DAA432C

Assignment-06

Group: 5

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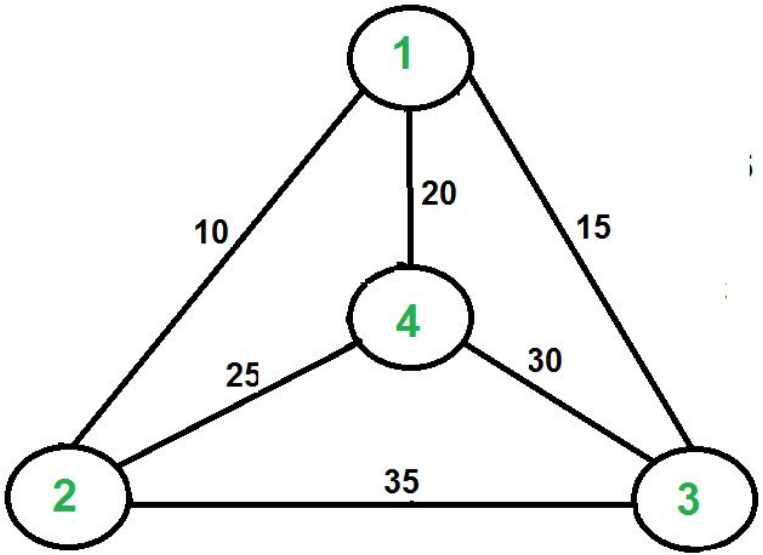
Problem Statement

Travelling Salesman Problem (TSP): Given a set of cities and distance between every pair of cities, the problem is to find the shortest possible route that visits every city exactly once and returns to the starting point.



For example, consider the graph shown in figure on right side. A TSP tour in the graph is 1-2-4-3-1. The cost of the tour is 10+25+30+15 which is 80.

Algorithm Discussion

Naive Solution:-

1. Consider city 1 as the starting and ending point.
2. Generate all (n-1)! Permutations of cities.
3. Calculate cost of every permutation and keep track of minimum cost permutation.
4. Return the permutation with minimum cost.

Time Complexity: Θ(n!)

Algorithm Discussion

Dynamic Programming solution:-

Let the given set of vertices be {1, 2, 3, 4,….n}. Let us consider 1 as starting and ending point of output. For every other vertex i (other than 1), we find the minimum cost path with 1 as the starting point, i as the ending point and all vertices appearing exactly once. Let the cost of this path be cost(i), the cost of corresponding Cycle would be cost(i) + dist(i, 1) where dist(i, 1) is the distance from i to 1. Finally, we return the minimum of all [cost(i) + dist(i, 1)] values.

To calculate cost(i) using Dynamic Programming, we need to have some recursive relation in terms of sub-problems. Let us define a term *C(S, i) be the cost of the minimum cost path visiting each vertex in set S exactly once, starting at 1 and ending at i*.
We start with all subsets of size 2 and calculate C(S, i) for all subsets where S is the subset, then we calculate C(S, i) for all subsets S of size 3 and so on.

Note :1 must be present in every subset.

Algorithm Discussion

Dynamic Programming solution:-

If size of S is 2, then S must be {1, i},
C(S, i) = dist(1, i)
Else if size of S is greater than 2.
C(S, i) = min { C(S-{i}, j) + dis(j, i)} where j belongs to S, j != i and j != 1.

For a set of size n, we consider n-2 subsets each of size n-1 such that all subsets don't have nth in them.

Using the above recurrence relation, we can write dynamic programming based solution. There are at most $O(n*2^n)$ subproblems, and each one takes linear time to solve. The total running time is therefore $O(n^2*2^n)$. The time complexity is much less than O(n!), but still exponential. Space required is also exponential. So this approach is also infeasible even for slightly higher number of vertices.

Pseudo Code

Algorithm 1: Dynamic Approach for TSP

```
Data: s: starting point; N: a subset of input cities; dist():
      distance among the cities
Result: Cost : TSP result
Visited[N] = 0;
Cost = 0;
Procedure TSP(N, s)
    Visited[s] = 1;
    if |N| = 2 and k ≠ s then
        Cost(N, k) =dist(s, k);
        Return Cost;
    else
        for j ∈ N do
            for i ∈ N and visited[i] = 0 do
                if j ≠ i and j ≠ s then
                    Cost(N, j) = min ( TSP(N − {i}, j) + dist(j, i))
                    Visited[j] = 1;
                end
            end
        end
    end
    Return Cost;
end
```

Time and Space Complexity

Naive Solution:-
Time Complexity: Θ(n!)

Dynamic Programming solution:-
Since, there are at most $O(n*2^n)$ subproblems, and each one takes linear time to solve. The total running time is therefore $O(n^2*2^n)$. The time complexity is much less than O(n!), but still exponential. Space required is also exponential.

Conclusion

TSP is a popular NP-Hard problem, but depending on the size of the input cities, it is possible to find an optimal or a near-optimal solution using various algorithms.

Thank You