Homework 5

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3.4.8

a. negative binomial

b.

$$S = \{5, 6, 7, 8, 9, ...\}$$

$$X = \text{nbinom}(r = 5, p = 0.05)$$

$$p(X = x) = \binom{x-1}{4} 0.05^5 (0.95)^{x-5}$$

c.

```r
1 - pnbinom(34, size = 5, prob = 0.05)
```

```
[1] 0.9562407
```

3.4.19

a.

$$X_1 = \text{pois}(\lambda = 2.6)$$
$$X_2 = \text{pois}(\lambda = 3.8)$$

$$\text{var}(X_1) = 2.6$$
$$\text{var}(X_2) = 3.8$$

b.

$$0.6P(X_1 = 0) + 0.4P(X_2 = 0) = 0.6e^{-2.6} + 0.4e^{-3.8} = 0.0535$$

c.

$$P(X_2 \mid \text{no error}) = \frac{P(\text{no error} \mid X_2)P(X_2)}{P(\text{no error})} = \frac{e^{-3.8} \cdot 0.4}{0.0535} = 0.167$$

3.5.1

a.

$$B = \text{Exp}\left(\lambda = \frac{1}{6}\right)$$

P(B > 4) = e^(-2 / 3) = 0.513

b.

$$\text{var}(B) = \frac{1}{\frac{1}{36}} = 36$$

$$\begin{aligned} 0.95 &= 1 - e^{-\frac{1}{6}x} \\ \ln(0.05) &= -\frac{1}{6}x \\ -6\ln(0.05) &= x \\ 17.97 &= x_{0.95} \end{aligned}$$

c.

i.

$$P(B > 5) = e^{-\frac{5}{6}} = 0.435$$

ii. $E(B) = 6$, so 6 more years (memoryless).

3.5.8

a.

```
college_a_percent <- 1 - pnorm(600, mean = 500, sd = 80)
```

The percentage of people who can get into College A is 10.5649774%.

b.

```
min_score_college_b <- qnorm(0.99, mean = 500, sd = 80)
```

The minimum score to get into college B is 686.1078299.

4.2.3

a.

$$P(X \leq 10, Y \leq 2) = 0.3 + 0.12 + 0.15 + 0.135 = 0.705$$

$$P(X \leq 10, Y = 2) = 0.135 + 0.12 = 0.255$$

b.

$$\begin{aligned} f_{X(8)} &= 0.42 \\ f_{X(10)} &= 0.15 + 0.135 + 0.025 = 0.31 \\ f_{X(12)} &= 0.03 + 0.15 + 0.09 = 0.27 \end{aligned}$$

$$f_{Y(1.5)} = 0.3 + 0.15 + 0.03 = 0.48$$
$$f_{Y(2)} = 0.12 + 0.135 + 0.15 = 0.405$$
$$f_{Y(2.5)} = 0.115$$

c.

$$P(X \leq 10 \mid Y = 2) = \frac{0.135 + 0.12}{0.12 + 0.135 + 0.15} = 0.63$$

4.2.8

a.

$$2\int_0^{1.5} e^{-x} \int_x^{3-x} e^{-y} dy dx$$
$$-2\int_0^{1.5} e^{-x}(e^{x-3} - e^{-x}) dx$$
$$-2e^{-3} - 2(e^{-3} - 1)$$
$$-4e^{-3} + 1$$

b.

$$f_x(x) = 2e^{-x} \int_x^{\infty} e^{-4} dy$$
$$= 2e^{-2x} \text{ for } x \geq 0$$
$$0 \text{ otherwise}$$

$$f_y(y) = 2e^{-y} \int_0^{y} e^{-x} dx$$
$$= -2e^{-y}(e^{-y} - 1) \text{ for } y \geq 0$$
$$0 \text{ otherwise}$$