Calculus in Brief with Python, Colab, GitHub, and $\LaTeX$ CiB - Version 0.41

November 12, 2023

Available on GitHub at:
https://GitHub.com/nicholaskarlson/CiB

Contents

Preface

This text, *Calculus in Brief - CiB* aspires to be more than just another math book. This book strives to foster collaborative math writing. Note that this book has very few references. The reader is encouraged to use resources available on the Web to fact-check. This book's view on "causation" and facts is heavily influenced by Mosteller and Tukey [MT77].

Redefining the Role of the Reader

Calculus in Brief (CiB) is an endeavor to reshape how math is written, understood, and studied. It's not just a passive read but an open-source approach to math, aiming to encourage students to become proactive learners.

This project strives to break the traditional mold of math education and invites readers and professional mathematicians to participate actively.

A Dynamic Relationship with Math

Calculus in Brief is not just a book but a movement and methodology, heralding a new era in how we approach, consume, and interact with math. By positioning the reader as an integral part of the math-book process, CiB fosters a dynamic relationship with math, making mathematics more accessible, proactive, and relevant. In this shifting paradigm, we are all potential mathematicians, creators of interesting and relevant ways to learn and study math.

Please fork the LaTeX source code for CiB (available on GitHub) and create your own book that chooses the facts and exercises most relevant to you! Also, starring the CiB project on GitHub would be greatly appreciated! Thanks for reading CiB!

Chapter 1

Introduction to CiB

Welcome to CiB on GitHub

Calculus in Brief, abbreviated CiB, isn't merely a passive read. It's an endeavor to reshape how math is written, studied, and taught. By presenting an open-source approach to math, the goal is to encourage everyone to become proactive readers and writers of math.

Fostering a Proactive Engagement with Math

Calculus in Brief is a call for a renewed engagement with mathematics. CiB is an endeavor to reshape how math is written, understood, and studied. It's not just a passive read but an open-source approach to math, aiming to encourage students to become proactive learners.

This project strives to break the traditional mold of math education and encourages readers and professional mathematicians to participate actively.

Please fork the LaTeX source code for CiB (available on GitHub) and create your own book on Calculus that chooses the content most relevant to you! Also, starring the CiB project on GitHub would be greatly appreciated! Thanks for reading CiB!

Chapter 2

Open-Source Ethos

The Spirit of Shared Knowledge and Collaboration

Math, like software, is better when it's open. CiB draws inspiration from the open-source software movement; this section elucidates how a collaborative, transparent, and shared approach can enhance our understanding of math. Here, we look at the philosophy behind open-source and how it beautifully combines with the study of mathematics.

Open-Source Math: Preserving Tradition Through Collaborative Exploration

Mathematics, like software, thrives when it embraces openness and transparency. CiB takes a leaf from the proven benefits of the open-source software model; this section highlights how a collaborative and transparent method can improve and deepen our grasp of math and its texts. Here, we explore the principles of open source and how these principles align with the development of mathematics and its texts.

Understanding the Open-Source Ethos

The open-source paradigm revolves around shared ownership, collaboration, and the free exchange of knowledge. In the software realm, this approach has led to groundbreaking innovations built and enhanced by a global community of skilled contributors. United by a mutual objective, these individuals pool their diverse talents and insights to improve and share software solutions for broader public benefit.

Advantages of the Open-Source Framework in Math

Collective Insight

Mirroring the collaborative essence of open-source software, many individuals can offer their perspectives and knowledge, making math texts more robust and varied.

Enhancement and Accuracy

Open platforms foster an environment of constructive criticism, ensuring prompt identification and correction of inaccuracies. This meticulous peer review can help provide a credible and current mathematical text.

Universal Access

Much as open-source software promotes free access and modification, open-source math prioritizes universal accessibility. This ensures mathematics knowledge isn't restricted to a select few but is available to all curious minds.

Potential Challenges

Despite its advantages, melding open-source with math has potential pitfalls. The volume of contributions can complicate accuracy verification processes.

However, the very community championing this open-source approach to math can serve as its vigilant protector. They can ensure that contributions undergo rigorous evaluation and referencing, akin to the meticulous checks within the open-source software community.

Conclusion: Reinvigorating Our Experience with Math

Adopting an open-source perspective to the approach of math signifies a refreshed approach. It beckons a worldwide community to collaborate and forge a comprehensive and exciting math text. In this refreshed approach, every individual can play a part, both as a contributor and a learner. Math texts, through this lens, evolve and flourish, reflecting the collective input of active participants.

Chapter 3

Introduction to GitHub

The Hub for Modern Collaboration

Harnessing GitHub: A New Frontier in Collaborative Math Writing

At the heart of our collaborative math endeavor lies GitHub, a platform traditionally associated with code but now repurposed for our endeavor. This section provides a primer on GitHub, laying the foundation for those unfamiliar and offering insights into its transformative potential for collective math writing, learning, and teaching.

A Brief Introduction to GitHub

Originally conceptualized as a platform for developers, GitHub is a repository hosting service that facilitates version control using Git. At its core, it allows multiple users to work on a project simultaneously, tracking changes and ensuring that the latest version of a project is always accessible. Over the years, GitHub has grown beyond its initial software-centric confines, becoming a hub for all kinds of collaborative projects, from writing to data science and now to math.

Repurposing GitHub for Math Texts

Version Control

Math writing, like software, is dynamic and constantly evolving. As new sources or perspectives emerge, math texts may need revisions. GitHub's version control ensures that every change made to a document is tracked, enabling mathematicians to see how math texts evolve over time.

Collaborative Writing

Multiple contributors can work on a single math text simultaneously. This multi-user capability ensures diverse viewpoints can be seamlessly integrated, making the math text richer and more comprehensive.

Review and Feedback

Just as developers review and comment on code, mathematicians can provide feedback on written content. This feature encourages rigorous peer review, ensuring accuracy and credibility.

Open Access

Math texts on GitHub can be made public, granting anyone access to read, contribute, or fork the text into their own versions. This workflow democratizes math texts, making the creation process a collective endeavor rather than the domain of a select few.

Transparency

All changes and contributions are logged, providing a clear trail of the evolution of a mathematical text. This transparency bolsters the credibility of the text hosted on the platform.

Community Building

Beyond just writing, GitHub fosters a community of mathematicians, enthusiasts, and readers who can discuss, debate, and engage in meaningful dialogues about math and available math texts on GitHub.

Conclusion: Envisioning a Collaborative Mathematical Landscape

Embracing GitHub as a tool for collaborative math signifies more than just a shift in approach; it heralds a new era of inclusivity, transparency, and dynamism in writing, learning, and math teaching.

Chapter 4

Encouragement to Fork

Invitation to Dive Deep and Make It Your Own

CiB isn't a static entity. It thrives on evolution, adaptation, and diversification, much like math itself. We encourage readers to "fork" (a term soon to be discussed) and create their own versions of this book. Read this section to understand the essence of "forking" and how it can be the starting point of your unique math journey.

The Concept of Forking: A Brief Overview

In the realm of software development, particularly in platforms like GitHub, "forking" refers to the act of creating a copy of a project, allowing one to make changes independently of the original. In this context, forking CiB enables readers to take the base content and adapt, modify, and expand upon it, tailoring the narrative to resonate with their perspectives, insights, and understanding.

How to Begin Your Forking Journey

Start Small: You don't need to rewrite entire chapters. Begin by adding annotations, insights, or even footnotes to existing content. As you grow more confident, you can expand and modify larger sections.

Engage with the Community: Share your forked version with other readers. This encourages discourse, debate, and constructive feedback, allowing your text to be refined and enhanced.

Celebrate input: Encourage others around you to fork and create their own versions. The more in-depth the input, the deeper our collective understanding of math potentially becomes.

Conclusion: The Power of Collective Math

The invitation to fork CiB isn't just about creating different versions of a book. It's a call to embrace collective writing, learning, and teaching. By embracing the essence of forking, math is not just something we read but something we actively shape, share, and pass on.

Chapter 5

More About GitHub

Discovering the Power of Collaborative Tools

Diving deeper into the world of GitHub, this chapter provides a comprehensive overview. Beyond its technicalities, we explore how GitHub emerged as a revolutionary platform for collaboration and how it can be leveraged for those interested in writing, teaching, and learning about math.

The Genesis of GitHub

GitHub began as a platform designed for software developers to manage and track changes to their codebase. Launched in 2008, it swiftly gained traction due to its user-friendly interface and efficient version control system powered by Git. Over the years, it evolved from a mere repository hosting service to a dynamic hub of collaboration, housing millions of projects and engaging tens of millions of users worldwide.

GitHub: More than Just Code

While GitHub's origins are rooted in code collaboration, its adaptable nature has made it a favored platform for various non-code projects. Writers, designers, educators, and researchers have discovered the potential of GitHub as a tool for:

Document Collaboration

With its built-in version control, contributors can track changes, revert to previous versions, and seamlessly merge updates.

Project Management

With features like "issues" and "milestones," teams can organize tasks, set goals, and monitor progress.

Open Access & Transparency

Public repositories allow for open contributions, ensuring transparency and fostering a sense of collective ownership.

Collaborative Writing

Multiple contributors can simultaneously work on a single document, with every change being tracked and attributed, facilitating teamwork on extensive projects like books or research papers.

Engaging the Public

With the platform's inherent transparency, researchers can make their work-in-progress accessible to the public, inviting insights, corrections, and contributions.

Case Study: CiB's Use of GitHub

CiB's journey on GitHub is a testament to the platform's potential in mathematical endeavors. By hosting the book on GitHub, the following is possible:

Feedback Loop

Readers can raise "issues," pointing out inaccuracies, suggesting enhancements, or even recommending new sections or topics.

Forking

As previously discussed, readers can "fork" the repository, creating their unique versions of the book while staying connected to the original.

Regular Updates

With math being dynamic, the book can be regularly updated, with new versions being released as and when significant changes are incorporated.

Challenges and Considerations

While GitHub offers many advantages, it's essential to understand its limitations:

Learning Curve

For those unfamiliar with Git or version control, there can be an initial learning curve.

Data Overwhelm

With vast amounts of data and contributions, ensuring quality and accuracy can be challenging.

Diverse Audience Management

Catering to both tech-savvy and non-tech audiences might require creating additional resources or tutorials to ensure inclusivity.

Conclusion: GitHub – A Paradigm Shift in Collaboration

The rise of GitHub marks a significant shift in how we perceive and participate in collaborative projects. Its adaptability, transparency, and user-centric design make it a powerful tool, not just for coders but for anyone passionate about collective endeavors. In the realm of mathematics, GitHub promises a future where texts are continually refined, expanded, and enriched by a global community.

Chapter 6

Forking Process

The Heart of Collaboration on GitHub

The beauty of open-source lies in its democratization of content creation. In this section, we demystify the process of "forking" on GitHub, guiding you step-by-step on how to take CiB and create a version uniquely yours.

Understanding Forking

Before diving into the specifics, it's crucial to understand what "forking" means in the context of GitHub. In the simplest terms, to "fork" a project means to create a personal copy of someone else's project. Forking allows you to freely experiment with changes without affecting the original project. Forking is akin to taking a book you admire and making a copy to write your notes, edits, or additional chapters without altering the original book.

Why Fork?

Experimentation

It provides a safe space where you can test out ideas, make changes, or introduce new content.

Personalization

For projects like CiB, it allows readers to customize the content, tailor it to their perspectives, or even localize it for specific audiences.

Collaboration

If you believe your changes have broad appeal, you can propose that they be incorporated back into the original project, enriching it with your unique contributions.

Step-by-Step Forking Guide

Set Up Your GitHub Account

If you don't have an account on GitHub, you'll need to create one. Visit GitHub's official site and sign up.

Navigate to the CiB Repository

Once logged in, search for the CiB project or navigate to its URL directly.

Click the 'Fork' Button

The fork button is located at the top right corner of the repository page; this button will create a copy of CiB in your account.

Clone Your Forked Repository

Forking allows you to have a local copy on your computer, making editing and experimentation easier. Use the command: `git clone [URL of your forked repo]`.

Make Your Changes

Using your preferred tools, introduce the edits, additions, or modifications you desire.

Commit and Push Changes

Once satisfied, save these changes (known as a "commit") and then "push" them to your forked repository on GitHub.

Optional – Create a Pull Request

If you believe your changes should be incorporated into the original CiB repository, you can create a "pull request." A pull request notifies the original authors of your suggestions.

Things to Keep in Mind

Stay Updated

The original CiB project may undergo updates. It's a good practice to regularly "pull" from the original repo to keep your fork up-to-date.

Engage with the Community

Open-source thrives on community interactions. Engage in discussions, seek feedback, and please remain open to constructive criticism.

Conclusion: Embracing the Forking Culture

Forking is more than just a technical process; it symbolizes the ethos of open-source — a world where knowledge is not hoarded but shared, refined, and built upon collectively. By forking CiB or any other project, you're not just creating a personal copy; you're becoming a part of a global movement that values collaboration, innovation, and the shared pursuit of knowledge. So, embark on this journey, make your unique mark, and contribute to the ever-evolving corpus of collective wisdom.

Chapter 7

Editing and Customizing

Tailoring Repositories to Suit Your Needs

Now, let's build upon the forking process; this segment delves into the next steps. How can you edit and customize your version of CiB? What tools and techniques are available at your disposal? Embark on this informative journey as we guide you through the intricacies of editing on GitHub.

Understanding the GitHub Workspace

Before diving into the specifics of editing, it's essential to familiarize yourself with the GitHub workspace. Think of it as a digital toolshed where each tool serves a unique function:

- **Repository (Repo)**: This is the project's main folder where all your project's files are stored and where you track all changes.
- **Branches**: These are parallel versions of a repository, allowing you to work on features or edits without altering the main project.
- **Commits**: This is a saved change in the repository, akin to saving a file after making edits.
- **Pull Requests**: This is how you notify the main project of desired changes, proposing that your edits be merged with the original.

Editing Files Directly on GitHub

For minor changes, you might opt to edit directly on GitHub:

1. Navigate to the File: Within your forked CiB repository, find the file you want to edit.

2. Click the Pencil Icon: This button allows you to edit the file.

3. Make Your Edits: Modify the content as needed.

4. Save and Commit: Below the editing pane, you'll see a "commit changes" section. Add a brief note summarizing your changes and click 'Commit.'

Editing Files Locally

For extensive customization:

1. Clone Your Repository: Use a tool like Git to clone (download) your forked repo to your local computer.

2. Edit Using Your Preferred Tools: This could range from text editors to specialized software, depending on the file type.

3. Commit and Push: After making your changes, save them (commit) and then upload (push) them to your GitHub repository.

Utilizing Branches for Extensive Customization

Branches are especially useful for significant overhauls or when working on different versions:

1. Create a New Branch: From your main project page, use the branch dropdown to type in a new branch name and create it.

2. Switch to Your Branch: Ensure you're working in this new parallel environment.

3. Make and Commit Changes: As you would in the main project.

4. Merging: Once satisfied with your edits in the branch, you can merge these changes back into the main project or keep them separate as a different version.

Exploring Additional Tools and Extensions

GitHub's ecosystem is rich with tools and extensions to enhance your editing experience:

- **GitHub Desktop**: An application that simplifies the process of managing your repositories without using command-line tools.
- **Markdown Editors**: Since many GitHub files (like READMEs) are written in Markdown, tools like StackEdit or Dillinger can be invaluable.
- **Extensions for Browsers**: Tools like Octotree can help in navigating repositories more effortlessly.

Conclusion: The Art of Tailored Content

Editing and customizing on GitHub might seem daunting initially, but with practice, it transforms into a manageable workflow. Many people find that the ability to take a project like CiB and mold it into something uniquely theirs is empowering. It's a testament to the open-source community's ethos, where shared knowledge becomes the canvas and our collective edits, the brushstrokes, crafting an ever-evolving masterpiece. As you embark on your customization journey, remember that every edit, no matter how small, contributes to the project potentially in significant ways.

Chapter 8

Engaging with the Community

Joining the Global Conversation

The Significance of the GitHub Community

The digital age has bestowed upon us the gift of connectivity. On platforms like GitHub, this connectivity transcends borders, disciplines, and ideologies, culminating in a melting pot of diverse ideas and knowledge. For mathematicians and math enthusiasts, GitHub offers a space not only to store and manage content but also to engage with an audience that is passionate, informed, and eager to contribute.

1. Discussions and Debates

One of the most enriching aspects of the GitHub community is the plethora of discussions that unfold:

- **Issues**: A core feature of GitHub, "issues" allow users to raise questions, report problems, or propose enhancements.
- **GitHub Discussions**: A newer feature, Discussions, acts like a community forum. It's an excellent place for extended conversations, brainstorming, and sharing ideas or resources.

2. Collaborative Content Creation

Beyond solitary endeavors, GitHub shines in its collaborative capabilities:

- **Pull Requests**: If you have made an alteration to a math text or added a new perspective, pull requests are the way to propose these changes to the original repository owner. Pull requests foster a collaborative spirit, where content isn't static but continually evolving with community input.

- **Fork and Merge**: As you've learned, forking allows you to create your version of a repository. Engaging with the Community means you can merge changes from others into your fork, blending a mixture of diverse insights.

3. Building and Nurturing Networks

Connections made on GitHub often spill over into lasting professional relationships:

- **Following and Followers**: Like on social media platforms, you can follow contributors whose work resonates with you. Following contributors creates a curated feed of updates and also allows you to be part of a more extensive network.
- **GitHub Stars**: If a particular project or repository impresses you, give it a star! Starring not only bookmarks the project for you but also shows appreciation to the creator.

4. Learning and Growing Through Feedback

The Community's feedback is an invaluable asset:

- **Code Reviews**: Although traditionally for software, text writers can use this feature to receive feedback on their methodologies or approaches, refining their work.
- **Community Insights**: The "insights" tab on a repository provides analytics. For text writers, this can give a sense of which topics garner more attention and interest.

5. Participating in Community Events

GitHub often hosts and sponsors events:

- **Hackathons**: While traditionally for coders, these events can be repurposed for text writer content creation, where participants collaboratively tackle projects or themes.
- **Webinars and Workshops**: These events can range from mastering GitHub's technical side to thematic discussions on math topics.

A Project of Collective Wisdom

Math, in many ways, is a collective endeavor. GitHub can provide a dynamic Community. By engaging with this Community you can become an active participant in the creation of mathematical texts.

Chapter 9

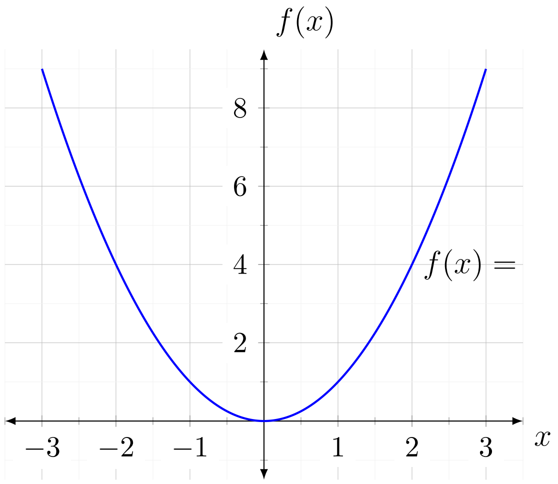
Functions

9.1 What is a Function?

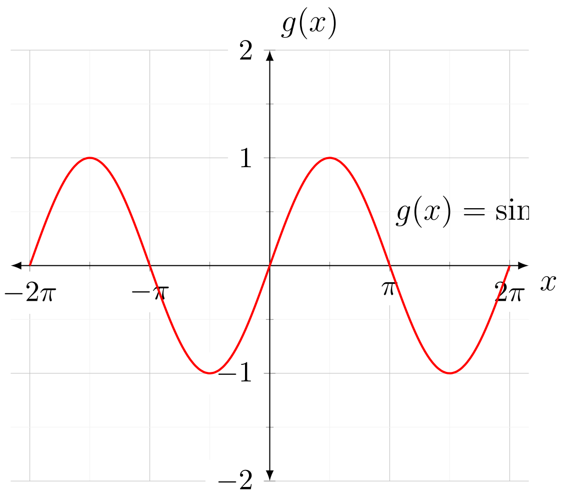
9.1.1 Definition and Examples

Definition 1 *A function is a relation between a set of inputs (domain) and a set of permissible outputs (range) with the property that each input is related to exactly one output.*

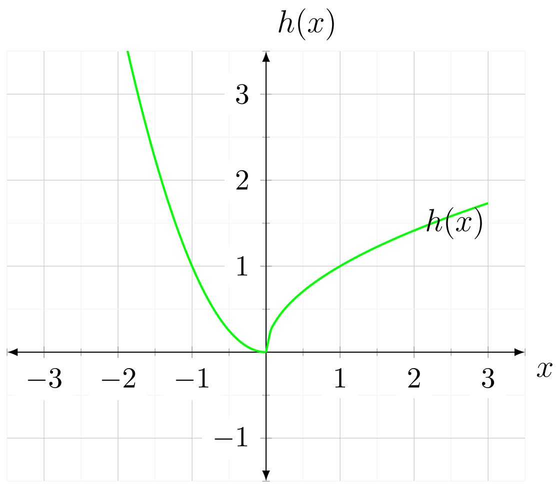
Example 1: Consider the function $f(x) = x^2$, which maps each real number to its square.



Example 2: Consider the function $g(x) = \sin(x)$, which maps real numbers to their sine values.



Example 3: Consider the piecewise function $h(x)$ defined as $h(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ \sqrt{x} & \text{if } x > 0 \end{cases}$.



9.1.2 Exercises

Exercise Graph the function $f(x) = 3x - 2$. Identify its slope and y-intercept.

Exercise Consider the function $g(x) = \frac{1}{x}$. For what values of x is $g(x)$ undefined?

Exercise Find the domain and range of the function $h(x) = \sqrt{x+4}$.

Exercise Determine if the function $f(x) = x^3 - x$ is even, odd, or neither. Justify your answer.

Exercise Sketch the graph of the piecewise function $p(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x+1 & \text{if } x \geq 0 \end{cases}$. Indicate any points of discontinuity.

Exercise Given the function $f(x) = 2x^2 - 5x + 3$, find $f(-1)$ and $f(2)$.

Exercise For the function $f(x) = |x - 3|$, find the x-coordinate where $f(x) = 0$.

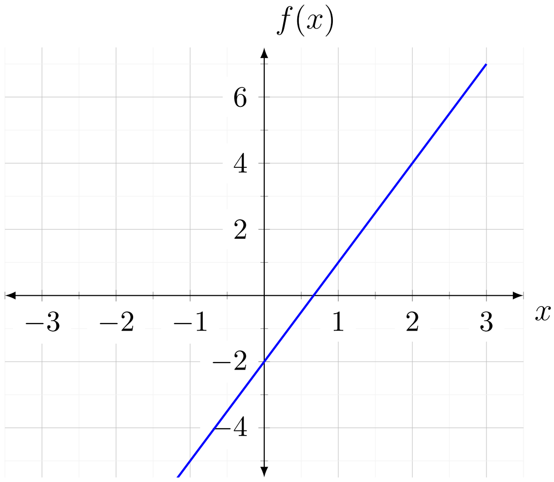
Exercise Create a function $f(x)$ that has a domain of all real numbers except $x = 2$ and a range of $y \geq 0$.

Exercise Determine the intervals on which the function $f(x) = -x^2+4x-3$ is increasing and decreasing.

Exercise For the function $g(x) = \cos(x)$, evaluate $g(\pi/2)$ and $g(\pi)$.

9.1.3 Solutions to Exercises

Solution: To graph $f(x) = 3x - 2$, we note that the slope (m) is 3 and the y-intercept (b) is -2. The graph is a straight line with these characteristics.

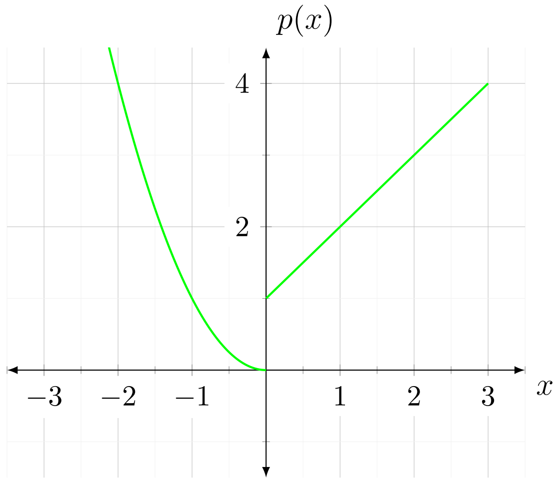


Solution: The function $g(x) = \frac{1}{x}$ is undefined when $x = 0$ because division by zero is not allowed.

Solution: For $h(x) = \sqrt{x+4}$, the domain is $x \geq -4$ (since the expression inside the square root must be non-negative), and the range is $y \geq 0$ (since the square root produces non-negative results).

Solution: The function $f(x) = x^3 - x$ is odd because $f(-x) = (-x)^3 - (-x) = -x^3 + x = -f(x)$.

Solution: The piecewise function $p(x)$ is sketched below. It is discontinuous at $x = 0$ since the left-hand limit (0) does not equal the right-hand limit (1).



Solution: For $f(x) = 2x^2 - 5x + 3$, we find $f(-1) = 2(-1)^2 - 5(-1) + 3 = 10$ and $f(2) = 2(2)^2 - 5(2) + 3 = -1$.

Solution: The function $f(x) = |x - 3|$ equals 0 when $x - 3 = 0$, i.e., $x = 3$.

Solution: One such function is $f(x) = \frac{1}{(x-2)^2}$. The denominator ensures that $x \neq 2$ (domain), and the square ensures all output values are positive (range).

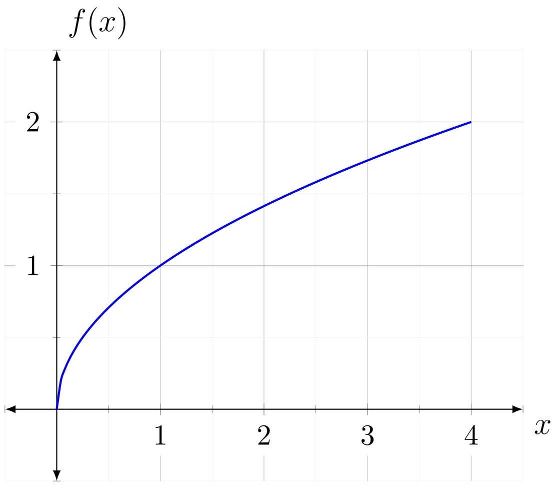
Solution: For $f(x) = -x^2 + 4x - 3$, the function is increasing where its derivative $f'(x) = -2x + 4$ is positive, i.e., for $x < 2$, and decreasing where $f'(x)$ is negative, i.e., for $x > 2$.

Solution: For $g(x) = \cos(x)$, we have $g(\pi/2) = \cos(\pi/2) = 0$ and $g(\pi) = \cos(\pi) = -1$.

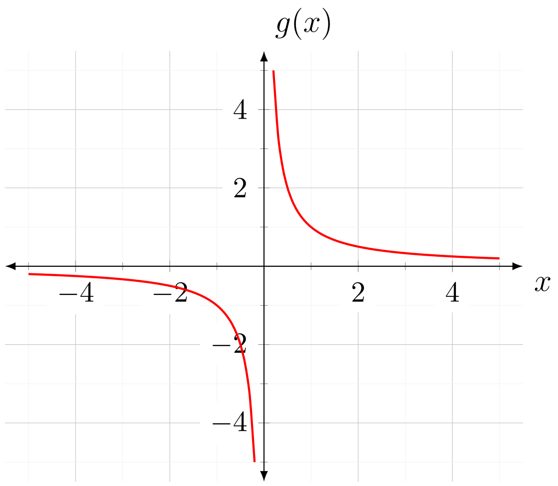
9.2 Domain and Range

Definition 2 *The domain of a function is the set of all possible inputs, while the range is the set of all possible outputs.*

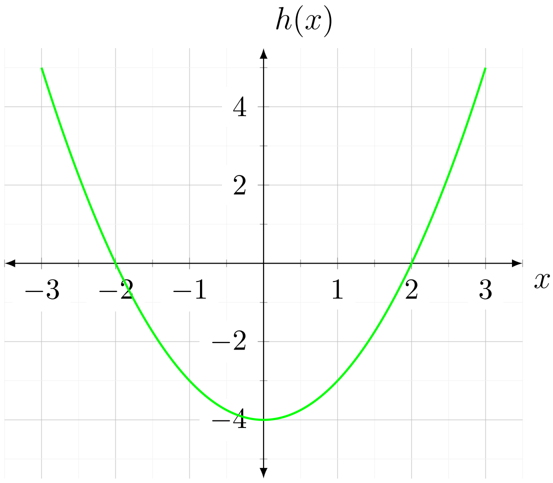
Example 4: For $f(x) = \sqrt{x}$, the domain is $x \geq 0$ and the range is $y \geq 0$. The graph of this function is as follows:



Example 5: Consider the function $g(x) = \frac{1}{x}$. Its domain is $x \neq 0$ and its range is $y \neq 0$. The graph is given by:



Example 6: For the function $h(x) = x^2 - 4$, the domain is all real numbers, but the range is $y \geq -4$. Its graph is:



9.2.1 Exercises on Domain and Range

Exercise Determine the domain and range of the function $f(x) = 2x + 3$.

Exercise Find the domain and range of the function $g(x) = \frac{1}{x-2}$.

Exercise Identify the domain and range for the quadratic function $h(x) = x^2 - 6x + 9$.

Exercise For the function $k(x) = \ln(x-1)$, determine its domain and range.

Exercise Consider the function $m(x) = \frac{x}{x^2-4}$. What are its domain and range?

Exercise Find the domain and range for the trigonometric function $n(x) = \sin(x)$.

Exercise Determine the domain and range of the absolute value function $p(x) = |x + 5|$.

Exercise For the piecewise function $q(x) = \begin{cases} x^2 & \text{if } x \leq 0 \\ 2x + 3 & \text{if } x > 0 \end{cases}$, identify the domain and range.

Exercise Consider the exponential function $r(x) = 2^x$. What is its domain and range?

Exercise Identify the domain and range for the cubic function $s(x) = x^3 - 3x$.

9.2.2 Solutions to Exercises on Domain and Range

Solution: For $f(x) = 2x + 3$, the domain is all real numbers since there are no restrictions on x. The range is also all real numbers because as x takes any real value, $2x + 3$ also covers all real numbers.

Solution: The function $g(x) = \frac{1}{x-2}$ is undefined when $x = 2$, so the domain is $x \neq 2$. The range is all real numbers except $y \neq 0$ because the function never touches the y-axis.

Solution: For the quadratic function $h(x) = x^2 - 6x + 9$, the domain is all real numbers. Since the vertex of this parabola is at $x = 3$ and it opens upwards, the minimum value is $h(3) = 0$, thus the range is $y \geq 0$.

Solution: The function $k(x) = \ln(x - 1)$ requires $x - 1 > 0$, so the domain is $x > 1$. The range of a natural logarithm function is all real numbers.

Solution: For $m(x) = \frac{x}{x^2-4}$, the function is undefined for $x = \pm 2$, so the domain is $x \neq \pm 2$. The range is all real numbers, as the function can take any y-value.

Solution: The domain of the trigonometric function $n(x) = \sin(x)$ is all real numbers. The range of the sine function is between -1 and 1, inclusive, so $y \in [-1, 1]$.

Solution: For the absolute value function $p(x) = |x+5|$, the domain is all real numbers. The range is $y \geq 0$ since absolute values are always non-negative.

Solution: The piecewise function $q(x)$ is defined for all real numbers, so the domain is all real numbers. To find the range, consider each piece: x^2 and $2x + 3$. The range is all non-negative values for x^2 and all real numbers for $2x + 3$, thus the range is all real numbers.

Solution: For the exponential function $r(x) = 2^x$, the domain is all real numbers, as exponents can take any real value. The range is $y > 0$ since exponential functions are always positive.

Solution: The cubic function $s(x) = x^3 - 3x$ has a domain of all real numbers. The range is also all real numbers because as a cubic function, it extends infinitely in both the positive and negative y-directions.

9.3 Types of Functions

9.3.1 Linear Functions

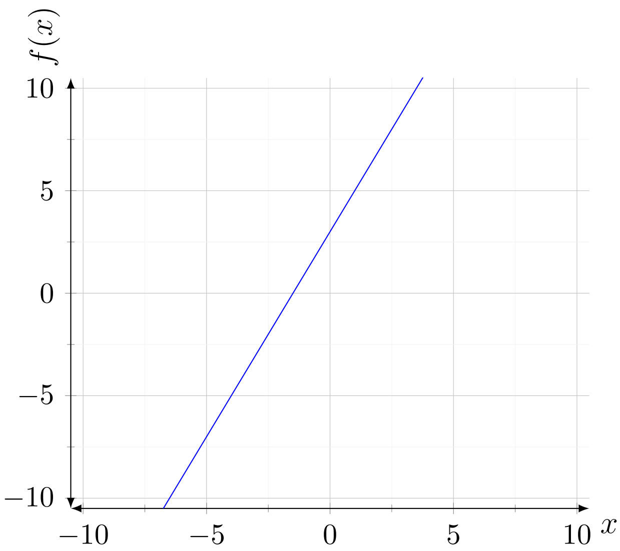
Definition 3 *A linear function has the form $f(x) = mx + b$, where m is the slope of the line and b is the y-intercept.*

Linear functions are fundamental in calculus and represent the simplest form of polynomial functions. Their graph is a straight line, and they have constant rates of change.

Properties of Linear Functions:

- The slope m determines the steepness of the line and its direction (increasing or decreasing).
- The y-intercept b is the point where the line crosses the y-axis.
- Linear functions have a constant rate of change, which is equal to the slope m.

Graphing a Linear Function: Consider the function $f(x) = 2x + 3$. Here, $m = 2$ and $b = 3$. The graph can be plotted as follows:



This line intersects the y-axis at (0, 3) and has a slope of 2, indicating it rises 2 units for every 1 unit it moves to the right.

Slope-Intercept Form: The equation $f(x) = mx + b$ is known as the slope-intercept form of a linear function, where the slope m and y-intercept b are easily identifiable.

Applications of Linear Functions: Linear functions model relationships with a constant rate of change and are widely used in various fields such as economics, physics, and social sciences.

Exercise Find the slope and y-intercept of the linear function $g(x) = -3x+7$ and sketch its graph.

Exercise If a linear function passes through the points (1, 2) and (3, 6), determine its equation.

Exercises on Linear Functions

Exercise Find the slope and y-intercept of the linear function $g(x) = -3x+7$ and sketch its graph.

Exercise If a linear function passes through the points (1, 2) and (3, 6), determine its equation.

Exercise Determine the equation of the line that is parallel to $f(x) = 4x-5$ and passes through the point (2, 3).

Exercise Find the point of intersection of the linear functions $f(x) = 2x+1$ and $g(x) = -x + 5$.

Exercise A rental car company charges a base fee of \$50 and an additional \$20 per day. Write a linear function representing the total cost C as a function of the number of days d rented.

Exercise Graph the linear function $h(x) = -\frac{1}{2}x + 4$ and identify where it intersects the x-axis and y-axis.

Exercise A phone company offers a monthly plan for \$30 with an additional charge of \$0.05 per minute of calls. Write a linear function representing the monthly cost M as a function of the number of minutes m used.

Exercise Determine whether the points (2, 4), (3, 6), and (5, 10) lie on the same linear function. If so, find the equation of the line.

Exercise Find the slope of the line that passes through the points (-1, -2) and (3, 4).

Exercise A company's profit P in thousands of dollars is linearly related to the number of units n sold, with a profit of \$4,000 for 500 units and a loss of \$2,000 for 200 units. Determine the linear function that models the profit.

Solutions to Exercises on Linear Functions

Solution: For $g(x) = -3x + 7$, the slope (m) is -3, and the y-intercept (b) is 7. The graph is a straight line with a downward slope, crossing the y-axis at (0, 7).

Solution: To find the equation of a line passing through (1, 2) and (3, 6), calculate the slope $m = \frac{6-2}{3-1} = 2$. Using the point-slope form: $y-2 = 2(x-1)$, the equation is $y = 2x$.

Solution: A line parallel to $f(x) = 4x-5$ has the same slope, m = 4. Passing through (2, 3), use point-slope form: $y - 3 = 4(x - 2)$, yielding $y = 4x - 5$.

Solution: To find the intersection of $f(x) = 2x + 1$ and $g(x) = -x + 5$, set them equal: $2x + 1 = -x + 5$. Solving gives $x = \frac{4}{3}$, and substituting back, $y = \frac{11}{3}$. Thus, they intersect at $(\frac{4}{3}, \frac{11}{3})$.

Solution: The total cost C is given by $C = 50 + 20d$, where d is the number of days rented.

Solution: For $h(x) = -\frac{1}{2}x + 4$, the graph intersects the y-axis at (0, 4) and intersects the x-axis at x where $h(x) = 0$. Setting the equation to 0 gives $x = 8$. So, it intersects the x-axis at (8, 0).

Solution: The monthly cost M is $M = 30 + 0.05m$, where m is the number of minutes used.

Solution: The slope between (2, 4) and (3, 6) is 2, and between (3, 6) and (5, 10) is also 2. Since the slope is consistent, they lie on the same line. The line equation, using point-slope form, is $y = 2x$.

Solution: The slope is $\frac{4-(-2)}{3-(-1)} = \frac{6}{4} = \frac{3}{2}$.

Solution: Let $P = mn + b$. Using given points, form equations: $4000 = 500m + b$ and $-2000 = 200m + b$. Solving these simultaneously gives $m = 20$ and $b = -6000$. So, $P = 20n - 6000$.

9.3.2 Polynomial Functions

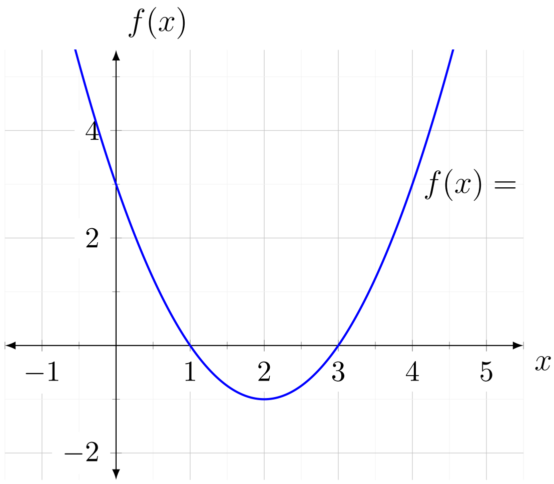
Definition 4 *A polynomial function is of the form $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_0$, where n is a non-negative integer, and $a_n, a_{n-1}, \ldots, a_0$ are constants.*

Polynomial functions are fundamental in calculus and algebra. They can range from simple linear functions to complex equations with high degrees. The highest power of x in the function (the degree of the polynomial) greatly influences the shape and behavior of its graph.

Characteristics of Polynomial Functions:

- The degree of the polynomial determines its basic shape and the number of roots.
- Polynomials are continuous and smooth functions.
- The leading coefficient a_n affects the end behavior of the polynomial.

Graphing Polynomial Functions: Consider the quadratic function $f(x) = x^2 - 4x + 3$. Its graph can be plotted as follows:



This graph represents a parabola opening upwards with its vertex and intercepts easily identifiable.

Real-World Applications: Polynomial functions model numerous real-world phenomena, including projectile motion, profit and loss calculations, population growth, and much more.

Exercise Sketch the graph of the cubic function $g(x) = x^3 - 6x^2 + 11x - 6$. Identify its roots and turning points.

Exercise Find a polynomial of degree 3 that has roots at $x = 1, 2,$ and 3.

Exercises on Polynomial Functions

Exercise Sketch the graph of the cubic function $g(x) = x^3 - 6x^2 + 11x - 6$. Identify its roots and turning points.

Exercise Find a polynomial of degree 3 that has roots at $x = 1, 2,$ and 3.

Exercise Determine the degree and leading coefficient of the polynomial $h(x) = 4x^4 - 3x^3 + 2x^2 - x + 7$.

Exercise For the quadratic function $f(x) = 2x^2 - 8x + 6$, find the vertex and axis of symmetry. Also, determine whether it opens upwards or downwards.

Exercise Write the polynomial function $p(x) = x^3 - 4x$ in factored form and identify its zeros.

Exercise A polynomial function has zeros at $x = -2, 0,$ and 4 with a leading coefficient of 1. Write the equation of this polynomial.

Exercise Given the polynomial $q(x) = x^4 - 10x^2 + 9$, find the y-intercept and x-intercepts (if any).

Exercise Graph the polynomial $r(x) = -x^3 + 3x^2 - 2x$ and determine the intervals where the function is increasing and decreasing.

Exercise For the polynomial function $s(x) = 3x^3 - 5x^2 + x - 2$, use synthetic division to determine whether $x = 1$ is a zero of s.

Exercise Consider the polynomial $t(x) = x^4 - 2x^2 - 3$. Find all the real zeros of t and sketch the graph of the function.

Solutions to Exercises on Polynomial Functions

Solution: For $g(x) = x^3 - 6x^2 + 11x - 6$, the roots are found by solving $g(x) = 0$. The roots are $x = 1, 2, 3$. Turning points occur where the derivative changes sign. Calculating $g'(x) = 3x^2 - 12x + 11$ and finding its roots gives the turning points at approximately $x \approx 1.57$ and $x \approx 2.43$.

Solution: A polynomial of degree 3 with roots at $x = 1, 2, 3$ can be written as $f(x) = (x - 1)(x - 2)(x - 3)$.

Solution: The degree of the polynomial $h(x) = 4x^4 - 3x^3 + 2x^2 - x + 7$ is 4, and the leading coefficient is 4.

Solution: For $f(x) = 2x^2 - 8x + 6$, the vertex form is $f(x) = 2(x - 2)^2 + 2$. The vertex is at (2, 2), and the axis of symmetry is $x = 2$. The parabola opens upwards since the coefficient of x^2 is positive.

Solution: The polynomial $p(x) = x^3 - 4x$ can be factored as $p(x) = x(x^2 - 4) = x(x - 2)(x + 2)$. The zeros are at $x = 0, 2, -2$.

Solution: A polynomial with zeros at $x = -2, 0$, and 4 and a leading coefficient of 1 is $f(x) = (x+2)x(x-4) = x^3 - 4x^2 - 2x$.

Solution: For $q(x) = x^4 - 10x^2 + 9$, the y-intercept is $q(0) = 9$. To find the x-intercepts, solve $x^4 - 10x^2 + 9 = 0$, which gives $x \approx \pm 3.16, \pm 0.84$.

Solution: Graphing $r(x) = -x^3 + 3x^2 - 2x$, the function increases where $r'(x) > 0$ and decreases where $r'(x) < 0$. Calculating $r'(x) = -3x^2 + 6x - 2$ and finding its roots gives the intervals of increase and decrease.

Solution: Using synthetic division to divide $3x^3 - 5x^2 + x - 2$ by $x - 1$, we find that the remainder is not zero. Hence, $x = 1$ is not a zero of $s(x)$.

Solution: To find the real zeros of $t(x) = x^4 - 2x^2 - 3$, solve $x^4 - 2x^2 - 3 = 0$. The real zeros are approximately $x \approx \pm 1.73, \pm 1.00$. Graphing $t(x)$ would show a polynomial crossing the x-axis at these points.

9.3.3 Rational Functions

Definition 5 *A rational function is the ratio of two polynomial functions.*

9.3.4 Trigonometric Functions

Definition 6 *Trigonometric functions include* $\sin(x)$, $\cos(x)$, *and* $\tan(x)$, *relating angles to ratios of sides in right triangles.*

9.3.5 Exponential and Logarithmic Functions

Definition 7 *Exponential functions have the form* $f(x) = a^x$ *and logarithmic functions are their inverses.*

9.4 Function Properties

9.4.1 Continuity

Definition 8 *A function is continuous at a point if the limit of the function as it approaches the point is equal to the function value at that point.*

9.4.2 Differentiability

Definition 9 *A function is differentiable at a point if it has a defined derivative at that point.*

9.4.3 Asymptotic Behavior

Definition 10 *Asymptotic behavior refers to the behavior of a function as it approaches infinity or a particular value.*

9.4.4 Periodicity and Symmetry

Definition 11 *A function is periodic if it repeats its values at regular intervals, and it is symmetric if it exhibits mirror symmetry about an axis or point.*

9.5 Function Transformations

9.5.1 Translation and Scaling

Definition 12 *Translation shifts a function horizontally or vertically, while scaling changes its size.*

9.5.2 Reflection and Rotation

Definition 13 *Reflection inverts a function across a line, and rotation turns it around a point.*

Chapter 10

Limits and Continuity

10.1 Introduction to Limits

10.1.1 Definition of a Limit

Definition 14 *The limit of $f(x)$ as x approaches a is L if for every $\epsilon > 0$, there exists a $\delta > 0$ such that $|f(x) - L| < \epsilon$ whenever $0 < |x - a| < \delta$.*

10.1.2 One-sided Limits

Definition 15 *The one-sided limits of $f(x)$ as x approaches a from the left (or right) are the values the function approaches as x approaches a from the left (or right).*

10.1.3 Limits Involving Infinity

Definition 16 *A limit involving infinity is the value that $f(x)$ approaches as x approaches infinity, or as $f(x)$ approaches infinity for some finite x.*

10.2 Properties of Limits

10.2.1 Limit Laws

Theorem 1 *Standard limit laws include the sum law, product law, quotient law, and power law.*

10.2.2 Squeeze Theorem

Theorem 2 *If $f(x) \leq g(x) \leq h(x)$ for all x near a and the limits of $f(x)$ and $h(x)$ as x approaches a are equal, then the limit of $g(x)$ as x approaches a is the same.*

10.2.3 Limits of Trigonometric Functions

Theorem 3 *Includes limits such as $\lim_{x \to 0} \frac{\sin x}{x} = 1$.*

10.3 Continuity

10.3.1 Definition and Examples

Definition 17 *A function is continuous at a point if it is defined at that point, its limit exists at that point, and the limit equals the function value.*

10.3.2 Types of Discontinuities

Definition 18 *Discontinuities can be classified as removable, jump, or infinite.*

10.3.3 Intermediate Value Theorem

Theorem 4 *If a function is continuous on a closed interval $[a, b]$ and N is any number between $f(a)$ and $f(b)$, then there exists a number c in the interval such that $f(c) = N$.*

Chapter 11

The Derivative

11.1 Understanding Derivatives

11.1.1 Definition and Notation

Definition 19 *The derivative of a function f at a point a is defined as $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$, if this limit exists.*

11.1.2 Interpreting Derivatives

Discussion 1 *The derivative at a point gives the slope of the tangent line to the function at that point. It represents the rate of change of the function with respect to its variable.*

11.1.3 Derivative as a Function

Definition 20 *The derivative of a function f is itself a function that assigns to each point x the slope of the tangent to f at x.*

11.2 Techniques of Differentiation

11.2.1 Product Rule and Quotient Rule

Theorem 5 *The product rule states that $(fg)' = f'g + fg'$ and the quotient rule states that $\left(\frac{f}{g}\right)' = \frac{f'g-fg'}{g^2}$, where f and g are functions of x.*

11.2.2 Chain Rule

Theorem 6 *If a function y is a function of u which is a function of x, then the derivative of y with respect to x is given by $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$.*

11.2.3 Higher-Order Derivatives

Definition 21 *The n-th order derivative of f, denoted $f^{(n)}$, is the derivative of $f^{(n-1)}$.*

11.2.4 Implicit Differentiation

Method 1 *Implicit differentiation is used when a function is given in an implicit form $F(x, y) = 0$ rather than the explicit form $y = f(x)$.*

11.3 Applications of Derivatives

11.3.1 Tangent Lines and Normal Lines

Application 1 *The equation of the tangent line to a curve $y = f(x)$ at a point $(a, f(a))$ is $y - f(a) = f'(a)(x - a)$.*

11.3.2 Increasing and Decreasing Functions

Theorem 7 *A function f is increasing (decreasing) on an interval if its derivative f' is positive (negative) on that interval.*

11.3.3 Maxima and Minima

Method 2 *Critical points, where $f'(x) = 0$ or $f'(x)$ does not exist, are potential locations of local maxima or minima.*

11.3.4 Concavity and Inflection Points

Discussion 2 *A function is concave up (down) where its second derivative f'' is positive (negative). Points where f'' changes sign are inflection points.*

11.3.5 Optimization Problems

Application 2 *Optimization involves finding the maximum or minimum values of a function subject to certain constraints, often using derivatives.*

11.3.6 Related Rates

Method 3 *Related rates problems involve finding the rate at which one quantity changes with respect to another using the chain rule.*

Chapter 12

Integration

12.1 Antiderivatives

12.1.1 Definition and Basic Techniques

Definition 22 *An antiderivative of a function $f(x)$ is a function $F(x)$ such that $F'(x) = f(x)$.*

Example 7: An antiderivative of $f(x) = 2x$ is $F(x) = x^2 + C$, where C is a constant.

12.1.2 Indefinite Integrals

Definition 23 *The indefinite integral of $f(x)$, denoted $\int f(x)\,dx$, represents the collection of all its antiderivatives.*

12.1.3 Initial Value Problems

Problem 1: Given a differential equation $f'(x) = g(x)$ and an initial condition $f(x_0) = y_0$, find the function $f(x)$.

12.2 Definite Integrals

12.2.1 Definition and Interpretation

Definition 24 *The definite integral of $f(x)$ from a to b, denoted $\int_a^b f(x)\,dx$, is the limit of the sum of areas of rectangles under the curve of $f(x)$ as the width of the rectangles approaches zero.*

12.2.2 Properties of Definite Integrals

Theorem 8 *Properties include linearity, additivity, and the fact that the integral from a to a is zero.*

12.2.3 Fundamental Theorem of Calculus

Theorem 9 *If F is an antiderivative of f on an interval, then $\int_a^b f(x)\,dx = F(b) - F(a)$.*

12.2.4 Numerical Integration Methods

Method 4 *Methods include the Trapezoidal Rule and Simpson's Rule, which provide approximations of definite integrals.*

12.3 Applications of Integrals

12.3.1 Area Between Curves

Application 3 *The area between curves $f(x)$ and $g(x)$ from a to b is $\int_a^b |f(x)-g(x)|\,dx$.*

12.3.2 Volumes of Solids of Revolution

Application 4 *The volume of a solid obtained by rotating a region about an axis can be found using the Disk or Washer methods.*

12.3.3 Arc Length and Surface Area

Method 5 *Formulas for the arc length of a curve and the surface area of a solid of revolution involve definite integrals.*

12.3.4 Work and Fluid Forces

Application 5 *Work done in moving an object and the force exerted by a fluid can be calculated using integration.*

Chapter 13

Advanced Topics in Calculus

13.1 Sequences and Series

13.1.1 Convergence and Divergence

Definition 25 *A sequence $\{a_n\}$ converges if it approaches a limit as n goes to infinity. A series $\sum a_n$ converges if the sequence of its partial sums converges.*

13.1.2 Tests for Convergence

Method 6 *Tests include the Integral Test, Comparison Test, Ratio Test, and Root Test.*

13.1.3 Power Series and Taylor Series

Definition 26 *A power series is a series of the form $\sum_{n=0}^{\infty} a_n(x-c)^n$. A Taylor series is a power series that represents a function.*

13.2 Multivariable Calculus

13.2.1 Functions of Several Variables

Definition 27 *A function of several variables is a function that takes several inputs and produces a single output.*

13.2.2 Partial Derivatives

Definition 28 *The partial derivative of a function with respect to one of its variables is its derivative with respect to that variable, holding the other variables constant.*

13.2.3 Multiple Integrals

Definition 29 *Multiple integrals involve integration over more than one variable, such as double and triple integrals.*

13.2.4 Vector Calculus

Discussion 3 *Vector calculus involves differentiation and integration of vector fields, including topics like gradient, divergence, and curl.*

13.3 Differential Equations

13.3.1 First Order Differential Equations

Method 7 *Methods for solving include separation of variables, integrating factors, and exact equations.*

13.3.2 Second Order Linear Differential Equations

Method 8 *Includes homogeneous and non-homogeneous equations, characteristic equations, and particular solutions.*

13.3.3 Systems of Differential Equations

Discussion 4 *Systems of differential equations involve several interrelated differential equations and can be solved using matrix methods and eigenvalues.*

13.4 Conclusion

This concludes the math currently included in CiB. Please fork the LaTeX source code for CiB (available on GitHub) and create your own book that chooses the facts and exercises most relevant to you! Also, starring the CiB project on GitHub would be greatly appreciated! Thanks for reading CiB!

Appendix I

Basic GitHub Guide

A Quick Start to Your GitHub Journey

Welcome to the fascinating world of GitHub, a platform that has revolutionized the way we collaborate on projects, share code, and build software together. Whether you are a programmer, a writer, or a mathematician, GitHub provides a set of powerful tools to help you collaborate with others, manage your projects, and contribute to the vast world of open-source software. In this guide, we will walk you through the foundational steps to get started with GitHub, helping you to navigate, contribute, and make the most out of this incredible platform.

Creating Your GitHub Account

The first step to joining the GitHub community is to create an account. Here's how you can do it:

1. Visit the GitHub website.
2. Click on the "Sign up" button.
3. Fill in the required information, including your username, email address, and password.
4. Verify your account and complete the sign-up process.

Once you have created your account, take a moment to explore your new GitHub dashboard. Here, you will find a variety of tools and features that will help you manage your projects, collaborate with others, and discover new and interesting repositories.

Creating Your First Repository

A repository (or "repo") is a digital directory where you can store your project files. Here's how you can create your first repository:

1. From your GitHub dashboard, click on the "New" button to create a new repository.

2. Give your repository a name and provide a brief description.

3. Initialize this repository with a README file. (This is an optional step, but it's a good practice to include a README file in every repository to explain what your project is about.)

4. Click "Create repository."

Congratulations! You have just created your first GitHub repository. You can now start adding files, collaborating with others, and managing your project right from GitHub.

Making Changes and Commits

GitHub uses Git, a version control system, to keep track of changes made to your project. Here's a quick guide on how to make changes and commits:

1. Navigate to your repository on GitHub.

2. Find the file you want to edit, and click on it.

3. Click the pencil icon to start editing.

4. Make your changes and then scroll down to the "Commit changes" section.

5. Provide a commit message that explains the changes you made.

6. Choose whether you want to commit directly to the main branch or create a new branch for your changes.

7. Click "Commit changes."

Your changes are now saved, and a new commit is created. Every commit has a unique ID, making it easy to track changes, revert to previous versions, and collaborate with others.

Collaborating with Others

One of the biggest strengths of GitHub is its collaborative nature. Here are some ways you can collaborate with others:

- **Forking:** You can fork a repository, create your own copy, make changes, and then propose those changes back to the original project.
- **Issues:** Use issues to report bugs, request new features, or start a discussion with the community.
- **Pull Requests:** Propose changes to a project by creating a pull request. This allows others to review your changes, discuss them, and eventually merge them into the project.

Conclusion: Embarking on Your GitHub Adventure

Now that you have a basic understanding of GitHub and how it works, you are ready to embark on your GitHub adventure. Explore repositories, contribute to open-source projects, collaborate with others, and build amazing things together. Remember, the GitHub community is vast and supportive, and there is a wealth of knowledge and resources available to help you along the way. Happy coding!

Appendix II

Basic Python and Colab Guide

Introduction to Python for Calculus

Python's versatility in mathematics, science, engineering, and data analysis stems from its simplicity and extensive library ecosystem. This guide will delve into Python packages essential for math and calculus, showcasing their utility in Google Colab notebooks.

Setting Up Python and Google Colab

Google Colab is a free, cloud-based platform enabling Python programming in a browser. It offers free computational resources, ideal for Python learning and experimentation.

Visit Google Colab to start. Create a new notebook, and use code cells to write and execute Python code with Shift+Enter.

Important Python Packages for Calculus

NumPy

NumPy, fundamental for scientific computing, offers support for large, multidimensional arrays and matrices, along with various mathematical functions.

SymPy

SymPy, a library for symbolic mathematics, allows algebraic manipulations and equation solving symbolically, perfect for calculus operations like differentiation and integration.

Matplotlib

Matplotlib, a Python plotting library, creates static, interactive, and animated visualizations, useful for graphing functions and data in calculus.

Pandas

Pandas provide high-performance, easy-to-use data structures, and data analysis tools, invaluable for handling and analyzing mathematical data.

SciPy

SciPy extends NumPy by adding a collection of algorithms and high-level commands for data manipulation and visualization.

Examples and Applications

Calculating Derivatives and Integrals with SymPy

Illustrate using SymPy to compute derivatives and integrals of functions.

Data Visualization with Matplotlib and Pandas

Demonstrate graphing functions and datasets, highlighting calculus concepts.

Solving Equations with SciPy

Show how to solve equations that commonly appear in calculus.

Numerical Methods in Python

Discuss Python's capabilities in numerical differentiation and integration, useful in calculus.

Using Python for Limits and Continuity

Examples showcasing how Python can help in understanding limits and continuity in functions.

Interactive Learning with Google Colab

Highlight the benefits of using Colab notebooks for interactive calculus learning, including real-time collaboration and easy sharing.

Creating a Colab Notebook for Practice Problems

In this section, we will guide you through creating a Google Colab notebook to solve calculus practice problems using Python.

Setting Up Your Colab Notebook

To start solving calculus problems with Python:

1. Open Google Colab.
2. Choose 'New Notebook' to create a blank notebook.
3. Rename the notebook to something descriptive, like 'Calculus Practice'.

Installing Necessary Libraries

At the beginning of your notebook, install any necessary Python libraries. For these exercises, ensure NumPy, SymPy, and Matplotlib are available:

```
Remove # if the following packages are not installed:
# !pip install numpy sympy matplotlib
```

Solving Exercise 1: Graphing a Linear Function

Let's solve the first exercise, which involves graphing a linear function.

```
import numpy as np
import matplotlib.pyplot as plt

# Define the function
def f(x):
    return 3*x - 2

# Generate x values
x = np.linspace(-10, 10, 400)

# Plot the function
plt.plot(x, f(x))
plt.xlabel('x')
plt.ylabel('f(x)')
plt.title('Graph of f(x) = 3x - 2')
plt.grid(True)
```

```
plt.show()

# Slope and y-intercept
print("Slope: 3")
print("Y-intercept: -2")
```

Solving Exercise 2: Identifying Undefined Points in a Function

Now, let's address the second exercise, which requires identifying for what values the function $g(x) = \frac{1}{x}$ is undefined.

```
import sympy as sp

x = sp.symbols('x')
g = 1/x

# Display the function
sp.plot(g, (x, -10, 10), title='Graph of g(x) = 1/x',
        xlabel='x', ylabel='g(x)', ylim=(-10, 10))

# Identifying undefined points
print("g(x) is undefined for x = 0")
```

Accessing the Completed Colab Notebook

The Colab notebook we've discussed is available for viewing and interaction. You can access it by clicking on the following link: Finished Colab Notebook on Graphing and Analysis. This link will take you directly to the notebook hosted on Google Colab, where you can view the complete code and run it yourself.

https://colab.research.google.com/drive/1HF-cmwqfIZ803i1i-CFyR7ssWsDt7WvV

Adding the Notebook to Your GitHub Repository

If you have downloaded the Colab notebook to your local machine and want to add it to your Git repository, follow these terminal commands on your Ubuntu machine:

```
# Navigate to your local Git repository directory
cd path/to/your/repo
```

```
# Add the Colab notebook file to the repository
git add name_of_the_notebook.ipynb

# Commit the addition with a descriptive message
git commit -m "Add Colab notebook for calculus exercises"

# Push the changes to the remote GitHub repository
git push origin main
```

Using Colab Notebooks for Problem Solving

These examples demonstrate how you can use Google Colab and Python to solve and visualize calculus problems. You can use similar steps to tackle other exercises, explore different functions, and deepen your understanding of calculus concepts.

Conclusion: Interactive Learning with Colab Notebooks

Google Colab notebooks offer an interactive and accessible way to explore calculus using Python. By integrating theoretical concepts with computational examples, students can gain a deeper understanding of calculus. We encourage you to use these notebooks to solve exercises, visualize mathematical concepts, and explore the vast possibilities that Python and Colab offer.

Conclusion: Python and Colab in Calculus

Python, with its comprehensive libraries, offers a powerful toolset for calculus exploration. Combined with Google Colab, it provides an accessible, interactive platform for learning and experimentation. Embrace Python and Colab to enhance your understanding of calculus and to explore mathematical problems creatively and efficiently.

Appendix III

Basic LaTeX Guide

A Quick Start to Your LaTeX Journey

Welcome to the immersive world of LaTeX, a typesetting system widely used for creating scientific and professional documents due to its powerful handling of formulas and bibliographies. This guide is designed to offer you the foundational steps to grasp the basics of LaTeX, enabling you to craft documents of high typographic quality akin to this book.

Setting Up Your LaTeX Environment

Before you can start creating documents with LaTeX, you need to set up a working LaTeX environment on your computer. Here's how you can do it:

1. Download and install a TeX distribution, which includes LaTeX. For Windows, MiKTeX is a popular choice, while Mac users might prefer MacTeX, and TeX Live is widely used on Linux.

2. Install a LaTeX editor. Some popular options include TeXShop (for Mac), TeXworks (cross-platform), and Overleaf (an online LaTeX editor).

3. Ensure that your TeX distribution and LaTeX editor are properly configured and integrated.

Creating Your First LaTeX Document

Once your LaTeX environment is set up, you are ready to create your first LaTeX document. Follow these steps:

1. Open your LaTeX editor and create a new document.
2. Insert the following code to set up a basic LaTeX document:

   ```
   \documentclass{article}
   \begin{document}
   Hello, \LaTeX\ world!
   \end{document}
   ```

3. Save your document with a .tex file extension.
4. Compile your document using your LaTeX editor. This process converts your .tex file into a PDF document.
5. View the output PDF and admire your first LaTeX creation.

Understanding LaTeX Commands and Environments

LaTeX documents are created using a series of commands and environments. Commands typically start with a backslash \ and are used to format text, insert special characters, or execute functions. Environments are used to define specific sections of your document that require special formatting.

- **Commands:** For example, \{italics} will render the word "italics" in italic font.
- **Environments:** To create a bulleted list, you would use the *itemize* environment:

  ```
  \begin{itemize}
      \item First item
      \item Second item
  \end{itemize}
  ```

Adding Structure to Your Document

LaTeX makes it easy to structure your documents with sections, subsections, and chapters. Here's how you can add structure:

```
\section{Introduction}
This is the introduction of your document.
\subsection{Background}
This subsection provides background information.
\subsubsection{Details}
This is a subsubsection for more detailed information.
```

Including Mathematical Formulas

LaTeX excels at typesetting mathematical formulas. Use the *equation* environment or the $ sign for inline formulas. For example:

```
The quadratic formula is \( x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \).
```

Adding Images and Tables

You can also include images and tables in your LaTeX documents:

- **Images:** Use the *graphicx* package and the *includegraphics* command.
- **Tables:** Use the *tabular* environment to create tables.

Compiling Your Document

LaTeX documents need to be compiled to produce a PDF. This can be done through your LaTeX editor. If your document includes bibliographies or cross-references, you may need to compile multiple times.

Conclusion: Embracing the Power of LaTeX

Congratulations! You have taken your first steps into the world of LaTeX. With practice, you will discover that LaTeX is a powerful tool for creating professional-quality documents, from simple articles to complex books. Embrace the learning curve, explore the vast array of packages available, and join the community of LaTeX users who are ready to help you on your journey. Happy typesetting!

Bibliography

[MT77] F. Mosteller and J. W. Tukey. *Data Analysis and Regression: A Second Course in Statistics*. Addison-Wesley Pub Co, Reading, MA, 1977.