Last-minute Exam 1 Review: QM217

Friday, November 1, 2024

1) EM Radiation as Particles (3.53)

An electron moving to the left at $0.8c$ collides with an incoming photon moving to the right. After the collision, the electron is moving to the right at $0.6c$ and an outgoing photon moves to the left. What was the wavelength of the incoming photon?

2) EM Radiation as Particles (3.47)

A photon has the same momentum as an electron moving at $10^6\ \frac{\mathrm{m}}{\mathrm{s}}$.

a.) Determine the photon's wavelength.

b.) What is the ratio of the kinetic energies of the two? (A photon is all kinetic energy.)

3) Matter as Waves (4.71)

a. Find the wavelength of a proton whose kinetic energy is equal to its internal energy.

b. The proton roughly has a radius of 10^{-15} m. Would this proton behave as a wave or as a particle?

4) Uncertainty Principle (4.61)

If a laser pulse is of short enough duration, it becomes rather superfluous to refer to its specific color. How short a duration must a light pulse be for its range of frequencies to cover the entire visible spectrum? (The visible spectrum is about $4.5 \times 10^{14}\ \mathrm{Hz} < f < 7.5 \times 10^{14}\ \mathrm{Hz}$).

5) Fourier Transform (4.69:)

A signal is described by

$$D(t) = Ce^{-\frac{|t|}{\tau}} \tag{1}$$

a.) Calculate the Fourier transform $A(\omega)$.

b.) How are $D(t)$ and $A(\omega)$ affected by a change in τ?

6) Break Time to Look at Cool Picture

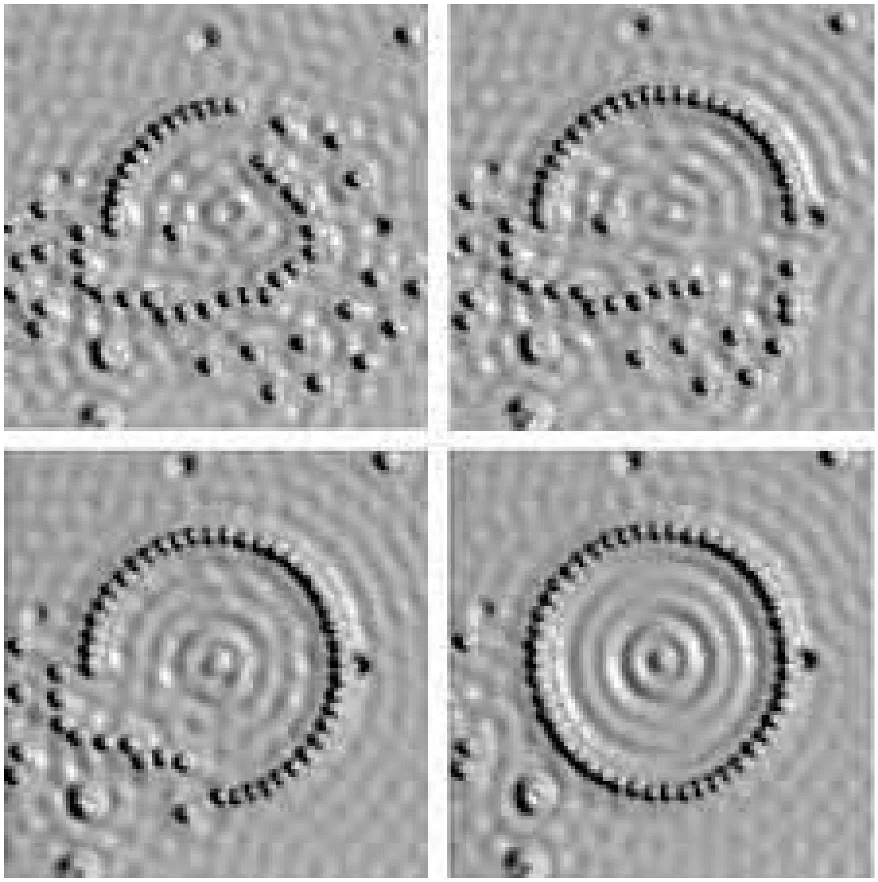

Figure 1: A standing wave in a quantum corral

"We used a scanning tunneling microscope, in which a metal needle under computer control is made to move along the contours of the surface being imaged. The height of the needle at each location on a square grid of points is recorded as a number in the computer. This sequence of numbers representing the height of the surface at each point on the grid is then rendered by the computer to look like a three-dimensional solid."

7) Bound States (5.5)

Just what is stationary in a stationary state? The particle? Something else?

8) Bound States: Delta Well (5.47)

Consider the delta well potential energy:

$$U(x) = \begin{cases} 0 & x \neq 0 \\ -\infty & x = 0 \end{cases} \tag{2}$$

Although not completely realistic, this potential energy is often a convenient approximation to a *very* strong, *very* narrow attractive potential energy well. It has only one allowed bound-state wave function, and because the top of the well is defined as $U = 0$, the corresponding bound-state energy is negative. Call its value $-E_0$.

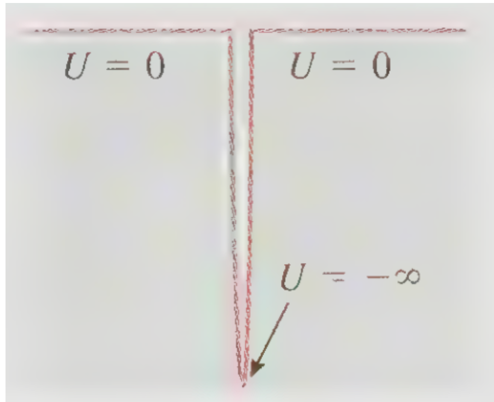

Figure 2: Delta Well

a. Applying the usual arguments and required continuity conditions (need it be smooth?), show that the wave function is given by

$$\psi(x) = \left(\frac{2mE_0}{\hbar^2}\right)^{\frac{1}{4}} e^{-\left(\sqrt{2mE_0}/\hbar\right)|x|} \tag{3}$$

b. Sketch $\psi(x)$ and $U(x)$ on the same diagram. Does this wave function exhibit the expected behavior in the classically forbidden region?