Mathematical Modelling

1 Decision Variables

$$x_{i,j,k,d} = 1 \tag{1}$$

If Delivery guy **i** picks dish **d** from restaurant **j** to customer **k**, 0 Otherwise.

2 Input variables

$$fav_{k,j} = 1 \tag{2}$$

If user **k** favors restaurant **j**, 0 Otherwise.

$$dishes_{j,d} = 1 \tag{3}$$

If restaurant **j** offers dish **d**, 0 Otherwise.

$$orders_{k,d} = 1 \tag{4}$$

If user **k** ordered dish **d**, 0 Otherwise.

$$dist_{a,b} \tag{5}$$

Distance between locations a,b.

$$v_c \tag{6}$$

Cost per meter for vehicle type **c**

$$vehicle_i = c \tag{7}$$

If delivery guy **i** has vehicle type **c**

$$max_j \tag{8}$$

Maximum orders for restaurant j

$$rl \tag{9}$$

Maximum road length that any delivery guys can drive from his location till delivering the order to the client.

$$L \tag{10}$$

Set representing all possible locations in the problem

$$lu_k \in L \tag{11}$$

Represents the location index of the user **k** in the locations set.

$$ld_i \in L \tag{12}$$

Represents the location index of the delivery guy **i** in the locations set.

$$lr_j \in L \tag{13}$$

Represents the location index of the restaurant **j** in the locations set.

3 Objective function

Maximize :

$$\sum_{i,j,k,d} x_{i,j,k,d} - w * \sum_{i,j,k,d} x_{i,j,k,d} * (dist_{ld_i,lr_j} + dist_{lr_j,lu_k}) * v_{vehicle_i} \tag{14}$$

4 constraints

1. Maximum dishes per restaurant

$$\sum_{i,d,k} x_{i,j,k,d} \leq max_j \qquad \forall_j \tag{15}$$

2. To assign an order to a delivery guy from a restaurant to go to a user, it must be that:
   - The dish is prepared in the restaurant.
   - The user favors that restaurant
   - the user ordered that dish

$$\sum_{i} x_{i,j,k,d} \leq dishes_{j,d} * favs_{k,j} * orders_{k,d} \qquad \forall_{d,j,k} \tag{16}$$

3. for the same delivery guys he cannot serve more than one customer.

$$\sum_{j,k,d} x_{i,j,k,d} \leq 1 \forall_i \tag{17}$$

4. Distance constraint

$$\sum_{i,j,k,d} x_{i,j,k,d} * (dist_{ld_i,lr_j} + dist_{lr_j,lu_k}) \leq rl \qquad \forall_{i,j,k,d} \tag{18}$$

5. orders can only be prepared by only one restaurant and delivered by one guy.

$$\sum_{i,j} x_{i,j,k,d} \leq 1 \qquad \forall_{k,d} \tag{19}$$

6. Variable Constraints.

$$x_{i,j,k,d} \in \{0,1\} \tag{20}$$