Physics 217 Introduction to Quantum Physics
Instructor: Maria Piarulli
Due Friday December 6 by 11.59pm

Problem Set 11 (100 points)

Suggested Reading: Randy Harris's book
Chapter 1 Secs. 1.1-1.2
Chapter 3 Secs. 3.1-3.2; Examples 3.1-3.2
Chapter 3 Secs. 3.4-3.6; Example 3.6
Chapter 4 Secs. 4.1-4.2
Chapter 4 Secs. 4.3-4.4; Examples 4.4-4.5
Chapter 4 Secs. 4.6-4.7; Example 4.7
Chapter 5 Secs. 5.1-5.9; Examples 5.4, 5.5
Chapter 5 Sec. 5.11; Example 5.6
Chapter 6 Secs. 6.1-6.2
Chapter 7 Secs. 7.1, 7.2, 7.4, 7.5, 7.6, 7.7

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1. **Wave-function and expectation values: (10 points)**: Consider a one-dimensional harmonic oscillator of mass m and frequency ω which is at instant $t = 0$ in a state determined by the following conditions:

   1) Every measure of energy gives values that satisfy the relation $\hbar\omega < E < 3\hbar\omega$;
   2) The average energy value is $E = \frac{11}{6}\hbar\omega$;

   Identify the wave function of the system.

2. **Three-dimensional cubic well: (20 points)** Consider a three-dimensional cubic well.

   a) How many different wave functions have the same energy as the one for which $(n_x, n_y, n_z) = (5, 1, 1)$.

   b) Into how many different levels would this split if the length of one side were increased by 5%?

   c) Make a scale diagram, similar to Figure 7.3 in the book, illustrating the energy splitting of the previously degenerate wave functions.

   d) Is any degeneracy left? If so, how might it be destroyed?

3. **Three-dimensional cubic well: (10 points)** An electron is trapped in a quantum dot, in which it is confined to a very small region in all three-dimensions. If the lowest-energy transition is to produce a photon of 450 nm wavelength, what should be the width of the well (assume cubic).

4. **Three-dimensional harmonic oscillator: (20 points)**: Consider the three-dimensional harmonic oscillator, for which the potential in cartesian coordinates is

   $$V(x, y, z) = \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)\ . \tag{1}$$

   (a) Write down the Hamiltonian of the system in cartesian coordinates.
   (b) Show that separation of variables in cartesian coordinates turn this into one-dimensional harmonic oscillators, and exploit your knowledge of the latter to determine the allowed energies.

(c) What is the energy of the ground state? Is this state degenerate? Write down the ground-state wave function in cartesian coordinates.
(d) What is the energy of the first excited state? Is this state degenerate? If yes, write down the corresponding eigenfunctions in cartesian coordinates.
(e) Write down the Hamiltonian of the system in spherical coordinates.
(f) What are the corresponding eigenfunctions of the ground-state and first-excited states in spherical coordinates?

5. **Spherical coordinates: (10 points)** The wave function for a stationary state of an atom is

$$\Psi(r, \theta, \phi) = A\, f(r) \sin\theta \cos\theta \, e^{i\phi} \tag{2}$$

where (r, θ, ϕ) are spherical coordinates and A is a normalization constant. Find (a) the value for the z-component of the angular momentum of the atom (b) the value of the square of the total angular momentum of the atom.

6. **Hydrogen atom: (15 points)**:
(a) Construct the wave function for the hydrogen in the state $n = 4$, $l = 3$, and $m = 3$. Express your answer as a function of the spherical coordinates r, θ, and ϕ.
(b) Find the expectation value of r in this state. (Look up any trivial integrals.)
(c) If you could somehow measure the observable $L_x^2 + L_y^2$ on an atom in this state, what value could you get?

7. **Hydrogen atom: (15 points)**:
(a) If the radial part of a particle's wavefunction is $R(r)$, what is the probability of finding the particle somewhere between radius r_1 and r_2?
(b) Write down the radial wavefunction $R_{10}(r)$ for the $n = 1$, $l = 0$ state of the hydrogen atom. The nucleus of the hydrogen atom is a proton, which has a radius $r_p = 10^{-15}$ m. Write down an approximate expression for $R_{10}(r)$ which is valid for $r \le r_p$. What is the probability of finding the electron inside the proton?
(c) Repeat part (b) for the $n = 2$, $l = 1$ state of hydrogen. Explain the difference between your results.