

Lecture 5: Degree distributions and power laws

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Sociology 204: Social Networks, Spring 2021
Princeton University

1/2: Scale-free networks



Review:

- ▶ simple model (ring lattice + rewiring) predicts that many networks will be “small-world” networks

Review:

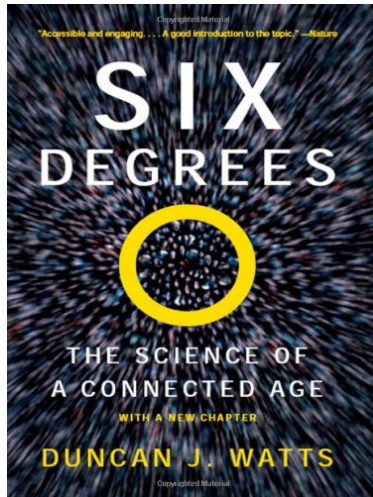
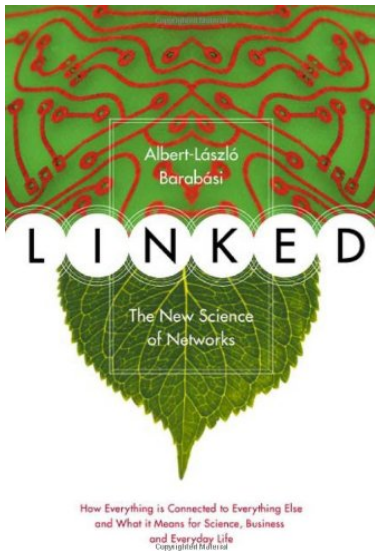
- ▶ simple model (ring lattice + rewiring) predicts that many networks will be “small-world” networks
- ▶ three real networks (movie actors, power grid, and worm brain) have high clustering coefficient (relative to Erdos-Renyi random graph) and similar characteristic path length to Erdos-Renyi random graph

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- ▶ abstract model helps us understand many types of networks

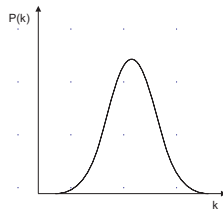
Review:

- ▶ simple model (ring lattice + rewiring) predicts that many networks will be “small-world” networks
- ▶ three real networks (movie actors, power grid, and worm brain) have high clustering coefficient (relative to Erdos-Renyi random graph) and similar characteristic path length to Erdos-Renyi random graph
- ▶ abstract model helps us understand many types of networks
- ▶ these network structural properties are important for dynamics happening on the network (e.g., disease spread)



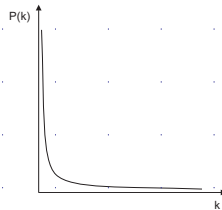
- ▶ degree: number of connections that a node has to other nodes (not related to degrees of separation)
- ▶ degree distribution: distribution of degrees

4.1



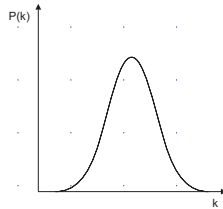
(a) Normal

4.2



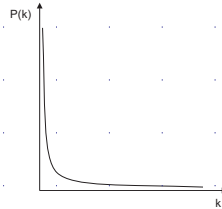
(b) Power law

4.1



(a) Normal

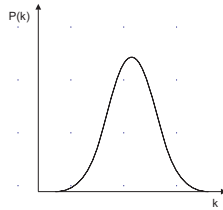
4.2



(b) Power law

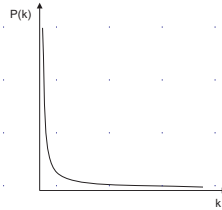
Is the distribution of heights more similar to normal or scale-free?

4.1



(a) Normal

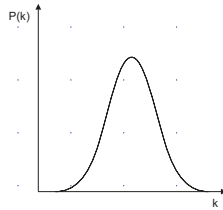
4.2



(b) Power law

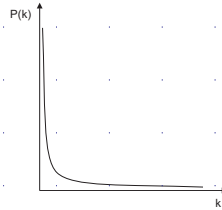
Is the distribution of heights more similar to normal or scale-free? normal

4.1



(a) Normal

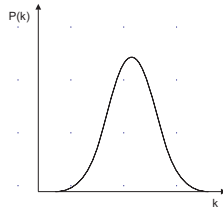
4.2



(b) Power law

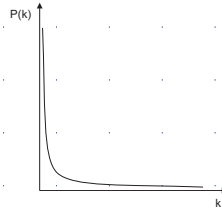
Is the distribution of wealth more similar to normal or scale-free?

4.1



(a) Normal

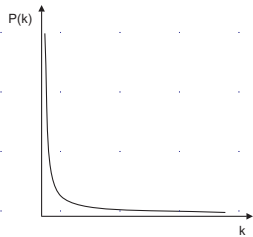
4.2



(b) Power law

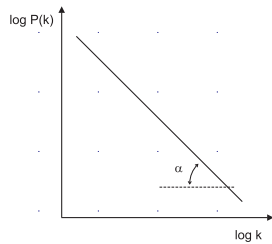
Is the distribution of wealth more similar to normal or scale-free? scale-free

4.2



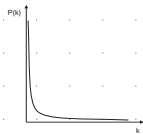
(a) Power law

4.3



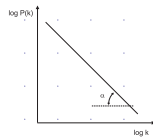
(b) log-log Power law

4.2



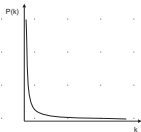
(a) Power law

4.3



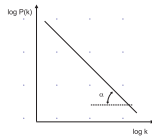
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4.2



(a) Power law

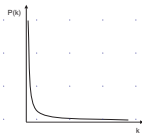
4.3



(b) log-log Power law

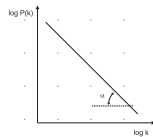
$$p(k) \propto \frac{1}{k^n}$$

4.2



(a) Power law

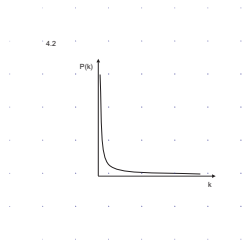
4.3



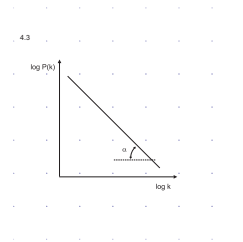
(b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$

$$\log p(k) \propto \log\left(\frac{1}{k^n}\right)$$



(a) Power law

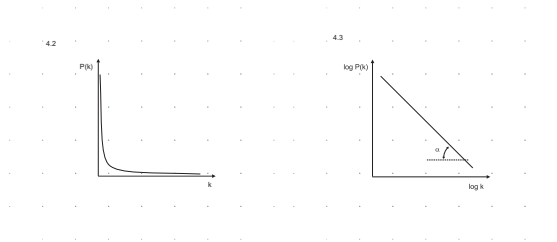


(b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$

$$\log p(k) \propto \log\left(\frac{1}{k^n}\right)$$

$$\log p(k) \propto \log(1) - \log(k^n)$$



(a) Power law

(b) log-log Power law

$$p(k) \propto \frac{1}{k^n}$$

$$\log p(k) \propto \log\left(\frac{1}{k^n}\right)$$

$$\log p(k) \propto \log(1) - \log(k^n)$$

$$\log p(k) \propto -n \log(k)$$

It turns out that many degree distributions follow a power law distribution (which Barabasi calls “scale-free”)

$$p(k) \sim \frac{1}{k^\gamma}$$

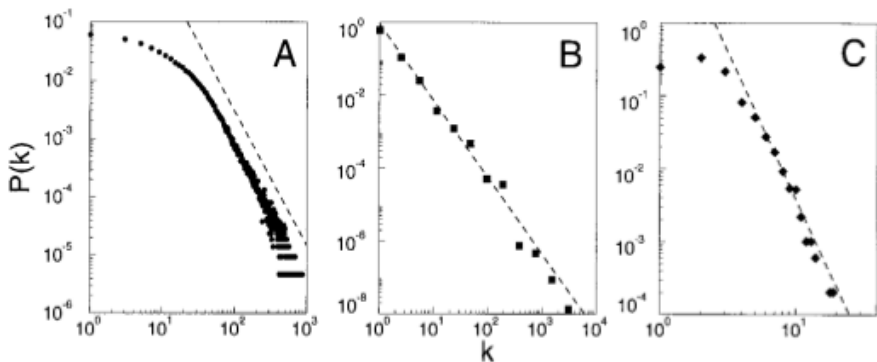
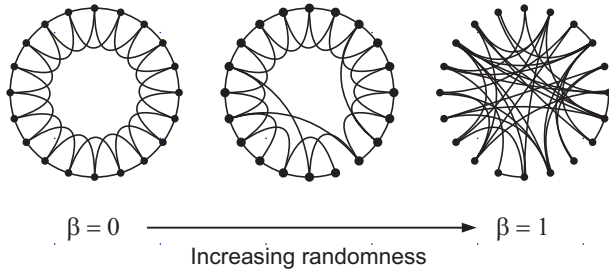


Fig. 1. The distribution function of connectivities for various large networks. (A) Actor collaboration graph with $N = 212,250$ vertices and average connectivity $\langle k \rangle = 28.78$. (B) WWW, $N = 325,729$, $\langle k \rangle = 5.46$ (6). (C) Power grid data, $N = 4941$, $\langle k \rangle = 2.67$. The dashed lines have slopes (A) $\gamma_{\text{actor}} = 2.3$, (B) $\gamma_{\text{www}} = 2.1$ and (C) $\gamma_{\text{power}} = 4$.

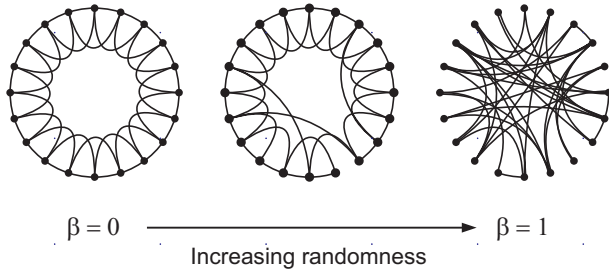
Does β model produce power law degree distribution?

3.6



Does β model produce power law degree distribution? No

3.6



Barabasi and Albert propose a very simple model that generates networks with power law degree distributions

- ▶ growth (new nodes enter the system)
- ▶ preferential attachment (more likely to connect to high degree nodes)

Demo

<http://www.netlogoweb.org/launch#http://ccl.northwestern.edu/netlogo/models/models/Sample%20Models/Networks/Preferential%20Attachment.nlogo>

