Assignment 1 - Bayesian Networks

Aleksander Skraastad

February 16, 2016

1 Counting and probability

1.1 5-card poker

1.1.1 A

The joint probability distribution uses "n choose r" (nCr), as we do not care about the ordering.

$$\binom{52}{5} = \frac{52!}{5!(52-5)!} = 2598960$$

1.1.2 B

Since we have the above number of possibilities, and each one is equally likely, the probability of one particular of these events occurring is $\frac{1}{2598960}$.

1.1.3 C

Since a royal straight flush incorporates all five cards, and is a unique hand for each of the four suits, the probability is simply $\frac{4}{2598960} = \frac{1}{649740}$.

In the case of four of a kind, the face of the cards can be each of the thirteen faces. In addition, each of these 4-card hands must be paired with the last remaining card, which can be any of the remaining cards in the deck. Hence, the total probability is $\frac{624}{2598960} = \frac{1}{4165}$.

1.2 Two cards in a deck

1.2.1 A

When the first card is drawn, there are only three remaining cards in the deck that can match it. The first card has a probability of 1 and can be ignored, since all cards are valid. Hence, the total probability is $\frac{3}{51}$.

1.2.2 B

This variant with a condition introduces additional information into the system. We assume or somehow know that the cards are of different suit. That implies that we have drawn a card from a particular suit, and can exclude the remaining cards from that suit. That means that after having drawn the first card, we have a target of 3 possible cards from a deck of 39 remaining cards, hence the probability is $\frac{3}{39} = \frac{1}{13}$.

1.3 Conditional Probability

In essence, the question is if $P(A|B) > P(A)$. To answer the question we use Bayes theorem: $P(X|Y) = \frac{P(X \wedge Y)}{P(Y)}$. We substitute and reformulate the equation:

$$\frac{P(A \wedge B)}{P(B)} > P(A)$$
$$P(A \wedge B) > P(A) * P(B)$$

Which means that we have

$$\frac{P(B \wedge A)}{P(A)} > P(B)$$

Thus, if the occurrence of B increases the probability of A, then the occurrence of A increases the probability of B.

2 Bayesian Network Construction

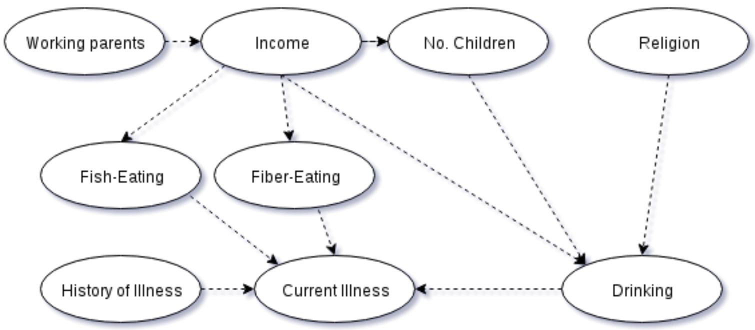

Figure 1: Bayesian Network

The conditional independence properties of the network are:

- Fish-Eating and Fiber-Eating
- Fish-Eating and No. Children

- Fish-Eating and Drinking
- Fiber-Eating and No. Children
- Fiber-Eating and Drinking

3 Monty Hall Problem Network

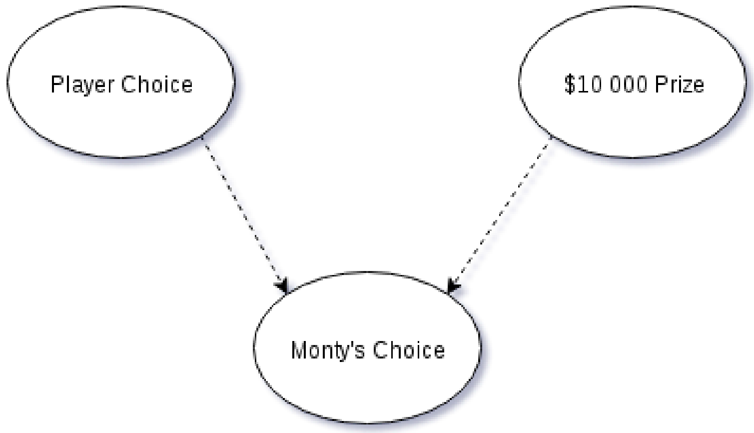

Figure 2: Bayesian Network

Figure 2 shows the Bayesian Network for the Monty Hall problem. Below are the probability tables for prize configuration:

	Choose 1	Choose 2	Choose 3
D1	0.0	0.0	0.0
D2	0.5	0.0	1.0
D3	0.5	1.0	0.0

Table 1: Probability table - Prize in Door 1

	Choose 1	Choose 2	Choose 3
D1	0.0	0.5	1.0
D2	0.0	0.0	0.0
D3	1.0	0.5	0.0

Table 2: Probability table - Prize in Door 2

While not always seen as very intuitive, this famous problem is not a difficult one. Initially, we have $\frac{1}{3}$ chance to choose the correct door. After the choice is made by the player, the host (Monty) then removes one of the options, which does not house the prize. Some people then assume that it

	Choose 1	Choose 2	Choose 3
D1	0.0	1.0	0.5
D2	1.0	0.0	0.5
D3	0.0	0.0	0.0

Table 3: Probability table - Prize in Door 3

is now 50% chance, since we only have two doors. But our choice has been made, and we cannot travel back in time and increase our odds just because a door has been removed. Naturally, it follows that since our door still only has a one-in-third chance, the remaining door must be $\frac{2}{3}$. This can be shown by utilizing Bayes theorem:

$$
\begin{aligned}
P(D_1|M) &= \tfrac{P(M|D_1)P(D_1)}{P(M)} = \tfrac{\frac{1}{2}*\frac{1}{3}}{\frac{1}{2}} = \tfrac{1}{3} \\
P(D_2|M) &= \tfrac{P(M|D_2)P(D_2)}{P(M)} = \tfrac{1*\frac{1}{3}}{\frac{1}{2}} = \tfrac{2}{3} \\
P(D_3|M) &= \tfrac{P(M|D_3)P(D_3)}{P(M)} = \tfrac{0*\frac{1}{3}}{\frac{1}{2}} = 0
\end{aligned}
$$