Assignment 2 - Hidden Markov Models

Aleksander Skraastad

March 2, 2016

1 Part A

The unobserved variables X_t in this world is simply $\{Rain\}$.

When it comes to the observed variables, the security guard can only assess whether or not the supervisor has an umbrella. Hence, the set of observable variables E_t is simply $\{Umbrella\}$.

X_t	X_{t-1}	$\neg X_{t-1}$
True	0.7	0.3
False	0.3	0.7

Table 1: Dynamic Model ($P(X_t|X_{t-1})$)

E_t	X_t	$\neg X_t$
True	0.9	0.2
False	0.1	0.8

Table 2: Observational Model ($P(E_t|X_t)$)

When it comes to the assumptions that are implied in this model, we have a Markov assumption. We assume that we are dealing with a fixed, finite set of previous states, to remove the problem of t being unbounded. We also assume that the state transitions of the world are caused by *stationary* processes. This removes the problem of having a possibly infinite amount of probability tables for rain since the probability of rain is the same for all t. Hence, our probability of rain is a first-order Markov assumption.

Additionally, we have a sensor Markov assumption, which in essence says that the world in its current state generates the sensor state E_t, regardless of possible previous dependencies. "Any sensor value worth it's salt should be able to generate the current sensor values".

2 Part B

Verification using $Umbrella = \{True, True\}$:

```
[2016-03-02 15:55:56,224] INFO main.<module>:53 --- Executing task B ---
[2016-03-02 15:55:56,224] INFO main.<module>:55 Executing task B with obs=[True, True]
[2016-03-02 15:55:56,268] INFO main.<module>:57 Day 1:
[[ 0.81818182]
 [ 0.18181818]]
[2016-03-02 15:55:56,270] INFO main.<module>:57 Day 2:
[[ 0.88335704]
 [ 0.11664296]]
```

Probability of rain on day five with $Umbrella = \{True, True, False, True, True\}$:

```
[2016-03-02 15:55:56,270] INFO main.<module>:60 Executing task B with
obs=[True, True, False, True, True]
[2016-03-02 15:55:56,273] INFO main.<module>:62 Day 1:
[[ 0.81818182]
 [ 0.18181818]]
[2016-03-02 15:55:56,275] INFO main.<module>:62 Day 2:
[[ 0.88335704]
 [ 0.11664296]]
[2016-03-02 15:55:56,276] INFO main.<module>:62 Day 3:
[[ 0.19066794]
 [ 0.80933206]]
[2016-03-02 15:55:56,278] INFO main.<module>:62 Day 4:
[[ 0.730794]
 [ 0.269206]]
[2016-03-02 15:55:56,279] INFO main.<module>:62 Day 5:
[[ 0.86733889]
 [ 0.13266111]]
```

3 Part C

Verification given $Umbrella = \{True, True\}$ (showing normalized vectors for each time step as well as the state of the forward message array of matrices before backwards propagation):

```
[2016-03-02 15:55:56,279] INFO main.<module>:65 Executing task C with obs=[True, True]
[2016-03-02 15:55:56,283] INFO main.<module>:67 Day 1:
[[ 0.81818182]
 [ 0.18181818]]
[2016-03-02 15:55:56,284] INFO main.<module>:67 Day 2:
[[ 0.88335704]
 [ 0.11664296]]
[matrix([[ 0.5],
        [ 0.5]]), matrix([[ 0.45],
        [ 0.1 ]]), matrix([[ 0.3105],
        [ 0.041 ]])]
[2016-03-02 15:55:56,288] INFO main.<module>:67 Day 2:
[[ 0.88335704]
 [ 0.11664296]]
[2016-03-02 15:55:56,290] INFO main.<module>:67 Day 1:
[[ 0.88335704]
 [ 0.11664296]]
```

Probability for rain on Day 1, given $Umbrella = \{True, True, False, False, False\}$ (Showing normalized vectors for each time step, as well as the state of the forward message array of matrices before backwards propagation):

```
[2016-03-02 16:14:55,446] INFO main.<module>:69 Executing task C with
obs=[True, True, False, True, True]
[2016-03-02 16:14:55,453] INFO main.<module>:71 Day 1:
[[ 0.81818182]
 [ 0.18181818]]
```

```
[2016-03-02 16:14:55,456] INFO main.<module>:71 Day 2:
[[ 0.88335704]
 [ 0.11664296]]
[2016-03-02 16:14:55,461] INFO main.<module>:71 Day 3:
[[ 0.19066794]
 [ 0.80933206]]
[2016-03-02 16:14:55,464] INFO main.<module>:71 Day 4:
[[ 0.730794]
 [ 0.269206]]
[2016-03-02 16:14:55,466] INFO main.<module>:71 Day 5:
[[ 0.86733889]
 [ 0.13266111]]
[matrix([[ 0.5],
        [ 0.5]]), matrix([[ 0.45],
        [ 0.1 ]]), matrix([[ 0.3105],
        [ 0.041 ]]), matrix([[ 0.022965],
        [ 0.09748 ]]), matrix([[ 0.04078755],
        [ 0.0150251 ]]), matrix([[ 0.02975293],
        [ 0.00455077]])]
[2016-03-02 16:14:55,474] INFO main.<module>:71 Day 5:
[[ 0.86733889]
 [ 0.13266111]]
[2016-03-02 16:14:55,476] INFO main.<module>:71 Day 4:
[[ 0.82041905]
 [ 0.17958095]]
[2016-03-02 16:14:55,479] INFO main.<module>:71 Day 3:
[[ 0.30748358]
 [ 0.69251642]]
[2016-03-02 16:14:55,482] INFO main.<module>:71 Day 2:
[[ 0.82041905]
 [ 0.17958095]]
[2016-03-02 16:14:55,485] INFO main.<module>:71 Day 1:
[[ 0.86733889]
 [ 0.13266111]]
```

4 Part D

See attached codebase.