Homework 7

Mark Schulist

1) 4.4.2

1.a)

$$X = \text{uniform}(0, 1) \quad Y = \text{uniform}(0, 1)$$

We can set $T = \max(X, Y)$ and solve for the CDF of T.

$$\begin{aligned} P(T \leq t) &= P(X \leq t)P(Y \leq t) \\ &= t \cdot t \\ &= t^2 \text{ for } 0 < t < 1 \end{aligned}$$

X and Y are independent so we can just multiply them.

$$F_T(t) = \begin{cases} 0 \text{ if } x < 0 \\ t^2 \text{ if } 0 \leq x \leq 1 \\ 1 \text{ if } x > 1 \end{cases}$$

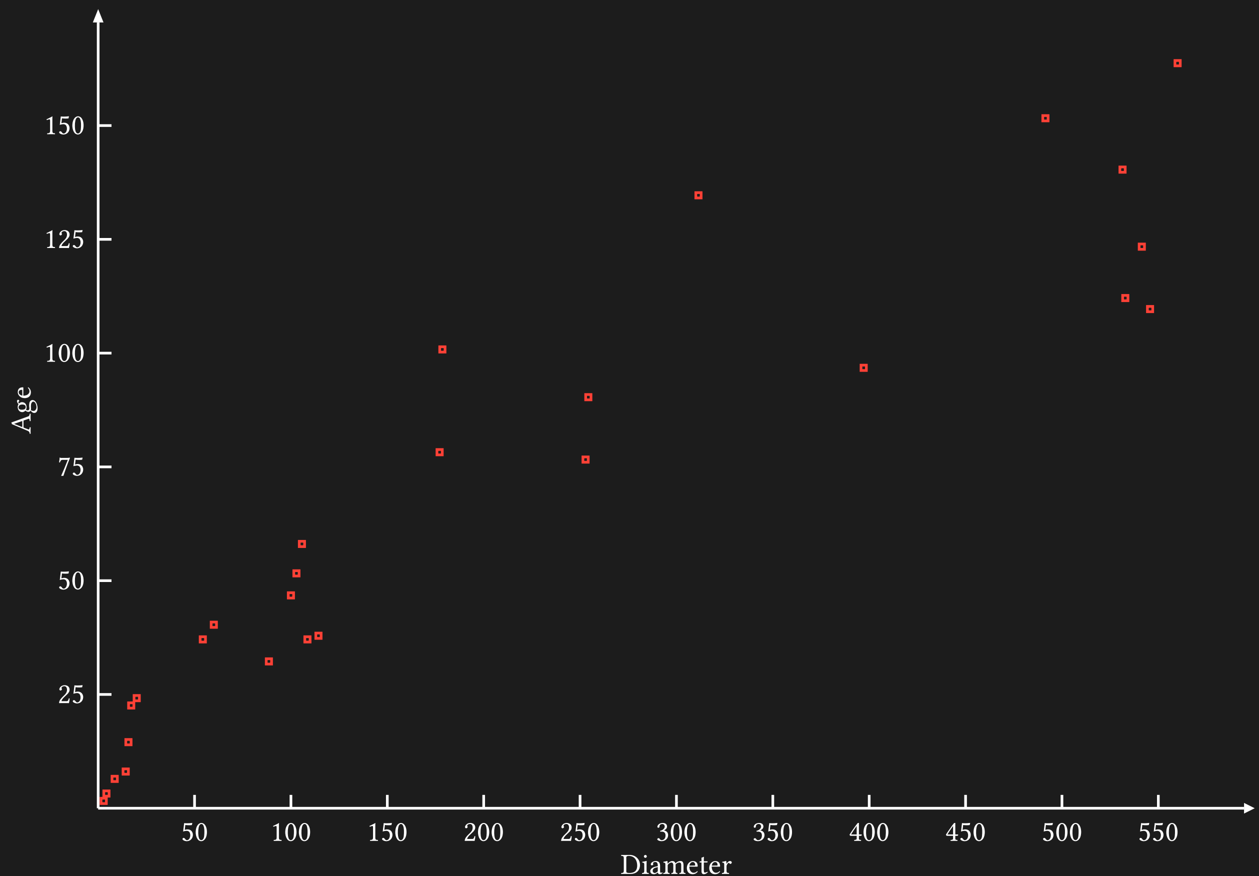
We can differentiate to get the PDF:

$$f_T(t) = \begin{cases} 0 & \text{if } x < 0 \\ 2t & \text{if } 0 \leq x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$$

2) 4.5.4

2.a)

I would expect a positive correlation because as a tree gets older it usually gets larger, which its diameter corresponds to.



Graph 1: Diameters and Ages of Trees

2.b)

```
cov(x, y)
[1] 9308.47

cor(x, y)
[1] 0.9262
```

Yes, it seems to decently capture the linear relationship between the data. In the plot, it is clear that there is a positive linear relationship between diameter and ages of trees.

3) 4.5.8

$$X = \text{uniform}(-1, 1) \quad Y = X^2$$

Show that $\rho_{X,Y} = 0$

$$\begin{aligned} E(XY) &= E(X^3) \\ &= \int_{-1}^{1} x^3 \,\mathrm{d}x \\ &= 0 \end{aligned}$$

$$
\begin{aligned}
E(Y) &= E(X^2) \\
&= \int_{-1}^{1} x^2 \, \mathrm{d}x \\
&= \frac{2}{3}
\end{aligned}
$$

$$
E(X) = 0
$$

$$
\begin{aligned}
\operatorname{Cov}(X, Y) &= E(XY) - E(X)E(Y) \\
&= 0 - 0 \\
&= 0
\end{aligned}
$$

4) 4.6.8

$$
\begin{aligned}
N_1 &= \text{recent}, & p_1 &= 0.6 \\
N_2 &= \text{moderately aged}, & p_2 &= 0.3 \\
N_3 &= \text{aged}, & p_3 &= 0.08 \\
N_4 &= \text{unusable}, & p_4 &= 0.02
\end{aligned}
$$

We will have 16 trials because $n = 16$.

4.a)

We can view this as a binomial with $p = 0.02, n = 16$.

```
dbinom(1, 16, 0.02)
[1] 0.2363421
```

4.b)

Here we can use the multinomial distribution. We want to know the probability of $x_1 = 10, x_2 = 4, x_3 = 2, x_4 = 0$.

$$
\frac{16!}{10!4!2!} 0.6^{10} 0.3^4 0.08^2 \approx 0.0377
$$

4.c)

```
dmultinom(c(10,4,2,0), prob=c(0.6,0.3,0.08,0.02))
[1] 0.03765241
```

4.d)

$$
\begin{aligned}
\operatorname{Cov}(N_1 + N_2, N_3) &= -16(0.6 + 0.3)(0.08) \\
&= -1.152
\end{aligned}
$$

It is negative because if one of the variables has a small value for N_i, then the others must have larger values. Therefore there is a negative relationship between the variables as adding “mass” to one of the N_i’s will reduce mass from another.

4.e)

We can treat this like a single binomial with $n = 16, p = 0.98$.

$$
\begin{aligned}
\mathrm{Var}(N_1 + N_2 + N_3) &= np(1-p) \\
&= 16(0.98 \cdot 0.02) \\
&= 0.3136
\end{aligned}
$$

5) 5.3.4

$$X_1 = \mathcal{N}(6,2) \quad X_2 = \mathcal{N}(7,3) \quad X_3 = \mathcal{N}(8,4)$$

5.a)

$$Y = X_1 + X_2 + X_3$$

$$
\begin{aligned}
Y &= \mathcal{N}(6+7+8, 2+3+4) \\
&= \mathcal{N}(21, 9)
\end{aligned}
$$

```
qnorm(0.95, 21, 3)
[1] 25.93456
```

5.b)

```
pnorm(25, 21, 3)
[1] 0.9087888
```

5.c)

$$Z = \mathrm{binom}(n=5, p=0.9087888)$$

```
dbinom(3,5,1 - 0.9087888)
[1] 0.006267155
```

6) Extra Problem

$$\mu_X = 2.8, \mu_Y = 110, \sigma_X^2 = 0.16, \sigma_Y^2 = 100, \rho = 0.6$$

6.a)

$$P(106 < Y < 124)$$

```
pnorm(124, 110, 10) - pnorm(106, 110, 10)
[1] 0.5746651
```

6.b)

$$P(106 < Y < 124 \mid X = 3.2)$$

$$
\begin{aligned}
Y|X = 3.2 &\sim \mathcal{N}\left(110 + \frac{0.6 \cdot 10}{0.4}(3.2 - 2.8), (1 - 0.36)100\right) \\
&\sim \mathcal{N}(116, 64)
\end{aligned}
$$

```
pnorm(124, 116, 8) - pnorm(106, 116, 8)
[1] 0.735695
```