Math 3890, Machine Problem 1: Due 2/2/2021

1) Write a function `v = horner(c,t)` based on Horner's algorithm that will evaluate a polynomial of the form $p(x) = \sum_{i=1}^{n+1} c_i x^{i-1}$ at all points in a column vector $t = (t_1, \ldots, t_N)^T$.

2) Write a script that
   a) reads in an integer n
   b) sets up interpolation points $a = x_1 < \cdots < x_{n+1} = b$
   c) finds the coefficient vector `c` of a polynomial of degree n that solves the interpolation problem $p(x_i) = f(x_i)$ $i = 1, \ldots, n+1$ for a given anonymous function `f`. Print these coefficients.
   d) uses Horner to evaluate this polynomial on equally-spaced points $a = t_1 < \ldots < t_N = b$ in the interval $[a, b]$.
   e) uses these values to plot both the interpolating polynomial p and the function f on the SAME graph
   f) computes the error $\max_{1 \le i \le N} |f(t_i) - p(t_i)|$.

3) Run your script with $f(x) = 1/(1 + x^2)$ on the interval $[a, b] := [-5, 5]$ with $N = 201$ and $n = 5$.

4) Repeat for $n = 9$ and $n = 17$ and turn in your listings together with the three figures annotated with the associated max errors.