Homework 3

Mark Schulist

2.3.15

Total number of unique hands: $\binom{52}{5} = 2598960$.

a.

$$\frac{48}{2598960} \approx 0.0000185 = 1.85 \times 10^{-5}$$

b.

$$\frac{\binom{4}{2}^2 \cdot 44}{\binom{52}{5}} \approx 0.000609$$

c.

$$\frac{4^2 \cdot \binom{4}{3} \cdot \binom{12}{2}}{\binom{52}{5}} \approx 0.00163$$

2.4.6

- E_1: machine A produces a batch with no defective components
- E_2: machine B produces a batch with no defective components
- $P(E_1) = 0.95$
- $P(E_2) = 0.92$
- $P(E_1 \cap E_2) = 0.88$

a.

$$P\left(E_1 \mid E_2^C\right) = P(E_1) - P(E_1 \cap E_2) = 0.95 - 0.88 = 0.07$$

b.

$$P\left(E_2 \mid E_1^C\right) = P(E_2) - P(E_1 \cap E_2) = 0.92 - 0.88 = 0.04$$

c.

$$P\left(E_2 \mid E_1^C\right) + P\left(E_1 \mid E_2^C\right) = 0.11$$

d.

$$P(E_1) + P(E_2) - P(E_1 \cap E_2) = 0.99$$

2.5.12

- $D =$ discovered airplane
- $L =$ has locator
- $P(D) = 0.7$
- $P(L \mid D) = 0.6$
- $P\big(L \mid D^C\big) = 0.1$

a.

$$P\big(L \cap D^C\big) = P\big(L \mid D^C\big)P\big(D^C\big) = 0.1 \cdot 0.3 = 0.03$$

b.

$$P(L) = P(D \cap L) + P\big(D^C \cap L\big) = 0.6 \cdot 0.7 + 0.03 = 0.45$$

c.

$$P\big(D^C \mid L\big) = \frac{P\big(L \mid D^C\big)P\big(D^C\big)}{P(L)} = \frac{0.1 \cdot 0.3}{0.45} \approx 0.0667$$

2.6.7

a.

Total-female: $1 - 0.65$

$P(F|T) = P(F)$

$P(M|T) = P(M)$

	Football	Basketball	Track	Total
Male	0.3	0.22	0.13	0.65
Female	0	0.28	0.07	0.35
Total	0.3	0.5	0.2	1

Table 1: Probilities of males and females playing certain sports

b.

$$P(F \mid B) = \frac{0.28}{0.5} = 0.56$$

c.

No they are not. $P(F \mid B) \neq P(F) \implies 0.56 \neq 0.35$.

3.2.4

x	$P(X = x)$
0	$\frac{7}{10} \cdot \frac{6}{9} \cdot \frac{5}{8} = 0.292$
1	$\frac{\binom{3}{1} \cdot \binom{7}{2}}{\binom{10}{3}} = 0.525$
2	$\frac{\binom{3}{2} \cdot \binom{7}{1}}{\binom{10}{3}} = 0.175$
3	$\frac{3}{10} \cdot \frac{2}{9} \cdot \frac{1}{8} = 0.00833$

Table 2: Probability mass function

$$\text{CDF}(x) = \begin{cases} 0 \text{ if } x < 0 \\ 0.292 \text{ if } 0 \leq x < 1 \\ 0.817 \text{ if } 1 \leq x < 2 \\ 0.992 \text{ if } 2 \leq x < 3 \\ 1 \text{ if } x \geq 3 \end{cases}$$

3.2.8

$$F(x) = \begin{cases} 0 \text{ if } x < 0 \\ \frac{x^2}{4} \text{ if } 0 \leq x \leq 2 \\ 1 \text{ if } x > 2 \end{cases}$$

a.

$$P(0.5 < X < 1)$$

$$F(1) - F(0.5) = \frac{1}{4} - \frac{1}{16} = \frac{3}{16} = 0.1875$$

b.

$$Y = 60X \Rightarrow X = \frac{Y}{60}$$

$$\frac{d}{dx} F(x) = \frac{x}{2}$$

$$f(x) = \begin{cases} 0 \text{ if } x \leq 0 \\ \frac{x}{2} \text{ if } 0 < x \leq 2 \\ 0 \text{ if } x > 2 \end{cases}$$

c.

$$\text{CDF} : F_y(y) = \frac{\left(\frac{y}{60}\right)^2}{4} = \frac{y^2}{14400}$$

$$\text{PDF} : \frac{d}{dy}F_y(y) = f_y(y) = \begin{cases} \frac{y}{60 \cdot 120} \text{ if } 0 < x < 120 \\ 0 \text{ otherwise} \end{cases}$$