ECE661: Computer Vision

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Answer 1:

All the points of the form $\begin{bmatrix} 0 \\ 0 \\ c \end{bmatrix}$ represent origin in physical space $\mathbb{R}^2$, where $c \neq 0$.

Answer 2:

No.
All the points at infinity in physical plane $\mathbb{R}^2$ can be viewed as intersection of all parallel lines.
If we take two pairs of parallel lines $\{\mathbf{l1}\ ,\ \mathbf{l2}\}$ and $\{\mathbf{l3}\ ,\ \mathbf{l4}\}$ which parametric representations are $\{\begin{bmatrix} a \\ b \\ c1 \end{bmatrix}, \begin{bmatrix} a \\ b \\ c2 \end{bmatrix}\}$ and $\{\begin{bmatrix} p \\ q \\ r1 \end{bmatrix}, \begin{bmatrix} p \\ q \\ r2 \end{bmatrix}\}$. Their intersections in representational space $\mathbb{R}^3$ will be different ideal points given by $\begin{bmatrix} b(c2 - c1) \\ a(c1 - c2) \\ 0 \end{bmatrix}$ and $\begin{bmatrix} q(r2 - r1) \\ p(r1 - r2) \\ 0 \end{bmatrix}$. So if we we consider all infinity points, **in representational space they will form a plane** $\begin{bmatrix} x1 \\ x2 \\ 0 \end{bmatrix}$ **(x-y plane)**.
If we take cross of two ideal points in $\mathbb{R}^3$ irrespective of points they all will map to $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ which is line at infinity in $\mathbb{R}^2$ from different directions. **So in physical** $\mathbb{R}^2$ **plane all points at infinity completes a line into a closed circle**.

Answer 3:

Degenerate conics $\mathbf{C}$ is represented by $\mathbf{lm^T} + \mathbf{ml^T}$ where $\mathbf{l}$ and $\mathbf{m}$ represent homogeneous coordinate of two vectors. Each term of $\mathbf{C}$ is a outer product of two vectors which means row/column space of outer product matrix will be linear combination of single $\mathbb{R}^3$ vector leading to rank 1. And as we know by identity in linear algebra rank of sum of matrices is bounded by sum of individual ranks. So rank of $\mathbf{C}$ can never exceed 2.

Answer 4:

(A)

Line passing through (0,0) and (3,5) $\mathbf{l_1}$:

$$\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix}$$

Line passing through (-3,4) and (7,5) $\mathbf{l_2}$:

$$\begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix} \times \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -4 \\ 13 \end{bmatrix}$$

Intersection of $\mathbf{l_1}$ and $\mathbf{l_2}$:

$$\begin{bmatrix} -5 \\ 3 \\ 0 \end{bmatrix} \times \begin{bmatrix} -1 \\ -4 \\ 13 \end{bmatrix} = \begin{bmatrix} 39 \\ 65 \\ 23 \end{bmatrix}$$

So point of intersection in $\mathbb{R}^2$ is $(\frac{39}{23}, \frac{65}{23})$.

(B)

Line passing through (-7,-5) and (7,5) $\mathbf{l_3}$:

$$\begin{bmatrix} -7 \\ -5 \\ 1 \end{bmatrix} \times \begin{bmatrix} 7 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} -10 \\ 14 \\ 0 \end{bmatrix}$$

As we can see line $\mathbf{l_3}$ has also third parameter 0 as similar to $\mathbf{l_1}$ which means both are passing through origin. So just by one more step we can tell intersection point is origin.

Answer 5:

Line passing through (0,0) and (5,-3) $\mathbf{l_1}$:

$$\begin{bmatrix}0\\0\\1\end{bmatrix} \times \begin{bmatrix}5\\-3\\1\end{bmatrix} = \begin{bmatrix}3\\5\\0\end{bmatrix}$$

Line passing through (-5,0) and (0,-3) $\mathbf{l_2}$:

$$\begin{bmatrix}-5\\0\\1\end{bmatrix} \times \begin{bmatrix}0\\-3\\1\end{bmatrix} = \begin{bmatrix}3\\5\\15\end{bmatrix}$$

Intersection of $\mathbf{l_1}$ and $\mathbf{l_2}$:

$$\begin{bmatrix}3\\5\\0\end{bmatrix} \times \begin{bmatrix}3\\5\\15\end{bmatrix} = \begin{bmatrix}75\\-45\\0\end{bmatrix}$$

As we can see also $\mathbf{l_1}$ and $\mathbf{l_2}$ both have same a and b parameter which means both are parallel. And all parallel lines in $\mathbb{R}^2$ meet at ideal points in representational space $\mathbb{R}^3$ which is the result of calculation also.

Answer 6:

Equation of ellipse is given by:

$$\frac{(x-3)^2}{1^2} + \frac{(y-2)^2}{\frac{1}{2^2}} = 1$$

$$x^2 + 4y^2 + -6x - 16y + 24 = 0$$

Writing ellipse in matrix form:

$$\mathbf{C} = \begin{bmatrix}1 & 0 & -3\\0 & 4 & -8\\-3 & -8 & 24\end{bmatrix}$$

$$\implies \mathbf{Cp} = \begin{bmatrix}1 & 0 & -3\\0 & 4 & -8\\-3 & -8 & 24\end{bmatrix}\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}-3\\-8\\24\end{bmatrix}$$

Intersection of $\mathbf{Cp}$ and Y-axis (x=0) in representational space $\mathbb{R}^3$:

$$\begin{bmatrix}-3\\-8\\24\end{bmatrix} \times \begin{bmatrix}1\\0\\0\end{bmatrix} = \begin{bmatrix}0\\24\\8\end{bmatrix}$$

Intersection of **Cp** and Y-axis (x=0) in physical space $\mathbb{R}^2$ is (0,3).

Intersection of **Cp** and X-axis (y=0) in representational space $\mathbb{R}^3$:

$$\begin{bmatrix} -3 \\ -8 \\ 24 \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -24 \\ 0 \\ -3 \end{bmatrix}$$

Intersection of **Cp** and X-axis (y=0) in physical space $\mathbb{R}^2$ is (8,0).

Answer 7:

Intersection of two line $x = 1/2$ and $y = -1/3$ is given by:

$$\begin{bmatrix} 1 \\ 0 \\ \frac{-1}{2} \end{bmatrix} \times \begin{bmatrix} 0 \\ 1 \\ \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{-1}{3} \\ 1 \end{bmatrix}$$

So line will intersect at $(\frac{1}{2}, \frac{-1}{3})$.