Homework 7

Mark Schulist

1) 5.4.5

$$X_i \sim U(35.5, 36.5)$$

$$H = \sum_{i=1}^{30} X_i$$

When we sum many random variables, they approach a normal distribution. So we can *approximate* the summed height to be normal.

We know that:

$$H \dot{\sim} \mathcal{N}(n\mu_X, n\sigma_X^2)$$

We know $n = 30$ by problem statement. We also know that $\mu_X = 36$ by uniform distribution. We can find σ_X^2:

$$\begin{aligned}\sigma_X^2 &= \frac{1}{12}(1)^2 \\ &= \frac{1}{12}\end{aligned}$$

Therefore, we know that:

$$H \dot{\sim} \mathcal{N}(1080, 2.5)$$

We can add another random variable (T) that is the difference between two H variables (one for each tower).

$$\begin{aligned}T &= H_1 - H_2 \\ T &\sim \mathcal{N}(0, 5)\end{aligned}$$

We would like to know the probability that $|T| < 4$.

$$P(|T| < 4) = P(T < 4) - P(T < -4)$$

```
pnorm(4, 0, sqrt(5)) - pnorm(-4, 0, sqrt(5))
[1] 0.9263617
```

2)

$$f_X(x) = e^{-x}, x > 0$$

$$Y = \ln(X)$$

$$f_Y(y) = f_X(g^{-1}(y))\left|\frac{\mathrm{d}}{\mathrm{d}y}g^{-1}(y)\right|$$

$$g^{-1}(y) = e^y$$
$$\left|\frac{\mathrm{d}}{\mathrm{d}y} g^{-1}(y)\right| = e^y$$

$$\begin{aligned} f_Y(y) &= f_X(e^y)e^y \\ &= e^{-e^y}e^y \\ &= e^{y-e^y} \end{aligned}$$

Support:

$$y > 0$$

3)

$$f_{X_1,X_2}(x_1, x_2) = x_1 + x_2 \text{ for } 0 < x_1 < 1, 0 < x_2 < 1$$

$$\begin{aligned} Y_1 &= X_1 X_2 \\ Y_2 &= X_2 \end{aligned}$$

$$\begin{aligned} X_1 &= \frac{Y_1}{Y_2} \\ X_2 &= Y_2 \end{aligned}$$

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial y_1} & \frac{\partial x_1}{\partial y_2} \\ \frac{\partial x_2}{\partial y_1} & \frac{\partial x_2}{\partial y_2} \end{vmatrix} = \begin{vmatrix} \frac{1}{y_2} & -\frac{y_1}{y_2^2} \\ 0 & 1 \end{vmatrix} = \frac{1}{y_2}$$

$$\begin{aligned} f_{Y_1,Y_2}(y_1, y_2) &= f_{X_1,X_2}\left(\frac{y_1}{y_2}, y_2\right)|J| \\ &= \left(\frac{y_1}{y_2} + y_2\right)\frac{1}{y_2} \\ &= \frac{y_1}{y_2^2} + 1 \end{aligned}$$

Support:

$$0 < \frac{y_1}{y_2} < 1, 0 < y_2 < 1$$

This shows that $y_2 > y_1$ because the fraction $\frac{y_1}{y_2}$ must be less than one.

$$0 < y_1 < y_2 < 1$$

4)

```
samples = 1e5
X1 = runif(samples)
X2 = runif(samples)

Y = apply(cbind(X1, X2), 1, 'max')
```

4.a)

```
cdf_y = function(y) {mean(Y < y)}

c(cdf_y(0.5), cdf_y(0.75))
[1] 0.25194 0.56338
```

4.b)

```
y_df = data.frame(yvals = Y)

ggplot(y_df, aes(x = yvals)) +
  geom_histogram(binwidth=0.05) +
  xlab('Y') +
  labs(title = 'Histogram of Y') +
  xlim(c(0,1)) +
  dark_theme_gray(base_size = 10) +
  theme(
    plot.background = element_rect(
      fill = '#1c1c1c', size=0,
      ),
      panel.background = element_rect(
        fill = '#1c1c1c', size=0,
      ),
    )
```