Homework 7

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1)

P	Q	$P \land Q$	$Q \Rightarrow (P \land Q)$	$P \Rightarrow (Q \Rightarrow (P \land Q))$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	T
F	F	F	T	T

2)

P	Q	R	$P \lor Q$	$R \land P$	$(P \lor Q) \Leftrightarrow (R \land P)$
T	T	T	T	T	T
T	T	F	T	F	F
T	F	T	T	T	T
F	T	T	T	F	F
F	T	F	T	F	F
T	F	F	T	F	F
F	F	T	F	F	T
F	F	F	F	F	T

3)

3.a)

$$P \land (Q \lor R) \equiv (P \land Q) \lor (P \land R)$$

We need to show that the truth tables are identical on both sides for all inputs (P, Q, R).

P	Q	R	$Q \lor R$	$P \land Q$	$P \land R$	$P \land (Q \lor R)$	$(P \land Q) \lor (P \land R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	F	F	F	F	F	F
T	F	T	T	F	T	T	T
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

As we can see, the last 2 columns are the same, so we can say that the two statements are equal.

3.b)

$$P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$$

P	Q	R	$Q \wedge R$	$P \vee Q$	$P \vee R$	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	F	F	T	T	T	T
T	F	T	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

Same as above, we can see that the last 2 columns are the same, therefore they are equal statements.

4)

$$(a, b) \sim (c, d) \Leftrightarrow a + d = c + b$$

Reflexive:

We need to show that $(a, b) \sim (a, b)$. Substituting in values shows:

$$a + b = a + b$$

Therefore, this relation is reflexive.

Symmetric:

We need to show that $(a, b) \sim (c, d) \Leftrightarrow (c, d) \sim (a, b)$.

$$a + d = c + b \Leftrightarrow c + b = a + d$$

Because the equals sign on $\mathbb{N}$ is symmetric, we can say that $\sim$ is also symmetric.

Transitive:

We need to show that if $(a, b) \sim (c, d)$ and $(c, d) \sim (e, f)$, then $(a, b) \sim (e, f)$.

$$\begin{aligned} a + d &= c + b \\ c + f &= e + d \\ \Longrightarrow \text{add both eqs and commutative} \quad a + f + (d + c) &= e + b + (d + c) \\ \Longrightarrow \text{prop 6} \quad a + f &= e + b \end{aligned}$$

The final line is the definition of $(a, b) \sim (e, f)$. Therefore, we have shown that $\sim$ is transitive.

5)

$$(a, b) \sim (c, d) \Leftrightarrow ad = bc$$

Reflexive:

$$\begin{aligned} &(a, b) \sim (a, b) \\ \Longrightarrow \quad & ab = ba \end{aligned}$$

This is the definition of our relation, so it is reflexive.

Symmetric:

$$(a, b) \sim (c, d) \Leftrightarrow (c, d) \sim (a, b)$$
$$\Longrightarrow ad = bc \Leftrightarrow cb = da$$

Multiplication is commutative, so this property holds (it is symmetric).

Transitive:

Want to show:

$$(a, b) \sim (c, d) \wedge (c, d) \sim (e, f) \Rightarrow (a, b) \sim (e, f)$$

We can use definition of the relation:

$$\begin{aligned} & ad = bc \\ & cf = de \\ \Longrightarrow \text{mul both eqs and commutative} \quad & af(dc) = be(dc) \\ \Longrightarrow \text{prop } 7, 0 \neq dc \quad & af = be \end{aligned}$$

This last line is the definition of the relation we want to show, so we have proven that $\sim$ is transitive.