Job Market Papers

This is the first of two job market papers. My first job market paper, “Regulation of Wages and Hours”, is applied theory. My second job market paper, “Asymmetric All-Pay Auctions with Spillovers”, is a more technical pure theory paper.

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“Regulation of Wages and Hours”

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Regulation of Wages and Hours

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Abstract

This paper studies the problem of a labor market regulator who knows that workers prefer to work fewer hours at their current wage, but lacks specific knowledge of production, labor disutility, and the bargaining protocol. We show that for a large class of bargaining protocols, moderate regulation (such as a small minimum wage) is counterproductive in that it results in hours that exceed the efficient quantity. We find that a combination of the minimum wage, overtime pay, and a cap on hours is optimal in a novel robust regulatory setting where the regulator has neither a prior nor exogenous bounds on model parameters.

Keywords: Overtime pay, regulation design, delegation, non-Bayesian, robust.
JEL Codes: D81, D82, D86, J08

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1 Introduction

Most workers cannot freely set their hours.[1] This restriction is not studied in the theoretical literature on labor regulation, where hours are assumed to be either fixed or chosen by workers.[2]

However the practice of regulation recognizes this issue. Many policies are designed to reduce the hours of individual workers. For example, the European Union imposes a sharp cap of 48 hours per week. In the United States, overtime pay requires companies to pay “time and a half” (1.5 times the wage) for each hour worked above 40 hours per week. Japan uses a lesser amount of overtime, “time and a quarter”, combined with a sharp cap of 55 hours per week. These policies are depicted in Figure 1.

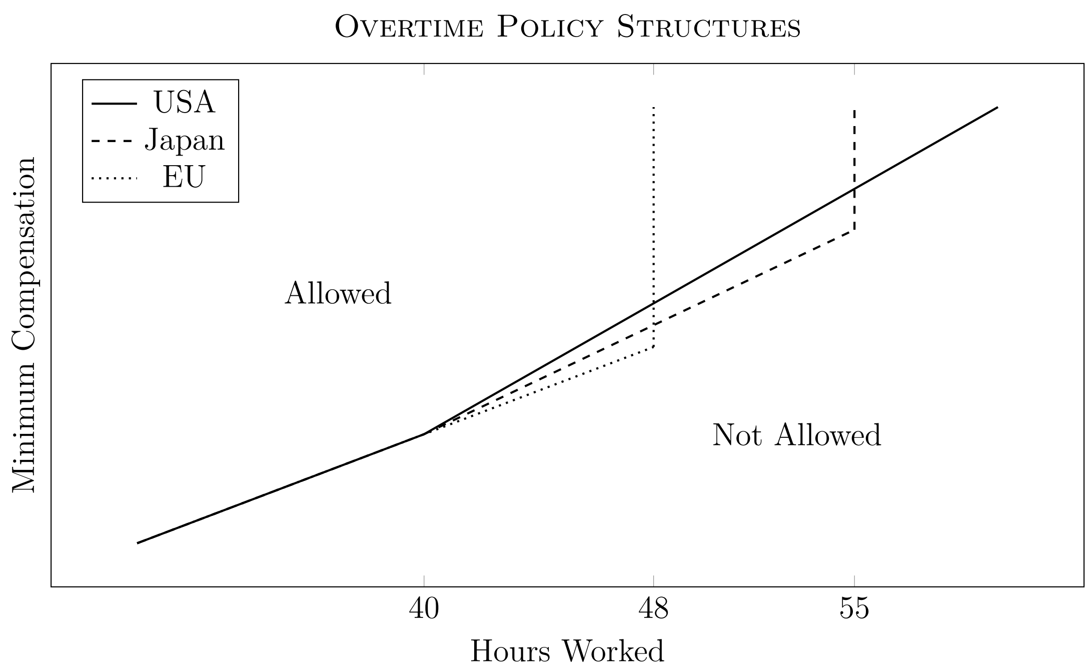

Figure 1: Regulations for minimum total compensation for each number of hours in the United States, Japan, and the European Union with a normalized minimum wage. Contracts with hours and compensation above the line are allowed. Contracts below the line are not.

[1] For example, less than 5% of hourly paid respondents to the 2016 United States General Social Survey said that they had full control of their hours compared to 47% who responded that their employer decides unilaterally. See Table 1 in Appendix for more details.

[2] It is, however, considered outside the context of regulation. For example Manning (2005) shows differences between the canonical model of monopsony and one where the employer chooses hours.

This paper addresses this gap in the theoretical literature by analyzing the effects of regulation in a model in which workers and firms bargain over both wages and hours of work.

Consider a fully-informed monopsonist firm that submits a “take it or leave it” offer of compensation and hours to a single risk-neutral worker with convex disutility from labor. We refer to this as the “ultimatum model” of monopsony to distinguish it from the traditional model.[3] In the absence of regulation, the proposed contract will maximize total surplus, which will be entirely appropriated by the firm, and the worker’s hours will be longer than she would like to work at the implied wage (i.e., total compensation divided by hours of work). Suppose that a regulator maximizes a weighted sum of worker utility and firm profits with more than half of the weight placed on the utility of the worker. The regulator would thus implement a regulation that (1) benefits the worker and (2) is not Pareto dominated.

We show that, when the regulator has complete information, minimum wage policies dominate overtime pay and caps on hours. This is because workers only want to reduce their labor if their implied wage is too low; if the wage is increased sufficiently, the problem vanishes. Moreover, an efficient minimum wage exists where both the firm and worker receive their preferred hours.

If the regulator has limited information about the market, he may be unable to implement the efficient minimum wage. In the standard model of monopsony, which we will refer to as the “flexible-hours” model, the worker and firm agree to each hour worked at a wage set by the firm. This flexible-hours model yields a strictly increasing relationship between agreed-upon hours and total surplus. As a result, any minimum wage below the efficient minimum wage is “better than nothing” in the sense that it benefits workers and increases total surplus above the market level. Thus, even if the regulator is unable to implement the first-best regulation, he still prefers any reasonable lower bound on the minimum wage to the free-market outcome.

This is not the case in the ultimatum model. If the minimum wage is binding but below the efficient minimum wage, the firm will select more hours than is efficient in an attempt to “claw back” the additional surplus the worker derives from higher wages. Consequently, the minimum wage that maximizes hours actually *minimizes* total surplus locally.[4] This implies a suboptimal minimum wage is not necessarily

[3]Ultimatum bargaining is just a special case of the bargaining studied in this paper.

[4]It is not the global minimum. Total surplus is positive at this point, yet a sufficiently large

"better than nothing": total surplus is lower, worker surplus may be lower, and the overall allocation may be strictly Pareto dominated by that of the unregulated market.

This fragility of the minimum wage as a welfare improving policy motivates our interest in robust regulation that always increases the worker's utility even when the regulator's information is quite limited.

We consider a framework where the regulator has no prior knowledge of production and costs, but: (1) observes the contract that prevails in the market before regulating, and (2) knows a specific reduction in hours at the implied wage which benefits the worker. Given this information, it is clear that reducing hours to this specific quantity at the current wage would make the worker better off, regardless of the true production and disutility from labor.

However, this reduction in hours is actually weakly dominated: that is, alternative policies exist which provide weakly greater payoffs for both the worker and firm in every possible state and strictly larger payoffs in some. Two insights demonstrate this observation. First, observing the market state enables the regulator to bound the worker's disutility from labor. Second, an inflexible reduction in hours is weakly dominated by any policy that guarantees the worker enough additional pay to compensate for the additional hours. Intuitively, if the firm is willing to pay the worker sufficiently in exchange for an additional hour, it does not make sense to block this transaction.

The question then is to determine what rate of compensation is "sufficiently large". The idea is to combine a minimum wage, overtime pay, and hours cap calibrated such that worker surplus will not be harmed. The minimum wage is set at the current implied wage, overtime pay begins at a preferred reduced number of hours, and the hours cap is set at the hours that prevailed in the market before regulation. We show that this is the unique policy that both dominates the inflexible reduction in hours and is never ex-post Pareto dominated by any other policy with this same property.

Policies with this described shape are common. For example, Japan and France both combine overtime pay with a cap on hours. Overtime pay in countries with no caps on hours, such as the United States are also fundamentally similar because there is a natural cap on the number of hours that one can work in a week. Some real-world regulations are plotted in Figure 1 and can be compared to our optimal regulation in Figure 2.

minimum wage will result in zero labor (and therefore zero surplus).

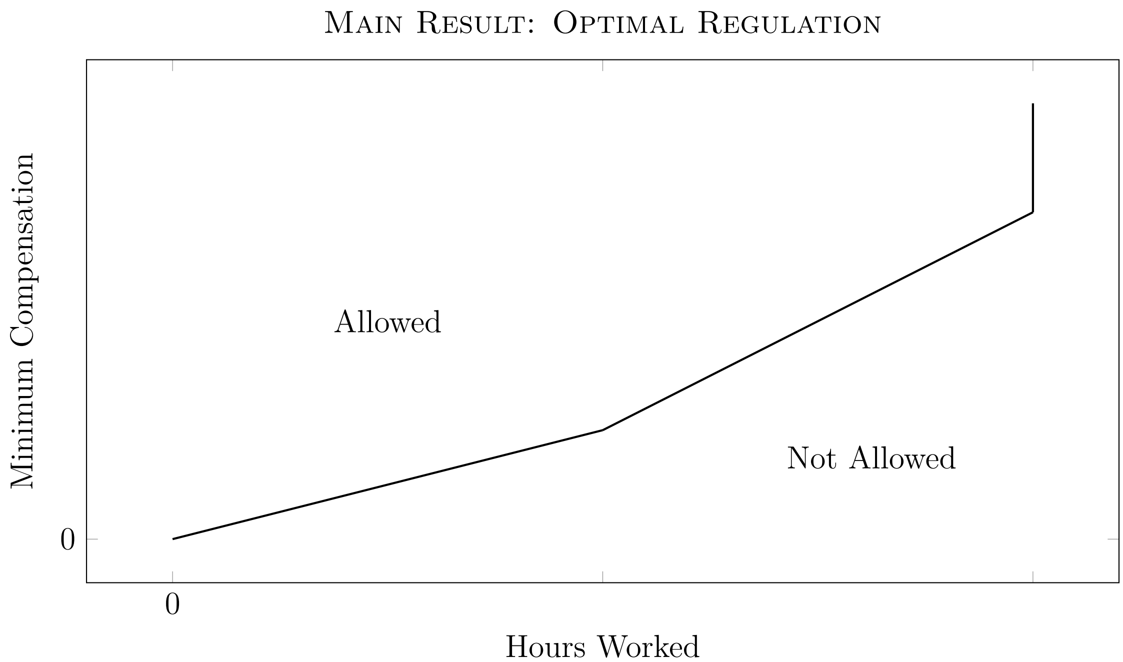

Figure 2: The never Pareto dominated satisficing policy combines a minimum wage, overtime pay, and a cap on hours. This policy guarantees the worker at least as much utility as the contract at the "kink".

This paper is the first to theoretically study the effects of regulation in a setting where hours and compensation are jointly contracted. Our analysis contributes to a large theoretical and empirical literature on labor regulation. We additionally build on a small literature on the joint determination of hours and compensation. In particular, Manning (2005) analyzes the ultimatum model of monopsony without regulation.

This study also joins a growing literature on robust contracting. Our informational setting shares key elements with Carroll (2015), which considers a robust moral hazard problem where the principal: (1) knows at least one of the agent's available actions and (2) obtains an optimal contract which exploits the alignment of incentives between the principal and agent. In our setting, the regulator instead: (1) knows one *regulation* which benefits the worker, instead of an action, and (2) obtains an optimal regulation which exploits efficiency,[5] which is analogous to aligned incentives in Carroll's context.[6]

[5]That is, bargaining between the worker and firm is Pareto optimal under the constraints of the regulation.

[6]Our framework can be applied to any robust delegation context. See Appendix 8.3.

The paper is organized as follows. In Section 2 we introduce a simple leading example to help fix ideas and build intuition about the main results. We present the formal model in Section 3. In Section 4, we obtain comparative statics results that can be applied to the regulation problem. The complete information setting is analyzed in Section 5.1. The case where the regulator lacks information about production and disutility is presented in Section 5.2. We explore extensions in Section 6 including the extension to heterogeneous workers in Section 6.1. In Section 7, we review the related literature and discuss the results.

2 Example: monopsony and minimum wage

To illustrate the our approach and the significance of our results for regulation, we present the model of monopsony that is common in the literature and contrast it with a simplified version of our model.

A monopsonist firm (it) contracts with a worker (she) to obtain hours of labor, ℓ, in exchange for a transfer, τ. From any given contract (ℓ, τ), the firm receives profits, $f(\ell) - \tau$, where f is a strictly concave, differentiable production function. The worker receives payoff $\tau - c(\ell)$, where c is a strictly convex, differentiable, and increasing labor cost function. Without loss, $f(0) = c(0) = 0$. We additionally make the standard assumption that $c'(x)x$ is convex.[7]

The hourly wage is the ratio of transfers to hours, or $w \equiv \tau/\ell$. The worker is *overworked* if she is working more hours than she would prefer to at the given wage: $w < c'(\ell)$. The worker is *underworked* if she would prefer to work more hours at the given wage, or $w > c'(\ell)$. The unique Pareto efficient number of hours, ℓ^*, equates marginal productivity and marginal cost: $f'(\ell^*) = c'(\ell^*)$. The worker could be overworked, underworked, or neither at the efficient quantity of labor.

Flexible-hours model The canonical Stigler (1946), henceforth flexible-hours, model of monopsony under a minimum wage is the standard model used in the literature. The firm makes some quota of employment hours, $\bar{\ell}$, available to the worker at an hourly wage, w. The worker then chooses to provide $\ell \leq \bar{\ell}$ hours of labor.

[7] This assumption is important for the standard model of monopsony considered below. Without this assumption, Loertscher and Muir (2021) show that a menu of stochastic wages may be optimal for the firm. The assumption is not relevant to our model.

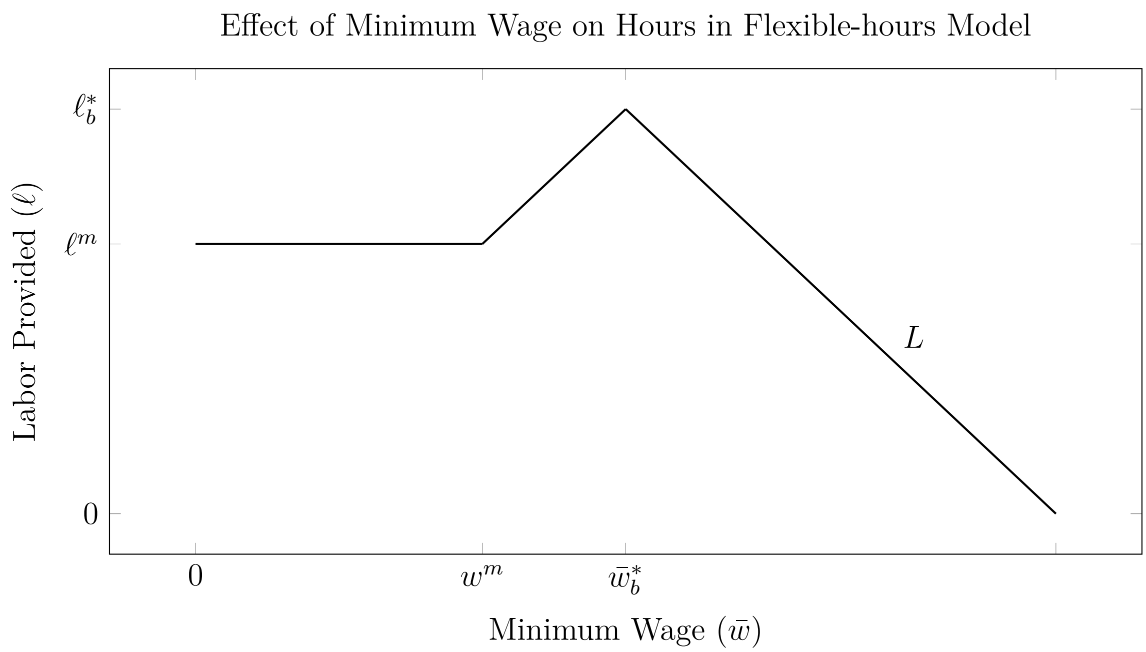

Figure 3: Plot of labor response curve, L for the flexible-hours model. For any minimum wage $\bar{w} \in (c'(\ell^m), c'(\ell_b^*))$, the worker is employed for a quantity of hours between the market employment and the efficient level of employment (the inverse of the marginal cost). The minimum wage $\bar{w}_b^* = c'(\ell_b^*) = f'(\ell_b^*)$ implements efficient employment. Labor is decreasing in the minimum wage for wages above $\bar{w}_b^*$ because the firm's limit on work hours is binding.

Overwork is impossible in this model because the worker can always choose to work fewer hours.

In the absence of a minimum wage, the firm equates marginal productivity and marginal expenditure, yielding ℓ^m work hours – a quantity notably below the efficient, surplus maximizing, amount. The introduction of a minimum wage $\bar{w}$ then improves total welfare by helping increase labor, as the higher compensation incentivizes workers to supply longer hours. This movement happens up until the worker is paid her marginal productivity, or $\bar{w}^* \equiv c'(\ell^*) = f'(\ell^*)$. Only beyond this level do we have that further increases in $\bar{w}$ become counterproductive. This relationship between the minimum wage and labor (i.e., the labor response curve, L) is plotted in Figure 3.

Note that, in this model, the minimum wage that maximizes labor among any set of minimum wages also maximizes total surplus in that set. This suggests that, if the introduction of a minimum wage has increased the number of hours worked, then the total surplus has inevitably improved – even if no other information about costs and production processes is available. This is a powerful relationship in terms of policy evaluation, and offers a clear guide to regulators considering introducing or altering the minimum wage. However, as we will show next, this intuition depends crucially on accepting that the worker has total flexibility in reducing her own hours. Without this supposition, the conclusions may be reversed.

Ultimatum model We model bargaining where hours are determined jointly with compensation. A simple case of this is ultimatum bargaining where the firm submits a "take it or leave it" offer consisting of hours and compensation. If the worker rejects the offer, then she receives an outside option of zero. The timing is:

1. The regulator determines the minimum wage, $\bar{w}$.
2. The firm (which knows c) announces a "take it or leave it" offer to the worker (ℓ, τ) where ℓ denotes the total amount of hours and τ denotes total compensation.
3. The worker chooses to accept the contract, or take an outside option of $(0, 0)$.
4. The firm receives
$$\pi(\ell, \tau) = f(\ell) - \tau,$$
and the worker obtains payoff
$$u(\ell, \tau) = \tau - c(\ell).$$

This game has a unique subgame perfect Nash equilibrium. The equilibrium contract solves the firm's profit maximization problem:

$$\max_{\ell,\tau} f(\ell) - \tau \text{ s.t. } \tau \geq c(\ell) \text{ and } \tau \geq \bar{w}\ell. \tag{1}$$

If $\bar{w} = 0$ (i.e., if there is no minimum wage) the firm extracts all surplus from the worker, and labor hours are efficient. The wage is equal to the worker's average cost, $w^m = {}^{c(\ell^*)}/_{\ell^*}$, implying the worker would prefer fewer hours (since c is convex, average

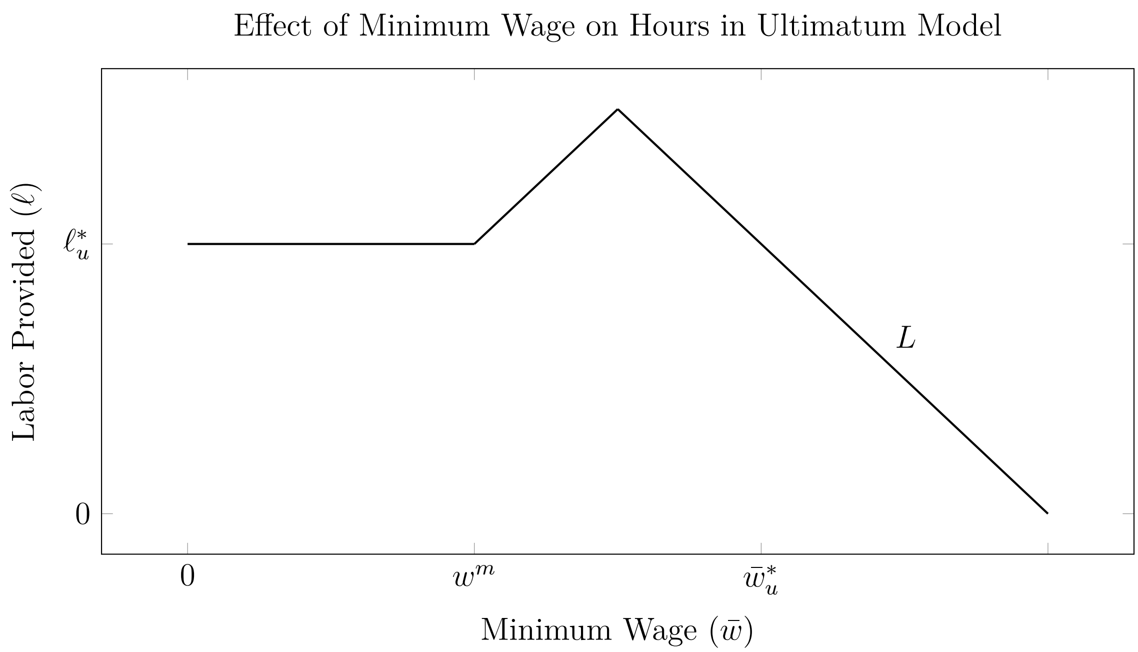

Figure 4: Plot of labor response curve, L for the ultimatum model. It has the same shape as the flexible-hours model in Figure 3. However, for any minimum wage $\bar{w} \in (c(\ell_u^*)/\ell_u^*, c'(\ell_u^*))$, the worker is employed for a number of hours that exceeds the efficient quantity. Therefore, the minimum wage that maximizes hours does not implement an efficient allocation.

costs are lower than marginal costs). In other words, she is overworked.[8]

The labor response curve, L, for the ultimatum model is shown in Figure 4. The curve for the flexible-hours model (plotted in Figure 3) has the same shape. In fact, the two curves are indistinguishable, as the following claim demonstrates.

Claim 1. *A flexible-hours model with cost, c_b, generates the same labor response curve as an ultimatum model with the same production function and cost, c_u, where*

$$c_u(x) \equiv c_b'(x)x.$$

This is significant because empirical studies on labor regulation generally focus on measuring some part of the labor response curve – often the change in labor after an

[8]In the more general model presented in Section 4, overwork in the absence of regulation is a necessary and sufficient condition for the market to behave similarly to the ultimatum model in response to regulation.

increase in the minimum wage. Suppose that the regulator has access to the *entire* labor response curve but not f, c, or even whether this labor response comes from the flexible-hours or ultimatum model. Due to Claim 1, the regulator cannot identify the true model without knowledge of the worker's disutility from labor, c. For any labor response curve, L, and production function, f, there is one disutility, c_b, that is consistent with the flexible-hours model and another disutility, c_u, that is consistent with the ultimatum model.

Of course, the two models are *not* equivalent for the worker, and the efficient minimum wage is markedly not the same. The unregulated market is already efficient in the ultimatum model: thus, a minimum wage that increases hours increases them *above* the efficient level. This is undesirable, an assertion we emphasize in the below claim.

Claim 2. *The efficient minimum wage for the flexible-hours model with cost c_b (i.e., $\bar{w}_b^*$) locally minimizes welfare in the ultimatum model with cost c_u.*

Claim 2 implies that a regulator who assumes the flexible-hours model and has sufficient knowledge of L to implement the efficient minimum wage under this model will locally minimize welfare in the case that the ultimatum model holds.

The next claim further demonstrates why knowledge of the labor response curve is not sufficient for optimal regulation.

Claim 3. *For any labor response curve, L:*

- $c_b(x) < c_u(x)\ \forall x > 0$*;*
- $\bar{w}_b^* < \bar{w}_u^*$*;*
- $\ell_b^* > \ell_u^*$*; and*
- *for $i \in \{b, u\}$ and all $\bar{w} \geq 0$,*

$$\left.\frac{d\, f(L(w)) - c_i(L(w))}{dw}\right|_{w=\bar{w}} > 0 \implies \left.\frac{d\, f(L(w)) - c_{-i}(L(w))}{dw}\right|_{w=\bar{w}} < 0.$$

The last point of Claim 3 implies that total surplus is always weakly decreasing in the minimum wage in at least one of the two models. This poses an impossibility for robust regulation using L. However, there is information other than L that can be used to regulate. For example, if the regulator knew that the worker was overworked, he could reject the flexible-hours model.

Overwork information is not only useful for determining the true model, but also for finding whether the current minimum wage is above or below the efficient one.

Claim 4. *For $i \in \{b, u\}$, w_i^* is the largest minimum wage such that the worker is not underworked in model i.*

Claim 4 demonstrates the importance of worker preferences in regulating labor. Knowing the actual level of labor that is achieved at each minimum wage is neither necessary nor sufficient for determining the efficient minimum wage. However, knowing whether the worker wants to work more or fewer hours at each minimum wage is sufficient.

3 Model

We adapt the model in Section 2 to allow for more general bargaining and regulation over hours and compensation. The players, payoffs, and definitions remain the same.

Contracts take the form $(\ell, \tau) \in \mathbb{R}_+^2$ where ℓ is hours of labor and τ is a gross payment to the worker. The worker's wage is defined as the average payment per hour: $w \equiv \tau/\ell$.

Players The firm's payoff from a contract (ℓ, τ) is

$$\pi(\ell, \tau) \equiv f(\ell) - \tau.$$

Failing to hire the worker results in no labor or payment.

The worker's utility function is $u(\ell, \tau) \equiv \tau - c(\ell)$ where τ is the gross payment from the firm for the worker's labor and c is the worker's disutility from labor. The worker's outside option is no labor or payment.

Without loss of generality, $f(0) = c(0) = 0$. We additionally assume f, c are differentiable, c is weakly increasing and strictly convex, f is strictly concave, and there exists a $t > 0$ such that $f'(t) < c'(t)$.

Regulation A regulator imposes a regulation to constrain the contracting space. A regulation a function, $\phi : \mathbb{R}_+ \to [0, \infty]$, such that the worker and firm are restricted

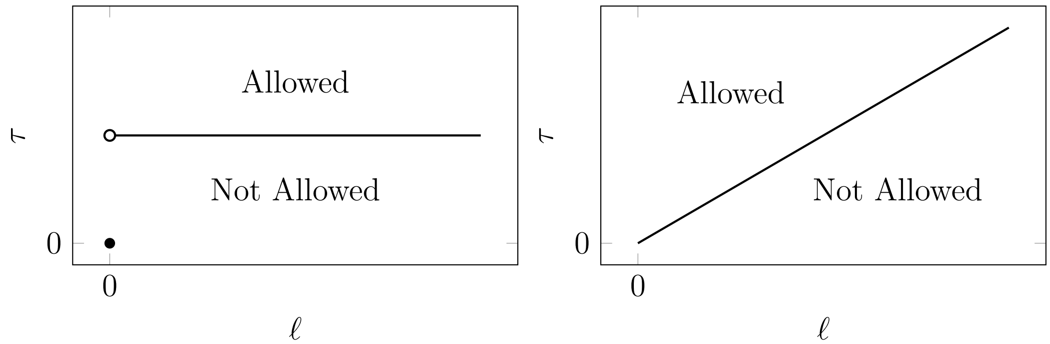

Figure 5: Two examples of policies. Only contracts above the lines are allowed. On the left, there is a minimum gross payment. On the right, there is a minimum wage.

to contracts with $\tau \geq \phi(\ell)$. We require that ϕ is weakly convex and $\phi(0) = 0$. In other words, the contracting space is a convex set which contains the disagreement point.[9] This allows for policies such as overtime pay and caps on hours.

If $\phi(\ell) \equiv 0$ for all ℓ, there is no regulation. A linear regulation, $\phi(\ell) \equiv \bar{w}\ell$, is a *minimum wage regulation* with *minimum wage*, $\bar{w}$.

Bargaining We assume that the contract (ℓ, τ) solves the following optimization problem:

$$\max_{\ell,\tau} M(f(\ell) - \tau, \tau - c(\ell)) \text{ s.t. } \tau \geq \phi(\ell)$$

where $M : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}$ is the *bargaining objective*. We make the following assumptions about M.

Assumption 1 (Weak monotonicity)**.** *For all $x, y, x', y' \in \mathbb{R}_+$ such that $x' > x$ and $y' > y$,*

$$M(x', y') > M(x, y).$$

Assumption 2 (Strict quasiconcavity)**.** *For all $x, y, x', y' \in \mathbb{R}_+$ such that $x' \neq x$ and $y' \neq y$ and for all $\lambda \in (0, 1)$,*

$$M(\lambda x' + (1 - \lambda)x, \lambda y' + (1 - \lambda)y) > \min\{M(x, y), M(x', y')\}.$$

Assumption 3 (Continuity)**.** *The function, M, is continuous in both arguments.*

[9]This is a standard assumption in the bargaining literature. If the bargaining set were not convex, then there would exist two permitted contracts such that some convex combination of them is not allowed. In this case, a real-world firm might find it optimal to switch between the two permitted contracts in alternating weeks to approximate the disallowed contract. This is not desirable.

These assumptions admit the most popular bargaining models including ultimatum bargaining, asymmetric Nash bargaining, egalitarian bargaining, and the more general proportional bargaining of Kalai (1977).[10]

Timing The game has two stages.

1. The regulator announces regulation ϕ.
2. The firm and worker negotiate a contract (ℓ, τ) according to M.

4 Comparative statics results

In this section, we show that minimum wages are an especially important class of regulations and that ultimatum bargaining typifies the properties of other bargaining protocols. Because of this, the claims established in Section 2 for minimum wage regulation in the ultimatum framework extend to this more general setting.

We say a regulation is *redistributive* if it is better for the worker than no regulation. The following proposition demonstrates that for every redistributive regulation, there is a minimum wage that gives the worker at least as much utility while maintaining the same surplus.

Proposition 1. *Let ϕ be a redistributive regulation that implements ℓ. There exists a minimum wage, $\bar{w}$, that implements ℓ such that $\bar{w}\ell \geq \phi(\ell)$.*

Because of this, minimum wages are without loss of optimality for any fully informed regulator with an objective that is increasing in τ.

In the analysis of ultimatum bargaining in Section 2, a minimum wage of zero and the minimum wage $\bar{w}^* \equiv f'(\ell^*)$ are both *efficient* regulations in that they both implement the efficient quantity of hours, ℓ^*. This remains true in the general case.

However, in the ultimatum model: (1) the worker is overworked without regulation and (2) the efficient minimum wage, $\bar{w}^*$, is always redistributive. For a general bargaining protocol, it may be that neither holds. The following proposition demonstrates that one cannot hold without the other.

[10] Our results also extend to the choice theoretic bargaining of Peters and Wakker (1991).

Theorem 1. *Let ϕ be an efficient regulation that implements τ. The worker is overworked under ϕ if and only if there exists another efficient regulation ψ such that $\psi(\ell^*) > \tau$.*

In other words, the worker is overworked under an efficient regulation if and only if there is another efficient regulation which redistributes more to the worker. Overwork, which is the setting under which a regulation such as overtime pay is appealing, is also a necessary and sufficient condition for there to exist some "costless" redistribution in the sense that we can redistribute to the worker without sacrificing any total surplus.

For the forward direction, if the worker is overworked, her wage is less than her marginal cost of labor, which is precisely the efficient minimum wage, $\bar{w}^* \equiv f'(\ell^*) = c'(\ell^*)$.

Intuitively, the proof of the converse uses the fact that any bargaining protocol responds to regulation in a manner similar to the ultimatum model when labor hours are efficient. Because marginal productivity and marginal disutility are equal at the efficient labor, hours are an almost perfect substitute for the constrained transfers. Because of this, if a regulation ψ is redistributive and $\psi'(\ell^*) \neq f'(\ell^*)$, some total surplus will always be traded off to increase firm profits. By convexity, a regulation with $\psi'(\ell^*) = f'(\ell^*)$ cannot result in a wage that exceeds the workers marginal productivity, $f'(\ell^*)$.

Two convenient choices of ϕ in Theorem 1 allow us to obtain new results and provide intuition for the theorem. First, if we set ϕ to zero, we obtain the following corollary.

Corollary 1.1. *There exists an efficient, redistributive regulation if and only if the worker is overworked under the free-market contract.*

Corollary 1.1 is a special case of Theorem 1 which uses the fact that the free-market is efficient. Using the efficient minimum wage $\bar{w}^*$ as ϕ yields a second corollary.

Corollary 1.2. *If ψ is an efficient, redistributive regulation, then $\psi(\ell^*) \leq f'(\ell^*)\ell^*$.*

Corollary 1.2 implies that $\bar{w}^*$ maximizes worker utility in the class of efficient policies. Corollary 1.2 follows from the fact that $f'(\ell^*)$ is equal to (and thus not less than) $c'(\ell^*)$. This characterizes the maximal efficient, redistributive minimum wage. However, we have yet to describe the effects of any other minimum wages.

In Section 2, any minimum wage between the free-market wage and $f'(\ell^*)$ implement labor that exceeds the efficient quantity, ℓ^*. This holds true in the general model when the worker is overworked in the absence of regulation. Otherwise, these minimum wages are below the free-market wage and thus do not bind.

In the case of overwork, the shape of the labor response curve is similar to that of the ultimatum model. That is, overwork is necessary and sufficient for the market to behave "like ultimatum bargaining" in response to regulation.

Corollary 1.3. *Let $L : \mathbb{R}_+ \to \mathbb{R}_+$ define the level of labor at each minimum wage and $w_0 \in [c(\ell^*)/\ell^*, c'(\ell^*))$ be the wage in the absence of regulation. Then, the labor response function, L, is continuous and*

$$L(x) \begin{cases} = \ell^* & \textit{if } x \in [0, w_0] \\ > \ell^* & \textit{if } x \in (w_0, f'(\ell^*)) \\ = \ell^* & \textit{if } x = f'(\ell^*) \\ < \ell^* & \textit{if } x \in (f'(\ell^*), f'(0)) \\ = 0 & \textit{if } x \geq f'(0). \end{cases}$$

First, labor is constant when the minimum wage is too low to bind. Second, hours exceed the efficient quantity when the minimum wage is between the market wage and $\bar{w}^*$. Therefore, the minimum wage that maximizes labor is inefficient. Finally, $\bar{w}^*$ is the unique efficient, redistributive minimum wage. So, any larger minimum wage implements fewer hours than is efficient. Therefore, the analysis in Section 2 is relevant for any bargaining protocol that results in overwork.

The general setting allows for new effects on the surplus of workers and firms. For example, it is possible for a minimum wage below $\bar{w}^*$ to *strictly* decrease the surplus of both the worker and the firm.

Example 1 (Egalitarian bargaining)**.** The worker and firm split the market surplus evenly. So, the market is described by

$$\max_{\ell,\tau} \min\{f(\ell) - \tau, \tau - c(\ell)\} \text{ s.t. } \tau \geq \phi(\ell).$$

Let $-c$ be "more concave" than f on $[0, \ell^*]$ in the sense that $f(\ell^*) - f'(\ell^*)\ell^* < c'(\ell^*)\ell^* - c(\ell^*)$. This condition is necessary and sufficient for overwork.

In the absence of regulation, $\tau_0 = \frac{f(\ell^*)+c(\ell^*)}{2}$ and profits and worker utility are both equal to half the maximum total surplus, $\frac{f(\ell^*)-c(\ell^*)}{2}$.

By assumption, the worker is overworked in equilibrium. As the minimum wage increases above the free-market wage, $w_0 = \frac{f(\ell^*)+c(\ell^*)}{2\ell^*}$, labor will increase to keep profits and worker utility equal. This will occur until the minimum wage reaches $f'(z)$ where z is the smallest solution to $\frac{f(z)+c(z)}{2z} = f'(z)$. For all minimum wages between w_0 and $f'(z)$, the worker's welfare and the profits of the firm are lower than in the unregulated state. This is because each is taking the same share of a smaller pie.

For any minimum wage above $f'(z)$, the worker obtains more than half of the total surplus. So, labor is set to maximize profits. As a result, the equilibrium contract at each minimum wage is the same as in the ultimatum model. $\triangle$

Example 1 shows the effect that minimum wages below the efficient minimum wage can have on the market. If a binding minimum wage benefits the worker, it is because it increases the worker's share of total surplus enough to compensate for the weak reduction in total surplus imposed by the policy.[11] In Example 1, any minimum wage in $(w_0, f'(z)]$ reduces total surplus without affecting the worker's share. As a result, both the worker and firm are strictly worse off under these regulations.

5 Optimal regulation

We now apply the results of Section 4 to the problem of a regulator. We restrict attention to settings where the worker is overworked in the absence of regulation. By Theorem 1, this is the same as restricting attention to the setting where the worker's wage is below her marginal cost.

5.1 Regulation with complete information

Suppose that the regulator knows f, c, and M and maximizes a weighted sum of worker utility and firm profits with more weight on workers. That is, the regulator's objective is to choose the ϕ that maximizes

$$\alpha u(\ell, \tau) + (1-\alpha)\pi(\ell, \tau) = \alpha(\tau - c(\ell)) + (1-\alpha)(f(\ell) - \tau)$$

[11] It is a strict reduction for any minimum wage other than $f'(\ell^*)$.

for $\alpha \in (0.5, 1]$. The case where $\alpha \to 0.5$ is of special interest. In this case, the regulator is not willing to sacrifice any total surplus to improve the welfare of the worker.

In this setting, Proposition 1 guarantees that any regulation can be weakly improved upon with a minimum wage. With this, it is straightforward to find an optimal policy.

Theorem 2. *Any minimum wage in $w^*(\alpha)$ where*

$$\begin{aligned} w^*(\alpha) \equiv \underset{w \geq f'(\ell^*)}{\arg\max} \quad & \alpha(w\ell - c(\ell)) + (1-\alpha)(f(\ell) - w\ell) \\ s.t. \quad & \ell = \arg\max_{l} M(f(l) - wl, wl - c(l)) \end{aligned}$$

is a redistributive optimal regulation for $\alpha \in (0.5, 1]$.

The assumption that the worker is overworked ensures that the optimal minimum wage is redistributive. If the worker is not overworked, minimum wage regulation is still without loss of optimality. However, it may be optimal not to regulate.[12]

Note that Theorem 2 does not imply that the optimal minimum wage is unique. This is further explored in Example 2.[13]

Example 2. Suppose that bargaining is proportional as in Kalai (1977) with proportion $\beta \in [0, 1]$ of total surplus going to the worker. The equilibrium contract solves:

$$\max_{\ell,\tau} \min\{(1-\beta)(f(\ell) - \tau), \beta(\tau - c(\ell))\} \text{ s.t. } \tau \geq \phi(\ell).$$

Note that this admits ultimatum bargaining ($\beta = 0$) and egalitarian bargaining ($\beta = 0.5$) as special cases.

Let $-c$ be sufficiently "more concave" than f on $[0, \ell^*]$ in the sense that $(1 - \beta)(f(\ell^*) - f'(\ell^*)\ell^*) < \beta(c'(\ell^*)\ell^* - c(\ell^*))$. This condition is necessary and sufficient for overwork.

Because the free-market is efficient, regulation cannot increase total surplus. So, any redistributive regulation must give the worker more than β of the total surplus.

[12] For example, if the worker chooses both hours and pay, the worker will extract all of the surplus in the market. The free-market outcome is clearly optimal in this case.

[13] Example 3 (in the Appendix) demonstrates another sort of multiplicity where there are multiple workers and different wages benefit different workers.

Therefore, as in Example 1, labor is chosen to maximize profits in any redistributive regulation.

Because the worker is overworked in the absence of regulation, the minimum wage $f'(\ell^*)$ is redistributive. Therefore, labor is chosen to maximize profits for all minimum wages greater than or equal to $f'(\ell^*)$.

Therefore, the equation in Theorem 2 is

$$\begin{aligned} \arg\max_{w \geq f'(\ell^*)} \quad & \alpha(w\ell - c(\ell)) + (1-\alpha)(f(\ell) - w\ell) \\ \text{s.t.} \quad & \ell = \arg\max_{l} f(l) - wl. \end{aligned}$$

The constraint simplifies to $w = f'(\ell)$. So, any optimal minimum wage satisfies $w^*(\alpha) = f'(\ell_\alpha)$ where

$$\ell_\alpha \in \arg\max_{l < \ell^*} (2\alpha - 1)f'(l)l + (1-\alpha)f(l) - \alpha c(l). \tag{2}$$

△

There may be multiple maxima that satisfy (2). Consequently, more assumptions are required to ensure that the optimal minimum wage for a given α is unique. In the case of Example 2, it is sufficient to assume that $f'(x)x$ is concave.

5.2 Robust regulation

Suppose the regulator does not observe production, costs, or the bargaining protocol. Moreover, he has no prior over these objects. Instead, the regulator observes: (1) the equilibrium contract, (ℓ_0, τ_0), that prevails prior to the regulation and (2) reduced hours $\hat{\ell} < \ell_0$ that the worker would prefer at the average wage, $w_0 \equiv \tau_0/\ell_0$.

Therefore, the regulator is aware of a contract, $(\hat{\ell}, w_0\hat{\ell})$, which the worker prefers to the status quo. We are interested in regulations that are *satisficing* in that they guarantee at least as much utility for the worker as this contract.

Definition 1 (Satisficing)**.** A regulation is satisficing if

$$\begin{aligned} \inf_{(M,f,c) \in I(\ell_0,\tau_0)} \quad & (\tau - c(\ell)) - (w_0\hat{\ell} - c(\hat{\ell})) \geq 0 \\ \text{s.t.} \quad & (\ell, \tau) = \arg\max_{l,t} M(f(l) - t, t - c(l)) \text{ s.t. } t \geq \phi(l) \end{aligned}$$

where $I(\ell_0, \tau_0)$ is the set of possible M, f, c that are consistent with ℓ_0, τ_0.

The satisficing property ensures that the worker weakly benefits from the market over the outcome where $(\hat{\ell}, w_0\hat{\ell})$ is the only contract allowed. A cap of $\hat{\ell}$ hours combined with a minimum wage of w_0 is clearly satisficing, but it is not the only such policy. There is an opportunity for improvement through refinement.

Intuitively, a more flexible policy – one permitting the worker to work additional hours when she is paid enough to find this additional labor desirable – is better than a sharp cap. The following definition formalizes this rationale.

Definition 2 (Never Pareto Dominated)**.** A regulation, ϕ, is Never Pareto Dominated (NPD) by ψ if there does not exist an $(M, f, c) \in I(\ell_0, \tau_0)$ such that the outcome of ϕ Pareto dominates the outcome of ψ.

Because bargaining satisfies weak Pareto, a more flexible regulation allowing more hours in exchange for compensation is NPD by the sharp cap on hours in any state of the world. However, there are many states of the world under which the more flexible policy Pareto dominates the cap.

We are interested in finding a policy that is both satisficing and NPD by any other satisficing policy. The following theorem identifies such a policy and guarantees that it is unique.

Theorem 3. *There is a unique policy which is satisficing and NPD by any satisficing regulation. It is*

$$\phi^*(x) \equiv \begin{cases} w_0 x & \textit{if } x \leq \hat{\ell} \\ w_0\hat{\ell} + \frac{w_0\ell_0}{\ell_0 - \hat{\ell}}(x - \hat{\ell}) & \textit{if } \hat{\ell} < x \leq \ell_0 \\ \infty & \textit{if } x > \ell_0. \end{cases}$$

Theorem 3 establishes that the unique NPD satisficing regulation, ϕ^*, is a combination of a minimum wage, overtime-pay, and a cap on hours. This regulation is plotted in Figure 2. Each of the three policies are very common and a similar combination of the three exists in Japan and France.

The NPD satisficing regulation is the most flexible satisficing policy as it is the pointwise minimum of all satisficing policies.[14] Intuitively, the NPD satisficing regulation allows $(\hat{\ell}, w_0\hat{\ell})$ as well as all contracts which the regulator *knows* that the

[14]Put another way, the bargaining set is the union of all satisficing bargaining sets. For a general delegation setting, we demonstrate that the union of satisficing sets is satisficing in Appendix 8.3.

worker prefers. This knowledge comes from the fact that the regulator can bound the worker's disutility from labor. The bound is obtained from the worker's individual rationality constraint at the initial contract, (ℓ_0, τ_0). That is, the regulator can bound the amount of additional income that the worker needs to work additional hours because he saw the worker work these hours in exchange for pay.

We now give a sketch of the main technique used in the proof to construct the upper bound on the worker's disutility from labor. Any violation of satisficing would come from contracts with labor in $(\hat{\ell}, \ell_0]$ that the worker likes less than $(\hat{\ell}, w_0\hat{\ell})$. We will show that any such contract is forbidden.

The worker weakly prefers a contract (x, y) to $(\hat{\ell}, w_0\hat{\ell})$ where $x \in (\hat{\ell}, \ell_0]$ if and only if the additional payment she receives offsets the increase in work hours,

$$y - w_0\hat{\ell} \geq c(x) - c(\hat{\ell}). \tag{3}$$

We are able to bound the right hand side (the increase in labor costs) using convexity and individual rationality of (ℓ_0, τ_0).

$$c(x) - c(\hat{\ell}) < \frac{x - \hat{\ell}}{\ell_0 - \hat{\ell}}\left[c(\ell_0) - c(\hat{\ell})\right] < \frac{w_0\ell_0}{\ell_0 - \hat{\ell}}(x - \hat{\ell})$$

Where the last step uses $c(\hat{\ell}) > 0$, and the fact that individual rationality implies $c(\ell_0) \leq \tau_0 = w_0\ell_0$. Combining the above with (3) yields that the worker prefers (x, y) to $(\hat{\ell}, w_0\hat{\ell})$ if

$$y \geq w_0\hat{\ell} + \frac{w_0\ell_0}{\ell_0 - \hat{\ell}}(x - \hat{\ell}) = \phi^*(x).$$

The fact that this regulation is never Pareto dominated comes from the fact that it is the minimal satisficing regulation. Because bargaining satisfies weak Pareto, the least restrictive regulation is never Pareto dominated by more restrictive policies. We show in Appendix 8.3 that the set of satisficing delegation sets in general delegation problems forms an upper semi-lattice. As a result, the least restrictive regulation exists and is unique.

Note that in our complete information setting in Section 5.1, we found NPD satisficing regulations. Under complete information, maximizing a weighted sum of payoffs yields all of the policies that are NPD by *any* policy. Moreover, any policy that maximizes the objective for $\alpha > 0.5$ is satisficing. To see this, note that the efficient minimum wage, $\bar{w}^*$ is necessarily larger than w_0 because the worker is overworked. Recall that the worker obtains her preferred hours at $\bar{w}^*$. Therefore, $\bar{w}^*$ is satisficing because the utility that the worker obtains at this higher wage with her most preferred hours exceeds that of any hours, $\hat{\ell}$, at the lower wage. Recall that $\bar{w}^*$ is optimal for $\alpha \to 0.5$. Clearly, if $\alpha > 0.5$, more weight is placed on the worker. So, the minimum wages associated with these larger weights are also satisficing.[15]

6 Extensions

6.1 Heterogeneous workers

Thus far, we have assumed that a firm acquires the services of a single worker. This makes sense if: (1) regulation can be customized to each worker or (2) workers have similar labor preferences. The first is unusual. While the second is not necessarily true, it is nevertheless common to use a representative agent to represent players with heterogeneous preferences.

6.1.1 Complete information regulation under heterogeneity

Suppose that there are $N \geq 2$ types of workers employed by a firm. Let c_i denote the cost of the i-th worker type. The costs and the production function, f, satisfy A1-3. For convenience, order the types in terms of efficient labor hours. That is, for $j > k$, $\ell_j^* \geq \ell_k^*$ where $\ell_j^* \equiv \arg\max_z \{f(z) - c_j(z)\}$ and $\ell_k^* \equiv \arg\max_z \{f(z) - c_k(z)\}$. The workers contract in accordance with the ultimatum model.

Assume that the regulator treats all workers equally. That is, the objective of the regulator is

$$\alpha \left(\sum_{i=1}^{N} \tau_i - c_i(\ell_i) \right) + (1-\alpha) \left(\sum_{i=1}^{N} f(\ell_i) - \tau_i \right) = \sum_{i=1}^{N} (2\alpha - 1)\tau_i + (1-\alpha) f(\ell_i) - \alpha c_i(\ell_i)$$

where $\alpha \in (0.5, 1]$.

We analyze this case of heterogeneous workers under complete information in depth in Appendix 8.4. Here we highlight and discuss two significant results.

[15] The only other never Pareto dominated, satisficing regulations are some that maximize the objective for $\alpha = 0.5$. These are efficient regulations that redistribute less than $\bar{w}^*$.

The first result is that if only the utility of the worker matters to the regulator, heterogeneity essentially has no effect on the problem.

Proposition 2. *For $\alpha = 1$, there is at least one optimal regulation that is a minimum wage. Any optimal minimum wage is the same as the optimal minimum wage in a single worker problem. This single worker has the average cost of all workers with positive utility under the regulation.*

Proposition 2 means that the optimal minimum wage in the heterogeneous case is the optimal minimum wage for a worker with costs that are averaged across some subset of the workers. We can interpret this virtual worker with averaged costs as a representative agent.

The proof contains an algorithm to find all optimum minimum wages by checking all possible subsets of workers. Example 3 in the Appendix shows how to use the algorithm in practice. In the example, there are two optimal minimum wages which benefit different subsets of workers.

Intuitively, Proposition 2 holds because a redistributive regulation implements a contract that does not depend on workers' costs. As a result, all workers who benefit from a regulation receive the same contract.

Of course, there may be some workers who receive a different contract because the regulation is not redistributive for them. However, because of the ultimatum bargaining, these workers are not negatively impacted by the policy.

The firm, on the other hand, may be negatively impacted. Because of this, if the regulator cares about firms or efficiency, the optimal policy may not be a minimum wage.

Proposition 3. *Let $\alpha \to 0.5$. Suppose $\ell_N^* \neq \ell_{N-1}^*$. If $\min_{i=1}^{N-1} \{c_i(\ell_i^*)/\ell_i^*\} > f'(\ell_N^*)$, the optimal regulation is a minimum wage of $f'(\ell_N^*)$. Otherwise, consider*

$$\phi(x) \equiv \begin{cases} Conv(x) & \text{if } x \leq \ell_{N-1}^* \\ Conv(\ell_{N-1}^*) + f'(\ell_N^*)(x - \ell_{N-1}^*) & \text{if } x > \ell_{N-1}^*, \end{cases}$$

and Conv is the largest convex function to fit under $\{(\ell_i^*, c_i(\ell_i^*))\}_{i=1}^{N-1}$ with the restriction that the slope is capped at $f'(\ell_N^*)$. If $\phi(\ell_N^*) > c_N(\ell_N^*)$, then ϕ is an optimal regulation.

Proposition 3 demonstrates two important properties of these problems. First, the optimal regulation may not be a minimum wage. The reason that a piecewise linear regulation may be optimal in the heterogeneous worker setting is that it reduces the effects of regulation the contracts of workers with zero payoff. Such regulation only affects firms. So, the issue becomes less relevant as α increases. The second property is that minimum wage regulation is also optimal in settings where heterogeneity is sufficiently large. In this case, the problem is separable and the lowest cost worker can be regulated alone. In Appendix 8.4, Proposition 7 shows that a minimum wage is also optimal for all $\alpha \in (0.5, 1]$ (but not in the limit) when heterogeneity is sufficiently small.

6.1.2 Robust regulation under heterogeneity

The previous analysis uses the market state from just one worker to regulate. As a result, the regulation is only necessarily satisficing for this one worker. Extending the analysis to multiple workers is simple. One can construct the never Pareto dominated satisficing regulation for each and take the maximum pointwise.

Proposition 4. *Let ϕ_i^* be the never Pareto dominated satisficing regulation for worker i. Suppose that the request, $\hat{\ell}$, is below the labor of each worker and that all workers are overworked. The unique never Pareto dominated regulation which is satisficing for all workers is*

$$\phi^*(x) = \max_i \phi_i^*(x).$$

The proof is in Appendix 8.1.7. For $\ell > \hat{\ell}$, each regulation only allows contracts which the worker prefers to the request. If we take the maximum, all of the points which are allowed are preferred to the request. The only trick is to ensure that $\ell < \hat{\ell}$ is satisficing. This comes from weak Pareto and overwork combined with Theorem 1.

The same proof also demonstrates that an hours cap at τ which uses the maximum of all workers wages is also satisficing. The assumption that all workers are overworked is important to establish that ϕ^* is satisficing. If some workers are not overworked, then ϕ^* may not be satisficing. However, for the same reason, an hours cap would also not be satisficing. The argument that ϕ^* is a Pareto improvement over an hours cap remains true.

6.2 Manipulation in robust regulation

In Section 5.2, we assumed that the firm and worker do not foresee the regulation that will be imposed. Moreover, we assume that the regulator knows some lower level of labor, $\hat{\ell}$, which the worker prefers to the current level of labor. This level could be arbitrary or be an internal belief of the regulator. However, it may be appealing to elicit this value.

In practice, manipulation of regulation is typically prevented through *grandfathering*, the use of information which predates the discussion of regulation. For example, in most cap-and-trade systems, permits are allocated based on historical energy usage that predates the discussion of these environmental policies.[16] This section considers what can happen when this practice is infeasible.

6.2.1 Manipulation by workers

Suppose that workers have complete information and interact with firms according to the ultimatum model. If the regulator asks the worker how many hours she wants to work, the worker wants to solve

$$\begin{aligned} \max_{\hat{\ell}\leq\ell_0} \quad & w_0\hat{\ell} + \frac{w_0\ell_0}{\ell_0-\hat{\ell}}(\ell-\hat{\ell}) - c(\ell) \\ \text{s.t.} \quad & \ell = \arg\max_{l\in[0,\hat{\ell}]} f(l) - \frac{w_0\ell_0}{\ell_0-\hat{\ell}}(l-\hat{\ell}). \end{aligned}$$

The solution is interior because setting the request, $\hat{\ell}=0$ or $\hat{\ell}=\ell_0$ results in no binding regulation being implemented. This can be solved using standard envelope theorem arguments. We instead consider two extreme cases for intuition: (1) when total surplus is small and (2) when total surplus is large.

If total surplus is small, both marginal cost and marginal productivity are low. The firm will not pay overtime unless the overtime pay multiplier is sufficiently small. Because the worker prefers all of the overtime points to the preferred point by construction, the worker wants to make a report such that she can earn overtime. To make the overtime pay multiplier sufficiently small, the worker will request a low $\hat{\ell}$.

[16]This also applies to regulations considered for individuals. For example, the 2022 student loan forgiveness policy in the U.S. does not apply to any loans taken out less than two months before its announcement.

As a result, the regulator imposes little regulation when there is not much surplus to redistribute.

If total surplus is sufficiently large, both marginal cost and marginal productivity are high. The firm will pay overtime even when the overtime pay multiplier is large. To extract a larger payment, the worker will request a large $\hat{\ell}$. As a result, the regulator imposes a large overtime payment multiplier when there is a lot of surplus to redistribute, but most hours are worked without overtime.

The regulation does not require the worker to be strategic or have information about production. However, if the worker does have access to these inputs, they can be used to make the regulation better. In both cases, the worker makes a strategic decision which makes the regulator's bound on the disutility of labor more accurately reflect the worker's marginal costs.

6.2.2 Manipulation by firms

Manipulation before regulation Suppose that the firm predicts the regulation and adjusts the contract in the preexisting regulation to interfere with the mechanism. For simplicity, suppose that $\hat{\ell}$ is fixed.

The firm cannot benefit from paying the worker more than her costs. This makes the regulation more restrictive. As a result, the firm can only manipulate by adjusting labor. The firm solves

$$\max_{z \geq \hat{\ell}, \ell \leq z} f(\ell) - \frac{c(z)}{z}\hat{\ell} - \frac{c(z)}{z - \hat{\ell}}(\ell - \hat{\ell}).$$

We consider the same two extreme cases. If total surplus is small, both marginal productivity and marginal cost are low. The firm cannot make the overtime payment multiplier arbitrarily small without making the wage arbitrarily large. Therefore, the firm will not pay overtime. This implies $\ell = \hat{\ell}$. In this case, the firm wants to set z as small as possible to reduce the hourly wage that must be paid. In equilibrium, $z = \ell = \hat{\ell}$.

If total surplus is large, both marginal productivity and marginal cost are large. In this case, a large choice of labor in the preexisting market is costly. The firm will end up paying overtime for all available hours.[17] In this case, the firm will set $\ell = z$.

[17] Note that this case also applies when $\hat{\ell}$ is sufficiently small relative to ℓ^*. This is because the firm's objective is decreasing in z for $\hat{\ell}$ sufficiently small.

The firm's problem can then be rewritten as

$$\max_z \; f(z) - \frac{c(z)}{z}\hat{\ell} - c(z).$$

Therefore, the firm hires the worker for fewer hours than is optimal in advance of the regulation. Intuitively, the firm takes into account the effect that an increase in labor will have on the overtime multiplier.

Manipulation after regulation Suppose that the firm is regulated but wants to prevent future regulation. When it chooses the contract, it takes into account that the regulator will obtain information from the prevailing contract.

In the ultimatum model, paying overtime to a worker reveals that the marginal productivity of the worker is $w_0 \frac{\ell_0}{\ell_0 - \hat{\ell}}$. As a result, the regulator knows that a minimum wage of $w_0 \frac{\ell_0}{\ell_0 - \hat{\ell}}$ will not affect total surplus, but will increase the surplus of the worker.

As a concrete example, suppose a firm in the U.S. hires a worker for ten dollars per hour for more than sixty hours per week. As a result, the firm must pay the worker time and a half for the last twenty hours that she works. If the regulator sees this, he knows that the firm would be willing to pay time and a half for the first forty hours as well. The regulator can use this information to impose a minimum wage of fifteen dollars per hour. This regulation costs the firm the equivalent of twenty hours of work each week.

This suggests that the regulator has a limited ability to improve on a labor cap if he cannot commit to the mechanism. The firm may be unwilling to offer any overtime to workers if doing so invites regulation.

7 Conclusion

We have explored regulation under very general assumptions in a setting where workers are overworked. We show that a minimum wage is the best tool that a fully informed regulator can use to alleviate this issue. Through a comparative statics exercise, we demonstrate that the minimum wage can hurt both workers and firms when it is set either too low or too high. We show that this is particularly important if regulators assume the flexible-hours model and try to interpret the effects of regulation on hours.

This issue of interpretation relates to the empirical literature on measuring the effects of minimum wage policies on hours (e.g., Jardim et al., 2022) and the effects of other policies such as overtime (Hamermesh and Trejo, 2000; Quach, 2020; Trejo, 1991). This paper proposes a framework which can be tested by and used in a welfare analysis of these policies using these empirical estimates.

Our study also cautions that the intensive margin (i.e., hours) and extensive margin must be treated differently with regards to regulation. For example, Jardim et al. (2022) shows that the 2014 increase of the minimum wage in Seattle did not significantly reduce employment, but did significantly reduce hours. Most would say that this is a bad sign. However, a reduction in hours may be good if these workers wanted their hours to be reduced. The objective of minimum wage regulation is not to maximize hours (or even take-home pay). The goal, broadly, is to improve the lives of workers. We demonstrate that this is at odds with hours maximization.

We find the overall optimal minimum wages when the regulator has complete information. Even if the regulator is not willing to sacrifice any total surplus to increase the worker's welfare, there exists a minimum wage that achieves this goal. This policy ensures an efficient market equilibrium where both the firm and worker receive their preferred number of hours.

This analysis joins two theoretical strands in labor economics. First, this paper is connected to the large literature on optimal regulation under imperfect competition with hours set by workers (e.g., Berger et al., 2022) or with fixed hours (e.g., Flinn, 2006; Loertscher and Muir, 2021). Secondly, our work is also related to the smaller literature on labor hours and overwork outside of a regulatory context (Feather and Shaw, 2000; Manning, 2005). The most novel innovation of our approach lies in combining these two strains by studying regulation in a setting with labor hours and overwork.

In addition to the complete information setting, we consider a regulator who has no prior over production and disutility from labor. This regulator instead observes the current market state and knows that a specific reduction in labor hours at the existing wage will benefit the worker.

This analysis contributes to the literature on robust implementation. Following Carroll (2015), this literature focuses on finding policies that can be implemented without any prior on the space of parameters. Guo and Shmaya (2019) study the problem of regulating an inefficient monopolist seller without any prior over supply and demand. Unlike our study, Guo and Shmaya (2019) consider a regulator who knows bounds on supply and demand.

In any robust analysis, the regulator must be able to somehow bound the unknown objects. In Carroll (2015), the principal is able to create an endogenous lower bound on the agent's technology from partial knowledge of the agent's set of available actions. Typically, these bounds are exogenous. This is troubling because the objects may be difficult for the regulator to bound in a reasonable way, and extremely permissive bounds generally produce unreasonable outcomes (e.g., arbitrarily large or small minimum wages).

We demonstrate a method for creating endogenous bounds on supply and demand when the regulator is able to observe the price and quantity that prevail in the market. To use this bound, we develop a new robust objective, the never Pareto dominated satisficing criterion. This objective is natural in any delegation problem. In general, the principal chooses the largest delegation set for which the principal does not regret the decision to delegate (i.e., the payoff is at least as high as the principal's preferred singleton delegation set). It is common for the principal and agent to have some inherent alignment of incentives such that this is beneficial. In our case, this alignment stems from the fact that our contract bargaining is Pareto efficient.

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8 Appendix

8.1 Proofs

8.1.1 Useful lemmas

Lemma 1. *The functional,*

$$\mathcal{L}[\phi] \equiv \arg\max_{l,t} M(f(l) - t, t - c(l)) \;\; s.t. \;\; t \geq \phi(l),$$

is sequentially continuous in the sense that $\phi_n \to \phi$ (under the Hausdorff metric) implies $\mathcal{L}[\phi_n] \to \mathcal{L}[\phi]$.

Proof. Suppose that M is continuous and satisfies weak Pareto, then $\mathcal{L}$ is continuous by Berge's theorem of the maximum. Continuity implies sequential continuity because the set of convex functions is a metric space under the Hausdorff metric.

In the setting of Peters and Wakker (1991), sequential continuity of bargaining ensures that the payoffs are sequentially continuous in ϕ. Continuity of hours and payment is immediate from continuity and strict concavity of the payoffs. □

Lemma 2. *A point on the unconstrained Pareto frontier of M lies above all points on the Pareto interior in some neighborhood of itself if and only if it is the overall optimum.*

Proof. This is immediate from continuity and strict quasiconcavity of M. The purpose of this lemma is to show this result under the assumptions of Peters and Wakker (1991): Pareto optimality, IIA, and sequential continuity.

First, note that the overall Pareto frontier is the curve in (π, u) space where total surplus is maximized. Because of transferable utility, this is a line with slope negative one. Without loss, say that the space of feasible payoffs is the simplex.

Let z be a point on the Pareto frontier that is above all points in the interior contained in an ϵ-ball centered at z for some $\epsilon > 0$. Then, construct an $\epsilon/2$-ball such that the point is at the right with a triangle removed as in the left of Figure 6. As δ decreases towards zero, the bargaining protocol must still choose z as it is the most preferred point. Continuity of the protocol implies that z must be chosen in the limit as well. Because this limit contains this segment of the Pareto frontier, z must exceed all points above it (preferred by the worker) on the frontier. The same argument can be applied for the points below using the $\epsilon/2$-ball to the right of Figure 6.

Therefore, z is a local maximum on the frontier. By quasiconcavity of M, it is the global maximum on the frontier. By monotonicity, it is the overall optimum. □

8.1.2 Proof of Proposition 1

Proof. Because ϕ improves the welfare of the worker, the policy is binding. Then, ℓ maximizes $M^\phi(x) \equiv M(f(x) - \phi(x), \phi(x) - c(x))$.

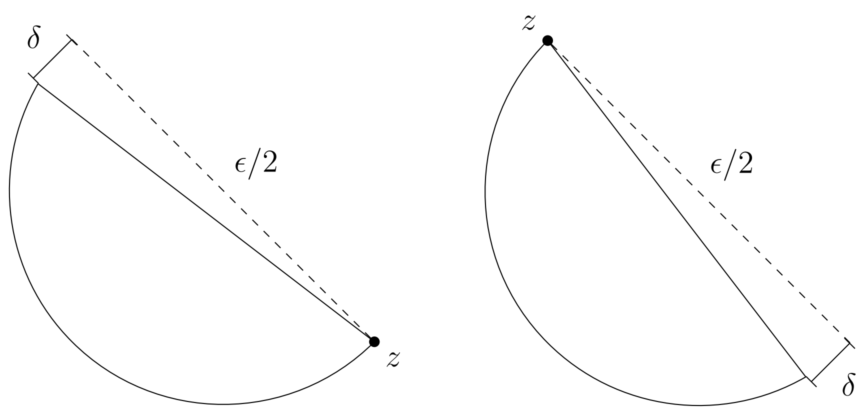

Figure 6: Figure of $\epsilon/2$-balls for Lemma 2. As $\delta \to 0$, this sequence approaches the full half circle, which contains a segment of the Pareto frontier.

By Lemma 1, the function,

$$L(w) \equiv \arg\max_{l} M(f(l) - wl, wl - c(l)),$$

is continuous. Because $L(f'(0)) = 0$, if a minimum wage implements labor hours, z, we can find a minimum wage to implement any labor hours less than z.

There are two cases.

In the first case, $\ell = \ell^*$. We established that $\bar{w} \equiv c'(\ell^*)$ implements ℓ^* and that $\phi'_-(\ell^*) \leq c'(\ell^*)$. The conclusion follows from convexity.

In the second case, $\ell \neq \ell^*$. Consider the minimum wage $\phi(\ell)/\ell$. Clearly, we are done if this also implements ℓ. If it implements labor greater than ℓ, continuity of L guarantees that an even larger minimum wage implements ℓ. Then, the only case left to check is that this minimum wage implements labor less than ℓ. This is impossible. Suppose by way of contradiction that the minimum wage does reduce labor. Then,

$$\arg\max_{x} M\left(f(x) - \frac{\phi(\ell)}{\ell}x, \frac{\phi(\ell)}{\ell}x - c(x)\right) < \ell.$$

However, this level of labor and transfer were available under ϕ (by convexity) and $(\ell, \phi(\ell))$ is available in the above. That both are the unique optima of their respective problems is a violation of IIA. $\square$

8.1.3 Proof of Theorem 1

Proof. The converse is trivial. If $\tau_0/\ell^* < c'(\ell^*) = f'(\ell^*)$, $\phi(x) = f'(\ell^*)x$ binds and implements ℓ^*.

For the forward statement, we know that the policy ϕ implements ℓ^* and improves the welfare of the worker. Therefore, $z \equiv (f(\ell^*)-\phi(\ell^*), \phi(\ell^*)-c(\ell^*))$ is the constrained optimum but is not the overall optimum. By Lemma 2, For any $\epsilon > 0$, there is an ϵ-ball around z such that z is not maximal. We want to show that that $f'(\ell^*) \geq \phi'_-(\ell^*)$.

Suppose, by way of contradiction, that $f'(\ell^*) < \phi'_-(\ell^*)$. Intuitively, decreasing ℓ in a neighborhood of ℓ^* is the same as relaxing the policy constraint because the marginal effect on profit and worker surplus are equal. As a result, there is an ϵ-ball around z available under ϕ, which is a contradiction. Formally, we can access the upper part of the ϵ-ball by transferring ϵ to the worker (because the constraint is in only one direction). We can access the lower part of the epsilon ball by decreasing ℓ by $\frac{\epsilon}{\phi'_-(\ell^*)-f'(\ell^*)}$:

$$f\left(\ell^* - \frac{\epsilon}{\phi'_-(\ell^*) - f'(\ell^*)}\right) - \phi\left(\ell^* - \frac{\epsilon}{\phi'_-(\ell^*) - f'(\ell^*)}\right) = f(\ell^*) + \phi(\ell^*) + \epsilon - \mathcal{O}(\epsilon^2)$$
$$\phi\left(\ell^* - \frac{\epsilon}{\phi'_-(\ell^*) - f'(\ell^*)}\right) - c\left(\ell^* - \frac{\epsilon}{\phi'_-(\ell^*) - f'(\ell^*)}\right) = \phi(\ell^*) - c(\ell^*) - \epsilon - \mathcal{O}(\epsilon^2).$$

This comes from taking a Taylor approximation near ℓ^* and using the fact that $f'(\ell^*) = c'(\ell^*)$. For ϵ sufficiently small, we can access any angle in the ϵ-ball on the Pareto interior. This is a contradiction.

Therefore, $c'(\ell^*) \geq \phi'_-(\ell^*)$. Therefore, for all $x \leq \ell^*$, $\phi(x) \leq c'(\ell^*)x$ by convexity. Because $\tau_o < \phi(\ell^*)$, we conclude $\tau_0/\ell^* < c'(\ell^*)$. Therefore, the worker is overemployed.

□

8.1.4 Proof of Corollary 1.3

Proof. Continuity comes from Lemma 1 and constant before the constraint binds is immediate from IIA.

We show the third point. The proof is similar to Theorem 1. At w_0, consider a small increase in the minimum wage. If the worker's incentive compatibility constraint binds, then the effect of the wage on the equilibrium is the same as under monopsony.

Therefore, $\psi'_+(w_0) = \frac{\ell^*}{c'(\ell^*)-w_0} > 0$.

Then, suppose the constraint does not bind, we know that labor cannot decrease because this is Pareto dominated. Therefore, it weakly increases at this point.

As in the proof of Theorem 1, increasing the labor perfectly counteracts an increase in the minimum wage in a neighborhood around the point where it first binds. We want to show that $w \in (w_0, f'(\ell^*))$ leads to $\ell > \ell^*$.

Suppose, by way of contradiction, a $w \in (w_0, f'(\ell^*))$ supports a contract $z \equiv (\ell^*, w\ell^*)$. By Lemma 2, for every $\epsilon > 0$, there is an ϵ-ball around z such that it is not maximal over all Pareto interior points. Therefore, there must be some angle within this ϵ-ball that cannot be accessed under minimum wage, w. However, this is not true. We can access the upper part of the ϵ-ball by transferring ϵ to the worker and can access the lower part of the ϵ-ball by increasing ℓ by $\frac{\epsilon}{f'(\ell^*)-w}$:

$$f\left(\ell^* + \frac{\epsilon}{f'(\ell^*)-w}\right) - \left(\ell^* + \frac{\epsilon}{f'(\ell^*)-w}\right)w = f(\ell^*) + w\ell^* + \epsilon - \mathcal{O}(\epsilon^2)$$
$$\left(\ell^* + \frac{\epsilon}{f'(\ell^*)-w}\right)w - c\left(\ell^* + \frac{\epsilon}{f'(\ell^*)-w}\right) = w\ell^* - c(\ell^*) - \epsilon - \mathcal{O}(\epsilon^2).$$

For ϵ sufficiently small, we can access any angle in the ϵ-ball on the Pareto interior. This is a contradiction.

The other points are justified in the statement of the theorem. □

8.1.5 Proof of Theorem 2

The proof consists of two steps. We first find the optimal minimum wages (rather than the optimal policies). We then use Proposition 1 to show that any optimal minimum wage is also an optimal policy.

First, find the optimal minimum wages. The expression for an optimal minimum wage is, by definition,

$$\begin{aligned} w^*(\alpha) \equiv \arg\max_w \quad & \alpha(\tau - c(\ell)) + (1-\alpha)(f(\ell) - \tau) \\ \text{s.t.} \quad & \ell, \tau = \arg\max_{l,t}\{M(f(l) - t, t - c(l)) \text{ s.t. } t \geq wl\} \end{aligned}$$

Second, simplify the expression. We would like to get rid of the minimum wage constraint and substitute $\tau = w\ell$. However, the minimum wage is not always binding. However, in optimizing, we can restrict attention to minimum wages greater than or equal to $f'(l^*)$ because: (1) this minimum wage maximizes total surplus and (2) a lower minimum wage redistributes less surplus to the worker because it involves less pay and more hours than desired by the worker.

Theorem 1 establishes that $f'(l^*)$ is a binding minimum wage. Therefore, the minimum wage constraint is binding on this restricted domain. Therefore, we can rewrite our expression to the one in the Theorem.

Finally, show that any optimal minimum wage is an optimal policy. By way of contradiction, suppose that an optimal minimum wage is not an optimal policy. Then, there exists a "superior" policy that strictly dominates the optimal minimum wage (in that it makes the objective larger). By Proposition 1, this "superior" policy is, itself, weakly dominated by a (possibly different) minimum wage. This is a contradiction because it implies that there is a minimum wage which strictly dominates an optimal minimum wage.

8.1.6 Proof of Theorem 3

Proof. We showed in the body of the paper that ϕ^* is satisficing. To repeat the logic, note that for all $x \in (\hat{\ell}, \ell_0]$,

$$c(x) - c(\hat{\ell}) < \frac{x - \hat{\ell}}{\ell_0 - \hat{\ell}} \left[c(\ell_0) - c(\hat{\ell}) \right] < \frac{w_0 \ell_0}{\ell_0 - \hat{\ell}} (x - \hat{\ell}) = \phi(x) - \phi(\hat{\ell}).$$

The increased transfers make up for the extra work. So, ϕ^* is satisficing.

We first show that any satisficing policy is greater or equal to ϕ^*. For this part, we can use the ultimatum model because it must be robust to any bargaining framework. The functions $f_\varepsilon(x) \equiv (w_0 + \varepsilon)x$ and

$$c_\varepsilon(x) \equiv \begin{cases} w_0 x - \frac{\varepsilon \ell_0}{2\hat{\ell}} x & \text{if } x \leq \hat{\ell} \\ w_0 x + \frac{\varepsilon}{2} (x - \ell_0) & \text{if } \hat{\ell} < x \leq \ell_0 \\ w_0 x + 2\varepsilon (x - \ell_0) & \text{if } x > \ell_0 \end{cases}$$

are feasible for all $\epsilon > 0$. Satisficing implies $\phi(x) \geq w_0 x$ for all $x < \hat{\ell}$ and $\phi(x) > w_0 x$ for all $x > \hat{\ell}$. However, if $\phi(\ell) > w_0 \ell$, for all ℓ there exists an ε such that the firm shuts down. So there must be some t such that $\phi(t) \leq w_0 t$ to be chosen in this case. For satisficing to hold, this implies $\phi(\hat{\ell}) = w_0 \hat{\ell}$. Now, consider $f_N(x) \equiv Nx$ and

$$c_N(x) \equiv \begin{cases} 0 & \text{if } x \leq \hat{\ell} \\ \frac{w_0 \ell_0}{\ell_0 - \hat{\ell}}(x - \hat{\ell}) & \text{if } \hat{\ell} < x \leq \ell_0 \\ w_0 \ell_0 + (N+1)(x - \ell^\star) & \text{if } x > \ell_0 \end{cases}$$

which are feasible. If there exists an $x > \hat{\ell}$ such that $\phi(x) < w_0 x + c_N(x)$, then there exists an N such that it will be chosen by the firm. Thus satisficing requires $\phi(x) \geq w_0 x + c_N(x)$ for all N. Therefore, satisficing implies that there must be a cap at ℓ_0. $\square$

8.1.7 Proof of Proposition 4

Proof. The function, $\phi^*(x)$, is convex (because it is the maximum of convex functions) and satisfies $\phi^*(0) = 0$. Therefore, it is a regulation. We now show that it is satisficing.

For $x \leq \hat{\ell}$, there is a minimum wage equal to the maximum wage paid to any worker. Because the worker is overworked, this wage is less than $f'(\ell^*) < f'(\hat{\ell})$. Therefore, the firm prefers $\hat{\ell}$ to any point below x. If the worker works less than $\hat{\ell}$ as a result of the regulation, it is because she prefers this (by weak Pareto). Therefore, any contract with $\ell \leq \hat{\ell}$ is satisficing under this policy.

For $x > \hat{\ell}$, there is a region that is the maximum of all of the policies for each worker. These policies were designed such that all allowed contracts in this region were weakly preferred to the requested contract for all workers. By taking the maximum, we ensure that every allowed contract is weakly preferred by all workers.

This policy is never Pareto dominated because it is minimal. Suppose, by way of contradiction, that a smaller policy existed which was satisficing for all workers, then it would be smaller at some points than the minimal satisficing policy of at least one worker. This is a contradiction.

The proof that the minimal policy is uniquely never Pareto dominated follows the same argument as Theorem 3. $\square$

8.2 Robust maxmin regulation

Suppose that a regulator with no knowledge of f, c regulates an ultimatum monopsony. The regulator knows that the assumptions on f, c hold. The objective of the regulator is

$$\begin{aligned} \max_{\phi} \inf_{f,c} \quad & \tau - c(\ell) \\ \text{s.t.} \quad & \ell = \arg\max_x f(x) - \max\{c(x), \phi(x)\} \\ & \tau = \max\{c(x), \phi(x)\}. \end{aligned}$$

The regulator has a lower bound on marginal productivity, $f'(x) \geq \bar{f}'(x)$, and an upper bound on marginal cost, $c'(x) \leq \bar{c}'(x)$ for all x. The regulator can make, $\bar{f}'$ weakly decreasing and $\bar{c}'$ weakly increasing using his knowledge that the underlying functions are concave. Assume that both bounds are left continuous.

Proposition 5. *A minimum wage with a labor cap is a maxmin policy. One such policy is defined by*

$$\phi(x) = \begin{cases} \bar{f}'(t)x & \text{if } x \leq t \\ \infty & \text{if } x > t \end{cases}$$

where $t \in \arg\max_z \bar{f}'(z)z - \bar{c}'(z)$.

Proof. If $\bar{f}'(0) \leq \bar{c}'(0)$, the infimum in the regulator's objective is zero. The worker's payoff is at least zero by the individual rationality constraint. So, any policy is maxmin.

Suppose that $\bar{f}'(0) > \bar{c}'(0)$. By left continuity, there exists at least one t that solves $t \in \arg\max_z \bar{f}'(z)z - \bar{c}'(z)$. A minimum wage of $\bar{f}'(t)$ maximizes total surplus when the bound holds exactly by granting the worker the contract $(t, \bar{f}'(t)t)$. In this state, this gives the worker a surplus of $\bar{f}'(z)z - \bar{c}'(z)$. It is not possible to achieve more than this in maxmin because the minimal state is at least as bad as this one.

To guarantee this return, we need to ensure that the firm does not choose a larger level of labor, $\ell > t$, such that $\bar{f}'(t) < \bar{c}'(\ell)$. We did not place an upper bound on marginal productivity. So, this needs to be guaranteed with some sort of convex policy. We could, for example, place a labor cap at t or at the largest q which satisfies $\bar{c}'(q) \leq \bar{f}'(t)$. It's also possible to integrate $\bar{c}'$ and add this to the wage after t. □

8.3 General robust delegation

The purpose of this section is to develop a general theory of never Pareto dominated satisficing delegation sets. We show that such a delegation set always exists and that a refinement can be used to obtain uniqueness. In particular, the least restrictive never Pareto dominated satisficing delegation set is unique.

Let $\mathcal{A}$ be a set of alternatives that a principal (e.g., regulator) and agent (e.g., aggregated labor market) may choose from. There is some state, $\theta \in \Theta$, that is known to the agent but unknown to the principal. This state may affect the payoff of both the principal and agent.

In order to elicit this information, the principal may choose a subset, $D \in \mathbb{D} \subset 2^{\mathcal{A}}$, to provide to the agent such that the agent makes a choice from D. This choice is defined by choice function, $\mathcal{C}_\theta : \mathbb{D} \to 2^{\mathcal{A}}$. This choice function is nonempty and satisfies the weak axiom of revealed preference, which we report in the form of Sen's α and β conditions, for each $\theta \in \Theta$.

Assumption 4 (Sen's α, IIA)**.** *If $x \in A \subseteq B$ and $x \in \mathcal{C}_\theta(B)$, then $x \in \mathcal{C}_\theta(A)$.*

Assumption 5 (Sen's β)**.** *If $x, y \in \mathcal{C}_\theta(A)$, $x \in \mathcal{C}_\theta(B)$, and $A \subseteq B$, then $y \in \mathcal{C}_\theta(B)$.*

These assumptions are typically used to obtain a revealed preference binary relation. In this case, we only impose the conditions on $\mathbb{D} \subset 2^{\mathcal{A}}$. This is less restrictive. For example, in this paper, Sen's β is vacuously true because all allowed delegation sets yield a unique choice.

The principal has a complete, but not necessarily transitive, weak relation: $\succeq_\theta$. He wants to choose a delegation set with two properties.

Definition 3 (Satisficing)**.** A delegation set, $D \in \mathbb{D}$ is satisficing with respect to $\hat{x} \in \mathcal{A}$ if for each $\theta \in \Theta$, there exists a $z \in \mathcal{C}_\theta(D)$ such that $z \succeq_\theta \hat{x}$.

Definition 4 (Never dominated)**.** A satisficing delegation set, $D \in \mathbb{D}$ is never Pareto dominated if for all satisficing delegation sets, $S \in \mathbb{D}$, and all $\theta \in \Theta$, $x \in \mathcal{C}_\theta(D) \implies x \in \mathcal{C}_\theta(D \cup S)$.

The satisficing criterion ensures that the outcome is at least as good for the principal and the never dominated condition imposes a refinement that we give the agent as much surplus as possible. In the setting of this paper, weak Pareto of the bargaining protocol implies that any never dominated delegation set is never Pareto dominated.

We want to show that there exists a delegation set that satisfies Definitions 3 and 4. We will do this by showing that there is a *least restrictive* satisficing set which contains all other satisficing delegation sets. This set is never dominated because the agent prefers larger delegation sets to smaller ones.

Lemma 3. *If the set of available delegation rules, $\mathbb{D}$, is closed under unions, then the set of satisficing delegation sets, $\mathbb{S} \subseteq \mathbb{D}$, is an upper semi-lattice under unions.*

Proof. We need to show that if S_1 and S_2 are satisficing, then $S_1 \cup S_2$ is satisficing. For each θ, consider each $x \in \mathcal{C}_\theta(S_1 \cup S_2)$. Because $x \in S_1 \cup S_2$, it is either in S_1, S_2, or both. Without loss, say it is contained in S_1. By Sen's α, $x \in \mathcal{C}_\theta(S_1)$. By Sen's β, $\mathcal{C}_\theta(S_1) \succeq \mathcal{C}_\theta(S_1 \cup S_2)$.

Because S_1 is satisficing, there exists $z \in \mathcal{C}_\theta(S_1)$ such that $z \succeq \hat{x}$. Because $z \in \mathcal{C}_\theta(S_1 \cup S_2)$, $S_1 \cup S_2$ is also satisficing. □

Proposition 6. *Suppose the set of available delegation rules, $\mathbb{D}$, is closed under unions and set of satisficing delegation sets, $\mathbb{S} \subseteq \mathbb{D}$ is nonempty. There exists a never dominated satisficing delegation set. Moreover, there exists a unique least restrictive never dominated satisficing delegation set.*

Proof. If there is only one satisficing delegation set, then we are done. It is never dominated.

If there is more than one, by Lemma 3, there exists an $S^* \in \mathbb{S}$ such that for all $S \in \mathbb{S}$, $S \subseteq S^*$. This delegation set is uniquely least restrictive and is never dominated because $S^* \cup S = S^*$. So, $x \in \mathcal{C}(S^*) \implies x \in \mathcal{C}(S^* \cup S)$. □

We assume in Proposition 6 that there is at least one satisficing delegation set. In many delegation settings, this will be $\{\hat{x}\}$. In the setting of this paper, it was the hours cap.

8.4 Complete information regulation under heterogeneity

Suppose that there are $N \geq 2$ types of workers employed by a firm. Let c_i denote the cost of the i-th worker type. The costs and the production function, f, satisfy A1-3. For convenience, order the types in terms of efficient labor hours. That is, for

$j > k$, $\ell_j^* \geq \ell_k^*$ where $\ell_j^* \equiv \arg\max_z f(z) - c_j(z)$ and $\ell_k^* \equiv \arg\max_z f(z) - c_k(z)$. The workers contract in accordance with the ultimatum model.

Assume that the regulator treats all workers equally. That is, the objective of the regulator is

$$\alpha \left(\sum_{i=1}^{N} \tau_i - c_i(\ell_i) \right) + (1-\alpha) \left(\sum_{i=1}^{N} f(\ell_i) - \tau_i \right) = \sum_{i=1}^{N} (2\alpha - 1)\tau_i + (1-\alpha) f(\ell_i) - \alpha c_i(\ell_i)$$

where $\alpha \in (0.5, 1]$.

Lemma 4. *All workers who receive a positive surplus under regulation, ϕ, have the same contract: $(\tilde{\ell}, \tilde{\tau})$. This contract is defined by $f'(\tilde{\ell}) = \phi'(\tilde{\ell})$ and $\tilde{\tau} = \phi(\tilde{\ell})$. If that contract is a minimum wage, any worker, i, with zero surplus has $\ell_i \leq \tilde{\ell}$.*

Proof. If a worker, i, has a positive surplus, $\phi(\ell_i) > c_i(\ell_i)$. Therefore, the IR constraint does not bind and c_i has no effect on the firm's problem. Therefore, all workers with positive surplus must have the same contract defined by first order conditions $f'(\tilde{\ell}) = \phi'(\tilde{\ell})$ and $\tilde{\tau} = \phi(\tilde{\ell})$.

If a worker, k, has zero surplus under minimum wage, $\bar{w}$, $\tau_k = c_k(\ell_k) \geq \bar{w}\ell_k$. Suppose, by way of contradiction that $\ell_k > \tilde{\ell}$. By convexity,

$$\bar{w} = f'(\tilde{\ell}) > f'(\ell_k).$$

Because the regulation exceeds, the worker's marginal productivity, the firm would prefer to hire the worker for fewer hours if this regulation were to bind. Therefore, the regulation must be strictly below the cost of worker k. Then, $f'(\ell_k)$ is efficient and this minimum wage is above the efficient minimum wage of k. This is a contradiction. □

Lemma 4 shows that the homogeneous and heterogeneous worker problems are fundamentally similar. All workers who benefit from a regulation receive the same contract. So, there is no way to design a regulation that provides different redistributive contracts to different workers.

If only the utility of the worker matters to the regulator, heterogeneity essentially has no effect on the problem.

Proposition 2. *For $\alpha = 1$, there is at least one optimal regulation that is a minimum wage. Any optimal minimum wage is the same as the optimal minimum wage in a* *single worker problem. This single worker has the average cost of all workers with positive utility under the regulation.*

Proof. When $\alpha = 1$, the regulator maximizes the average utility of all workers. By Lemma 4, all workers with positive payoffs have the same contract. For every subset of workers $S \subseteq 2^{1,\dots,N}$, the regulator can solve

$$\max_{\phi} \sum_{i \in S} \phi(\tilde{\ell}) - c_i(\tilde{\ell}) \text{ s.t. } f'(\tilde{\ell}) = \phi'(\tilde{\ell}) \tag{4}$$

which is equivalent to

$$\max_{\phi} \phi(\tilde{\ell}) - \frac{1}{|S|} \sum_{i \in S} c_i(\tilde{\ell}) \text{ s.t. } f'(\tilde{\ell}) = \phi'(\tilde{\ell}).$$

This is the same as the single worker problem where c is replaced by the average of c_i for all $i \in S$. Therefore, the optimum is always a minimum wage.

Some of these problems may be *invalid* in the sense that the regulation does not actually benefit all workers in S. Lemma 4 ensures $\ell_i \leq \tilde{\ell}$ for any worker, i, with zero utility. As a result, there is no way to reach these workers with a regulation. Therefore, the problem for S is invalid only if there is no optimal regulation that benefits all workers in S. Therefore, the optimal minimum wages are valid solutions to (4). The regulator chooses the set of S that maximize the objective. □

Proposition 2 suggests that multiple optimal minimum wages may exist. This is because the set of workers affected by the regulation may differ across optimal policies.

Example 3 (Two optimal minimum wages)**.** Suppose a firm with $f(x) \equiv x - \frac{x^2}{2}$ contracts the services of two workers. Worker 1 has cost $c_1(x) \equiv \frac{7x^2}{2}$ and worker 2 has cost $c_2(x) \equiv \frac{x^2}{2}$.

The three candidate optimal regulation problems are

$$\max_{x,\bar{w}} \bar{w}x - \frac{1}{|S|} \sum_{i \in S} c_i(x) \text{ s.t. } \bar{w} = f'(x).$$

The solutions to the three candidate problems are $\ell_1 = \frac{1}{9}, \ell_2 = \frac{1}{3}, \ell_{1,2} = \frac{1}{6}$ with wages

$\bar{w}_1 = \frac{8}{9}, \bar{w}_2 = \frac{2}{3}, \bar{w}_{1,2} = \frac{5}{6}$. The utility of worker 2 under $\bar{w}_1$ is

$$\bar{w}_1 \ell_1 - c_2(\ell_1) = \frac{8}{9}\frac{1}{9} - \frac{(1/9)^2}{2} = \frac{5}{54} > 0.$$

This means the regulation is invalid because it should benefit only worker 1. The second benefits worker 2, but does not benefit worker 1 because

$$\bar{w}_2 \ell_2 - c_1(\ell_2) = \frac{2}{3}\frac{1}{3} - 7\frac{(1/3)^2}{2} = -\frac{1}{6} < 0.$$

Therefore, it is valid and the benefit of this regulation is

$$\bar{w}_2 \ell_2 - c_2(\ell_2) = \frac{2}{3}\frac{1}{3} - \frac{(1/3)^2}{2} = \frac{1}{6}.$$

The combined benefit of the joint regulation is

$$\begin{aligned}(\bar{w}_{1,2}\ell_{1,2} - c_1(\ell_{1,2})) + (\bar{w}_{1,2}\ell_{1,2} - c_2(\ell_{1,2})) &= \left(\frac{5}{6}\frac{1}{6} - 7\frac{(1/6)^2}{2}\right) + \left(\frac{5}{6}\frac{1}{6} - \frac{(1/6)^2}{2}\right) \\ &= \frac{1}{24} + \frac{1}{8} = \frac{1}{6}.\end{aligned}$$

This is the same as the effect of the optimal minimum wage for worker 2. Therefore, both $\bar{w}_2 = \frac{2}{3}$ and $\bar{w}_{1,2} = \frac{5}{6}$ are optimal minimum wage policies. While both are optimal, their distributive effects are different. △

The heterogenous worker problem has different implications for $\alpha \in (0.5, 1)$. The largest effect comes in the case of total surplus maximization ($\alpha \to 0.5$). An immediate implication from Lemma 4 is that a regulation can be efficient and benefit more than one worker only if the workers who benefit have the same efficient hours. If efficient labor hours are strictly ranked, at most one worker can benefit from a regulation that maximizes total surplus.

However, if heterogeneity is small, the problems for $\alpha \in (0.5, 1)$ are always similar to the homogeneous case.

Proposition 7. *For each $\alpha \in (0.5, 1]$, there exists an $\epsilon > 0$ such that $\max_x |c_i(x) - c_k(x)| < \epsilon$ for all $i, k \leq N$ implies there is at least one optimal regulation that is a minimum wage. Any optimal minimum wage is the same as the optimal minimum wage in a single worker problem. This single worker has the average cost of all* *workers.*

Proof. We only need to show that for any $\alpha \in (0.5, 1]$, there exists an ϵ such that the regulator gives every worker a positive payoff. Giving worker i a positive payoff requires $\phi(\ell_i) - c_i(\ell_i) > 0$. Therefore, any regulation that increases the payoff of one worker increases the payoff of all workers for ϵ sufficiently small. Therefore, we just need to show that there exists a binding optimal regulation.

The fact that there is such a regulation is obvious. Consider a minimum wage of $f'(\ell_N^*)$. This regulation gives strictly positive benefit to all players. Loss in total surplus can be made arbitrarily small. Therefore, this dominates not regulating for any $\alpha \in (0.5, 1]$. $\square$

From this proof, we can see that the required bounds on heterogeneity become more strict as α gets closer to 0.5. From Proposition 2, we know that no conditions are needed when $\alpha = 1$.

Lemma 7 does not hold for $\alpha \to 0.5$. In this case, there is no exactly efficient regulation that increases the utility of workers when heterogeneity in costs is arbitrarily small. However, an exactly efficient regulation that increases worker utility may exist when heterogeneity is not small.

Proposition 3. *Let $\alpha \to 0.5$. Suppose $\ell_N^* \neq \ell_{N-1}^*$. If $\min_{i=1}^{N-1} \{c_i(\ell_i^*)/\ell_i^*\} > f'(\ell_N^*)$, the optimal regulation is a minimum wage of $f'(\ell_N^*)$. Otherwise, consider*

$$\phi(x) \equiv \begin{cases} Conv(x) & \text{if } x \leq \ell_{N-1}^* \\ Conv(\ell_{N-1}^*) + f'(\ell_N^*)(x - \ell_{N-1}^*) & \text{if } x > \ell_{N-1}^*, \end{cases}$$

and Conv is the largest convex function to fit under $\{(\ell_i^*, c_i(\ell_i^*))\}_{i=1}^{N-1}$ with the restriction that the slope is capped at $f'(\ell_N^*)$. If $\phi(\ell_N^*) > c_N(\ell_N^*)$, then ϕ is an optimal regulation.

Proof. Lemma 4 and efficiency ensure minimum wage redistribution can only be used to benefit worker N. Affecting any other worker at all will reduce efficiency. Therefore, the regulation must be nonbinding for all other workers. If if the first condition holds, the efficient minimum wage is optimal because it is the largest efficient regulation for N.

If the second condition holds, any regulation for any other worker, i with slope $f'(\ell_i^*)$ at ℓ_i^* will also affect player N. Regulation, ϕ, is as large as possible while lying below all other points and having slope $f'(\ell_N^*)$ at ℓ_N^*. □

Proposition 3 demonstrates two important properties of these problems. First, the optimal regulation may not be a minimum wage. The reason that a piecewise linear regulation may be optimal in the heterogeneous worker setting is that it reduces the effects of regulation the contracts of workers with zero payoff. Such regulation only affects firms. So, the issue becomes less relevant as α increases. The second property is that minimum wage regulation is also optimal in settings where heterogeneity is sufficiently large. In this case, the problem is separable and the lowest cost worker can be regulated alone.

8.5 Oligopsony

We now add an entrant firm with production function g such that $f'(x) \geq g'(x)$ for all x. The incumbent must provide the worker enough surplus so that the entrant cannot make any profitable offer to the worker. Therefore, the profit-maximization problem of the incumbent under oligopsony is

$$\max_{\ell,\tau} f(\ell) - \tau \text{ s.t. } \tau \geq \max\{c(\ell) + u[\phi; g], \phi(\ell)\} \tag{5}$$

where u is maximum surplus that the entrant can offer. That is,

$$u[\phi; g] = \max_{\ell} g(\ell) - c(\ell) \text{ s.t. } g(\ell) \geq \phi(\ell). \tag{6}$$

Note that the regulation, ϕ, enters twice into the incumbent's problem. As in the monopsony case, it pushes the worker's salary up. However, it also constrains the maximum in (6). This means that the regulation reduces competitive pressure. This tension between regulation and competition is the main difference between oligopsony and monopsony.

8.5.1 Pre-regulation benchmark

In the absence of regulation, the incumbent offers the efficient level of labor and matches the best offer of the entrant. So, the incumbent offers contract (ℓ^*, τ_g^*) with

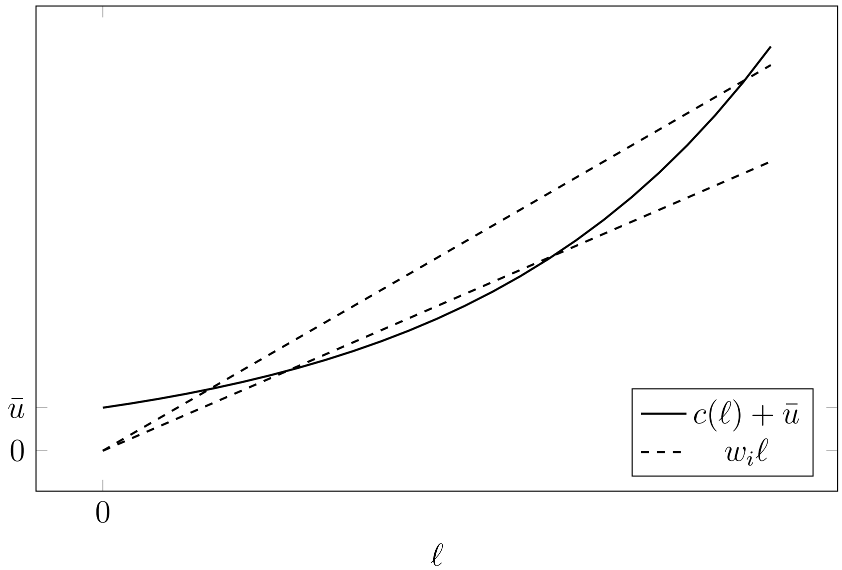

Figure 7: A plot of labor holding the worker's utility constant. The lower intersections are contracts that underemploy the worker while the intersections at the upper part of the curve overemploy the worker at the same wages.

$\ell^* \equiv \arg\max_x f(x) - c(x)$ and $\tau_g^* \equiv c(\ell^*) + u[0; g]$ where u is defined by (6).

Unlike under monopsony, it is now possible that the worker is underemployed. That is, she might prefer to work more hours at the average wage offered by the incumbent.

Lemma 5. *Suppose an incumbent with production function, f offers labor quantity, ℓ and receives profits, Π. Then, the worker is underemployed if and only if the incumbent earns profit, $\Pi < f(\ell) - c'_+(\ell)\ell$ and is overemployed if and only if $\Pi > f(\ell) - c'_-(\ell)\ell$.*

For the incumbent, the right hand sides of the above inequalities do not depend on the production function of the entrant, g. However, the equilibrium profit is weakly decreasing in g. Therefore, markets with more competitive entrants (larger g) have underemployment and markets with less competitive entrants (lower g) have overemployment. Because the entrant receives zero profit, its best offer would underemploy the worker.

Because the entrant has lower marginal productivity than the incumbent, the entrant's best offer involves weakly less labor than the incumbent's. This fact combined with Figure 7 demonstrate the incumbent pays a lower wage than the entrant if the employee is underemployed. This is because the incumbent is compensating the worker with more labor, and therefore can pay less. If the worker is overemployed, the incumbent's wage may be greater than the entrant's.

8.5.2 Minimum wage regulation

The competitive constraint ensures that $\tau - c(\ell) \geq u[\phi; g]$. If this condition is binding for some policy, ϕ, it is impossible for the policy to increase the welfare of the worker over the pre-regulation benchmark because the worker's surplus is the left hand side of the constraint and u is weakly decreasing in phi. When the competitive constraint does not bind, the entrant is irrelevant.

Therefore, for any policy that increases the welfare of the worker, the oligopsony outcome and monopsony outcome are the same. Because of this, the justification for restricting attention to minimum wage policies under Monopsony, Proposition 1, also holds under Oligopsony.

On the other hand, the effects of minimum wages that do not improve the welfare of workers are very different under Monopsony and Oligopsony. The most apparent difference between the two is that a minimum wage can strictly reduce worker welfare because workers have strictly positive welfare in the pre-regulation benchmark.

Because of this, it is not obvious that the market can be efficiently regulated.

Proposition 8. *Let (ℓ, τ) be the contract offered by the incumbent under minimum wage, $\bar{w} \geq 0$, and let ℓ^* be the efficient level of labor. Assume f is differentiable at ℓ^*. Then, there exists a larger minimum wage $\bar{w}' > \bar{w}$ that implements ℓ^* if and only if (ℓ, τ) overemploys the worker.*

Section 2 shows that, as in the neoclassical model, there is a binding minimum wage that achieves the efficient level of labor under a monopsony. It is well known that this is impossible under perfect competition when labor demand is a function.[18]

For $\bar{w} = 0$, Proposition 8 demonstrates that if a worker is overemployed in the absence of regulation, the market is uncompetitive enough for a minimum wage regulation to be efficient. If the worker is underemployed in the absence of regulation, the market is competitive enough that efficient minimum wage regulation is impossible.

[18]When f is not differentiable, demand is a correspondence. That is, there may be an interval of wages assigned to any level of labor. In this case, imagine that supply intersects this demand at the bottom of this interval. Then, it's clear that you can impose a minimum wage to the top of the interval.

This means that a regulator need only know if workers desired hours exceed their actual hours in the pre-regulation benchmark to determine if efficient minimum wage regulation is possible. The proposition goes further to say that if there is already a minimum wage in place, a regulator can tell whether this existing regulation is above or below the efficient minimum wage just by observing whether employees are underemployed or overemployed.

Proposition 9. *There f, g, c and ϕ such that, relative to the pre-regulation benchmark,*

- *worker hours are greater;*
- *the wage offered by the incumbent is strictly lower;*
- *the incumbent's profits are strictly larger.*

However, if the worker is underemployed in the pre-regulation benchmark, then none of the above are possible.

8.6 Tables and figures

Table 1: Answers to "Which of the following statements best describes how your working hours are decided? In this question, working hours refers to the total number of hours you work each week, not the time you start and finish work each day." in the 2016 General Social Survey (Smith et al., 2018).

	How worker is paid:		
	All Workers	Non Hourly	Hourly
Employer decides	40.77%	32.40%	46.72%
Employer decides with some input	25.96%	18.80%	31.05%
Worker decides within limits	18.30%	26.40%	12.54%
Worker free to decide	8.32%	13.60%	4.56%
Outside of worker and employer's control	6.49%	8.40%	5.13%
Observations	601	250	351