THE MANGA GUIDE TO

CALCULUS

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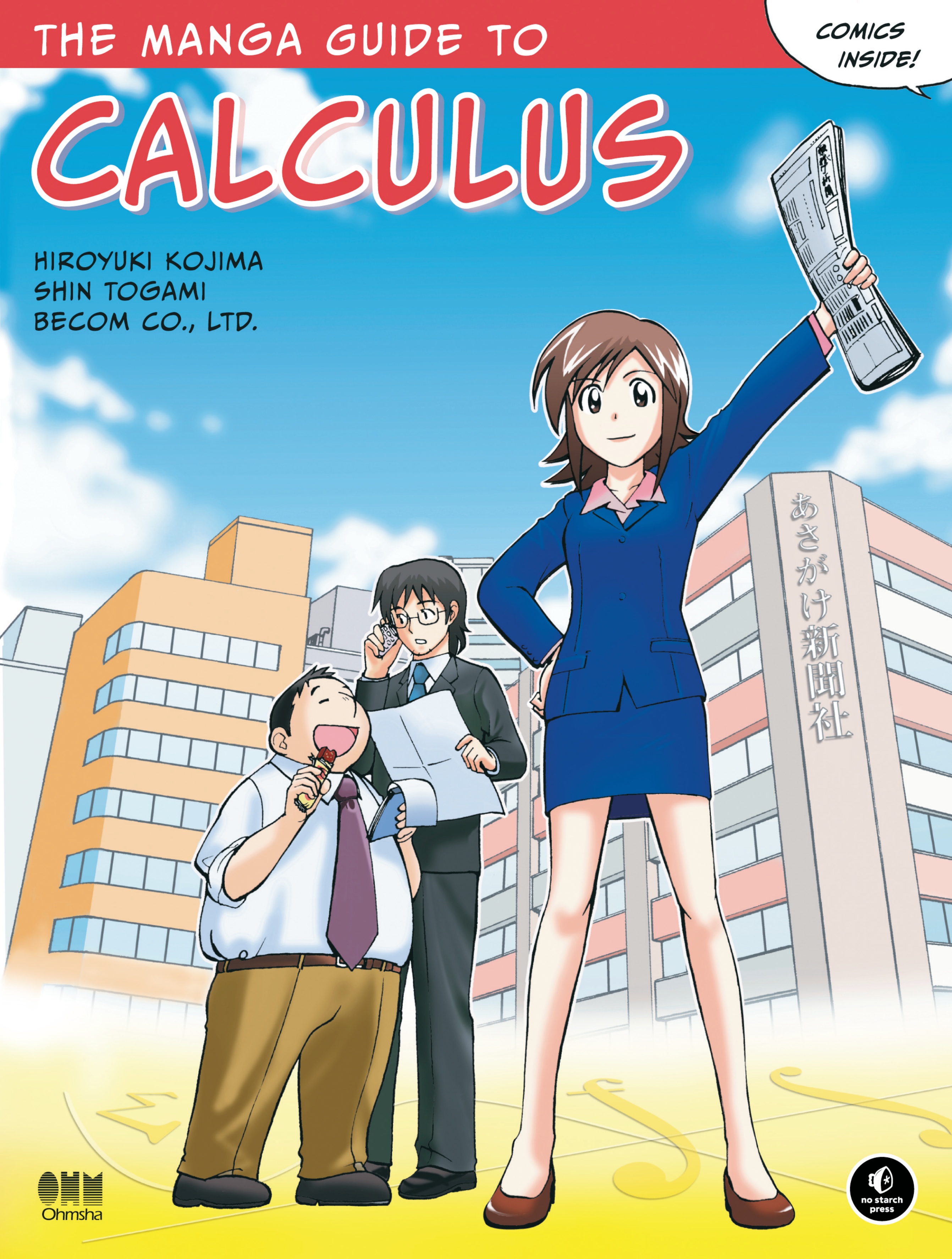

Ohmsha
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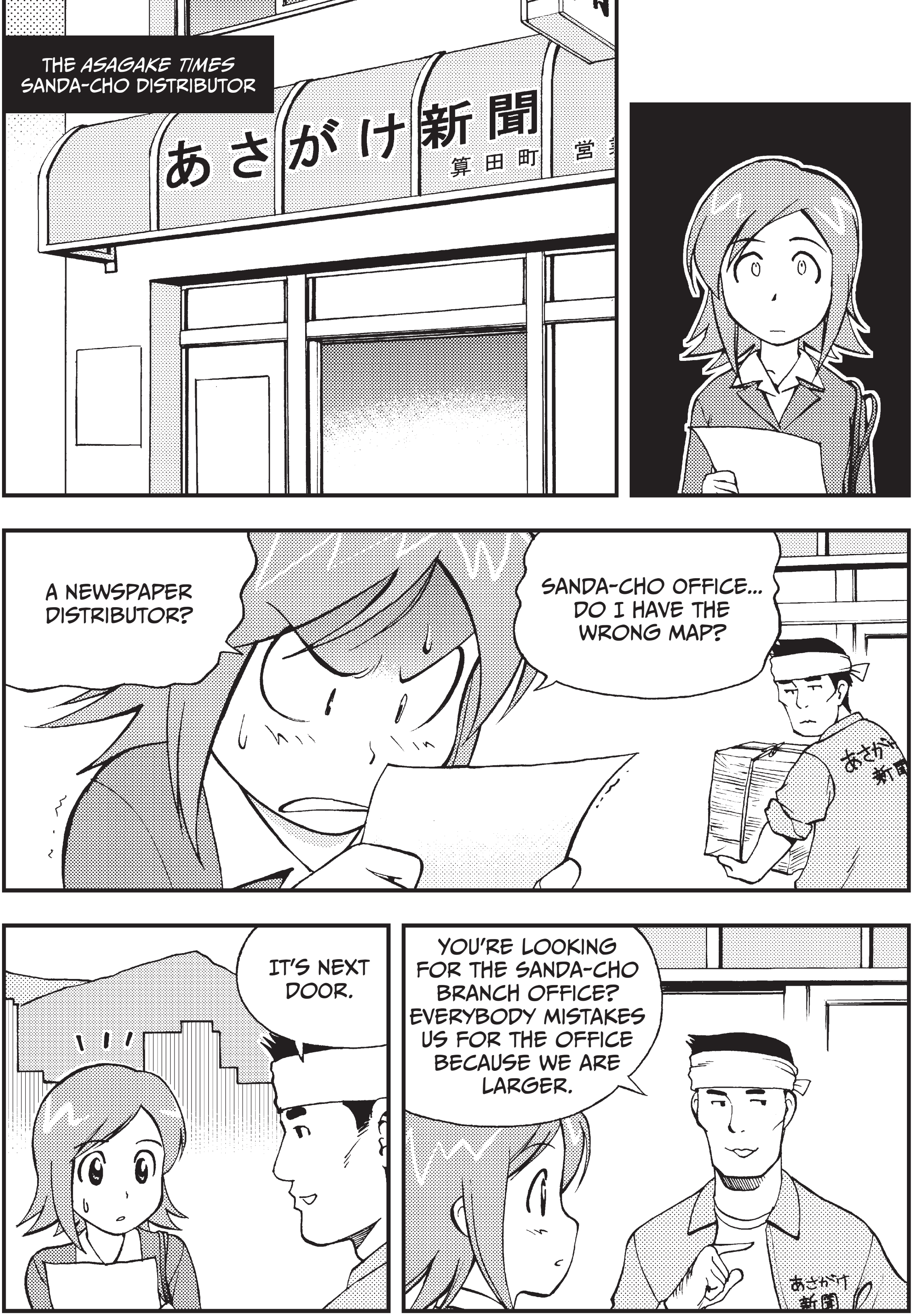
THE *ASAGAKE TIMES*
SANDA-CHO DISTRIBUTOR
あさがけ新聞
算田町
A NEWSPAPER DISTRIBUTOR?
SANDA-CHO OFFICE... DO I HAVE THE WRONG MAP?
IT'S NEXT DOOR.
YOU'RE LOOKING FOR THE SANDA-CHO BRANCH OFFICE? EVERYBODY MISTAKES US FOR THE OFFICE BECAUSE WE ARE LARGER.
あさがけ新聞

CALCULATING THE DERIVATIVE OF A CONSTANT, LINEAR, OR QUADRATIC FUNCTION

1. Let's find the derivative of constant function $f(x) = \alpha$. The differential coefficient of $f(x)$ at $x = a$ is

$$\lim_{\varepsilon \to 0} \frac{f(a+\varepsilon) - f(a)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{\alpha - \alpha}{\varepsilon} = \lim_{\varepsilon \to 0} 0 = 0$$

Thus, the derivative of $f(x)$ is $f'(x) = 0$. This makes sense, since our function is constant—the rate of change is 0.

NOTE The *differential coefficient* of $f(x)$ at $x = a$ is often simply called the derivative of $f(x)$ at $x = a$, or just $f'(a)$.

2. Let's calculate the derivative of linear function $f(x) = \alpha x + \beta$. The derivative of $f(x)$ at $x = \alpha$ is

$$\lim_{\varepsilon \to 0} \frac{f(\alpha+\varepsilon) - f(a)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{\alpha(a+\varepsilon) + \beta - (\alpha a + \beta)}{\varepsilon} = \lim_{\varepsilon \to 0} \alpha = \alpha$$

Thus, the derivative of $f(x)$ is $f'(x) = \alpha$, a constant value. This result should also be intuitive—linear functions have a constant rate of change by definition.

3. Let's find the derivative of $f(x) = x^2$, which appeared in the story. The differential coefficient of $f(x)$ at $x = a$ is

$$\lim_{\varepsilon \to 0} \frac{f(a+\varepsilon) - f(a)}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{(a+\varepsilon)^2 - a^2}{\varepsilon} = \lim_{\varepsilon \to 0} \frac{2a\varepsilon + \varepsilon^2}{\varepsilon} = \lim_{\varepsilon \to 0} (2a + \varepsilon) = 2a$$

Thus, the differential coefficient of $f(x)$ at $x = a$ is $2a$, or $f'(a) = 2a$. Therefore, the derivative of $f(x)$ is $f'(x) = 2x$.

SUMMARY

- The calculation of a limit that appears in calculus is simply a formula calculating an error.
- A limit is used to obtain a derivative.
- The derivative is the slope of the tangent line at a given point.
- The derivative is nothing but the rate of change.