z-scores and z-tests

What is a z-score?

- Shows how one piece of data fits with a distribution
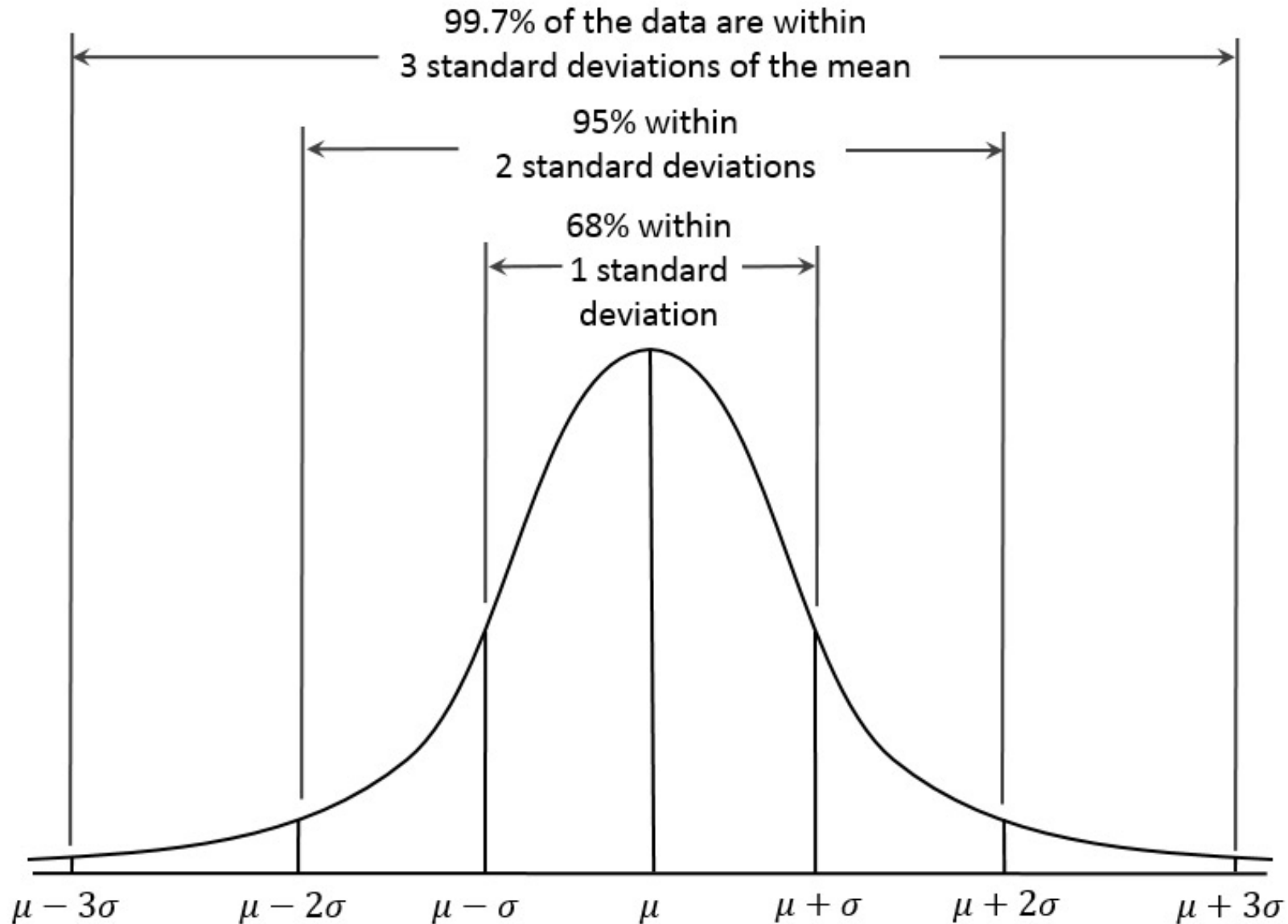
$$z = \frac{x - \mu}{\sigma}$$

What is a z-test?

- Tells you the probability that a particular score will fall in a specific spot on a distribution

- Must use the normal probability applet
  - http://davidmlane.com/hyperstat/z_table.html

Percentiles

- The percent of data BELOW the number

  - 55th percentile = 55% of data below your score
  - 99th percentile = 99% of data below your score
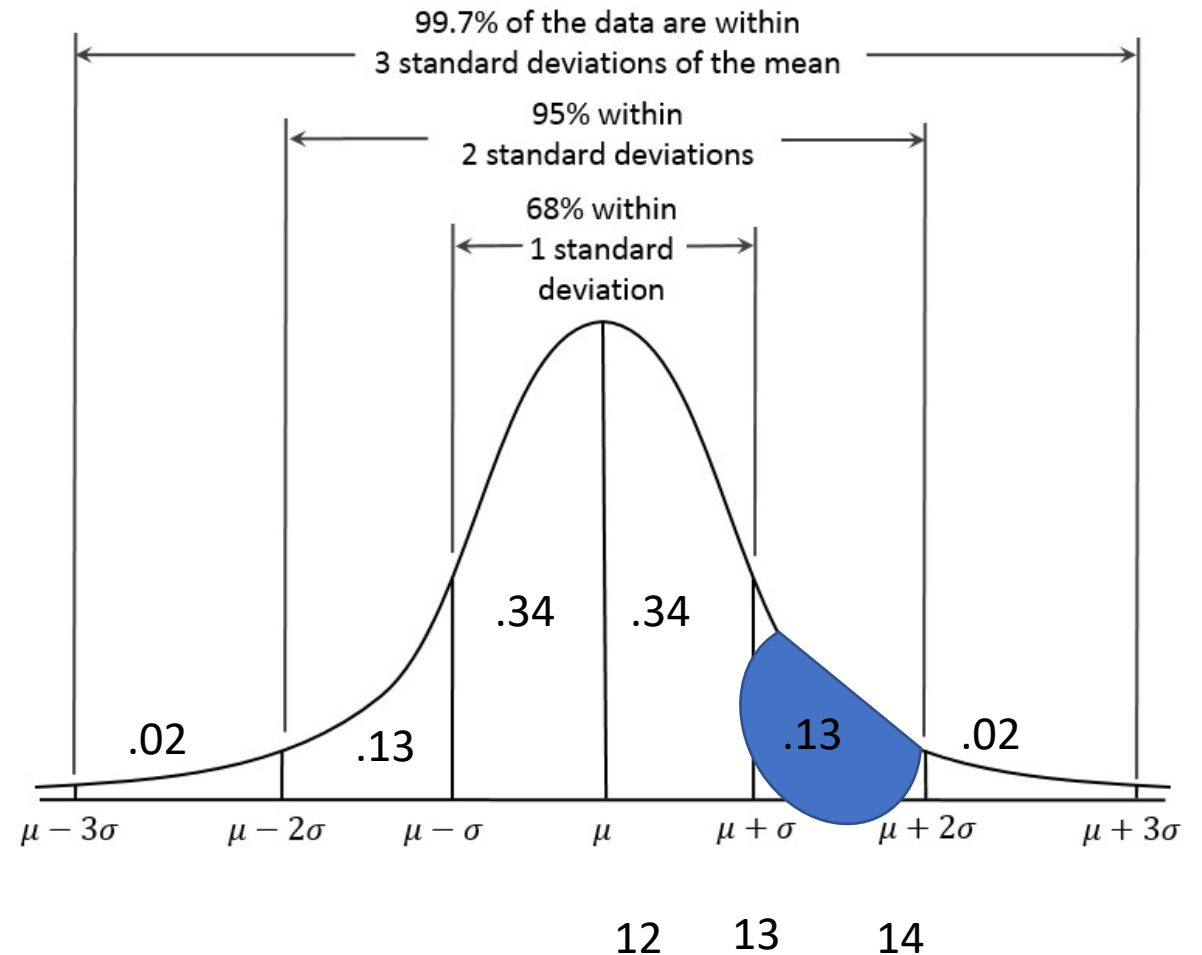
- Often seen on standardized tests

The 68-95-99 Rule

99.7% of the data are within
3 standard deviations of the mean

95% within
2 standard deviations

68% within
1 standard
deviation

$\mu - 3\sigma$ $\mu - 2\sigma$ $\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$ $\mu + 3\sigma$

A Few Examples

Using "The Rule"

- For a normally distributed variable with a mean of 12 and a standard deviation of 1, how much of the distribution is between 13 and 14 according to the rule?

99.7% of the data are within 3 standard deviations of the mean

95% within 2 standard deviations

68% within 1 standard deviation

.34 .34

.02 .13 .13 .02

$\mu - 3\sigma$ $\mu - 2\sigma$ $\mu - \sigma$ μ $\mu + \sigma$ $\mu + 2\sigma$ $\mu + 3\sigma$

12 13 14

Calculate the z-score

- Always need three pieces of information:
    - Mu = 19
    - Sigma = 3
    - X =17

- Z = (17 – 19) / 3
- Z = -.066

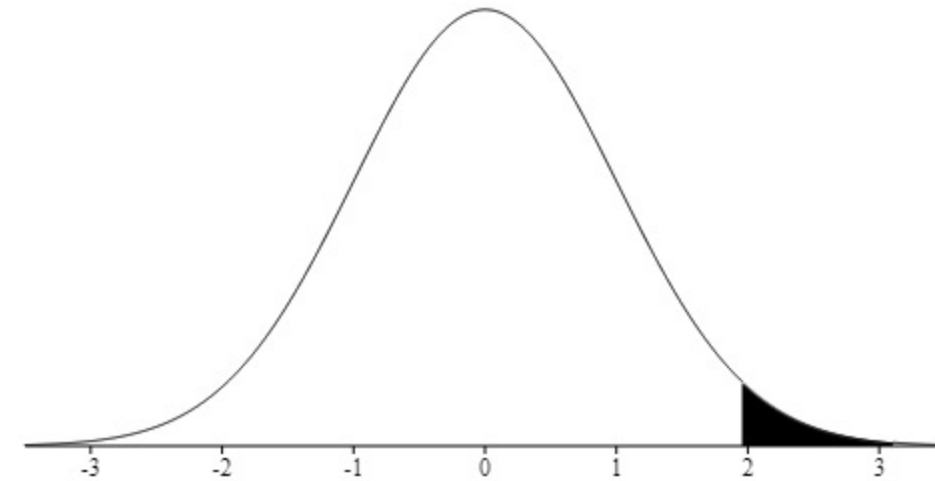$$z = \frac{x - \mu}{\sigma}$$

Find sigma using the z-score formula

- X = 22
- Mu = 11
- Z = 2.3

- Plug it in, then re-arrange to find sigma

- 2.3 = (22 – 11) / sigma
- 2.3 sigma = 22 – 11
- Sigma = (22-11) / 2.3
- Sigma = 4.78

$$z = \frac{x - \mu}{\sigma}$$

Z-tests

- Use the applet!

- What is the probability of randomly selecting a value above 92 for a distribution whose mean is 141 and sigma is 39?

- p = .89

○ Area from a value (Use to compute p from Z)
○ Value from an area (Use to compute Z for confidence intervals)

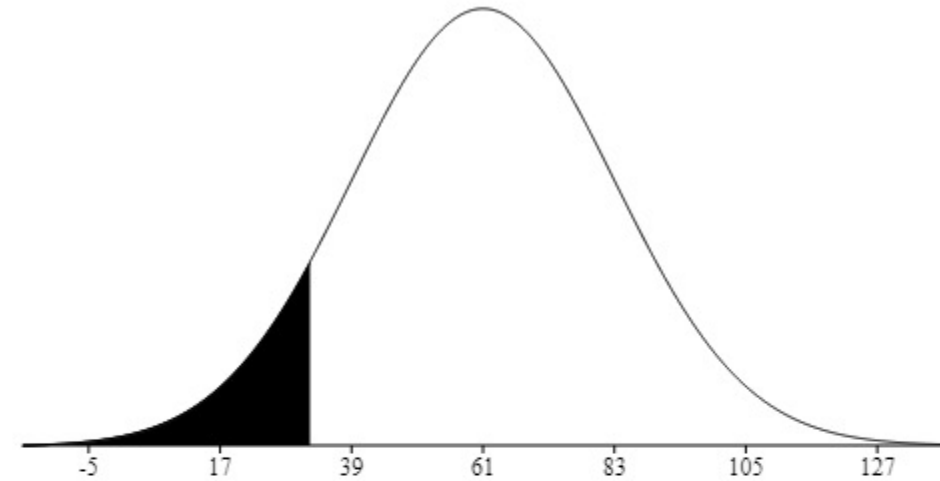Specify Parameters:
Mean 141
SD   39
● Above 92
○ Below 1.96
○ Between -1.96   and 1.96
○ Outside -1.96   and 1.96

Results:
Area (probability) 0.8955
Recalculate

Z-tests

- Use the applet!

- What is the probability of randomly selecting a value below 32 for a distribution whose mean is 61 and sigma is 22?

- p = .99

Area from a value (Use to compute p from Z)
Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:
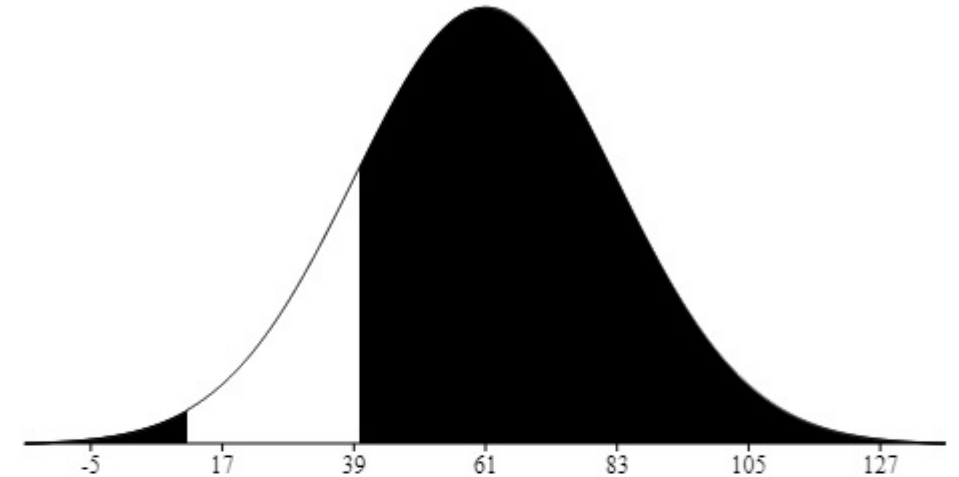Mean 61
SD   22
○ Above
◉ Below 32
○ Between -1.96 and 1.96
○ Outside -1.96 and 1.96

Results:
Area (probability) 0.0937
Recalculate

Z-tests

- Use the applet!

- What is the probability of randomly selecting a value that less than 11 or greater than 40 for a distribution whose mean is 61 and sigma is 22?

- p = .84

○ Area from a value (Use to compute p from Z)
○ Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:
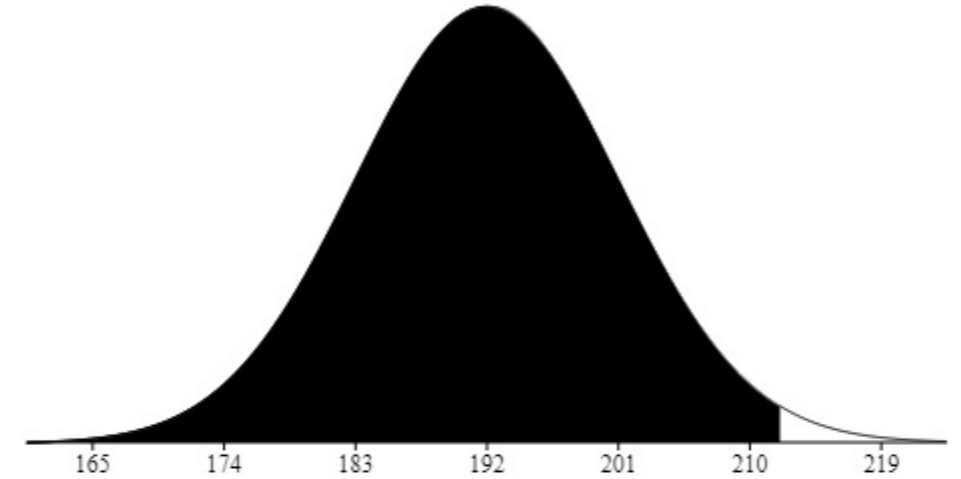Mean 61
SD   22
○ Above
○ Below 32
○ Between -1.96 and 1.96
● Outside 11 and 40

Results:
Area (probability) 0.8416
Recalculate

Score to a Percentile

- You score a 212 on an exam, that has a distribution mean of 192 and a standard deviation of 9. What is your percentile?

- 99% percentile

Area from a value (Use to compute p from Z)
Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:
Mean 192
SD 9
Above
Below 212
Between -1.96 and 1.96
Outside 11 and 40

Results:
Area (probability) 0.9869
Recalculate
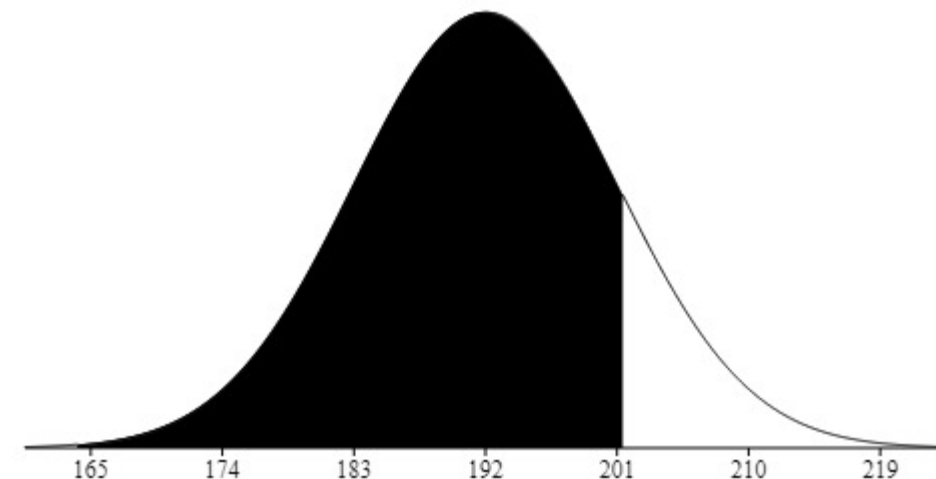
Percentile to a Score

- You score in the 85th percentile on an exam with a distribution mean of 192 and a standard deviation of 9. What is your score?

- Score of 201

O Area from a value (Use to compute p from Z)
● Value from an area (Use to compute Z for confidence intervals)

Specify Parameters:
Area [.85]
Mean [192]
SD [9]

Results:
[Recalculate]
O Above [   ]
● Below [201.328]
O Between [   ]
O Outside [   ]