Lecture 5: Degree distributions and power laws

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Sociology 204: Social Networks, Spring 2021
Princeton University

1/2: Scale-free networks

Review:

- simple model (ring lattice + rewiring) predicts that many networks will be "small-world" networks

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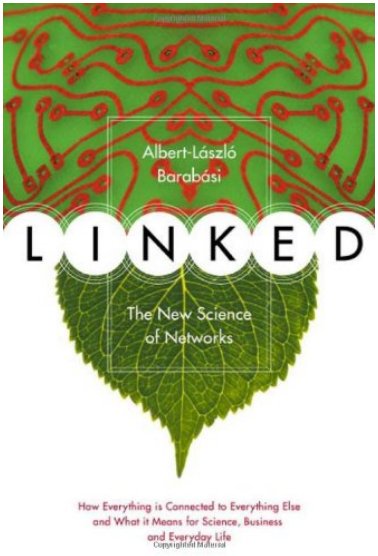
- abstract model helps us understand many types of networks
- these network structural properties are important for dynamics happening on the network (e.g., disease spread)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LINKED
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How Everything is Connected to Everything Else
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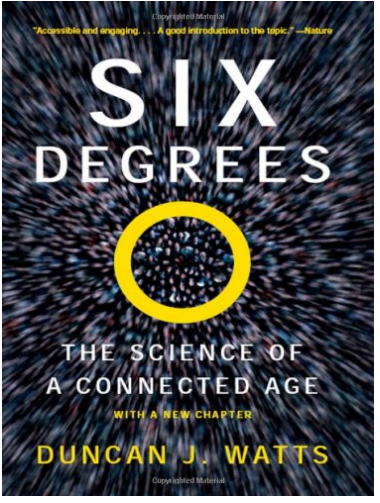
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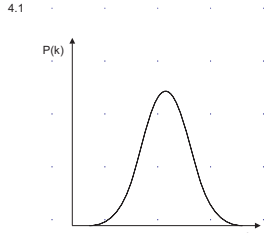
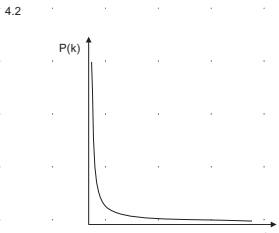
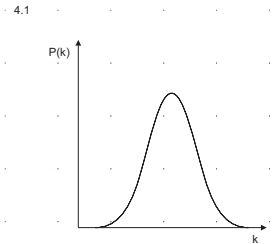
- degree: number of connections that a node has to other nodes (not related to degrees of separation)
- degree distribution: distribution of degrees



(a) Normal



(b) Power law



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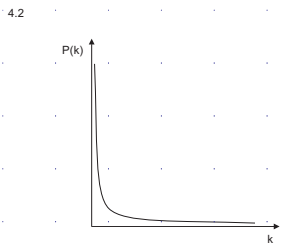
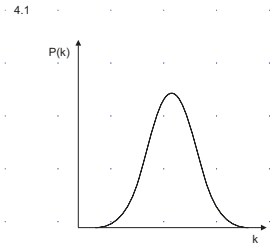


(b) Power law

Is the distribution of heights more similar to normal or scale-free?



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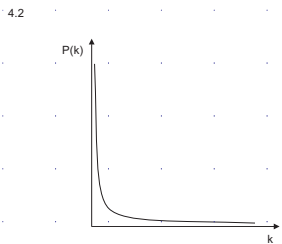
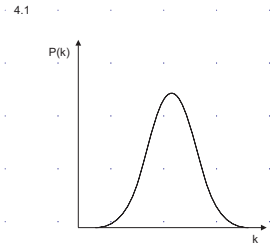


(b) Power law

Is the distribution of heights more similar to normal or scale-free? normal



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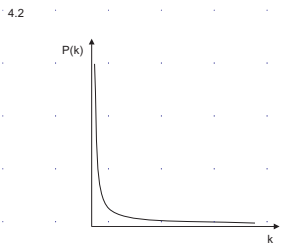
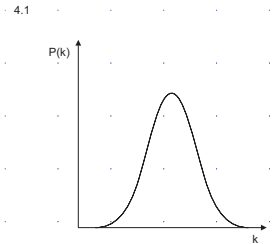


(b) Power law

Is the distribution of wealth more similar to normal or scale-free?



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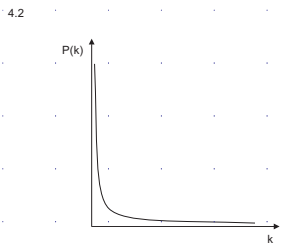
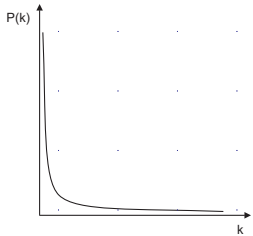


(b) Power law

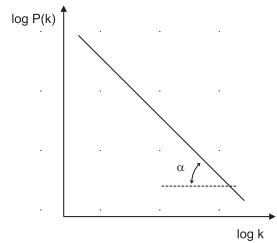
Is the distribution of wealth more similar to normal or scale-free? scale-free

4.2

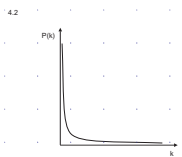


(a) Power law

4.3



(b) log-log Power law



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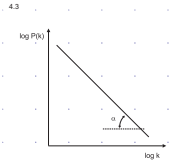
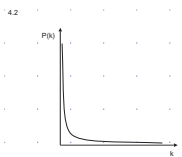


(b) log-log Power law



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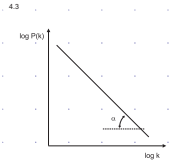
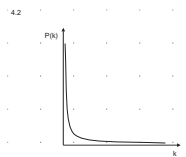


(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$



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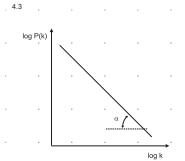
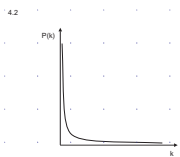


(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$
$logp(k) \propto log(\frac{1}{k^n})$



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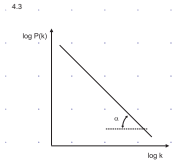

(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$
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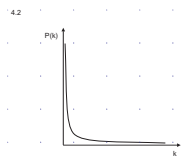
$logp(k) \propto log(1) - log(k^n)$



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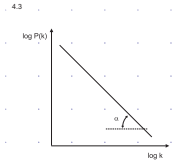


(b) log-log Power law

$p(k) \propto \frac{1}{k^n}$
$logp(k) \propto log(\frac{1}{k^n})$
$logp(k) \propto log(1) - log(k^n)$
$logp(k) \propto -nlog(k)$

It turns out that many degree distributions follow a power law distribution (which Barabasi calls "scale-free")

$p(k) \sim \frac{1}{k^{\gamma}}$

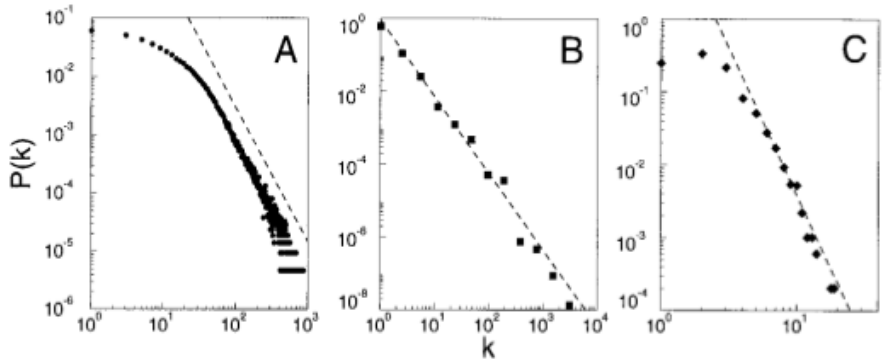
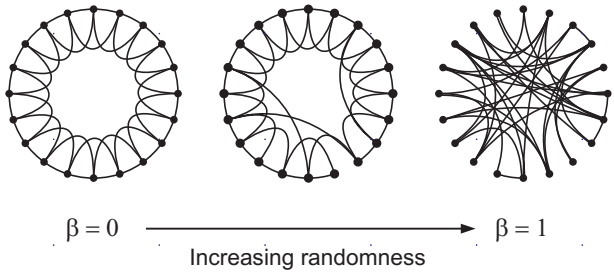


Fig. 1. The distribution function of connectivities for various large networks. (**A**) Actor collaboration graph with $N = 212{,}250$ vertices and average connectivity $\langle k \rangle = 28.78$. (**B**) WWW, $N = 325{,}729$, $\langle k \rangle = 5.46$ (*6*). (**C**) Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{\text{actor}} = 2.3$, (B) $\gamma_{\text{www}} = 2.1$ and (C) $\gamma_{\text{power}} = 4$.

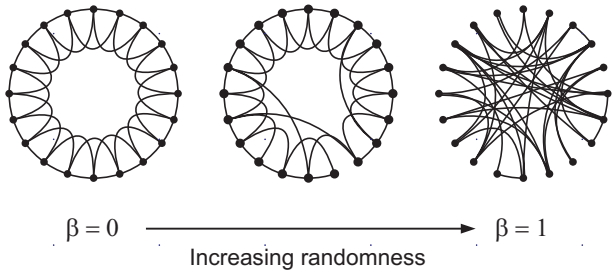
Does β model produce power law degree distribution?

3.6



Does β model produce power law degree distribution? No

3.6



Barabasi and Albert propose a very simple model that generates networks with power law degree distributions

- growth (new nodes enter the system)
- preferential attachment (more likely to connect to high degree nodes)

Demo
http://www.netlogoweb.org/launch#http://ccl.northwestern.edu/netlogo/models/models/Sample%20Models/Networks/Preferential%20Attachment.nlogo