Job Market Papers

This is the first of two job market papers. My first job market paper, “Regulation of Wages and Hours”, is applied theory. My second job market paper, “Asymmetric All-Pay Auctions with Spillovers”, is a more technical pure theory paper.

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Regulation of Wages and Hours

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November 8, 2022

Abstract

When hours and wages are bargained jointly, workers may not receive their preferred hours. This paper studies labor market regulation when it is known that workers prefer to work fewer hours at their wage, but specific knowledge of production, labor disutility, and bargaining protocol is absent. We show that for a large class of bargaining protocols, there is regulation that increases worker utility without sacrificing total surplus. We derive the optimality of overtime pay in a novel robust regulatory setting where the regulator has neither a prior nor exogenous bounds on model parameters.

Keywords: Overtime pay, regulation design, delegation, non-Bayesian, robust.
JEL Codes: D81, D82, D86, J08

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1 Introduction

It is known that most workers cannot freely set their hours.[1] This friction is not studied in the theoretical literature on labor regulation, where hours are assumed to be either fixed or chosen by workers.[2]

However, in the practice of regulation, the importance of this issue is recognized. Many policies are designed to reduce the hours of individual workers. For example, the European Union imposes a sharp cap of 48 hours per week. In the United States, overtime pay requires companies to pay "time and a half" (1.5 times the wage) for each hour worked above 40 hours per week. Japan uses a lesser amount of overtime, "time and a quarter", combined with a sharp cap of 55 hours per week.

To study policies such as these, we assume that wages and hours are determined jointly through bargaining between the worker and firm. For example, consider a monopsonist firm that proposes a "take it or leave it" offer of compensation and hours to a single risk-neutral worker. In the absence of regulation, the proposed contract will maximize total surplus, which will be entirely appropriated by the firm, and the worker's hours will be longer than she would like to work at the imputed wage (total compensation divided by hours of work).

We examine the problem of a regulator who maximizes a weighted sum of worker utility and firm profits with more weight placed on the utility of workers. This means that the regulator implements a regulation that benefits the worker and is not Pareto dominated. The regulator can obviously increase the utility of the worker by simply capping the hours while leaving the wage at its original imputed level. However, it's not obvious that this maximizes the regulator's payoff. Indeed, we find that the regulator can do better even when only limited information is available.

Under complete information, we show that minimum wage policies are without loss of optimality. That is, overtime pay and caps on hours are not useful policy tools. This is due to the fact that workers only want to reduce their labor because their imputed wage is too low. If the wage is increased sufficiently, the problem vanishes. Moreover, an efficient minimum wage exists which maximizes total surplus

[1]For example, less than 5% of hourly paid respondents to the 2016 General Social Survey said that they had full control of their hours compared to 47% who responded that their employer decides unilaterally. See Table 1 in Appendix for more details.

[2]It is, however, considered outside the context of regulation. For example Manning (2013) shows differences between the canonical model of monopsony and one where the employer chooses hours.

and transfers surplus to the worker. In addition, both the firm and worker receive their preferred hours at this wage.

Of course, if information is imperfect, the regulator may be unable to implement the optimal policy. This paper provides novel intuitions for welfare analysis when the minimum wage is suboptimal. In the standard model of monopsony, where workers choose hours, there is a monotone relationship between hours and total surplus. As a result, hours maximization is equivalent to total surplus maximization. In addition, any suboptimal minimum wage is "better than nothing" in the sense that it benefits workers and increases total surplus above the market level. As a result, a regulator that does not know the optimal minimum wage, but can bound it from below, prefers this lower bound to the market outcome.

This is not the case in our model. If the minimum wage is binding, but is set below the efficient minimum wage, the firm will select more hours than is efficient. This is because increasing hours allows the firm to "claw back" the additional surplus that the worker derives from higher wages. As a result, the minimum wage that maximizes hours actually *minimizes* total surplus locally. Additionally, In our setting, a suboptimal minimum wage is not necessarily "better than nothing". Total surplus is lower, worker surplus may be lower, and the overall allocation may be strictly Pareto dominated by the unregulated market.

This fragility of the minimum wage as a welfare improving policy motivates an exercise in robust regulation. It is appealing for the regulator to implement a policy that is *always* successful in increasing worker utility in some optimal way without requiring exogenous knowledge of production and disutility from labor. We develop an analogue of our complete information framework in this setting.

Consider a framework where the regulator has no prior over production and costs, but: (1) observes the contract that prevails in the market before regulating, and (2) knows that a particular reduction in hours at the imputed wage benefits the worker. Given this information, it is clear that reducing hours to this preferred quantity at the current wage would make the worker better off under every possible state.

However, this reduction in hours is dominated.[3] This is demonstrated with two insights. First, observing the market state enables the regulator to bound the worker's disutility from labor. Second, an inflexible reduction in hours is dominated by any

[3]That is, alternative policies exist which provide weakly greater payoffs for both players in every possible state and strictly larger payoffs in some.

policy that guarantees the worker enough additional pay to compensate for the additional hours. Intuitively, if the firm is willing to pay the worker a sufficiently large sum of money to work an additional hour, it does not make sense to block this transaction.

It is obvious that it is better to impose overtime pay of, say, $1 million for an additional hour than to set hours inflexibly. In the off-chance that the additional hour is this valuable, the regulator might as well allow the transaction. However, this policy is ex-post Pareto dominated if the hour is even slightly less valuable. We show that there is a unique policy which both dominates the inflexible reduction in hours and is never ex-post Pareto dominated by any other policy with this property. This regulation is an initially non-binding minimum wage and hours cap combined with overtime pay. The minimum wage is set at the current imputed wage, overtime pay begins at the preferred reduction in hours, and an hours cap is set at the hours that prevailed in the market before regulation. Policies with this shape are common. For example, Japan and France both combine overtime pay with a cap on hours.[4] Some real-world regulations designed to affect hours are plotted in Figure 1 which can be compared to our optimal regulation in Figure 2.

One immediate implication of our analysis is that any sharp cap on hours can be improved with a "softer" regulation that includes overtime pay. The only informational requirement for this analysis is that the regulator can observe the market before implementing the policy. Intuitively, the regulator should allow the worker to work some hours above the cap if and only if it is apparent that the worker receives enough compensation to benefit from doing so.

Our robust regulation framework translates the core ideas of Carroll (2015), which considers a robust moral hazard problem, to the setting of regulation. In Carroll (2015), the principal: (1) knows at least one of the agent's available actions and (2) obtains an optimal contract which exploits the alignment of incentives between the principal and agent. In our setting, the regulator: (1) knows one *regulation* which benefits the worker instead of an action and (2) obtains an optimal regulation which exploits efficiency,[5] which is analogous to aligned incentives in our context. Our novel framework can be applied to any robust delegation context.[6]

[4]Overtime pay in countries with no caps on hours, such as the United States are also fundamentally similar because there is a natural cap on the number of hours that one can work in a week.

[5]That is, bargaining between the worker and firm is Pareto optimal under the constraints of the regulation.

[6]A solution exists as long as the agent has rational preferences. A refinement is required to

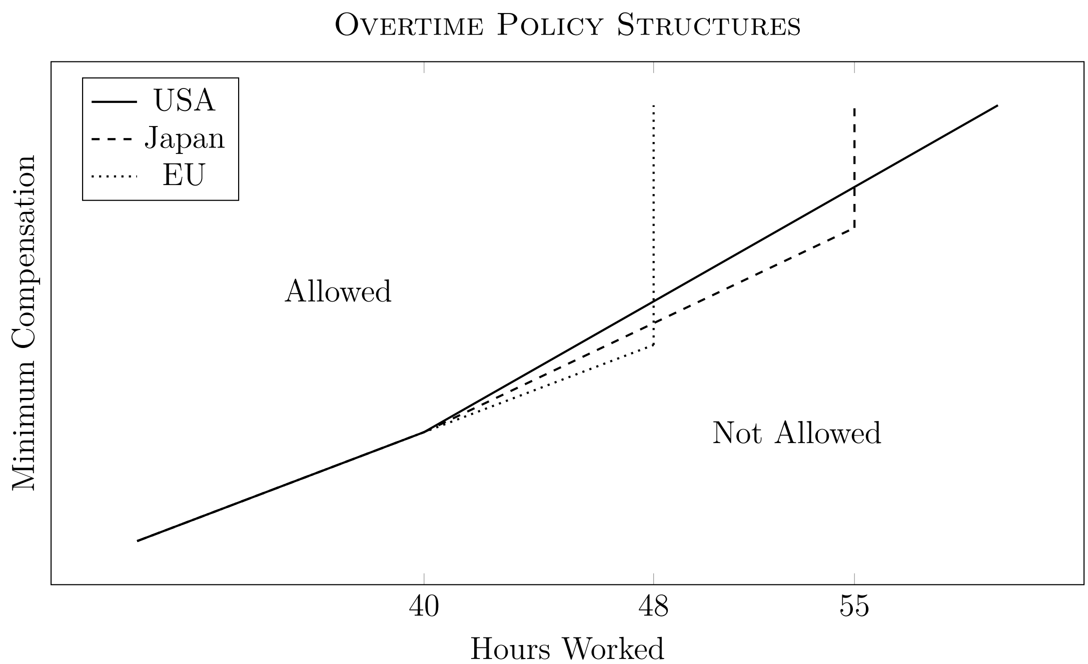

Figure 1: Regulations in the United States, Japan, and the European Union with a normalized minimum wage. Contracts with hours and compensation above the line are allowed. Contracts bellow the line are not.

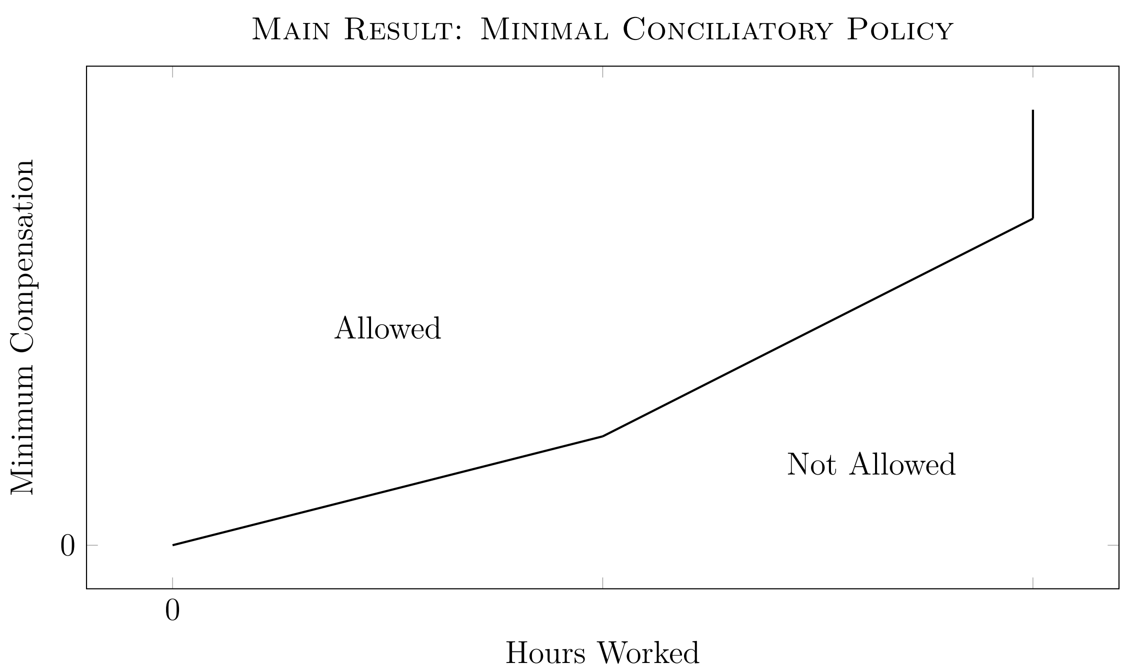

Figure 2: The never Pareto dominated satisficing policy combines overtime pay with a labor cap.

The paper is organized as follows. In Section 2 we introduce a simple leading example to help fix ideas and build intuition about the main results. We present the model in Section 3. In Section 4, we obtain comparative statics results that can be applied to the regulation problem. The regulation problem is considered under complete information in Section 5.1 and is discussed without information about costs or production in Section 5.2. We explore extensions in Section 6 including the extension to heterogeneous workers in Section 6.1. In Section 7, we review the related literature and discuss the results.

2 Example: monopsony and minimum wage

We begin with a comparison of the typical approach taken by the literature and the simplest possible example of our model. Our aim is to make our departure from the literature clear, show what effects this has, and demonstrate why our results are important for regulation.

A monopsonist firm contracts with a worker to obtain hours of labor, ℓ, in exchange for a transfer, τ. A contract is a tuple, (ℓ, τ). From any given contract, the firm receives profits, $f(\ell) - \tau$, where f is a strictly concave, differentiable production function. The worker receives payoff, $\tau - c(\ell)$, where c is a strictly convex, differentiable, and increasing labor cost function. Without loss, $f(0) = c(0) = 0$. We additionally make the standard assumption that $c'(x)x$ is convex.[7]

The hourly wage is the ratio of transfers to hours, $w \equiv {}^{\tau}/_{\ell}$. The worker is *overworked* if she is working more hours than she would prefer to at the given wage: $w < c'(\ell)$. The worker is *underworked* if she would prefer to work more hours at the given wage. The unique Pareto efficient number of hours, ℓ^*, where marginal productivity is equal to marginal cost: $f'(\ell^*) = c'(\ell^*)$. The worker could be overworked, underworked, or neither at the efficient quantity of labor.

Flexible-hours model The Stigler (1946), henceforth flexible-hours, model of monopsony under a minimum wage is analogous to the standard model of monopoly with a price cap. The firm makes some quota of employment hours, $\bar{\ell}$, available to the

guarantee uniqueness.

[7]This assumption is important for the standard model of monopsony considered below. Without this assumption, Loertscher and Muir (2021) show that a menu of stochastic wages may be optimal for the firm. The assumption is not relevant to our alternative model.

Effect of Minimum Wage on Hours in Flexible-hours Model

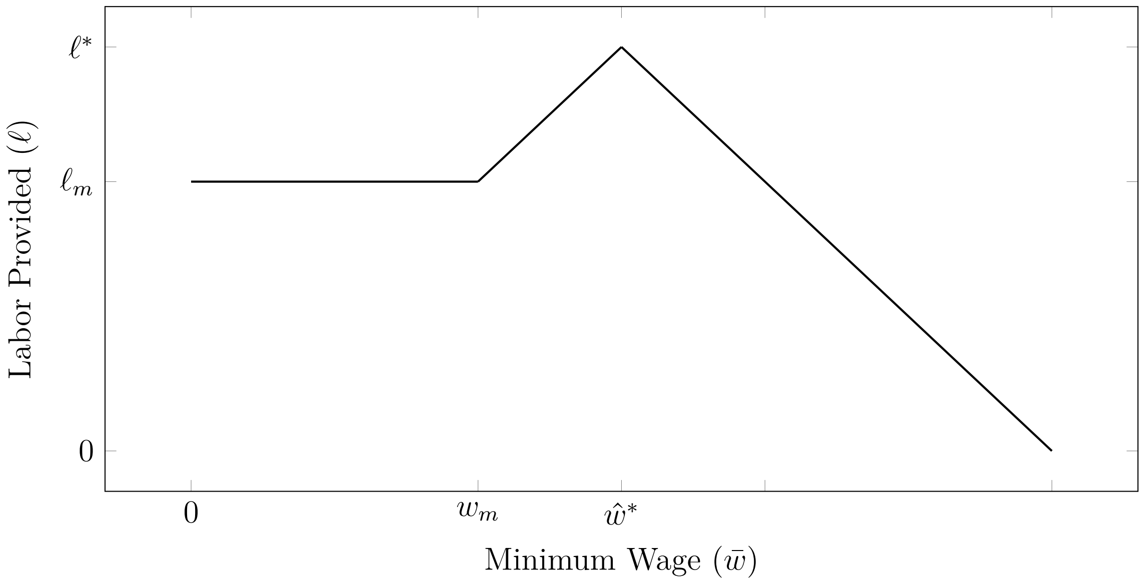

Figure 3: Relationship between the minimum wage and employment hours in the flexible-hours model. For any minimum wage $\bar{w} \in (c'(\ell_m), c'(\ell^*))$, the worker is employed for a quantity of hours between the market employment and the efficient level of employment (the inverse of the marginal cost). The minimum wage $\bar{w} = c'(\ell^*) = f'(\ell^*)$ implements efficient employment.

worker at some hourly wage, w, which is greater than or equal to the minimum wage, $\bar{w}$. The worker then chooses to provide $\ell \leq \bar{\ell}$ hours of labor. Overwork is impossible in this model because the worker can always choose to work fewer hours.

Without a minimum wage, it is well known that the firm equates marginal productivity and marginal expenditure. The market quantity, ℓ_m, of work hours is below the efficient quantity because the firm takes into account the effect that each additional hour has on the wage. Labor is increasing in the minimum wage until the efficient minimum wage, $\hat{w}^* \equiv c'(\ell^*) = f'(\ell^*)$, and decreases thereafter.

The labor hours, $L(w)$, at each minimum wage, w, are plotted in Figure 3 and are given by

$$L(x) = \begin{cases} \ell_m & \text{if } x < c'(\ell_m) \\ (c')^{-1}(x) & \text{if } x \in [c'(\ell_m), c'(\ell^*)] \\ (f')^{-1}(x) & \text{if } x > c'(\ell^*), \end{cases}$$

where ℓ_m solves $f'(\ell_m) = c'(\ell_m) + c''(\ell_m)\ell_m$, and $(f')^{-1}, (c')^{-1}$ are the inverses of c', f' respectively (i.e., inverse supply and and demand).

Incomplete knowledge of L still allows for welfare improving regulation because there is a strictly monotone relationship between labor hours and total surplus. The minimum wage that maximizes labor among any set of minimum wages also maximizes total surplus in that set.

Ultimatum model The firm no longer allows the worker to select the labor hours. The firm instead proposes a "take it or leave it" contract to the worker. If the worker rejects the contract, she receives an outside option of zero. The timing now is:

1. The regulator determines the minimum wage, $\bar{w}$.
2. The firm (who knows c) announces a "take it or leave it" contract to the worker (ℓ, τ) where ℓ denotes the total amount of hours and τ denotes total compensation.
3. The worker chooses to accept the contract, or take an outside option of $(0, 0)$. If the worker is indifferent, she takes the offer.
4. The firm receives
$$\pi(\ell, \tau) = f(\ell) - \tau,$$
and the worker obtains payoff
$$u(\ell, \tau) = \tau - c(\ell).$$

The game has a unique subgame perfect Nash equilibrium. The equilibrium contract solves the firm's profit maximization problem,

$$\max_{\ell,\tau} f(\ell) - \tau \text{ s.t. } \tau \geq c(\ell) \text{ and } \tau \geq \bar{w}\ell. \tag{1}$$

If $\bar{w} = 0$ (i.e., if there is no minimum wage) the firm extracts all surplus from the worker. In this case, labor hours are efficient. Because the full surplus is extracted, the wage is equal to the average cost, $w = {c(\ell^*)}/{\ell^*}$. Since c is convex, average costs are lower than marginal costs, implying the worker would prefer to work fewer hours at

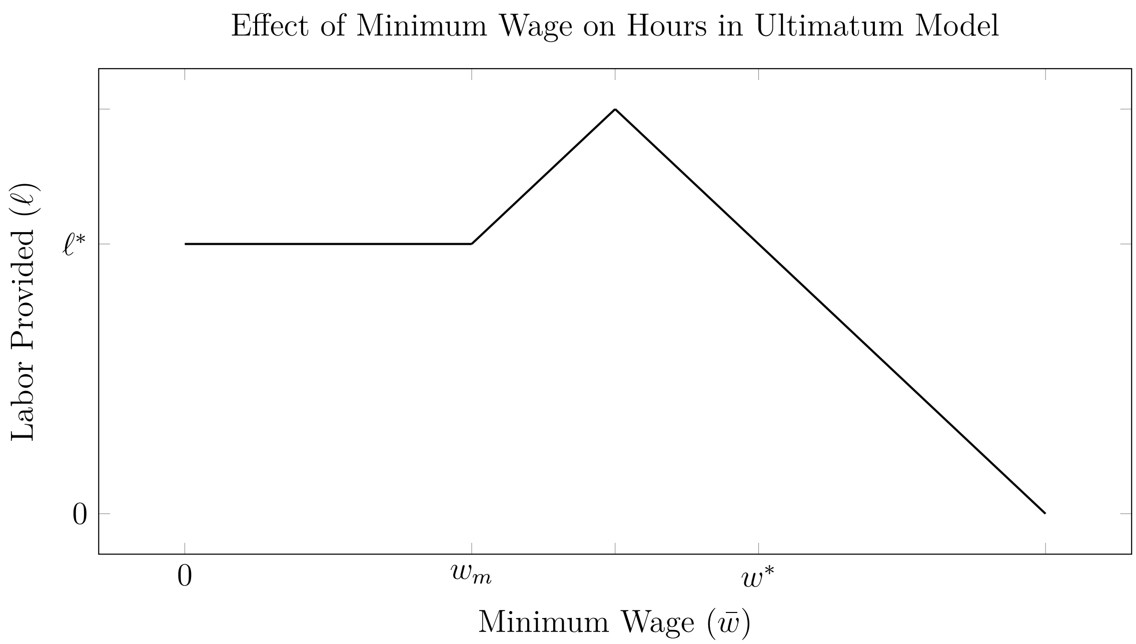

Figure 4: Relationship between the minimum wage and employment hours in the ultimatum model. For any minimum wage $\bar{w} \in (c(\ell^*)/\ell^*, c'(\ell^*))$, the worker is employed for a quantity of hours that exceeds the efficient level of employment (the inverse of the marginal cost). The minimum wage $\bar{w} = c'(\ell^*) = f'(\ell^*)$ implements efficient employment.

the imputed wage. That is, she is overworked.[8]

Suppose that the regulator has access to the entire labor response curve, L. That is, the regulator knows the labor, ℓ, which maximizes (1) for each minimum wage, $\bar{w}$. However, the regulator does not know f or c and does not know whether this labor response comes from the flexible-hours or ultimatum model.

The labor response curve for the ultimatum model is shown in Figure 4. The curve for the flexible-hours model (plotted in Figure 3) has the same shape. In fact, the two models are indistinguishable.

Remark 1. *A flexible-hours model with cost, c, generates the same labor response*

[8]In the more general model presented in Section 4, overwork in the absence of regulation is a necessary and sufficient condition for the market to behave similarly to the ultimatum model in response to regulation.

curve, L, as an ultimatum model with the same production function and cost, d, where

$$d(x) \equiv c'(x)x.$$

Due to Remark 1, the regulator identify the true model without knowledge of the worker's disutility from labor, c. For any labor response curve, L, and production function, f, there is one disutility, c, that is consistent with the flexible-hours model and another disutility, d, that is consistent with the ultimatum model.

Of course, the two models are *not* equivalent for the worker. As a result, the efficient minimum wage is not the same. The unregulated market is already efficient in the ultimatum model. A minimum wage that increases labor increases hours above the efficient level. This poses an issue for a regulator who assumes the flexible-hours model when the ultimatum model is the true model.

Remark 2. *The welfare-maximizing (efficient) minimum wage in the flexible-hours model locally minimizes welfare in the ultimatum model.*

Remark 2 implies that a regulator who assumes the flexible-hours model and has sufficient knowledge of L to implement the efficient minimum wage under this model will locally minimize welfare in the case that the ultimatum model holds. This bias is signed.

Remark 3. *The following statements are true:*

- *The efficient minimum wage is larger in the ultimatum model than in the flexible-hours model.*
- *The efficient number of hours is larger in the flexible-hours model than in the ultimatum model.*
- *If total welfare is increasing in the minimum wage in one model, it is decreasing in the other.*

Remark 3 implies that total welfare is always weakly decreasing in the minimum wage in at least one of the two models. This poses an impossibility for robust regulation using L. However, there is information other than L that can be used to regulate. For example, if the regulator knew that the worker were overworked, he could reject the flexible-hours model.

Overwork information is not only useful for determining the true model, but also for finding whether the current minimum wage is above or below the efficient one.

Remark 4. *Let the efficient minimum wage in model i be w_i^*. Then, w_i^* is the largest minimum wage such that the worker is not underworked in model i.*

Remark 4 demonstrates the importance of worker preferences in regulating labor. Knowing the actual level of labor that is achieved at each minimum wage is neither necessary nor sufficient for determining the efficient minimum wage. However, knowing whether the worker wants to work more or fewer hours at each minimum wage is sufficient.

3 Model

We now develop a model that generalizes the example in Section 2. In particular, we model more general bargaining and an extension of the minimum wage that allows for overtime and caps on hours. The players, payoffs, and definitions remain the same.

A worker sells hours to a firm. The contracts take the form $(\ell, \tau) \in \mathbb{R}_+^2$ where ℓ is hours of labor and τ is a gross payment to the worker. The worker's wage is defined as the average payment per hour: $w \equiv \tau/\ell$.

Firm The firm offers a contract to the worker. When the firm receives ℓ units of labor and pays τ, it has the following payoff:

$$\pi(\ell, \tau) \equiv f(\ell) - \tau.$$

Failing to hire the worker results in no labor or payment.

Worker The worker has a utility function defined by $u(\ell, \tau) \equiv \tau - c(\ell)$ where τ is a gross payment from the firm for the workers' labor and c defines the worker's disutility from labor. The worker has an outside option with zero labor and transfers. A case where the worker's outside option is an endogenous contract is considered in Appendix 8.5.

Regulation A regulator imposes a regulation to constrain the bargaining space. A regulation a function, $\phi : \mathbb{R}_+ \to [0, \infty]$, such that the worker and firm are restricted to

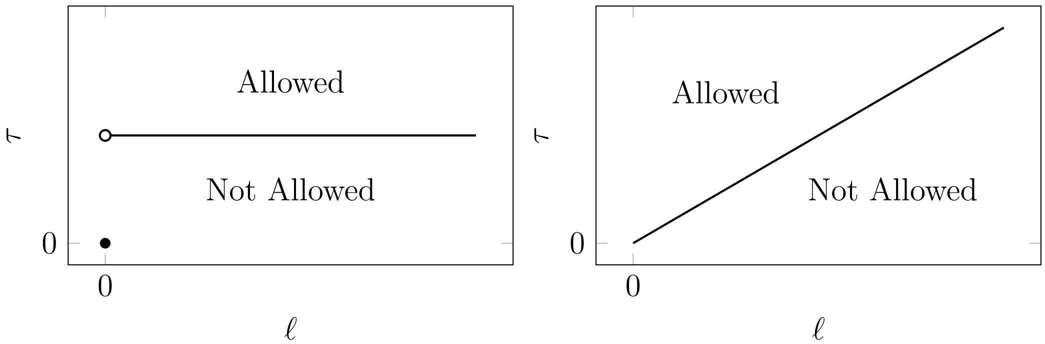

Figure 5: Two examples of policies. Only contracts above the lines are allowed. On the left, there is a minimum gross payment. On the right, there is a minimum wage.

contracts with $\tau \geq \phi(\ell)$. We require that the bargaining space is convex and contains the disagreement point. Therefore, ϕ is convex and $\phi(0) = 0$.

We say a market is *unregulated* if $\phi(\ell) \equiv 0$ for all ℓ. That is, if the firm is unconstrained. We call a linear regulation, $\tau \geq \phi(\ell) \equiv \bar{w}\ell$, a *minimum wage regulation* and refer to the slope, $\bar{w}$, as a *minimum wage*.

In Section 4, we build results about the effects of exogenous regulation. We use these results to implement optimal regulation in Section 5. The regulator has complete information in Section 5.1 and has no knowledge about f or c in Section 5.2. We defer discussion of the regulator's objective in those sections.

Bargaining We assume that the contract, (ℓ, τ) is chosen according to:

$$\arg\max_{\ell,\tau} M(f(\ell) - \tau, \tau - c(\ell)) \text{ s.t. } \tau \geq \phi(\ell)$$

where $M : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ is denoted the *bargaining objective*. This representation is axiomatised in Peters and Wakker (1991). We discuss this modelling choice in Section 3.1.

Timing The game has two stages.

1. The regulator announces regulation, ϕ.
2. The firm and worker negotiate a contract according to M. The contract is a tuple (ℓ, τ).

3.1 Assumptions

Without loss of generality, $f(0) = c(0) = 0$. We additionally assume the following about the production and cost functions.

Assumption 1 (Monotonicity)**.** *The function c is weakly increasing.*

Assumption 2 (Smoothness)**.** *The functions f, c are differentiable.*

Assumption 3 (Interiority)**.** *The functions $f, -c$ are strictly concave, $f'(0) > c'(0)$, and there exists a $t > 0$ such that $f'(t) < c'(t)$.*

The first assumption ensures that the worker weakly prefers less work to more given the same payment. The second assumption, smoothness, makes the analysis cleaner. The third assumption ensures that there exists at least a unique efficient level of labor.

We also make the following two assumptions about the bargaining objective, M.

Assumption 4 (Weak monotonicity)**.** *For all $x, y, x', y' \in \mathbb{R}_+$ such that $x' \neq x$ and $y' \neq y$,*

$$M(x', y') > M(x, y).$$

Assumption 5 (Strict quasiconcavity)**.** *For all $x, y, x', y' \in \mathbb{R}_+$ such that $x' \neq x$ and $y' \neq y$ and for all $\lambda \in (0, 1)$,*

$$M(\lambda x' + (1 - \lambda)x, \lambda y' + (1 - \lambda)y) > \min\{M(x, y), M(x', y')\}.$$

Assumption 6 (Continuity)**.** *The function, M, is continuous in both arguments.*

Peters and Wakker (1991) show that any single valued bargaining solution which satisfies Pareto optimality, independence of irrelevant alternatives, and sequential continuity can be represented as the maximum to some function, M, which satisfies *strong* monotonicity and strict quasiconcavity.[9] We assume a weaker version of monotonicity, but add the assumption that M admits a continuous representation. We additionally prove the results under these three axioms without assuming continuity of M.[10]

[9] This result is also proven by Bossert (1994).

[10] See Lemmas 1 and 2 in the Appendix.

The ultimatum game, egalitarian bargaining, and the more general proportional bargaining of Kalai (1977) do not fit the assumptions of Peters and Wakker (1991) because they can not be represented with a strictly monotone M. They can, however, be represented with a continuous, weakly monotone bargaining objective (A4). To allow for these models, we extend beyond Pareto optimality and study a larger class of representable bargaining solutions that satisfy weak Pareto.

4 Comparative statics results

As in the ultimatum case, monotonicity of the bargaining objective (Assumption 4) ensures that the market is efficient in the absence of regulation. Unlike in the ultimatum case, the worker may not be overworked in the absence of regulation. Instead, overwork in the absence of regulation is a necessary and sufficient condition for there to be an efficient regulation that increases the welfare of the worker.

Theorem 1. *Let τ_0 be the payment to the worker under some regulation, ϕ_0, which implements the efficient level of labor. There exists a regulation, ϕ, that also implements the efficient level of labor, ℓ^*, and larger payment, $\phi(\ell^*) > \tau_0$, if and only if $\frac{\tau_0}{\ell^*} < c'(\ell^*)$.*

Theorem 1 establishes that overwork under efficient regulation, ϕ_0, is a necessary and sufficient condition for there to exist another efficient regulation that the worker prefers. A convenient choice of ϕ_0 is zero (no regulation) because the market is efficient in the absence of regulation. Therefore, overwork in the absence of regulation is a necessary and sufficient condition for there to exist a regulation that is both efficient and beneficial for workers.

Another convenient choice is the minimum wage regulation, $\phi_0(x) \equiv f'(\ell^*)x$. This minimum wage maximizes total surplus because ℓ^* is the preferred number of hours for both the worker and the firm whenever it binds. Because $f'(\ell^*)$ is equal to (and thus not less than) $c'(\ell^*)$, Theorem 1 implies that this regulation is maximal.

Corollary 1.1. *If ϕ implements the efficient regulation, then a minimum wage of $f'(\ell^*)$ does as well and $\phi(\ell^*) \leq f'(\ell^*)\ell^*$.*

This pins down the efficient, convex regulation that maximizes surplus for the worker. It is a minimum wage. The assumption that labor hours are efficient is not critical. A minimum wage maximizes worker surplus for any quantity of labor hours that can be achieved by regulation.

Theorem 2. *Let ϕ be a regulation that implements ℓ. If the worker has greater utility under ϕ than in the unregulated state, then there is a minimum wage, $\bar{w}$, that implements ℓ and $\bar{w}\ell \geq \phi(\ell)$.*

Theorem 2 ensures that minimum wage policies are without loss of optimality within the class of convex policies for any fully informed regulator with an objective that is increasing in τ. This is true, for example, if the regulator maximizes the weighted sum, $\alpha u(\ell,\tau)+(1-\alpha)\pi(\ell,\tau)$ for some $\alpha \in (0.5, 1]$. This is the case considered in Section 5.1. A minimum wage may not be optimal if the regulator is not perfectly informed (Section 5.2) or if workers are heterogeneous (Section 6.1.1).

In the monopsony case of Section 2, the minimum wage that maximizes labor is inefficient. When the worker is overworked in the absence of regulation, this holds generally.

Theorem 3. *Let $L : \mathbb{R}_+ \to \mathbb{R}_+$ define the level of labor at each minimum wage and $w_0 \in [c(\ell^*)/\ell^*, c'(\ell^*))$ be the wage in the absence of regulation. The following properties hold:*

- *The labor response function, L, is continuous.*
- *For all $x \in [0, w_0]$, $L(x) = \ell^*$.*
- *For all $x \in (w_0, f'(\ell^*))$, $L(x) > \ell^*$.*
- *$L(f'(\ell^*)) = \ell^*$ because this minimum wage is TS maximizing.*
- *For all $x > f'(\ell^*)$, $L(x) < \ell^*$ by Corollary 1.1.*
- *For all $x > f'(\ell^*)$, $L(x) = 0$ by individual rationality.*

Theorem 3 implies that the efficient minimum wage is unique. The only minimum wages that implement the efficient number of labor hours are $f'(\ell^*)$ and non-binding minimum wages at or below w_0. Under overwork, the data in the general model follows the same basic shape as the data in the ultimatum model and the interpretation is the same. Labor is constant when the minimum wage is too low to bind. It is above the efficient level when the minimum wage is between the market wage and the efficient minimum wage. Any minimum wage above the minimum wage will result in labor below the efficient level.

One important feature of the ultimatum model is that there is an interval of minimum wages that are dominated by no regulation. In this case, the worker's welfare remains the same, at zero, but the firm's profits decrease. In the general case, there need not be any minimum wages that are dominated by no regulation. On the other hand, it is possible for the surplus of the worker and firm to both decrease.

Example 1 (Egalitarian bargaining)**.** The worker and firm split the market surplus evenly. The market is described by

$$\max_{\ell,\tau} \min\{f(\ell)-\tau, \tau-c(\ell)\} \text{ s.t. } \tau \geq \phi(\ell).$$

Let $-c$ be "more concave" than f on $[0,\ell^*]$ in the sense that $f(\ell^*) - f'(\ell^*)\ell^* < c'(\ell^*)\ell^* - c(\ell^*)$. This condition is necessary and sufficient for overwork.

In the absence of regulation, $\tau_0 = \frac{f(\ell^*)+c(\ell^*)}{2}$ and profits and worker welfare are both equal to half the maximum total surplus, $\frac{f(\ell^*)-c(\ell^*)}{2}$.

By assumption, the worker is overworked in equilibrium. As the minimum wage increases above $w_0 \equiv \frac{f(\ell^*)+c(\ell^*)}{2\ell^*}$, labor will increase to keep profits and worker welfare equal. This will occur until the minimum wage reaches $f'(z)$ where z is the smallest solution to $\frac{f(z)+c(z)}{2z} = f'(z)$. For all minimum wages between w_0 and $f'(z)$, the worker's welfare and the profits of the firm are lower than in the unregulated state. This is because they are evenly dividing the total surplus and this total surplus is lower because labor is inefficient.

For larger minimum wages, the equilibrium will be the same as in the ultimatum model. Once the worker's utility exceeds the firm's profits, the objective is equivalent to profit maximization. △

Example 1 shows the effect that minimum wages below the efficient minimum wage can have on the market. If a binding minimum wage benefits the worker, it is because it increases the worker's share of total surplus enough to compensate for the weak reduction in total surplus imposed by the policy. In Example 1, any minimum wage in $(w_0, f'(z)]$ reduces total surplus without affecting the worker's share. As a result, both the worker and firm are strictly worse off under these regulations.

5 Optimal regulation

We now apply the results of Section 4 to the problem of a regulator. In Section 5.1, the regulator has complete knowledge of production, costs, and the bargaining protocol. In Section 5.2, the regulator has no prior over these objects. To avoid unnecessary complexity, we introduce a different objective in each section. The complete information objective is a refinement of the robust objective.

5.1 Regulation with complete information

Suppose that the regulator knows f, c, and M and maximizes a weighted sum of worker utility and firm profits with more weight on workers. That is, the regulator's objective is to choose the ϕ that maximizes

$$\alpha u(\ell,\tau) + (1-\alpha)\pi(\ell,\tau) = \alpha(\tau - c(\ell)) + (1-\alpha)(f(\ell) - \tau)$$

for $\alpha \in (0.5, 1]$. The case where $\alpha \to 0.5$ is of special interest. In this case, the regulator is not willing to sacrifice any total surplus to improve the welfare of the worker.

Proposition 1. *The optimal regulation is a minimum wage. For $\alpha \to 0.5$, the optimal minimum wage is $f'(\ell^*)$. For all $\alpha \in (0.5, 1]$, there is an optimal regulation defined by*

$$\begin{aligned} \arg\max_{w} \quad & \alpha(w\ell - c(\ell)) + (1-\alpha)(f(\ell) - w\ell) \\ s.t. \quad & \ell = \arg\max_{l} M(f(l) - wl, wl - c(l)) \end{aligned}$$

which is strictly larger than $f'(\ell^*)$.

Proof. Immediate from Corollary 1.1 and Theorem 2. Non-binding minimum wages are irrelevant because the free-market wage is preferable to these values. □

Note that we did not assume overwork in the absence of regulation because Theorem 2 does not require this condition. However, if the worker is not overworked in the absence of regulation, $f'(\ell^*)$ is not a binding minimum wage. So, there is no guarantee that this optimal minimum wage binds in the case of underwork. In the case that an optimal minimum wage does not bind, there is no policy which increases the objective function. A good example of this is the "inverse" ultimatum model where the worker chooses both hours and payment. In this case, the worker receives all surplus. So, all optimal regulations will be a nonbinding minimum wage at the market wage.

Theorem 1 does not necessarily give a unique optimal minimum wage without additional assumptions on f, c, and M.

Example 2 (Proportional bargaining, Kalai, 1977)**.** Suppose that bargaining is proportional. This is a generalization of the egalitarian bargaining in Example 1 and ultimatum bargaining where the market is described by

$$\max_{\ell,\tau} \min\{(1-\beta)(f(\ell)-\tau), \beta(\tau - c(\ell))\} \text{ s.t. } \tau \geq \phi(\ell)$$

for some $\beta \in [0,1]$.

This bargaining is *proportional* because, in the absence of regulation, it grants the worker proportion β of the total surplus. This is ultimatum bargaining when $\beta = 0$ and is egalitarian bargaining when $\beta = 0.5$. When $\beta = 1$, this is "inverse" ultimatum bargaining where the worker chooses both hours and payment – and thus obtains the full surplus.

Because regulation cannot increase total surplus, any regulation that benefits the worker must give the worker more than β of the total surplus. Therefore, the firm is the minimum in the above expression. This means that labor will be chosen to maximize worker surplus.

Case 1. If the worker is overworked in the absence of regulation, then, the minimum wage $f'(\ell^*)$ increases the surplus of the worker. Therefore, labor optimizes the firm's surplus. As a result, the optimal minimum wage solves

$$\begin{aligned} \arg\max_{w \geq f'(\ell^*)} \quad & \alpha(w\ell - c(\ell)) + (1-\alpha)(f(\ell) - w\ell) \\ \text{s.t.} \quad & \ell = \arg\max_{l} f(l) - wl. \end{aligned}$$

Therefore, we set $w = f'(\ell_\alpha)$ where

$$\ell_\alpha \equiv \arg\max_{l<\ell^*} (2\alpha - 1)f'(l)l + (1-\alpha)f(l) - \alpha c(l). \tag{2}$$

Case 2. If the worker is not overworked in the absence of regulation, the analysis in Case 1 still applies to any regulation that benefits the worker. Because the market is efficient without regulation, the objective cannot be increased by any policy that does not benefit the worker. So, Equation 2 is a necessary (but not sufficient) condition for an optimal policy. To find the optimal policy, the regulator needs to check that the candidate optimum is not dominated by the free market. $\triangle$

From Equation 2, we can see that more assumptions are required to ensure that a *unique* optimal minimum wage exists for $\alpha \in (0.5, 1]$. In the case of Example 2, it would be sufficient to assume that $f'(x)x$ is concave.

5.2 Robust regulation

A cap on hours is a robust solution to the problem of overemployment. The regulation reduces hours under even the weakest of assumptions and requires no knowledge of production, f, costs, c, or the bargaining protocol, M. In Appendix 8.2, we demonstrate that a labor cap combined with a minimum wage is a maxmin policy. That is, if the regulator has access to exogenous bounds on marginal productivity and marginal cost, a cap on hours combined with a minimum wage maximizes the worst case worker utility.

In this section, we show that we can improve over any hours cap in a Pareto sense by observing the state of the market before regulating. In doing so, we establish a novel objective which can be applied to general delegation problems. This procedure does not require exogenous bounds on parameters. We will instead construct bounds endogenously by allowing the regulator to observe some existing market state.

Suppose the regulator does not observe production, costs, or the bargaining protocol. Moreover, he has no prior over these objects. Instead, the regulator observes a preexisting market state, (ℓ_0, τ_0). For our purposes, this state will be the equilibrium of the market without any regulation. However, the regulator may observe the state under any regulation.[11]

To motivate the hours cap, suppose that the regulator knows that the worker

[11] When observing the market state and preexisting regulation, the regulator cannot determine whether the policy is binding. For example, a preexisting minimum wage could be set at the market wage. As a result, a preexisting regulation provides no additional information to the regulator. A case where the regulator knows that a preexisting regulation is binding is considered in Section 6.2.2. This also does not affect the problem if the regulation is a minimum wage.

prefers to work $\hat{\ell}$ at preexisting wage, $w_0 \equiv \tau_0/\ell_0$, to the preexisting state. We refer to $\hat{\ell}$ as the *requested hours*. Because this is a model of overwork, $\hat{\ell} < \ell_0$. The requested hours may be requested by workers, may be inferred from survey data, or may just be a strong belief held by the regulator.

In this section, we presuppose that the existing market state is truthful and that the requested hours are indeed preferred by the worker to the labor hours in the current market state. This modelling setting is reasonable in regulation because data used to set regulations is typically taken from some period before the regulation could be anticipated. The issue of manipulation is discussed in Section 6.2.

A hours cap at $\hat{\ell}$ consists of a minimum wage of the preexisting wage (w_0) up to the requested hours $\hat{\ell}$ and an infinite required payment thereafter. That is, the hours cap is $\bar{\phi}$, where

$$\bar{\phi}_{\hat{\ell}}(x) \equiv \begin{cases} w_0 x & \text{if } x \leq \hat{\ell} \\ \infty & \text{if } x > \hat{\ell}. \end{cases}$$

The kink point in this regulation is always individually rational for both the worker and firm because it is the convex combination of two individually rational contracts. Note that the minimum wage set at the preexisting wage is important. For example, in the ultimatum model, the firm would like to reduce the wage after hours are reduced because the wage is the average cost, and this average cost is increasing in labor hours by convexity.

The hours cap satisfies the requested hours in the sense that the hours cap implements $\hat{\ell}$ and a payment of at least $w_0\hat{\ell}$.

Definition 1 (Satisficing)**.** A regulation is satisficing if

$$\begin{aligned} \inf_{(M,f,c)\in I(\ell_0,\tau_0)} \quad & (\tau - c(\ell)) - (w_0\hat{\ell} - c(\hat{\ell})) \geq 0 \\ \text{s.t.} \quad & (\ell, \tau) = \left(\arg\max_{l,t} M(f(l) - t, t - c(l)) \text{ s.t. } t \geq \phi(l)\right) \end{aligned}$$

where $R[\phi|f,c] \equiv U[\phi|f,c] - \left(w_0\hat{\ell} - c(\hat{\ell})\right)$ and $I(\ell_0, \tau_0, \hat{\ell})$ is the set of possible M, f, c that are consistent with $\ell_0, \tau_0, \hat{\ell}$.

Proposition 2. *The labor cap, $\bar{\phi}_{\hat{\ell}}$, is satisficing.*

Proof. Immediate from strict quasiconcavity of M and unconstrained optimality of (ℓ_0, τ_0). □

Because the regulator knows that $(\hat{\ell}, w_0\hat{\ell})$ is preferred to the preexisting contract, a cap on hours benefits workers in a *robust* way. This robust benefit is captured by the satisficing criterion. The hours cap, and by extension the satisficing property, are appealing because it is simple and benefits worker under very weak assumptions.

The regulation is, however, Pareto dominated under many states: M, f, c. Intuitively, the infinite slope at $\hat{\ell}$ is unnecessarily restrictive. There must be some (perhaps large) amount that the firm can pay such that the worker is willing to work more. If an additional hour of labor is worth millions of dollars to the firm and the worker is willing to work this additional hour, then everyone benefits from permitting this transaction.

We refine the satisficing criterion using the condition that our chosen regulation not be Pareto dominated by any satisficing regulation under any state.

Definition 2 (Never Pareto dominated)**.** A regulation, ϕ, is never Pareto dominated if for all $(M, f, c) \in I(\ell_0, \tau_0, \hat{\ell})$ and all satisficing policies, ψ,

$$U[\psi|M, f, c] > U[\phi|M, f, c] \implies \Pi[\psi|M, f, c] > \Pi[\phi|M, f, c]$$

and

$$\Pi[\psi|M, f, c] > \Pi[\phi|M, f, c] \implies U[\psi|M, f, c] > U[\phi|M, f, c]$$

where $I(\ell_0, \tau_0, \hat{\ell})$ is the set of possible f, c, M that are consistent with $\ell_0, \tau_0, \hat{\ell}$.

Note that the analysis in Sections 5.1 and 6.1.1, the objective of the regulator is a never Pareto dominated satisficing objective. For example, the total surplus maximizing minimum wage, $f'(\ell^*)$ is satisficing because it involves a larger wage and assigns the worker her preferred hours at that wage. The larger wages even better for the worker and thus are also satisficing. The never Pareto dominated condition is weak in the complete information case because there is only one possible M, f, c under which the outcome need not Pareto dominated.

In the robust setting, where many states are possible, never Pareto dominated is a very strong condition. It is not obvious that such a regulation exists. We show in Appendix 8.3 that a never Pareto dominated satisficing regulation exists in a large class of delegation problems. An additional refinement can be applied to ensure uniqueness in this general setting.

Theorem 4. *There is a unique never Pareto dominated satisficing regulation defined by,*

$$\phi^*(x) = \begin{cases} w_0 x & \text{if } x \leq \hat{\ell} \\ w_0\hat{\ell} + \frac{w_0\ell_0}{\ell_0-\hat{\ell}}(x-\hat{\ell}) & \text{if } \hat{\ell} < x \leq \ell_0 \\ \infty & \text{if } x > \ell_0. \end{cases}$$

Theorem 4 establishes that the unique never Pareto dominated satisficing regulation, ϕ^*, is an overtime regulation with a cap on hours. This regulation is plotted in Figure 2. The proof, which is contained in Appendix 8.1.5, can also be applied to the situation where the regulator knows that workers and firms operate under the ultimatum model. Here, we will outline the main technique that we use to show that this regulation is satisficing.

By IIA, any violation of the satisficing criterion would come from the market contracting on a point allowed under ϕ^* that is not allowed under the hours cap, $\bar{\phi}_\ell$. So, any violation of satisficing would come from contracts to with labor in $(\hat{\ell}, \ell_0]$ that the worker likes less than $(\hat{\ell}, w_0\hat{\ell})$. We will show that any such contract is forbidden.

The worker weakly prefers a contract (x, y) to $(\hat{\ell}, w_0\hat{\ell})$ where $x \in (\hat{\ell}, \ell_0]$ if and only if the additional payment she receives offsets the increase in work hours,

$$y - w_0\hat{\ell} \geq c(x) - c(\hat{\ell}). \tag{3}$$

We are able to bound the right hand side (the increase in labor costs) using convexity and individual rationality of (ℓ_0, τ_0).

$$c(x) - c(\hat{\ell}) < \frac{x-\hat{\ell}}{\ell_0-\hat{\ell}}\left[c(\ell_0) - c(\hat{\ell})\right] < \frac{w_0\ell_0}{\ell_0-\hat{\ell}}(x-\hat{\ell})$$

Where the last step uses $c(\hat{\ell}) > 0$, and the fact that individual rationality implies $c(\ell_0) \leq \tau_0 = w_0\ell_0$. Combining the above with (3) yields that the worker prefers (x, y) to $(\hat{\ell}, w_0\hat{\ell})$ if

$$y \geq w_0\hat{\ell} + \frac{w_0\ell_0}{\ell_0-\hat{\ell}}(x-\hat{\ell}) = \phi^*(x).$$

The fact that this regulation is never Pareto dominated comes from the fact that it is the minimal satisficing regulation. Because bargaining is efficient, the least restrictive regulation is never Pareto dominated by more restrictive policies. We show in Appendix 8.3 that the set of satisficing delegation sets in general delegation problems forms an upper semi-lattice. As a result, the least restrictive regulation exists and is unique.

6 Extensions

6.1 Heterogeneous workers

Thus far, we have assumed that a firm acquires the services of a single worker. This makes sense if: (1) regulation can be customized to each worker or (2) workers have similar labor preferences. The first is unusual. The second is not necessarily true.

6.1.1 Complete information regulation under heterogeneity

Suppose that there are $N \geq 2$ types of workers employed by a firm. Let c_i denote the cost of the i-th worker type. The costs and the production function, f, satisfy A1-3. For convenience, order the types in terms of efficient labor hours. That is, for $j > k$, $\ell_j^* \geq \ell_k^*$ where $\ell_j^* \equiv \arg\max_z f(z) - c_j(z)$ and $\ell_k^* \equiv \arg\max_z f(z) - c_k(z)$. The workers contract in accordance with the ultimatum model.

Assume that the regulator treats all workers equally. That is, the objective of the regulator is

$$\alpha\left(\sum_{i=1}^{N}\tau_i - c_i(\ell_i)\right) + (1-\alpha)\left(\sum_{i=1}^{N} f(\ell_i) - \tau_i\right) = \sum_{i=1}^{N}(2\alpha-1)\tau_i + (1-\alpha)f(\ell_i) - \alpha c_i(\ell_i)$$

where $\alpha \in (0.5, 1]$.

We analyze this case of heterogeneous workers under complete information in depth in Appendix 8.4. Here we highlight and discuss two significant results.

The first result is that if only the utility of the worker matters to the regulator, heterogeneity essentially has no effect on the problem.

Proposition 3. *For $\alpha = 1$, there is at least one optimal regulation that is a minimum wage. Any optimal minimum wage is the same as the optimal minimum wage in a single worker problem. This single worker has the average cost of all workers with positive utility under the regulation.*

Proposition 3 means that the optimal minimum wage in the heterogeneous case is the optimal minimum wage for a worker with costs that are averaged across some subset of the workers. The proof contains an algorithm to find all optimum minimum wages by checking all possible subsets of workers. Example 3 in the Appendix shows how to use the algorithm in practice. In the example, there are two optimal minimum wages which benefit different subsets of workers.

Intuitively, Proposition 3 holds because workers cannot be harmed by regulation in the ultimatum model. If the regulator does not care about firms, then the workers who are affected by regulation, but do not benefit are irrelevant. If the regulator cares about firms or efficiency, the optimal policy may not be a minimum wage.

Proposition 4. *Let $\alpha \to 0.5$. Suppose $\ell_N^* \neq \ell_{N-1}^*$. If $\min_{i=1}^{N-1} \{c_i(\ell_i^*)/\ell_i^*\} > f'(\ell_N^*)$, the optimal regulation is a minimum wage of $f'(\ell_N^*)$. Otherwise, consider*

$$\phi(x) \equiv \begin{cases} Conv(x) & \text{if } x \leq \ell_{N-1}^* \\ Conv(\ell_{N-1}^*) + f'(\ell_N^*)(x - \ell_{N-1}^*) & \text{if } x > \ell_{N-1}^*, \end{cases}$$

and Conv is the largest convex function to fit under $\{(\ell_i^*, c_i(\ell_i^*))\}_{i=1}^{N-1}$ with the restriction that the slope is capped at $f'(\ell_N^*)$. If $\phi(\ell_N^*) > c_N(\ell_N^*)$, then ϕ is an optimal regulation.

Proposition 4 demonstrates two important properties of these problems. First, the optimal regulation may not be a minimum wage. The reason that a piecewise linear regulation may be optimal in the heterogeneous worker setting is that it reduces the effects of regulation the contracts of workers with zero payoff. Such regulation only affects firms. So, the issue becomes less relevant as α increases. The second property is that minimum wage regulation is also optimal in settings where heterogeneity is sufficiently large. In this case, the problem is separable and the lowest cost worker can be regulated alone. In Appendix 8.4, Proposition 8 shows that a minimum wage is also optimal for all $\alpha \in (0.5, 1]$ (but not in the limit) when heterogeneity is sufficiently small.

6.1.2 Robust regulation under heterogeneity

The previous analysis uses the market state from just one worker to regulate. As a result, the regulation is only necessarily satisficing for this one worker. Extending the analysis to multiple workers is simple. You just construct the never Pareto dominated satisficing regulation for each and take the maximum pointwise.

Proposition 5. *Let ϕ_i^* be the never Pareto dominated satisficing regulation for worker i. Suppose that the request, $\hat{\ell}$, is below the labor of each worker and that all workers are overworked. The unique never Pareto dominated regulation which is satisficing for all workers is*

$$\phi^*(x) = \max_i \phi_i^*(x).$$

The proof is in Appendix 8.1.6. For $\ell > \hat{\ell}$, each regulation only allows contracts which the worker prefers to the request. If we take the maximum, all of the points which are allowed are preferred to the request. The only trick is to ensure that $\ell < \hat{\ell}$ is satisficing. This comes from weak Pareto and overwork combined with Theorem 1.

The same proof also demonstrates that an hours cap at τ which uses the maximum of all workers wages is also satisfying. The assumption that all workers are overworked is important to establish that ϕ^* is satisficing. If some workers are not overworked, then ϕ^* may not be satisficing. However, for the same reason, an hours cap would also not be satisficing. The argument that ϕ^* is a Pareto improvement over an hours cap remains true.

6.2 Manipulation in robust regulation

In Section 5.2, we assumed that the firm and worker do not foresee the regulation that will be imposed. Moreover, we assume that the regulator knows some lower level of labor, $\hat{\ell}$, which the worker prefers to the current level of labor. This level could be a arbitrary or be an internal belief of the regulator. However, it may be appealing to elicit this value.

In practice, manipulation of regulation is typically prevented through *grandfathering*, the use of information which predates the discussion of regulation. For example, in most cap-and-trade systems, permits are allocated based on historical energy usage that predates the discussion of these environmental policies.[12] This section considers what can happen when this practice is infeasible.

[12]This also applies to regulations considered for individuals. For example, the 2022 student loan forgiveness policy in the U.S. does not apply to any loans taken out less than two months before its announcement.

6.2.1 Manipulation by workers

Suppose that workers have complete information and interact with firms according to the ultimatum model. If the regulator asks the worker how many hours she wants to work, the worker wants to solve

$$\begin{aligned} \max_{\hat{\ell}\leq \ell_0} \quad & w_0\hat{\ell} + \frac{w_0\ell_0}{\ell_0-\hat{\ell}}(\ell-\hat{\ell}) - c(\ell) \\ \text{s.t.} \quad & \ell = \operatorname*{arg\,max}_{l\in[0,\hat{\ell}]} f(l) - \frac{w_0\ell_0}{\ell_0-\hat{\ell}}(l-\hat{\ell}). \end{aligned}$$

The solution is interior because setting the request, $\hat{\ell} = 0$ or $\hat{\ell} = \ell_0$ result in no binding regulation being implemented. This can be solved using standard envelope theorem arguments. We instead consider two extreme cases for intuition: (1) when total surplus is small and (2) when total surplus is large.

If total surplus is small, both marginal cost and marginal productivity are low. The firm will not pay overtime unless the overtime pay multiplier is sufficiently small. Because the worker prefers all of the overtime points to the requested point by construction, the worker wants to make a report such that she can earn overtime. To make the overtime pay multiplier sufficiently small, the worker will request a low $\hat{\ell}$. As a result, the regulator imposes little regulation when there is not much surplus to redistribute.

If total surplus is sufficiently large, both marginal cost and marginal productivity are high. The firm will pay overtime even when the overtime pay multiplier is large. To extract a larger payment, the worker will request a large $\hat{\ell}$. As a result, the regulator imposes a large overtime payment multiplier when there is a lot of surplus to redistribute, but most hours are worked without overtime.

The regulation does not require the worker to be strategic or have information about production. However, if the worker does have access to these inputs, they can be used to make the regulation better. In both cases, the worker makes a strategic decision which makes the regulator's bound on the disutility of labor more accurately reflect the worker's marginal costs.

6.2.2 Manipulation by firms

Manipulation before regulation Suppose that the firm predicts the regulation and adjusts the contract in the preexisting regulation to interfere with the mechanism. For simplicity, suppose that $\hat{\ell}$ is fixed.

The firm cannot benefit from paying the worker more than her costs. This makes the regulation more restrictive. As a result, the firm can only manipulate by adjusting labor. The firm solves

$$\max_{z \geq \hat{\ell}, \ell \leq z} f(\ell) - \frac{c(z)}{z}\hat{\ell} - \frac{c(z)}{z - \hat{\ell}}(\ell - \hat{\ell}).$$

We consider the same two extreme cases. If total surplus is small, both marginal productivity and marginal cost are low. The firm cannot make the overtime payment multiplier arbitrarily small without making the wage arbitrarily large. Therefore, the firm will not pay overtime. This implies $\ell = \hat{\ell}$. In this case, the firm wants to set z as small as possible to reduce the hourly wage that must be paid. In equilibrium, $z = \ell = \hat{\ell}$.

If total surplus is large, both marginal productivity and marginal cost are large. In this case, a large choice of labor in the preexisting market is costly. The firm will end up paying overtime for all available hours.[13] In this case, the firm will set $\ell = z$. The firm's problem can then be rewritten as

$$\max_{z} f(z) - \frac{c(z)}{z}\hat{\ell} - c(z).$$

Therefore, the firm hires the worker for fewer hours than is optimal in advance of the regulation. Intuitively, the firm takes into account the effect that an increase in labor will have on the overtime multiplier.

Manipulation after regulation Suppose that the firm is regulated but wants to prevent future regulation. When it chooses the contract, it takes into account that the regulator will obtain information from the prevailing contract.

In the ultimatum model, paying overtime to a worker reveals that the marginal productivity of the worker is $w_0 \frac{\ell_0}{\ell_0 - \hat{\ell}}$. As a result, the regulator knows that a minimum

[13] Note that this case also applies when $\hat{\ell}$ is sufficiently small relative to ℓ^*. This is because the firm's objective is decreasing in z for $\hat{\ell}$ sufficiently small.

wage of $w_0 \frac{\ell_0}{\ell_0 - \hat{\ell}}$ will not affect total surplus, but will increase the surplus of the worker.

As a concrete example, suppose a firm in the U.S. hires a worker for ten dollars per hour for more than sixty hours per week. As a result, the firm must pay the worker time and a half for the last twenty hours that she works. If the regulator sees this, he knows that the firm would be willing to pay time and a half for the first forty hours as well. The regulator can use this information to impose a minimum wage of fifteen dollars per hour. This regulation costs the firm the equivalent of twenty hours of work each week.

This suggests that the regulator has a limited ability to improve on a labor cap if he cannot commit to the mechanism. The firm may be unwilling offer any overtime to workers if doing so invites regulation.

7 Conclusion

We have explored regulation under very general assumptions in a setting where workers are overworked. We show that a minimum wage is the best tool that a fully informed regulator can use to alleviate this issue. Through a comparative statics exercise, we demonstrate that the minimum wage can hurt both workers and firms when it is set either too low or too high. We show that this is particularly important if regulators assume the flexible-hours model and try to interpret the effects of regulation on hours.

This issue of interpretation relates to the empirical literature on measuring the effects of minimum wage policies on hours (e.g., Jardim et al., 2022) and the effects of other policies such as overtime (Hamermesh and Trejo, 2000; Quach, 2020; Trejo, 1991). This paper proposes a framework which can can be tested by and used in a welfare analysis of these results.

Our study also cautions that the intensive margin (i.e., hours) and extensive margin must be treated differently with regards to regulation. For example, Jardim et al. (2022) shows that the 2014 increase of the minimum wage in Seattle did not significantly reduce employment, but did significantly reduce hours. Most would say that this is a bad sign. However, a reduction in hours may be good if these workers wanted their hours to be reduced. The objective of minimum wage regulation is not to maximize hours (or even take-home pay). The goal, broadly, is to improve the lives of workers. We demonstrate that this is at odds with hours maximization.

We find the overall optimal minimum wages when the regulator has complete information. Even if the regulator is not willing to sacrifice any total surplus to increase the worker's welfare, there exists a minimum wage that increases the utility of the worker. This policy ensures an efficient market equilibrium where both the firm and worker receive their preferred number of hours.

This analysis joins two theoretical strands in labor economics. First, this paper is connected to the large literature on optimal regulation under imperfect competition with hours set by workers (e.g., Berger et al., 2022) or with fixed hours (e.g., Flinn, 2006; Loertscher and Muir, 2021). Secondly, our work is also related to the smaller literature on labor hours and overwork outside of a regulatory context (Feather and Shaw, 2000; Manning, 2013). The most novel innovation of our approach lies in combining these two strains by studying regulation in a setting with labor hours and overwork.

In addition to the complete information setting, we consider a regulator who has no prior over production and disutility from labor. This regulator instead observes the current market state and knows that a specific reduction in labor hours at the existing wage will benefit the worker.

This analysis contributes to the literature on robust implementation. Following Carroll (2015), this literature focuses on finding policies that can be implemented without any prior on the space of parameters. Guo and Shmaya (2019) study the problem of regulating an inefficient monopolist seller without any prior over supply and demand. Unlike our study, Guo and Shmaya (2019) consider a regulator who knows bounds on supply and demand.

In any robust analysis, the regulator must be able to somehow bound the unknown objects. In Carroll (2015), the principal is able to create an endogenous lower bound on the agent's technology from partial knowledge of the agent's set of available actions. Typically, these bounds are exogenous. This is troubling because the objects may be difficult for the regulator to bound in a reasonable way, and extremely permissive bounds generally produce unreasonable outcomes (e.g., arbitrarily large or small minimum wages).

We demonstrate a method for creating endogenous bounds on supply and demand when the regulator is able to observe the price and quantity that prevail in the market. To use this bound, we develop a new robust objective, the never Pareto dominated satisficing criterion. This objective is natural in any delegation problem. In general, the principal chooses the largest delegation set for which the principal does not regret the decision to delegate (i.e., the payoff is at least as high as the principal's preferred singleton delegation set). It is common for the principal and agent to have some inherent alignment of incentives such that this is beneficial. In our case, this alignment stems from the fact that our contract bargaining is Pareto efficient.

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8 Appendix

8.1 Proofs

8.1.1 Useful lemmas

Lemma 1. *The functional,*

$$\mathcal{L}[\phi] \equiv \arg\max_{l,t} M(f(l) - t, t - c(l)) \;\; s.t. \;\; t \geq \phi(l),$$

is sequentially continuous in the sense that $\phi_n \to \phi$ *(under the Hausdorff metric) implies* $\mathcal{L}[\phi_n] \to \mathcal{L}[\phi]$.

Proof. Suppose that M is continuous and satisfies weak Pareto, then $\mathcal{L}$ is continuous by Berge’s theorem of the maximum. Continuity implies sequential continuity because the set of convex functions is a metric space under the Hausdorff metric.

In the setting of Peters and Wakker (1991), sequential continuity of bargaining ensures that the payoffs are sequentially continuous in ϕ. Continuity of hours and payment is immediate from continuity and strict concavity of the payoffs. □

Lemma 2. *A point on the unconstrained Pareto frontier of* M *lies above all points on the Pareto interior in some neighborhood of itself if and only if it is the overall optimum.*

Proof. This is immediate from continuity and strict quasiconcavity of M. The purpose of this lemma is to show this result under the assumptions of Peters and Wakker (1991): Pareto optimality, IIA, and sequential continuity.

First, note that the overall Pareto frontier is the curve in (π, u) space where total surplus is maximized. Because of transferable utility, this is a line with slope negative one. Without loss, say that the space of feasible payoffs is the simplex.

Let z be a point on the Pareto frontier that is above all points in the interior contained in an ϵ-ball centered at z for some $\epsilon > 0$. Then, construct an $\epsilon/2$-ball such that the point is at the right with a triangle removed as in the left of Figure 6. As δ decreases towards zero, the bargaining protocol must still choose z as it is the most preferred point. Continuity of the protocol implies that z must be chosen in the limit as well. Because this limit contains this segment of the Pareto frontier, z must exceed all points above it (preferred by the worker) on the frontier. The same argument can be applied for the points below using the $\epsilon/2$-ball to the right of Figure 6.

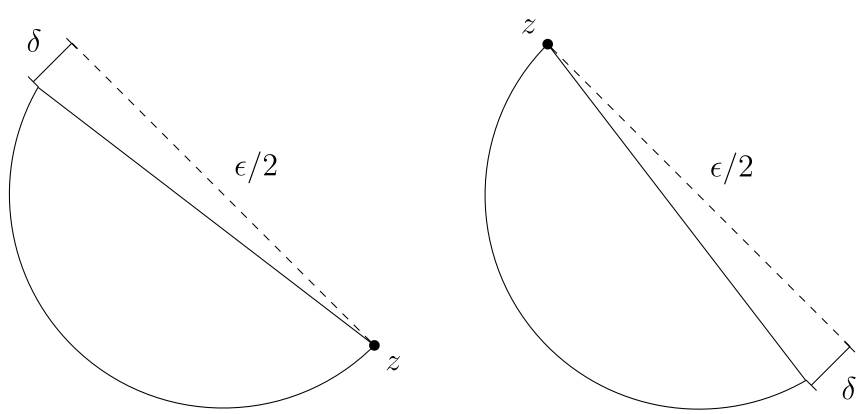

Figure 6: Figure of $\epsilon/2$-balls for Lemma 2. As $\delta \to 0$, this sequence approaches the full half circle, which contains a segment of the Pareto frontier.

Therefore, z is a local maximum on the frontier. By quasiconcavity of M, it is the global maximum on the frontier. By monotonicity, it is the overall optimum. $\square$

8.1.2 Proof of Theorem 1

Proof. The converse is trivial. If $\tau_0/\ell^* < c'(\ell^*) = f'(\ell^*)$, $\phi(x) = f'(\ell^*)x$ binds and implements ℓ^*.

For the forward statement, we know that the policy ϕ implements ℓ^* and improves the welfare of the worker. Therefore, $z \equiv (f(\ell^*) - \phi(\ell^*), \phi(\ell^*) - c(\ell^*))$ is the constrained optimum but is not the overall optimum. By Lemma 2, For any $\epsilon > 0$, there is an ϵ-ball around z such that z is not maximal. We want to show that that $f'(\ell^*) \geq \phi'_-(\ell^*)$.

Suppose, by way of contradiction, that $f'(\ell^*) < \phi'_-(\ell^*)$. Intuitively, decreasing ℓ in a neighborhood of ℓ^* is the same as relaxing the policy constraint because the marginal effect on profit and worker surplus are equal. As a result, there is an ϵ-ball around z available under ϕ, which is a contradiction. Formally, we can access the upper part of the ϵ-ball by transferring ϵ to the worker (because the constraint is in only one direction). We can access the lower part of the epsilon ball by decreasing ℓ by $\frac{\epsilon}{\phi'_-(\ell^*)-f'(\ell^*)}$:

$$f\left(\ell^* - \frac{\epsilon}{\phi'_-(\ell^*) - f'(\ell^*)}\right) - \phi\left(\ell^* - \frac{\epsilon}{\phi'_-(\ell^*) - f'(\ell^*)}\right) = f(\ell^*) + \phi(\ell^*) + \epsilon - \mathcal{O}(\epsilon^2)$$
$$\phi\left(\ell^* - \frac{\epsilon}{\phi'_-(\ell^*) - f'(\ell^*)}\right) - c\left(\ell^* - \frac{\epsilon}{\phi'_-(\ell^*) - f'(\ell^*)}\right) = \phi(\ell^*) - c(\ell^*) - \epsilon - \mathcal{O}(\epsilon^2).$$

This comes from taking a Taylor approximation near ℓ^* and using the fact that $f'(\ell^*) = c'(\ell^*)$. For ϵ sufficiently small, we can access any angle in the ϵ-ball on the Pareto interior. This is a contradiction.

Therefore, $c'(\ell^*) \geq \phi'_-(\ell^*)$. Therefore, for all $x \leq \ell^*$, $\phi(x) \leq c'(\ell^*)x$ by convexity. Because $\tau_o < \phi(\ell^*)$, we conclude $\tau_0/\ell^* < c'(\ell^*)$. Therefore, the worker is overemployed.

□

8.1.3 Proof of Theorem 2

Proof. Because ϕ improves the welfare of the worker, the policy is binding. Then, ℓ maximizes $M^\phi(x) \equiv M(f(x) - \phi(x), \phi(x) - c(x))$.

By Lemma 1, the function,

$$L(w) \equiv \arg\max_l M(f(l) - wl, wl - c(l)),$$

is continuous. Because $L(f'(0)) = 0$, if a minimum wage implements labor hours, z, we can find a minimum wage to implement any labor hours less than z.

There are two cases.

In the first case, $\ell = \ell^*$. We established that $\bar{w} \equiv c'(\ell^*)$ implements ℓ^* and that $\phi'_-(\ell^*) \leq c'(\ell^*)$. The conclusion follows from convexity.

In the second case, $\ell \neq \ell^*$. Consider the minimum wage $\phi(\ell)/\ell$. Clearly, we are done if this also implements ℓ. If it implements labor greater than ℓ, continuity of L

guarantees that an even larger minimum wage implements ℓ. Then, the only case left to check is that this minimum wage implements labor less than ℓ. This is impossible. Suppose by way of contradiction that the minimum wage does reduce labor. Then,

$$\arg\max_x M\left(f(x) - \frac{\phi(\ell)}{\ell}x, \frac{\phi(\ell)}{\ell}x - c(x)\right) < \ell.$$

However, this level of labor and transfer were available under ϕ (by convexity) and $(\ell, \phi(\ell))$ is available in the above. That both are the unique optima of their respective problems is a violation of IIA. □

8.1.4 Proof of Theorem 3

Proof. Continuity comes from Lemma 1 and constant before the constraint binds is immediate from IIA.

We show the third point. The proof is similar to Theorem 1. At w_0, consider a small increase in the minimum wage. If the worker's incentive compatibility constraint binds, then the effect of the wage on the equilibrium is the same as under monopsony. Therefore, $\psi'_+(w_0) = \frac{\ell^*}{c'(\ell^*)-w_0} > 0$.

Then, suppose the constraint does not bind, we know that labor cannot decrease because this is Pareto dominated. Therefore, it weakly increases at this point.

As in the proof of Theorem 1, increasing the labor perfectly counteracts an increase in the minimum wage in a neighborhood around the point where it first binds. We want to show that $w \in (w_0, f'(\ell^*))$ leads to $\ell > \ell^*$.

Suppose, by way of contradiction, a $w \in (w_0, f'(\ell^*))$ supports a contract $z \equiv (\ell^*, w\ell^*)$. By Lemma 2, for every $\epsilon > 0$, there is an ϵ-ball around z such that it is not maximal over all Pareto interior points. Therefore, there must be some angle within this ϵ-ball that cannot be accessed under minimum wage, w. However, this is not true. We can access the upper part of the ϵ-ball by transferring ϵ to the worker and can access the lower part of the ϵ-ball by increasing ℓ by $\frac{\epsilon}{f'(\ell^*)-w}$:

$$f\left(\ell^* + \frac{\epsilon}{f'(\ell^*) - w}\right) - \left(\ell^* + \frac{\epsilon}{f'(\ell^*) - w}\right)w = f(\ell^*) + w\ell^* + \epsilon - \mathcal{O}(\epsilon^2)$$
$$\left(\ell^* + \frac{\epsilon}{f'(\ell^*) - w}\right)w - c\left(\ell^* + \frac{\epsilon}{f'(\ell^*) - w}\right) = w\ell^* - c(\ell^*) - \epsilon - \mathcal{O}(\epsilon^2).$$

For ϵ sufficiently small, we can access any angle in the ϵ-ball on the Pareto interior. This is a contradiction.

The other points are justified in the statement of the theorem. $\square$

8.1.5 Proof of Theorem 4

Proof. We showed in the body of the paper that ϕ^* is satisficing. To repeat the logic, note that for all $x \in (\hat{\ell}, \ell_0]$,

$$c(x) - c(\hat{\ell}) < \frac{x - \hat{\ell}}{\ell_0 - \hat{\ell}} \left[c(\ell_0) - c(\hat{\ell})\right] < \frac{w_0 \ell_0}{\ell_0 - \hat{\ell}}(x - \hat{\ell}) = \phi(x) - \phi(\hat{\ell}).$$

The increased transfers make up for the extra work. So, ϕ^* is satisficing.

We first show that any satisficing policy is greater or equal to ϕ^*. For this part, we can use the ultimatum model because it must be robust to any bargaining framework. The functions $f_\varepsilon(x) \equiv (w_0 + \varepsilon)x$ and

$$c_\varepsilon(x) \equiv \begin{cases} w_0 x - \frac{\varepsilon \ell_0}{2\hat{\ell}} x & \text{if } x \leq \hat{\ell} \\ w_0 x + \frac{\varepsilon}{2}(x - \ell_0) & \text{if } \hat{\ell} < x \leq \ell_0 \\ w_0 x + 2\varepsilon(x - \ell_0) & \text{if } x > \ell_0 \end{cases}$$

are feasible for all $\epsilon > 0$. Satisficing implies $\phi(x) \geq w_0 x$ for all $x < \hat{\ell}$ and $\phi(x) > w_0 x$ for all $x > \hat{\ell}$. However, if $\phi(\ell) > w_0 \ell$, for all ℓ there exists an ε such that the firm shuts down. So there must be some t such that $\phi(t) \leq w_0 t$ to be chosen in this case. For satisficing to hold, this implies $\phi(\hat{\ell}) = w_0 \hat{\ell}$. Now, consider $f_N(x) \equiv Nx$ and

$$c_N(x) \equiv \begin{cases} 0 & \text{if } x \leq \hat{\ell} \\ \frac{w_0 \ell_0}{\ell_0 - \hat{\ell}}(x - \hat{\ell}) & \text{if } \hat{\ell} < x \leq \ell_0 \\ w_0 \ell_0 + (N+1)(x - \ell^\star) & \text{if } x > \ell_0 \end{cases}$$

which are feasible. If there exists an $x > \hat{\ell}$ such that $\phi(x) < w_0 x + c_N(x)$, then there exists an N such that it will be chosen by the firm. Thus satisficing requires $\phi(x) \geq w_0 x + c_N(x)$ for all N. Therefore, satisficing implies that there must be a cap at ℓ_0. $\square$

8.1.6 Proof of Proposition 5

Proof. The function, $\phi^*(x)$, is convex (because it is the maximum of convex functions) and satisfies $\phi^*(0) = 0$. Therefore, it is a regulation. We now show that it is satisficing.

For $x \leq \hat{\ell}$, there is a minimum wage equal to the maximum wage paid to any worker. Because the worker is overworked, this wage is less than $f'(\ell^*) < f'(\hat{\ell})$. Therefore, the firm prefers $\hat{\ell}$ to any point below x. If the worker works less than $\hat{\ell}$ as a result of the regulation, it is because she prefers this (by weak Pareto). Therefore, any contract with $\ell \leq \hat{\ell}$ is satisficing under this policy.

For $x > \hat{\ell}$, there is a region that is the maximum of all of the policies for each worker. These policies were designed such that all allowed contracts in this region were weakly preferred to the requested contract for all workers. By taking the maximum, we ensure that every allowed contract is weakly preferred by all workers.

This policy is never Pareto dominated because it is minimal. Suppose, by way of contradiction, that a smaller policy existed which was satisficing for all workers, then it would be smaller at some points than the minimal satisficing policy of at least one worker. This is a contradiction.

The proof that the minimal policy is uniquely never Pareto dominated follows the same argument as Theorem 4. □

8.2 Robust maxmin regulation

Suppose that a regulator with no knowledge of f, c regulates an ultimatum monopsony. The regulator knows that Assumptions 1, 2, and 3 hold. The objective of the regulator is

$$\begin{aligned} \max_{\phi} \inf_{f,c} \quad & \tau - c(\ell) \\ \text{s.t.} \quad & \ell = \arg\max_{x} f(x) - \max\{c(x), \phi(x)\} \\ & \tau = \max\{c(x), \phi(x)\}. \end{aligned}$$

The regulator has a lower bound on marginal productivity, $f'(x) \geq \bar{f}'(x)$, and an upper bound on marginal cost, $c'(x) \leq \bar{c}'(x)$ for all x. By Assumption 3, $\bar{f}'$ is weakly decreasing and $\bar{c}'$ is weakly increasing. If not, the regulator can iron these bounds using concavity. Assume that both bounds are left continuous.

Proposition 6. *A minimum wage with a labor cap is a maxmin policy. One such policy is defined by*

$$\phi(x) = \begin{cases} \bar{f}'(t)x & \text{if } x \leq t \\ \infty & \text{if } x > t \end{cases}$$

where $t \in \arg\max_z \bar{f}'(z)z - \bar{c}'(z)$.

Proof. If $\bar{f}'(0) \leq \bar{c}'(0)$, the infimum in the regulator's objective is zero. The worker's payoff is at least zero by the individual rationality constraint. So, any policy is maxmin.

Suppose that $\bar{f}'(0) > \bar{c}'(0)$. By left continuity, there exists at least one t that solves $t \in \arg\max_z \bar{f}'(z)z - \bar{c}'(z)$. A minimum wage of $\bar{f}'(t)$ maximizes total surplus when the bound holds exactly by granting the worker the contract $(t, \bar{f}'(t)t)$. In this state, this gives the worker a surplus of $\bar{f}'(z)z - \bar{c}'(z)$. It is not possible to achieve more than this in maxmin because the minimal state is at least as bad as this one.

To guarantee this return, we need to ensure that the firm does not choose a larger level of labor, $\ell > t$, such that $\bar{f}'(t) < \bar{c}'(\ell)$. We did not place an upper bound on marginal productivity. So, this needs to be guaranteed with some sort of convex policy. We could, for example, place a labor cap at t or at the largest q which satisfies $\bar{c}'(q) \leq \bar{f}'(t)$. It's also possible to integrate $\bar{c}'$ and add this to the wage after t. □

8.3 General robust delegation

The purpose of this section is to develop a general theory of never Pareto dominated satisficing delegation sets. We show that such a delegation set always exists and that a refinement can be used to obtain uniqueness. In particular, the least restrictive never Pareto dominated satisficing delegation set is unique.

Let $\mathcal{A}$ be a set of alternatives that a principal (e.g., regulator) and agent (e.g., aggregated labor market) may choose from. There is some state, $\theta \in \Theta$, that is known to the agent but unknown to the principal. This state may affect the payoff of both the principal and agent.

In order to elicit this information, the principal may choose a subset, $D \in \mathbb{D} \subset 2^{\mathcal{A}}$, to provide to the agent such that the agent makes a choice from D. This choice is defined by choice function, $\mathcal{C}_\theta : \mathbb{D} \to 2^{\mathcal{A}}$. This choice function is nonempty and satisfies the weak axiom of revealed preference, which we report in the form of Sen's α and β conditions, for each $\theta \in \Theta$.

Assumption 7 (Sen's α, IIA)**.** *If $x \in A \subseteq B$ and $x \in \mathcal{C}_\theta(B)$, then $x \in \mathcal{C}_\theta(A)$.*

Assumption 8 (Sen's β)**.** *If $x, y \in \mathcal{C}_\theta(A)$, $x \in \mathcal{C}_\theta(B)$, and $A \subseteq B$, then $y \in \mathcal{C}_\theta(B)$.*

These assumptions are typically used to obtain a revealed preference binary relation. In this case, we only impose the conditions on $\mathbb{D} \subset 2^{\mathcal{A}}$. This is less restrictive. For example, in this paper, Sen's β is vacuously true because all allowed delegation sets yield a unique choice.

The principal has a complete, but not necessarily transitive, weak relation: $\succeq_\theta$. He wants to choose a delegation set with two properties.

Definition 3 (Satisficing)**.** A delegation set, $D \in \mathbb{D}$ is satisficing with respect to $\hat{x} \in \mathcal{A}$ if for each $\theta \in \Theta$, there exists a $z \in \mathcal{C}_\theta(D)$ such that $z \succeq_\theta \hat{x}$.

Definition 4 (Never dominated)**.** A satisficing delegation set, $D \in \mathbb{D}$ is never Pareto dominated if for all satisficing delegation sets, $S \in \mathbb{D}$, and all $\theta \in \Theta$, $x \in \mathcal{C}_\theta(D) \implies x \in \mathcal{C}_\theta(D \cup S)$.

The satisficing criterion ensures that the outcome is at least as good for the principal and the never dominated condition imposes a refinement that we give the agent as much surplus as possible. In the setting of this paper, weak Pareto of the bargaining protocol implies that any never dominated delegation set is never Pareto dominated.

We want to show that there exists a delegation set that satisfies Definitions 3 and 4. We will do this by showing that there is a *least restrictive* satisficing set which contains all other satisficing delegation sets. This set is never dominated because the agent prefers larger delegation sets to smaller ones.

Lemma 3. *If the set of available delegation rules, $\mathbb{D}$, is closed under unions, then the set of satisficing delegation sets, $\mathbb{S} \subseteq \mathbb{D}$, is an upper semi-lattice under unions.*

Proof. We need to show that if S_1 and S_2 are satisficing, then $S_1 \cup S_2$ is satisficing. For each θ, consider each $x \in \mathcal{C}_\theta(S_1 \cup S_2)$. Because $x \in S_1 \cup S_2$, it is either in S_1, S_2, or both. Without loss, say it is contained in S_1. By Sen's α, $x \in \mathcal{C}_\theta(S_1)$. By Sen's β, $\mathcal{C}_\theta(S_1) \succeq \mathcal{C}_\theta(S_1 \cup S_2)$.

Because S_1 is satisficing, there exists $z \in \mathcal{C}_\theta(S_1)$ such that $z \succeq \hat{x}$. Because $z \in \mathcal{C}_\theta(S_1 \cup S_2)$, $S_1 \cup S_2$ is also satisficing. □

Proposition 7. *Suppose the set of available delegation rules, $\mathbb{D}$, is closed under unions and set of satisficing delegation sets, $\mathbb{S} \subseteq \mathbb{D}$ is nonempty. There exists a never dominated satisficing delegation set. Moreover, there exists a unique least restrictive never dominated satisficing delegation set.*

Proof. If there is only one satisficing delegation set, then we are done. It is never dominated.

If there is more than one, by Lemma 3, there exists an $S^* \in \mathbb{S}$ such that for all $S \in \mathbb{S}$, $S \subseteq S^*$. This delegation set is uniquely least restrictive and is never dominated because $S^* \cup S = S^*$. So, $x \in \mathcal{C}(S^*) \implies x \in \mathcal{C}(S^* \cup S)$. □

We assume in Proposition 7 that there is at least one satisficing delegation set. In many delegation settings, this will be $\{\hat{x}\}$. In the setting of this paper, it was the hours cap.

8.4 Complete information regulation under heterogeneity

Suppose that there are $N \geq 2$ types of workers employed by a firm. Let c_i denote the cost of the i-th worker type. The costs and the production function, f, satisfy A1-3. For convenience, order the types in terms of efficient labor hours. That is, for $j > k$, $\ell_j^* \geq \ell_k^*$ where $\ell_j^* \equiv \arg\max_z f(z) - c_j(z)$ and $\ell_k^* \equiv \arg\max_z f(z) - c_k(z)$. The workers contract in accordance with the ultimatum model.

Assume that the regulator treats all workers equally. That is, the objective of the regulator is

$$\alpha\left(\sum_{i=1}^{N} \tau_i - c_i(\ell_i)\right) + (1-\alpha)\left(\sum_{i=1}^{N} f(\ell_i) - \tau_i\right) = \sum_{i=1}^{N} (2\alpha - 1)\tau_i + (1-\alpha)f(\ell_i) - \alpha c_i(\ell_i)$$

where $\alpha \in (0.5, 1]$.

Lemma 4. *All workers who receive a positive surplus under regulation, ϕ, have the same contract: $(\tilde{\ell}, \tilde{\tau})$. This contract is defined by $f'(\tilde{\ell}) = \phi'(\tilde{\ell})$ and $\tilde{\tau} = \phi(\tilde{\ell})$. If that contract is a minimum wage, any worker, i, with zero surplus has $\ell_i \leq \tilde{\ell}$.*

Proof. If a worker, i, has a positive surplus, $\phi(\ell_i) > c_i(\ell_i)$. Therefore, the IR constraint does not bind and c_i has no effect on the firm's problem. Therefore, all workers with positive surplus must have the same contract defined by first order conditions $f'(\tilde{\ell}) = \phi'(\tilde{\ell})$ and $\tilde{\tau} = \phi(\tilde{\ell})$.

If a worker, k, has zero surplus under minimum wage, $\bar{w}$, $\tau_k = c_k(\ell_k) \geq \bar{w}\ell_k$. Suppose, by way of contradiction that $\ell_k > \tilde{\ell}$. By convexity,

$$\bar{w} = f'(\tilde{\ell}) > f'(\ell_k).$$

Because the regulation exceeds, the worker's marginal productivity, the firm would prefer to hire the worker for fewer hours if this regulation were to bind. Therefore, the regulation must be strictly below the cost of worker k. Then, $f'(\ell_k)$ is efficient and this minimum wage is above the efficient minimum wage of k. This is a contradiction. □

Lemma 4 shows that the homogeneous and heterogeneous worker problems are fundamentally similar. All workers who benefit from a regulation receive the same contract. So, there is no way to design a regulation that provides different redistributive contracts to different workers.

If only the utility of the worker matters to the regulator, heterogeneity essentially has no effect on the problem.

Proposition 3. *For $\alpha = 1$, there is at least one optimal regulation that is a minimum wage. Any optimal minimum wage is the same as the optimal minimum wage in a single worker problem. This single worker has the average cost of all workers with positive utility under the regulation.*

Proof. When $\alpha = 1$, the regulator maximizes the average utility of all workers. By Lemma 4, all workers with positive payoffs have the same contract. For every subset of workers $S \subseteq 2^{1,\ldots,N}$, the regulator can solve

$$\max_{\phi} \sum_{i \in S} \phi(\tilde{\ell}) - c_i(\tilde{\ell}) \text{ s.t. } f'(\tilde{\ell}) = \phi'(\tilde{\ell}) \tag{4}$$

which is equivalent to

$$\max_{\phi} \phi(\tilde{\ell}) - \frac{1}{|S|} \sum_{i \in S} c_i(\tilde{\ell}) \text{ s.t. } f'(\tilde{\ell}) = \phi'(\tilde{\ell}).$$

This is the same as the single worker problem where c is replaced by the average of c_i for all $i \in S$. Therefore, the optimum is always a minimum wage.

Some of these problems may be *invalid* in the sense that the regulation does not actually benefit all workers in S. Lemma 4 ensures $\ell_i \leq \tilde{\ell}$ for any worker, i, with zero utility. As a result, there is no way to reach these workers with a regulation. Therefore, the problem for S is invalid only if there is no optimal regulation that benefits all workers in S. Therefore, the optimal minimum wages are valid solutions to (4). The regulator chooses the set of S that maximize the objective. $\square$

Proposition 3 suggests that multiple optimal minimum wages may exist. This is because the set of workers affected by the regulation may differ across optimal policies.

Example 3 (Two optimal minimum wages)**.** Suppose a firm with $f(x) \equiv x - \frac{x^2}{2}$ contracts the services of two workers. Worker 1 has cost $c_1(x) \equiv \frac{7x^2}{2}$ and worker 2 has cost $c_2(x) \equiv \frac{x^2}{2}$.

The three candidate optimal regulation problems are

$$\max_{x,\bar{w}} \bar{w}x - \frac{1}{|S|}\sum_{i \in S} c_i(x) \text{ s.t. } \bar{w} = f'(x).$$

The solutions to the three candidate problems are $\ell_1 = \frac{1}{9}, \ell_2 = \frac{1}{3}, \ell_{1,2} = \frac{1}{6}$ with wages $\bar{w}_1 = \frac{8}{9}, \bar{w}_2 = \frac{2}{3}, \bar{w}_{1,2} = \frac{5}{6}$. The utility of worker 2 under $\bar{w}_1$ is

$$\bar{w}_1\ell_1 - c_2(\ell_1) = \frac{8}{9}\frac{1}{9} - \frac{(1/9)^2}{2} = \frac{5}{54} > 0.$$

This means the regulation is invalid because it should benefit only worker 1. The second benefits worker 2, but does not benefit worker 1 because

$$\bar{w}_2\ell_2 - c_1(\ell_2) = \frac{2}{3}\frac{1}{3} - 7\frac{(1/3)^2}{2} = -\frac{1}{6} < 0.$$

Therefore, it is valid and the benefit of this regulation is

$$\bar{w}_2\ell_2 - c_2(\ell_2) = \frac{2}{3}\frac{1}{3} - \frac{(1/3)^2}{2} = \frac{1}{6}.$$

The combined benefit of the joint regulation is

$$\begin{aligned}(\bar{w}_{1,2}\ell_{1,2} - c_1(\ell_{1,2})) + (\bar{w}_{1,2}\ell_{1,2} - c_2(\ell_{1,2})) &= \left(\frac{5}{6}\frac{1}{6} - 7\frac{(1/6)^2}{2}\right) + \left(\frac{5}{6}\frac{1}{6} - \frac{(1/6)^2}{2}\right) \\ &= \frac{1}{24} + \frac{1}{8} = \frac{1}{6}.\end{aligned}$$

This is the same as the effect of the optimal minimum wage for worker 2. Therefore, both $\bar{w}_2 = \frac{2}{3}$ and $\bar{w}_{1,2} = \frac{5}{6}$ are optimal minimum wage policies. While both are optimal, their distributive effects are different. $\triangle$

The heterogenous worker problem has different implications for $\alpha \in (0.5, 1)$. The largest effect comes in the case of total surplus maximization ($\alpha \to 0.5$). An immediate implication from Lemma 4 is that a regulation can be efficient and benefit more than one worker only if the workers who benefit have the same efficient hours. If efficient labor hours are strictly ranked, at most one worker can benefit from a regulation that maximizes total surplus.

However, if heterogeneity is small, the problems for $\alpha \in (0.5, 1)$ are always similar to the homogeneous case.

Proposition 8. *For each $\alpha \in (0.5, 1]$, there exists an $\epsilon > 0$ such that $\max_x |c_i(x) - c_k(x)| < \epsilon$ for all $i, k \leq N$ implies there is at least one optimal regulation that is a minimum wage. Any optimal minimum wage is the same as the optimal minimum wage in a single worker problem. This single worker has the average cost of all workers.*

Proof. We only need to show that for any $\alpha \in (0.5, 1]$, there exists an ϵ such that the regulator gives every worker a positive payoff. Giving worker i a positive payoff requires $\phi(\ell_i) - c_i(\ell_i) > 0$. Therefore, any regulation that increases the payoff of one worker increases the payoff of all workers for ϵ sufficiently small. Therefore, we just need to show that there exists a binding optimal regulation.

The fact that there is such a regulation is obvious. Consider a minimum wage of $f'(\ell_N^*)$. This regulation gives strictly positive benefit to all players. Loss in total surplus can be made arbitrarily small. Therefore, this dominates not regulating for any $\alpha \in (0.5, 1]$. $\square$

From this proof, we can see that the required bounds on heterogeneity become more strict as α gets closer to 0.5. From Proposition 3, we know that no conditions are needed when $\alpha = 1$.

Lemma 8 does not hold for $\alpha \to 0.5$. In this case, there is no exactly efficient regulation that increases the utility of workers when heterogeneity in costs is arbitrarily small. However, an exactly efficient regulation that increases worker utility may exist when heterogeneity is not small.

Proposition 4. *Let $\alpha \to 0.5$. Suppose $\ell_N^* \neq \ell_{N-1}^*$. If $\min_{i=1}^{N-1} \{c_i(\ell_i^*)/\ell_i^*\} > f'(\ell_N^*)$, the optimal regulation is a minimum wage of $f'(\ell_N^*)$. Otherwise, consider*

$$\phi(x) \equiv \begin{cases} Conv(x) & \text{if } x \leq \ell_{N-1}^* \\ Conv(\ell_{N-1}^*) + f'(\ell_N^*)(x - \ell_{N-1}^*) & \text{if } x > \ell_{N-1}^*, \end{cases}$$

and Conv is the largest convex function to fit under $\{(\ell_i^*, c_i(\ell_i^*))\}_{i=1}^{N-1}$ with the restriction that the slope is capped at $f'(\ell_N^*)$. If $\phi(\ell_N^*) > c_N(\ell_N^*)$, then ϕ is an optimal regulation.

Proof. Lemma 4 and efficiency ensure minimum wage redistribution can only be used to benefit worker N. Affecting any other worker at all will reduce efficiency. Therefore, the regulation must be nonbinding for all other workers. If if the first condition holds, the efficient minimum wage is optimal because it is the largest efficient regulation for N.

If the second condition holds, any regulation for any other worker, i with slope $f'(\ell_i^*)$ at ℓ_i^* will also affect player N. Regulation, ϕ, is as large as possible while lying below all other points and having slope $f'(\ell_N^*)$ at ℓ_N^*. □

Proposition 4 demonstrates two important properties of these problems. First, the optimal regulation may not be a minimum wage. The reason that a piecewise linear regulation may be optimal in the heterogeneous worker setting is that it reduces the effects of regulation the contracts of workers with zero payoff. Such regulation only affects firms. So, the issue becomes less relevant as α increases. The second property is that minimum wage regulation is also optimal in settings where heterogeneity is sufficiently large. In this case, the problem is separable and the lowest cost worker can be regulated alone.

8.5 Oligopsony

We now add an Entrant firm with production function g. The incumbent must provide the worker enough surplus so that the Entrant cannot make any profitable offer to the worker. Therefore, the profit-maximization problem of the incumbent under oligopsony is

$$\max_{\ell,\tau} f(\ell) - \tau \text{ s.t. } \tau \geq \max\{c(\ell) + u[\phi; g], \phi(\ell)\} \tag{5}$$

where u is maximum surplus that the Entrant can offer. That is,

$$u[\phi; g] = \max_{\ell} g(\ell) - c(\ell) \text{ s.t. } g(\ell) \geq \phi(\ell). \tag{6}$$

Note that the regulation, ϕ, enters twice into the Incumbent's problem. As in the monopsony case, it pushes the worker's salary up. However, it also constrains the maximum in (6). This means that the regulation reduces competitive pressure. This tension between regulation and competition is the main difference between oligopsony and monopsony.

8.5.1 Pre-regulation benchmark

In the absence of regulation, the Incumbent offers the efficient level of labor and matches the best offer of the Entrant. So, the Incumbent offers contract (ℓ^*, τ_g^*) with $\ell^* \equiv \arg\max_x f(x) - c(x)$ and $\tau_g^* \equiv c(\ell^*) + u[0; g]$ where u is defined by (6).

Unlike under monopsony, it is now possible that the worker is underemployed. That is, she might prefer to work more hours at the average wage offered by the Incumbent.

Lemma 5. *Suppose a Incumbent with production function, f offers labor quantity, ℓ and receives profits, Π. Then, the worker is underemployed if and only if the Incumbent earns profit, $\Pi < f(\ell) - c'_+(\ell)\ell$ and is overemployed if and only if $\Pi > f(\ell) - c'_-(\ell)\ell$.*

For the Incumbent, the right hand sides of the above inequalities do not depend on the production function of the Entrant, g. However, the equilibrium profit is weakly decreasing in g. Therefore, markets with more competitive Entrants (larger g) have underemployment and markets with less competitive Entrants (lower g) have overemployment. Because the Entrant receives zero profit, its best offer would underemploy the worker.

Assumption A**??** ensures that the Entrant's best offer involves weakly less labor than the Incumbent's. This fact combined with Figure 7 demonstrate the Incumbent pays a lower wage than the Entrant if the employee is underemployed. This is because the Incumbent is compensating the worker with more labor, and therefore can pay less. If the worker is overemployed, the Incumbent's wage may be greater than the Entrant's.

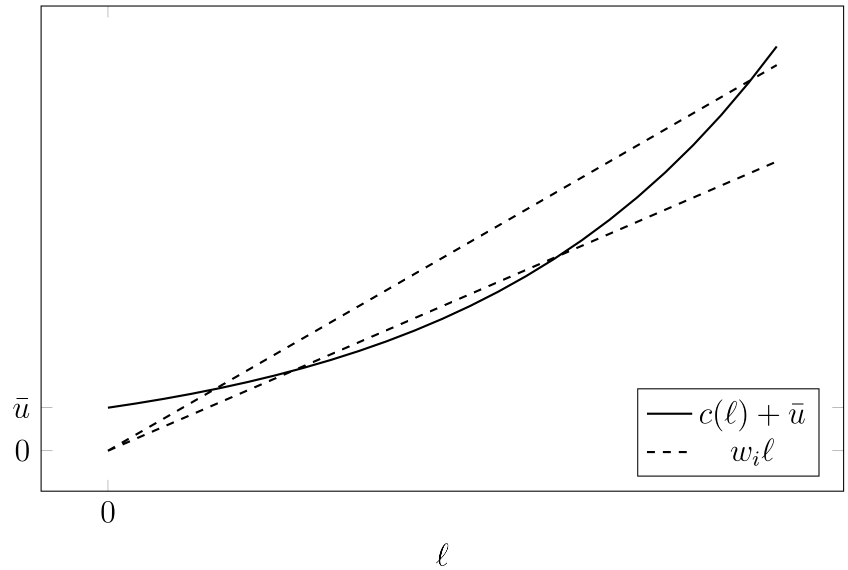

Figure 7: A plot of labor holding the worker's utility constant. The lower intersections are contracts that underemploy the worker while the intersections at the upper part of the curve overemploy the worker at the same wages.

8.5.2 Minimum wage regulation

The competitive constraint ensures that $\tau - c(\ell) \geq u[\phi; g]$. If this condition is binding for some policy, ϕ, it is impossible for the policy to increase the welfare of the worker over the pre-regulation benchmark because the worker's surplus is the left hand side of the constraint and u is weakly decreasing in *phi*. When the competitive constraint does not bind, the Entrant is irrelevant.

Therefore, for any policy that increases the welfare of the worker, the oligopsony outcome and monopsony outcome are the same. Because of this, the justification for restricting attention to minimum wage policies under Monopsony, Lemma **??**, also holds under Oligopsony.

On the other hand, the effects of minimum wages that do not improve the welfare of workers are very different under Monopsony and Oligopsony. The most apparent difference between the two is that a minimum wage can strictly reduce worker welfare because workers have strictly positive welfare in the pre-regulation benchmark.

Because of this, it is not obvious that the market can be efficiently regulated.

Proposition 9. *Let (ℓ, τ) be the contract offered by the Incumbent under minimum wage, $\bar{w} \geq 0$, and let ℓ^* be the efficient level of labor. Assume f is differentiable at*

ℓ^*. Then, there exists a larger minimum wage $\bar{w}' > \bar{w}$ that implements ℓ^* if and only if (ℓ, τ) overemploys the worker.

Section 2 shows that, as in the neoclassical model, there is a binding minimum wage that achieves the efficient level of labor under a monopsony. It is well known that this is impossible under perfect competition when labor demand is a function.[14]

For $\bar{w} = 0$, Proposition 9 demonstrates that if a worker is overemployed in the absence of regulation, the market is uncompetitive enough for a minimum wage regulation to be efficient. If the worker is underemployed in the absence of regulation, the market is competitive enough that efficient minimum wage regulation is impossible.

This means that a regulator need only know if workers desired hours exceed their actual hours in the pre-regulation benchmark to determine if efficient minimum wage regulation is possible. The proposition goes further to say that if there is already a minimum wage in place, a regulator can tell whether this existing regulation is above or below the efficient minimum wage just by observing whether employees are underemployed or overemployed.

Proposition 10. *There f, g, c and ϕ such that, relative to the pre-regulation benchmark,*

- *worker hours are greater;*
- *the wage offered by the Incumbent is strictly lower;*
- *the Incumbent's profits are strictly larger.*

However, if the worker is underemployed in the pre-regulation benchmark, then none of the above are possible.

8.6 Tables and figures

[14]When f is not differentiable, demand is a correspondence. That is, there may be an interval of wages assigned to any level of labor. In this case, imagine that supply intersects this demand at the bottom of this interval. Then, it's clear that you can impose a minimum wage to the top of the interval.

Table 1: Answers to "Which of the following statements best describes how your working hours are decided? In this question, working hours refers to the total number of hours you work each week, not the time you start and finish work each day." in the 2016 General Social Survey (Smith et al., 2018).

	How worker is paid:		
	All Workers	Non Hourly	Hourly
Employer decides	40.77%	32.40%	46.72%
Employer decides with some input	25.96%	18.80%	31.05%
Worker decides within limits	18.30%	26.40%	12.54%
Worker free to decide	8.32%	13.60%	4.56%
Outside of worker and employer's control	6.49%	8.40%	5.13%
Observations	601	250	351