For two vectors x, y cosine similarity is calculated as:-

$$\cos\_sim(x, y) = \frac{x \cdot y}{\|x\| \, \|y\|}$$

where $x \cdot y$ is the dot product & $\|x\|$, $\|y\|$ are their Euclidean norms. (L2-norm)

Dot product in 2D:-

if $x = (x_1, x_2)$ & $y = (y_1, y_2)$ then

$$x \cdot y = x_1 y_1 + x_2 y_2$$

norm:- $\|x\| = \sqrt{x_1^2 + x_2^2}$ & $\|y\| = \sqrt{y_1^2 + y_2^2}$

Dividing by norms removes length, so cosine similarity depends only on the angle.

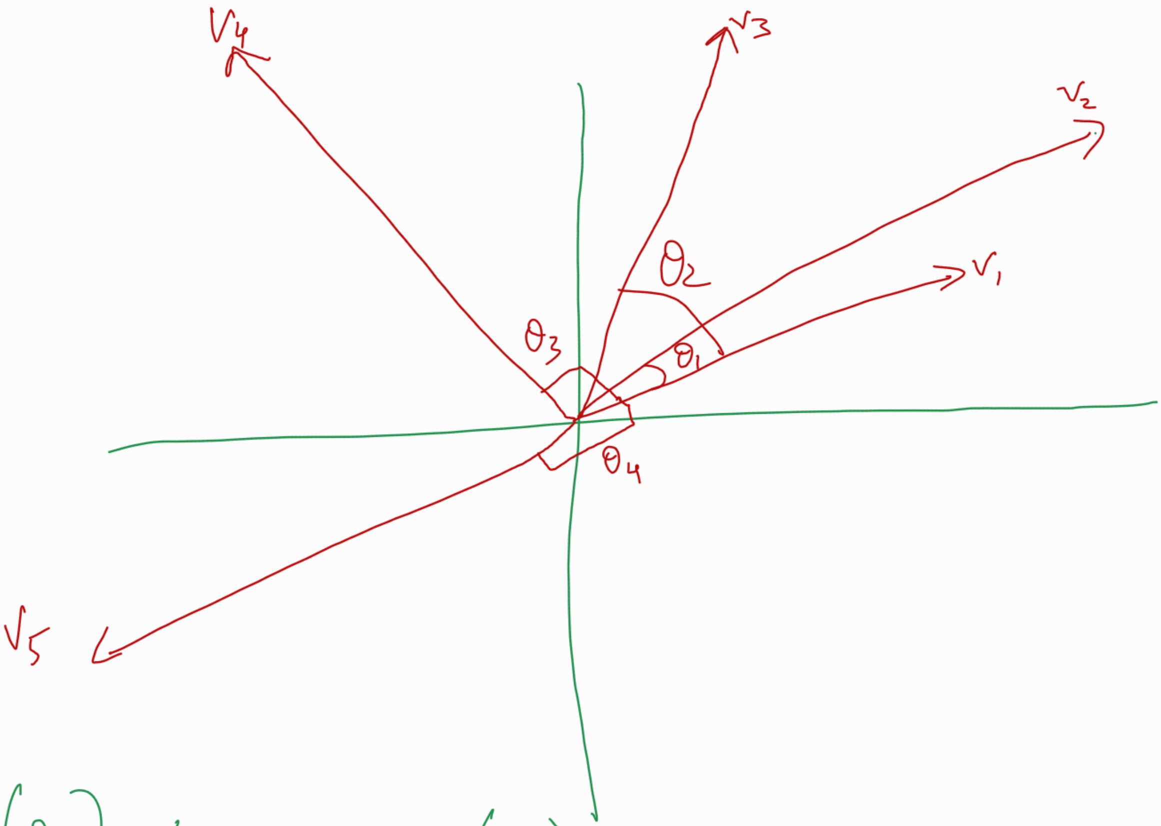
Same formula can be rewritten as:-

$$S_c(A, B) := \cos(\theta) = \frac{A \cdot B}{\|A\| \, \|B\|} =$$

$$= \frac{\sum_{i=1}^{n} A_i B_i}{\sqrt{\sum_{i=1}^{n} A_i^2} \cdot \sqrt{\sum_{i=1}^{n} B_i^2}}$$

for given vectors A, B.

Values range from '-1' to '+1'. A value of '1' means that the given vectors are identical, '0' means they are orthogonal (perpendicular) & '-1' means they're opposite in direction irrespective of length.



$v_1 - v_2 \ (\theta_1) < v_1 - v_3 \ (\theta_2)$

So v_1 & v_2 are much alike than v_1 & v_3.

v_1, v_4 are perpendicular ($\theta_3 = 90^\circ$)

v_1 & v_5 are opp. in direction since $\theta = 180^\circ$