Math 3890, Machine Problem 3: Due Tu., 2/9/21

1) Write a function `[y,c] = cubnat(t,z)` which computes the extended knot vector and coefficient vector for the natural cubic spline that interpolates the data values $z_1, \ldots, z_n$ at the points $a = t_1 < \cdots < t_n = b$.

2) Write a script to define an anonymous function `f`. Input a value of n, and let $t := \{t_1, \ldots, t_n\}$ be n equally-spaced points in $[a, b]$. Set $z = f(t)$. Then call on `cubnat` to produce the corresponding interpolating spline. Print out the extended knot vector and the coefficients of the interpolating spline. Compute and print the maximum and RMS errors on 501 equally-spaced points in $[a, b]$. Plot the spline and the test function on these 501 points (on the same figure).

3) Run your script with the function $f(x) = e^x \sin(2\pi x)$ on the interval $[-1, 1]$ with $n = 9$. Hand in listings of your function `cubnat` and the main script along with the output of the code and the plot.

4) Modify your script to create a table similar to the one in Example 1.10 for the clamped cubic spline.