Problem Set 6

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1)

1.a)

Baryons: 3 quarks, spin $\frac{1}{2}$, $l = 0$.

For the first 2: $s_a = 0, 1$. Now we add the third into the combinations.

$$\begin{aligned} s &= \left(\frac{1}{2} + s_a\right), \left|\frac{1}{2} - s_a\right| \\ s &= \frac{1}{2}, \frac{3}{2}, \frac{1}{2}, \frac{1}{2} \end{aligned} \tag{1}$$

So the total possible values of s are $\frac{1}{2}, \frac{3}{2}$.

1.b)

Mesons: 2 quarks, $s = \frac{1}{2}$, $l = 0$.

$s = s_1 + s_2, |s_1 - s_2|$.

So $s = 1, 0$.

2)

$l = 1, m_l = 0, s = \frac{1}{2}, m_s = -\frac{1}{2}$. We can look up in the table to find which uncoupled states correspond to this coupled state.

We can see that this state is a sum of $\sqrt{\frac{2}{3}}\left|\frac{3}{2}\ \frac{1}{\sqrt{3}}\right\rangle + \sqrt{\frac{1}{3}}\left|\frac{1}{2}\ \text{-}\frac{1}{2}\right\rangle$ by looking in the Clebsch-Gordan table.

The first ket has $j = \frac{3}{2}$, so we get J^2 with $j(j+1)\hbar^2$. So in this case, $J^2 = \frac{15}{4}\hbar^2$, and this occurs with probability $\frac{2}{3}$.

The second ket has $j = \frac{1}{2}$, so $J^2 = \frac{3}{4}\hbar^2$. This occurs with probability $\frac{1}{3}$.

3)

$s_1 = 1, m_1 = 1, s_2 = 2, m_2 = 2$

The total spin $m = 3$. $s_z = \hbar$.

If we look int the Clebsch-Gordan table for total spin = 3 with $s_z = \hbar$, we find that there are three possible values for the z component of angular momentum.

$|21\rangle$ has a z component of angular momentum of $2\hbar$, and this occurs with probability $\frac{1}{15}$.

$|10\rangle$ has a z component of angular momentum of $\hbar$, and this occurs with probability $\frac{8}{15}$.

$|01\rangle$ has a z component of angular momentum of $0\hbar$, and this occurs with probability $\frac{6}{15}$.

4) 11.16

$$I = 1, g_D = 0.857 \tag{2}$$

$j_1 = 1, j_2 = \frac{1}{2}$.

$$F^2|FM_F\rangle = F(F+1)\hbar^2|FM_F\rangle \tag{3}$$

We know that $m_1 \in \{-1, 0, 1\}$ and $m_2 \in \left\{-\frac{1}{2}, \frac{1}{2}\right\}$.

So now we can fill out the F^2 matrix using Equation 3.

$$F^2 \doteq \hbar^2 \begin{array}{cccccc} \left|\frac{3}{2}\ \frac{3}{2}\right\rangle & \left|\frac{3}{2}\ \frac{1}{2}\right\rangle & \left|\frac{3}{2}\ \text{-}\frac{1}{2}\right\rangle & \left|\frac{3}{2}\ \text{-}\frac{3}{2}\right\rangle & \left|\frac{1}{2}\ \frac{1}{2}\right\rangle & \left|\frac{1}{2}\ \text{-}\frac{1}{2}\right\rangle \\ \frac{15}{4} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{15}{4} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{15}{4} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{15}{4} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{3}{4} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{3}{4} \end{array} \tag{4}$$

We know that $S^2 \doteq \frac{3}{4}\hbar^2$ Id and $I^2 \doteq 2\hbar^2$ Id.

$$\begin{aligned} H'_{hf} &= \frac{A}{2\hbar^2}(F^2 - S^2 - I^2) \\ &= \frac{A}{2}\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 \end{pmatrix} \end{aligned} \tag{5}$$

So we can see $\Delta E = \frac{3}{2}A$.

To compute A we need to use the version for hydrogen and then replace the g_p with g_D.

$$\begin{aligned} \frac{A_p}{A_D} &= \frac{g_p}{g_D} \\ \implies \quad A_D &= \frac{A_p g_D}{g_p} = 217.7 \text{ MHz} \end{aligned} \tag{6}$$

$$\frac{3}{2}A_D = 326.5 \text{ MHz} \tag{7}$$

So the split is less compared to H.

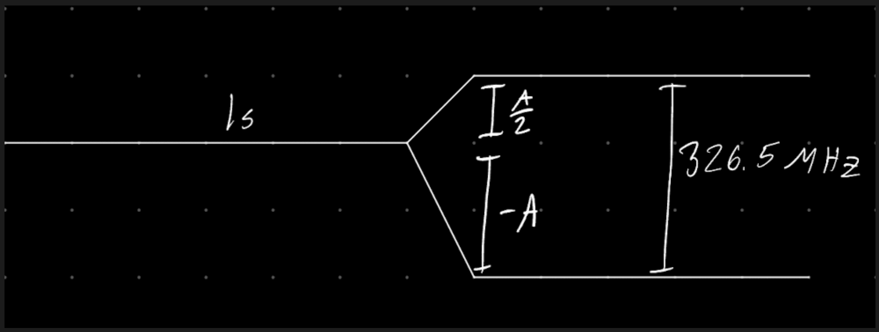

Figure 1: Energy Split of Deuterium Atom with Hyperfine Structure

In Figure 1, the top line refers to when $F = \frac{3}{2}$ and the bottom when $F = \frac{1}{2}$.