Homework 11

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1) 7.3.10

1.a)

$$\hat{p} = \frac{985}{1516}$$
$$= 0.645$$

$$\left(0.645 - 1.96\frac{\sqrt{0.645(1-0.645)}}{\sqrt{1516}}, 0.645 + 1.96\frac{\sqrt{0.645(1-0.645)}}{\sqrt{1516}}\right)$$
$$(0.621, 0.669)$$

1.b)

False. This interval is the confidence we have that the true proportion lies within it, not the probability.

2) 7.4.4

$$\hat{p} = \frac{75}{193}$$
$$= 0.389$$

2.a)

$$0.03 = 1.96 \cdot \frac{\sqrt{0.389(1-0.389)}}{\sqrt{n}}$$
$$\implies \quad n = 1015$$

2.b)

If we did not have any information about the variance from the sample proportion, we would have to estimate it at a values of 0.5. This will give us the largest spread (variance), and the true variance will probability be smaller.

3)

$$X_1, ..., X_{10} \sim \text{Bern}(\theta)$$

$$h_\theta(t) = 4t^3 \quad 0 < t < 1$$

$$p_{X|\theta=t} = t^x(1-t)^{1-x} \quad x \in \{0, 1\}$$

$$\boldsymbol{X} = \{1, 1, 1, 0, 1, 1, 0, 1, 1, 1\}$$

3.a)

$$p_{\boldsymbol{X}|\theta=t} = t^8(1-t)^2$$

$$g_{\theta|\boldsymbol{X}} = Kt^8(1-t)^2$$

$$\begin{aligned}
\frac{1}{K} &= \int_0^1 t^8(1-t)^2 \,\mathrm{d}t \\
&= \int_0^1 t^8(1-2t+t^2) \,\mathrm{d}t \\
&= \int_0^1 (t^8 - 2t^9 + t^{10}) \,\mathrm{d}t \\
&= \frac{t^9}{9} - \frac{1}{5}t^{10} + \frac{t^{11}}{11}\bigg|_{t=0}^{1} \\
&= \frac{1}{9} - \frac{1}{5} + \frac{1}{11} \\
&= \frac{1}{495} \\
\implies \quad K &= 495
\end{aligned}$$

$$g_{\theta|\boldsymbol{X}}(t) = 495t^8(1-t)^2$$

3.b)

$$\begin{aligned}
495\int_0^1 tt^8(1-t)^2 \,\mathrm{d}t &= 495\int_0^1 (t^9 - 2t^{10} + t^{11}) \,\mathrm{d}t \\
&= 495\left(\frac{1}{10} - \frac{2}{11} + \frac{1}{12}\right) \\
&= 0.75
\end{aligned}$$