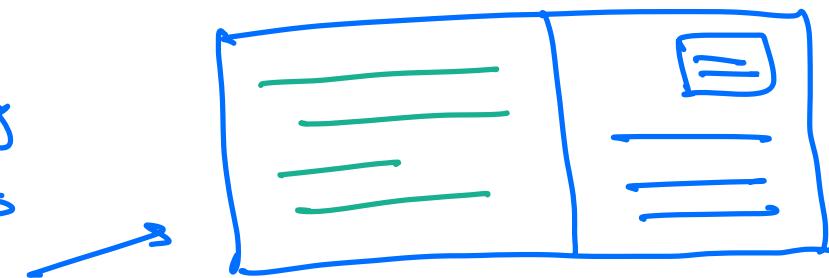
OS







$\downarrow$  32 bit  
OS  
 $\downarrow$  give me memory  
 $\downarrow$  B1-B4 to store your int



$\Rightarrow$  Postcards  
 $2^{32} - 1$

$2^{15} + 1$   
hexadecimal  
compact

