

# A Verified Implementation of B+-Trees in Isabelle/HOL

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## Abstract

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80 **2 Lorem ipsum dolor sit amet**

81 Lorem ipsum dolor sit amet, consectetur adipiscing elit [4]. Praesent convallis orci arcu, eu  
 82 mollis dolor. Aliquam eleifend suscipit lacinia. Maecenas quam mi, porta ut lacinia sed,  
 83 convallis ac dui. Lorem ipsum dolor sit amet, consectetur adipiscing elit. Suspendisse potenti.  
 84 Donec eget odio et magna ullamcorper vehicula ut vitae libero. Maecenas lectus nulla, auctor

■ **Listing 1** Useless code.

```
for i:=maxint to 0 do
begin
  j:=square(root(i));
end;
```

85 nec varius ac, ultricies et turpis. Pellentesque id ante erat. In hac habitasse platea dictumst.  
 86 Curabitur a scelerisque odio. Pellentesque elit risus, posuere quis elementum at, pellentesque  
 87 ut diam. Quisque aliquam libero id mi imperdiet quis convallis turpis eleifend.

88 ► **Lemma 1** (Lorem ipsum). *Vestibulum sodales dolor et dui cursus iaculis. Nullam ullam-*  
 89 *corper purus vel turpis lobortis eu tempus lorem semper. Proin facilisis gravida rutrum.*  
 90 *Etiam sed sollicitudin lorem. Proin pellentesque risus at elit hendrerit pharetra. Integer at*  
 91 *turpis varius libero rhoncus fermentum vitae vitae metus.*

92 **Proof.** Cras purus lorem, pulvinar et fermentum sagittis, suscipit quis magna.

93 **Just some paragraph within the proof.** Nam liber tempor cum soluta nobis eleifend option  
 94 congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum  
 95 dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut  
 96 laoreet dolore magna aliquam erat volutpat.

97 ▷ **Claim 2.** content...

98 **Proof.** content...

99 1. abc abc abc

100

101 ► **Corollary 3** (Curabitur pulvinar, [2]). *Nam liber tempor cum soluta nobis eleifend option*  
 102 *congue nihil imperdiet doming id quod mazim placerat facer possim assum. Lorem ipsum*  
 103 *dolor sit amet, consectetur adipiscing elit, sed diam nonummy nibh euismod tincidunt ut*  
 104 *laoreet dolore magna aliquam erat volutpat.*

105 ► **Proposition 4.** *This is a proposition*

106 Proposition 4 and Proposition 4 . . .

## 107 2.1 Curabitur dictum felis id sapien

108 Curabitur dictum Corollary 3 felis id sapien Corollary 3 mollis ut venenatis tortor feugiat.  
 109 Curabitur sed velit diam. Integer aliquam, nunc ac egestas lacinia, nibh est vehicula nibh, ac  
 110 auctor velit tellus non arcu. Vestibulum lacinia ipsum vitae nisi ultrices eget gravida turpis  
 111 laoreet. Duis rutrum dapibus ornare. Nulla vehicula vulputate iaculis. Proin a consequat  
 112 neque. Donec ut rutrum urna. Morbi scelerisque turpis sed elit sagittis eu scelerisque quam  
 113 condimentum. Pellentesque habitant morbi tristique senectus et netus et malesuada fames  
 114 ac turpis egestas. Aenean nec faucibus leo. Cras ut nisl odio, non tincidunt lorem. Integer  
 115 purus ligula, venenatis et convallis lacinia, scelerisque at erat. Fusce risus libero, convallis at  
 116 fermentum in, dignissim sed sem. Ut dapibus orci vitae nisl viverra nec adipiscing tortor  
 117 condimentum [1]. Donec non suscipit lorem. Nam sit amet enim vitae nisl accumsan pretium.

118 **2.2 Proin ac fermentum augue**

119 Proin ac fermentum augue. Nullam bibendum enim sollicitudin tellus egestas lacinia euismod  
 120 orci mollis. Nulla facilisi. Vivamus volutpat venenatis sapien, vitae feugiat arcu fringilla ac.  
 121 Mauris sapien tortor, sagittis eget auctor at, vulputate pharetra magna. Sed congue, dui  
 122 nec vulputate convallis, sem nunc adipiscing dui, vel venenatis mauris sem in dui. Praesent  
 123 a pretium quam. Mauris non mauris sit amet eros rutrum aliquam id ut sapien. Nulla  
 124 aliquet fringilla sagittis. Pellentesque eu metus posuere nunc tincidunt dignissim in tempor  
 125 dolor. Nulla cursus aliquet enim. Cras sapien risus, accumsan eu cursus ut, commodo vel  
 126 velit. Praesent aliquet consectetur ligula, vitae iaculis ligula interdum vel. Integer faucibus  
 127 faucibus felis.

- 128 ■ Ut vitae diam augue.
- 129 ■ Integer lacus ante, pellentesque sed sollicitudin et, pulvinar adipiscing sem.
- 130 ■ Maecenas facilisis, leo quis tincidunt egestas, magna ipsum condimentum orci, vitae  
 131 facilisis nibh turpis et elit.

132 ► Remark 5. content...

133 **3 Pellentesque quis tortor**

134 Nec urna malesuada sollicitudin. Nulla facilisi. Vivamus aliquam tempus ligula eget ornare.  
 135 Praesent eget magna ut turpis mattis cursus. Aliquam vel condimentum orci. Nunc congue,  
 136 libero in gravida convallis [3], orci nibh sodales quam, id egestas felis mi nec nisi. Suspendisse  
 137 tincidunt, est ac vestibulum posuere, justo odio bibendum urna, rutrum bibendum dolor sem  
 138 nec tellus.

139 ► **Lemma 6** (Quisque blandit tempus nunc). *Sed interdum nisl pretium non. Mauris sodales  
 140 consequat risus vel consectetur. Aliquam erat volutpat. Nunc sed sapien ligula. Proin faucibus  
 141 sapien luctus nisl feugiat convallis faucibus elit cursus. Nunc vestibulum nunc ac massa  
 142 pretium pharetra. Nulla facilisis turpis id augue venenatis blandit. Cum sociis natoque  
 143 penatibus et magnis dis parturient montes, nascetur ridiculus mus.*

144 Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam  
 145 vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit  
 146 amet neque.

147 **4 Morbi eros magna**

148 Morbi eros magna, vestibulum non posuere non, porta eu quam. Maecenas vitae orci risus,  
 149 eget imperdiet mauris. Donec massa mauris, pellentesque vel lobortis eu, molestie ac turpis.  
 150 Sed condimentum convallis dolor, a dignissim est ultrices eu. Donec consectetur volutpat  
 151 eros, et ornare dui ultricies id. Vivamus eu augue eget dolor euismod ultrices et sit amet nisi.  
 152 Vivamus malesuada leo ac leo ullamcorper tempor. Donec justo mi, tempor vitae aliquet non,  
 153 faucibus eu lacus. Donec dictum gravida neque, non porta turpis imperdiet eget. Curabitur  
 154 quis euismod ligula.

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## 166 A Styles of lists, enumerations, and descriptions

167 List of different predefined enumeration styles:

- 168 ■ `\begin{itemize}...\end{itemize}`  
 169 ■ ...  
 170 ■ ...
- 171 1. `\begin{enumerate}...\end{enumerate}`  
 172 2. ...  
 173 3. ...
- 174 (a) `\begin{alphaenumerate}...\end{alphaenumerate}`  
 175 (b) ...  
 176 (c) ...
- 177 (i) `\begin{romanenumerate}...\end{romanenumerate}`  
 178 (ii) ...  
 179 (iii) ...
- 180 (1) `\begin{bracketenumerate}...\end{bracketenumerate}`  
 181 (2) ...  
 182 (3) ...
- 183 **Description 1** `\begin{description} \item[Description 1] ... \end{description}`  
 184 **Description 2** Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui.  
 185 Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus  
 186 massa sit amet neque.
- 187 **Description 3** ...
- 188 Proposition 10 and Proposition 10 ...

## 189 B Theorem-like environments

190 List of different predefined enumeration styles:

- 191 ► **Theorem 7.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo  
 192 dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus  
 193 massa sit amet neque.*
- 194 ► **Lemma 8.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui.  
 195 Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa  
 196 sit amet neque.*

197 ► **Corollary 9.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui.*  
 198 *Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa*  
 199 *sit amet neque.*

200 ► **Proposition 10.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo*  
 201 *dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus*  
 202 *massa sit amet neque.*

203 ► **Conjecture 11.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo*  
 204 *dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus*  
 205 *massa sit amet neque.*

206 ► **Observation 12.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et*  
 207 *leo dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus*  
 208 *massa sit amet neque.*

209 ► **Exercise 13.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo*  
 210 *dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus*  
 211 *massa sit amet neque.*

212 ► **Definition 14.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo*  
 213 *dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus*  
 214 *massa sit amet neque.*

215 ► **Example 15.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo*  
 216 *dui. Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus*  
 217 *massa sit amet neque.*

218 ► **Note 16.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui.*  
 219 *Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa*  
 220 *sit amet neque.*

221 ► **Note.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam*  
 222 *vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit*  
 223 *amet neque.*

224 ► **Remark 17.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui.*  
 225 *Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa*  
 226 *sit amet neque.*

227 ► **Remark.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui.*  
 228 *Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa*  
 229 *sit amet neque.*

230 ▷ **Claim 18.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui.*  
 231 *Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa*  
 232 *sit amet neque.*

233 ▷ **Claim.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui.*  
 234 *Nam vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa*  
 235 *sit amet neque.*

236 ► **Proof.** *Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam*  
 237 *vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit*  
 238 *amet neque.* ◀

239 Proof. Fusce eu leo nisi. Cras eget orci neque, eleifend dapibus felis. Duis et leo dui. Nam  
240 vulputate, velit et laoreet porttitor, quam arcu facilisis dui, sed malesuada risus massa sit  
241 amet neque. 