Homework 4

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3.3.3

a.

$$X = \text{amount spent on flat screen TVs by two random customers}$$

x	$P(X = x)$
0	`dbinom(0, 2, 0.3)` $= 0.49$
400	$2 \cdot 0.7 \cdot 0.3 \cdot 0.4 = 0.168$
750	$2 \cdot 0.7 \cdot 0.3 \cdot 0.6 = 0.252$
800	`dbinom(2, 2, 0.3)` $\cdot 0.4^2 = 0.0144$
1150	`dbinom(1, 2, 0.4)` $\cdot 0.3^2 = 0.0432$
1500	`dbinom(2, 2, 0.3)` $\cdot 0.6^2 = 0.0324$

Table 1: PMF of X

b.

$$E(X) = 400 \cdot 0.168 + 750 \cdot 0.252 + 800 \cdot 0.0144 + 1150 \cdot 0.0432 + 1500 \cdot 0.0324 = 366$$

$$\text{var}(X) = 0.49(366)^2 + 0.168(400 - 366)^2 + 0.252(750 - 366)^2 + \\ 0.0144(800 - 366)^2 + 0.0432(1150 - 366)^2 + 0.0324(1500 - 366)^2 = 173922$$

3.3.7

a.

$$F(x) = \frac{x^2}{4}$$

Median is when CDF = 0.5

$$0.5 = \frac{x^2}{4}$$
$$2 = x^2$$
$$\sqrt{2} = x$$

Median is $\sqrt{2}$. We can ignore negative case because $F(x)$ is only valid $0 < x \leq 2$.

We can do the same for the 0.25 and 0.75 quantiles.

$$
\begin{aligned}
0.25 &= \frac{x^2}{4} \\
1 &= x
\end{aligned}
$$

$Q_1 = 1$

$$
\begin{aligned}
0.75 &= \frac{x^2}{4} \\
\sqrt{3} &= x
\end{aligned}
$$

$Q_3 = \sqrt{3}$

IQR = $Q_3 - Q_1 = 0.732$

b.

$$
\begin{aligned}
E(X) &= \int_0^2 \frac{x}{2} \cdot x dx \\
&= \frac{x^3}{6}\Big|_{x=0}^{2} \\
&= \frac{4}{3}
\end{aligned}
$$

$$
\begin{aligned}
\text{var}(X) &= E(X^2) - E(X)^2 \\
&= \int_0^2 \frac{x}{2} \cdot x^2 dx - \frac{16}{9} \\
&= \frac{x^4}{8}\Big|_{x=0}^{x=2} - \frac{16}{9} \\
&= 2 - \frac{16}{9} \\
&= \frac{2}{9}
\end{aligned}
$$

"sd"(X) = sqrt(2/9) = 0.471

3.4.4

a.

$$n = 10, p = 0.5$$

$$E(X) = np = 5$$

$$\text{var}(X) = np(1 - p) = 2.5$$

b.

$$P(X = 5) = \binom{10}{5} 0.5^5 \cdot 0.5^5 = 0.246$$

c.

```r
most_5 <- pbinom(5, 10, 0.5)
```

The $P(X \leq 5) = 0.6230469$.

d.

When $Y = 10 - X$, Y represents the number of questions incorrectly answered. There are 10 total questions, so the complement $(10 - X)$ is the number that are not included in X.

e.

We know the distribution is symmetric, so $P(2 \leq Y \leq 5) = P(2 \leq X \leq 5)$.

```r
sym <- pbinom(5, 10, 0.5) - pbinom(1, 10, 0.5)
actual <- pbinom(8, 10, 0.5) - pbinom(4, 10, 0.5)
sym == actual
```

```
[1] TRUE
```

We can see that the two methods are equivalent, and both return a probability of 0.6123047.

3.4.13

b.

$$S = 0, 1, 2, 3$$

$$\text{PMF } (X = x) = \frac{\binom{3}{x}\binom{17}{5-x}}{\binom{20}{5}}$$

c.

$$P(X = 1) = \frac{\binom{3}{1}\binom{17}{5-1}}{\binom{20}{5}} \approx 0.461$$

d.

$$E(X) = 5 \cdot \frac{3}{20} = \frac{3}{4}$$

$$\text{var}(X) = \frac{3}{4}\left(1 - \frac{3}{20}\right)\frac{15}{19} \approx 0.503$$

Additional Problem

$$f(x) = \begin{cases} 1 \text{ if } 0 \leq x \leq 1 \\ 0 \text{ elsewhere} \end{cases}$$

$$\begin{aligned} \mathrm{MGF}_X(t) &= E(e^{tx}) \\ &= \int_0^1 e^{tx} 1 dx \\ &= \left. \frac{e^{tx}}{t} \right|_{x=0}^{1} \\ &= \frac{e^t}{t} - \frac{1}{t} \\ &= \frac{e^t - 1}{t} \end{aligned}$$

When $t \neq 0$, the moment generating function of X is $\frac{e^t-1}{t}$.

When $t = 0$, the MGF is equal to one. This is because the MGF when $t = 0$ takes the form of $E(X^0) = E(1) = 1$.