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EXERCISE 2

TDT4136 - Intoduction to AI

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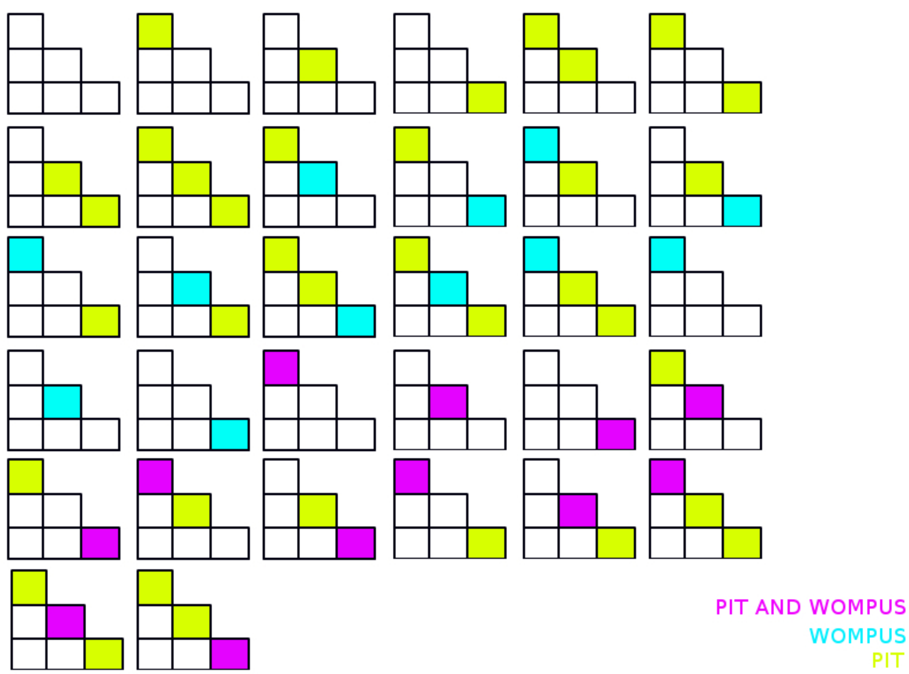
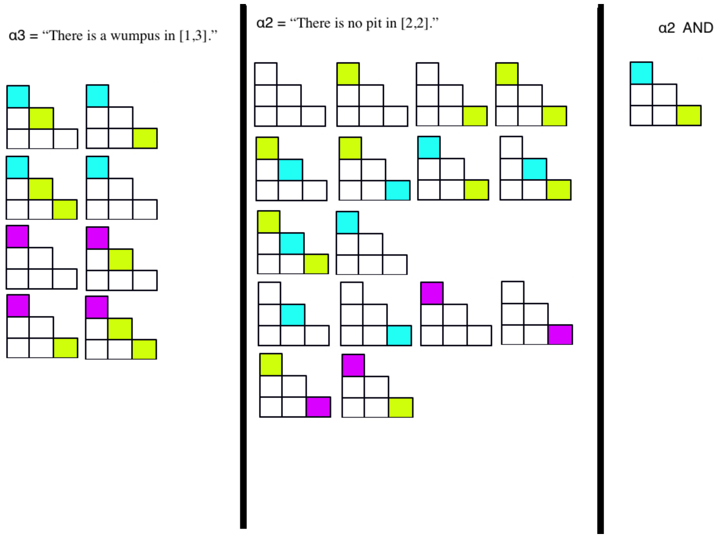
Chapter 1

Models and Entailment in Propositional Logic

1.1 Question 1





1.2 Question 2

1.2.1 Task A

$False \models True$

In this example, the only possible world is False. True cannot entail from False. Thus, incorrect.

1.2.2 Task B

$True \models False$

In this example, the only possible world is True. But α is false, thus it is incorrect.

1.2.3 Task C

$(A \wedge B) \models (A \iff B)$

In this example, the only world that is true is if both A and B are true (truth table). It therefore entails that A and B are bidirectional.

1.2.4 Task D

$(A \iff B) \models (A \vee B)$

Not correct. A = false, B = false. In this world $(A \iff B)$ is true, but $(A \vee B)$ is false. Thus the argument is not correct.

1.2.5 Task E

$(A \iff B) \models (\neg A \vee B)$

In this example, all cases where $(A \iff B)$ is true so are $(\neg A \vee B)$. A = false, B = false or A = true, B = true both states give us $(\neg A \vee B)$ = true.

1.2.6 Task F

$(A \wedge B) \implies C \models (A \implies C) \vee (B \implies C)$

In this statement, using a truth table, we can see that our KB is true for all permutations of True/False of A,B and C except one. KB is False when A = True, B = True, but C = False. Examining the right hand side, wee see that for the same permutations of True/False for A, B and C, all outcomes of α are True except for one, which happens to be the same as our KB. Thus α is True for all worlds where KB is True and the statement is correct.

1.2.7 Task G

$(C \vee (\neg A \wedge \neg B) \equiv ((A \implies C) \wedge (B \implies C))$

Using truth table we see that there is no state where $(C \vee (\neg A \wedge \neg B)$ is true and $((A \implies C) \wedge (B \implies C))$ is false. Both sides of the equation has the same value and are thus logical equivalent. Thus the statement is correct.

1.2.8 Task H

$(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B)$

We quickly see that α is True for all permutations except where both A and B are false. Since our KB has an identical expression in a conjuctional statement, we can deduce that our α will be true for all worlds where our KB is true and the statement is correct.

1.2.9 Task I

$(A \vee B) \wedge (\neg C \vee \neg D \vee E) \models (A \vee B) \wedge (\neg D \vee E)$

Using the same deduction from the above task, we immediatly see that the $(A \lor B)$ expression is used in a conjunction with the remaining expression on both sides, thus we know that we can eliminate them from our analysis. What remains is two almost identical expressions, where our lefthand side has one more element in its disjunctional expression. This, by truth table analysis leads to a larger set of True worlds for our KB than our α, leading us to conclude that the statement is incorrect.

1.2.10 Task J

$(A \lor B) \land \neg(A \implies B)$

A sentence is satisfiable if it is true in at least one model. Using a truth table, we can see that our expression is true when A = True and B = False. Hence, our sentence is satisfiable.

1.2.11 Task K

$(A \iff B) \land (\neg A \lor B)$

Enumerating the truth table for this gives us two models for which the expression holds true. It is when both A and B are either True or False. Hence, our sentence is satisfiable.

1.3 Question 3

A	B	C	D	$(B \vee C)$	$\neg A \vee \neg B \vee \neg C \vee \neg D$	$(A \implies B) \wedge A \wedge \neg B \wedge C \wedge D$
1	1	1	1	1	0	0
1	1	1	0	1	1	0
1	1	0	1	1	1	0
1	1	0	0	1	1	0
1	0	1	1	1	1	0
1	0	1	0	1	1	0
1	0	0	1	0	1	0
1	0	0	0	0	1	0
0	1	1	1	1	1	0
0	1	1	0	1	1	0
0	1	0	1	1	1	0
0	1	0	0	1	1	0
0	0	1	1	1	1	0
0	0	1	0	1	1	0
0	0	0	1	0	1	0
0	0	0	0	0	1	0

1.4 Question 4

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$ commutativity of $\wedge$

$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$ commutativity of $\vee$

$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$ associativity of $\wedge$

$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$ associativity of $\vee$

$\neg(\neg\alpha) \equiv \alpha$ double-negation elimination

$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$ contraposition

$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$ implication elimination

$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$ biconditional elimination

$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$ De Morgan

$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$ De Morgan

$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$ distributivity of $\wedge$ over $\vee$

$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$ distributivity of $\vee$ over $\wedge$

Figure 7.11 Standard logical equivalences. The symbols α, β, and γ stand for arbitrary sentences of propositional logic.

1.4.1 Task A

$Smoke \implies Smoke$

$\neg Smoke \vee Smoke$

$Smoke$	$Smoke \implies Smoke$
0	1
1	1

VALID

1.4.2 Task B

$Smoke \implies Fire$

$\neg Smoke \vee Fire$

$Smoke$	$Fire$	$Fire \implies Smoke$
0	0	1
0	1	1
1	0	0
1	1	1

NEITHER (SATISFIABLE)

1.4.3 Task C

$(Smoke \implies Fire) \implies (\neg Smoke \implies \neg Fire)$

$(\neg Smoke \vee Fire) \implies (Smoke \vee \neg Fire)$

$\neg(\neg Smoke \vee Fire) \vee (Smoke \vee \neg Fire)$

$(\neg(\neg Smoke) \wedge \neg Fire)) \vee (Smoke \vee \neg Fire)$

$(Smoke \wedge \neg Fire) \vee (Smoke \vee \neg Fire)$

$(Smoke \wedge \neg Fire) \vee Smoke \vee \neg Fire$

$Smoke$	$Fire$	$\neg Smoke$	$\neg Fire$	$(Smoke \implies Fire)$	$(\neg Smoke \implies Fire)$	HELE
0	0	1	1	1	1	1
0	1	1	0	1	0	0
1	0	0	1	0	1	1
1	1	0	0	1	1	1

NEITHER (SATISFIABLE)

1.4.4 Task D

$Smoke \vee Fire \vee \neg Fire$

$Smoke$	$Fire$	$\neg Fire$	$Smoke \vee Fire \neg Fire$
0	0	1	1
0	1	0	1
1	0	1	1
1	1	0	1

VALID

1.4.5 Task E

$((Smoke \wedge Heat) \iff ((Smoke \implies Fire) \vee (Heat \implies Fire))$ VALID

1.4.6 Task F

$(Smoke \implies Fire) \implies ((Smoke \wedge Heat) \implies Fire)$ VALID

1.4.7 Task G

$Big \lor Dumb \lor (Big \implies Dumb)$ VALID

1.5 Question 5

1.5.1 Task A

$A_1 \lor A_{73}$

The possible models in this situation is all combinations $A_1,...,A_{100}$ where $A_1 \lor A_{73}$ is true. There is only one situation which this is not the case, $\neg A_1 \land \neg A_{73}$. This gives us $3 * 2^{98}$ models.

1.5.2 Task B

$A_7 \lor (A_{19} \land A_{33})$

$\neg A_7 \land \neg(A_{19} \land A_{33})$

We have 8 permutations of the given statment, 5 of them is true. The number of models in the set $A_1,...,A_{100}$ which is valid:
$5 * 2^{97}$

1.5.3 Task C

$A_{11} \implies A_{22}$

Three possible models makes this expression valid. The only combination of $A_{11}\&A_{22}$ that is not true is if A_{11} is $True$ while A_{22} is $False$.

This gives us $3 * 2^{98}$ models in our knowledge base that will satisfy the statement.

1.5.4 Task D

$(A_{11} \implies A_{22}) \lor (A_{55} \implies A_{66})$

$A_{11} \land \neg A_{22} \land A_{55} \land \neg A_{66}$

16 permutations, 15 of them valid. $15 * 2^{96}$

1.5.5 Task E

$(A_{11} \implies A_{22}) \wedge (A_{55} \implies A_{66})$

$A_{11} \wedge \neg A_{22} \vee A_{55} \wedge \neg A_{66}$

Using our results from Task C, we can infer that our number of models using the v, since each of the braced subexpressions have 3 models that hold true, and both subexpressions have to be true in order for the whole expression to be true. Hence, we have $3 * 3 * 2^{96} = 9/16 * 2^{100}$

1.5.6 Task F

$(A_1 \wedge A_2 \wedge ... \wedge A_{50}) \wedge (\neg A_{51} \wedge \neg A_{52} \wedge ... \wedge \neg A_{100})$

One valid model.

1.5.7 Task G

$2 * 2^{50} - 1$ models. One valid for each disjunction-block.

1.5.8 Task H

$(A_1 \wedge A_2 \wedge ... \wedge A_{50}) \vee (A_{51} \wedge A_{52} \wedge ... \wedge A_{100})$

$2^{51} - 1$

1.5.9 Task I

$(A_1 \wedge A_2 \wedge ... \wedge A_{50}) \vee [(A_{51} \wedge A_{52} \wedge ... \wedge A_{99}) \wedge \neg A_{100}]$

$2^{51} - 1$

Chapter 2

Resolution in Propositional Logic

2.1 Question 1

Convert each of the following sentences to Conjunctive Normal Form

2.1.1 Task A

$A \wedge B \wedge C$

Already in CNF and DNF

2.1.2 Task B

$A \vee B \vee C$

Already in CNF and DNF

2.1.3 Task C

$\neg A \vee (B \vee C)$

2.2 Question 2

KB:

$(A \vee \neg B) \implies \neg C$

$D \wedge E \implies C$

$A \wedge D$

Create the logical equivalences:

$(A \vee \neg B) \implies \neg C \equiv (\neg A \vee \neg C) \wedge (B \vee \neg C)$

$D \wedge E \implies C \equiv D \wedge \neg E \vee C$

Combine with $A \wedge D$

$((\neg A \vee \neg C) \wedge (B \vee \neg C) \wedge A \wedge D \equiv (B \vee \neg C) \wedge A \wedge D \equiv \neg C \wedge A \wedge D$

$(D \wedge (\neg E \vee C)) \wedge A \wedge D \equiv D \wedge A \wedge (\neg E \vee C)$

Resolve:

$(\neg C \wedge A \wedge D) \wedge (D \wedge A \wedge (\neg E \vee C)) \equiv \neg C \wedge A \wedge D \wedge \neg E$

$\neg C \wedge A \wedge D \wedge \neg E \subseteq \neg E \implies KB \models \neg E$

Chapter 3

Representations in First-Order Logic

There were errors in the exercise numbers in the version updated 15.09 aswell. The green and blue book exercise numbers were mixed up. We had initially done those for the blue book (which we have), without noticing the start of the exercise text, since the exercise numbers for the blue book were correct for the rest of the assignment. We later noticed that the numbers were reversed, and had to do the exercises for stated as "green book", which were in fact for the blue one.

3.1 Question 1

a. Emily is either a surgeon or a lawyer.

$Occupation(Emily, Surgeon) \lor Occupation(Emily, Lawyer)$

b. Joe is an actor, but he also holds another job.

$Occupation(Joe, Actor) \land Occupation(Joe, o)$

c. All surgeons are doctors.

$\forall Occupation(p, Surgeon) \implies Occupation(p, Doctor)$

e. Emily has a boss who is a lawyer.

$Boss(Emily, p2) \land Occupation(p2, Lawyer)$

3.2 Question 1.wrong assignment number!

3.2.1 A

- (2) is syntactically invalid and therefore meaningless
- (1) correctly expresses the English sentence
- (3) is syntactically valid but does not express the meaning of the English sentence

3.2.2 B

- (1) correctly expresses the English sentence
- (3) is syntactically valid but does not express the meaning of the English sentence
- (2) is syntactically invalid and therefore meaningless
- (2) is syntactically invalid and therefore meaningless

3.2.3 C

- (1) correctly expresses the English sentence
- (1) correctly expresses the English sentence
- (3) is syntactically valid but does not express the meaning of the English sentence
- (3) is syntactically valid but does not express the meaning of the English sentence

3.3 Question 2

Arithmetic assertions can be written in first-order logic with the predicate symbol $<$, the function symbols $+$ and $\times$, and the constant symbols 0 and 1. Additional predicates can also be defined with biconditionals.

3.3.1 Task A

$\mathbb{U} = \mathbb{N}$

$$\exists y(x = y + y)$$

3.3.2 Task B

$\mathbb{U} = \mathbb{N}$

$$(1 < x) \land \forall y((y|x) \rightarrow ((y = 1) \lor (y = x)))$$

3.3.3 Task C

$\mathbb{U} = \mathbb{N}$

E = Even($\mathbb{U}$)

P = Prime($\mathbb{U}$)

$$\forall n \in E \quad \exists p \in P \quad \exists q \in P \quad n = p + q$$

3.4 Question 2.wrong assignment number!

Because we are only interested in the color of WA. We need to specify this with $ColorOf(WA) = red$, if we do not we could conclude that anything that is red is equal to WA.

3.5 Question 3

Either:

$\forall x Key(x) \implies Lost(Key(x), Before(Now, t2)) \land$
$\forall x, y Pair(Sock(x), Sock(y)) \implies (Lost(Sock(x), Before(Now, t2))) \lor Lost(Sock(y), Before(Now, t2))$

Or:

$\forall x : Lost(Key(x), Before(Now, t2)) \land \forall y, z : PairSock(y, z) \land (Lost(Sock(y) \lor Sock(z)), Before(Now, t2))$

We are simply not quite sure.

3.6 Question 3.wrong assignment number!

a. No two people have the same social security number.
$\neg\exists$ x, y, n $Person(x) \land Person(y) \implies [HasSS\#(x, n) \land HasSS\#(y, n)]$
Need to check that x and y is not the same person.
$\neg\exists$ x, y, n $Person(x) \land Person(y) \land \neg(x = y) \implies [HasSS\#(x, n) \land HasSS\#(y, n)]$

b. John's social security number is the same as Mary's.
$\exists n HasSS\#(John, n) \land HasSS\#(Mary, n)$
This is a correct statement.

c. Everyone's social security number has nine digits.
$\forall x, n Person(x) \implies [HasSS\#(x, n) \land Digits(n, 9)]$
This is a correct statement.

3.7 Question 4

“Everyone’s DNA is unique and is derived from their parents’ DNA.”

DNA(x) - DNA string for person x.

DerivedFrom(u,v,w), this means u is given by v and w.

$\forall x, y\ (\neg(x = y) \implies \neg(DNA(x) = DNA(y))) \land DerivedFrom(DNA(x), DNA(Mother(x)), DNA(Father(x)))$

Chapter 4

Resolution in First-Order Logic

4.1 Question 1

4.1.1 Task B

Owner(Leo, Rocky) . . . Owner(x, y)

$\theta = \{x/Leo, y/Rocky\}$

4.1.2 Task C

Owner(Leo, x) . . . Owner(y, Rocky)

$\theta = \{x/Rocky, y/Leo\}$

4.1.3 Task D

Owner(Leo, x) . . . Rides(Leo, Rocky)

$\theta = \{x/Rocky\}$

4.2 Question 2

$Hybrid(Prius)$

$Drives(Linn, Prius)$

$\forall x : Green(x) \iff Bikes(x) \vee [\exists y : Drives(x, y) \wedge Hybrid(y)]$

Please help.