Representação Matricial de Operadores

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Outline

Operadores

$$L^2|lm\rangle = \hbar^2 l(l+1)|lm\rangle$$

$$L_z|lm\rangle = \hbar m|lm\rangle$$

$$|n\rangle = \frac{1}{\sqrt{n!}}(A^\dagger)^n|0\rangle$$

$$H|n\rangle = \hbar\omega\left(n+\frac{1}{2}\right)|n\rangle$$

Operadores

$$A^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$A|n\rangle = \sqrt{n}|n-1\rangle$$

$$\langle m|n\rangle = \delta_{mn}$$

$$I = \sum_{n=0}^{\infty} |n\rangle\langle n|$$

Sanduíches

$$\langle m|H|n\rangle = \hbar\omega\left(n + \frac{1}{2}\right)\delta_{nm}$$

$$\langle m|A^\dagger|n\rangle = \sqrt{n+1}\,\delta_{m,n+1}$$

$$\langle m|A|n\rangle = \sqrt{n}\,\delta_{m,n-1}$$

Representação Matricial-P

$$P = \begin{pmatrix} \langle 1|P|1\rangle & \langle 1|P|2\rangle & \cdots \\ \langle 2|P|1\rangle & \langle 2|P|2\rangle & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & \langle m|P|n\rangle & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

Hamiltoniano

$$H = \hbar\omega \begin{pmatrix} 1/2 & 0 & 0 & 0 & \cdots \\ 0 & 3/2 & 0 & 0 & \cdots \\ 0 & 0 & 5/2 & 0 & \cdots \\ 0 & 0 & 0 & 7/2 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

$A^\dagger$

$$A^\dagger = \begin{pmatrix} 0 & 0 & 0 & 0 & \cdots \\ \sqrt{1} & 0 & 0 & 0 & \cdots \\ 0 & \sqrt{2} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{3} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

A

$$A = \begin{pmatrix} 0 & \sqrt{1} & 0 & 0 & \cdots \\ 0 & 0 & \sqrt{2} & 0 & \cdots \\ 0 & 0 & 0 & \sqrt{3} & \cdots \\ 0 & 0 & 0 & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

L_z

$$\langle l'm'|L_z|lm\rangle = m\hbar\langle l'm'|lm\rangle = \hbar m\delta_{mm'}\delta_{ll'}$$

$$L_z = \hbar\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$L_{\pm}$

$$\langle l'm'|L_{\pm}|lm\rangle = \hbar\sqrt{l(l+1)-m(m\pm 1)}\langle l'm'|lm\pm 1\rangle$$

$$= \hbar\sqrt{l(l+1)-m(m\pm 1)}\delta_{ll'}\delta_{mm'}$$

$$L_{+} = \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$L_\pm$

$$L_- = \hbar \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

L_x

$$L_x = \frac{L_+ + L_-}{2} = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

L_y

$$L_y = \frac{-i(L_+ - L_-)}{2} = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

Exemplo

Calcule usando matrizes $[L_x, L_y] = i\hbar L_z$

Bras e Kets

Em uma base $|n\rangle$ podemos expressar um ket $|\psi\rangle$

$$\langle n|\psi\rangle \rightarrow \begin{pmatrix} \langle 1|\psi\rangle \\ \langle 2|\psi\rangle \\ \langle 3|\psi\rangle \\ \vdots \end{pmatrix} \rightarrow \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \vdots \end{pmatrix}$$

Bras e Kets

De forma análoga:

$$\langle n|\phi\rangle \rightarrow \begin{pmatrix} \langle 1|\phi\rangle \\ \langle 2|\phi\rangle \\ \langle 3|\phi\rangle \\ \vdots \end{pmatrix} \rightarrow \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \end{pmatrix}$$

Bras e Kets

$$\langle \psi | n \rangle \longrightarrow (\alpha_1^*, \alpha_2^*, \cdots)$$

$$\langle \psi | \phi \rangle = \sum_n \langle \psi | n \rangle \langle n | \phi \rangle = (\alpha_1^*, \alpha_2^*, \cdots) \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \end{pmatrix} = \sum_n \beta_n \alpha_n^*$$

Sanduíches

$$\langle \phi | Q | \psi \rangle = \sum_n \langle \psi | n \rangle \langle n | \phi \rangle = (\alpha_1^*, \alpha_2^*, \cdots) \begin{pmatrix} Q_{11} & Q_{12} & \cdots \\ Q_{21} & Q_{22} & \cdots \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \vdots \end{pmatrix}$$

$$= \sum_{nm} \alpha_n^* Q_{nm} \beta_m^*$$

Exercícios

Obtenha a representação matricial dos seguintes operadores:

- x
- x^2
- p
- px
- xp