Given an NxN chessboard and a Knight at position (x,y). The Knight has to take exactly K steps, where at each step it chooses any of the 8 directions uniformly at random. What is the probability that the Knight remains in the chessboard after taking K steps, with the condition that it can’t enter the board again once it leaves it? Solve using Dynamic programming.

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Abstract — In this paper we have approached the problem of finding the probability that the knight remains in the chessboard after k number of steps.This paper shows how this problem can be solved using dynamic programming.

Keywords

- Dynamic programming

Algorithm

Step 1:

Define direction vectors for the knight.

```
            Ex:- dx[ ] = {1, 2, 2, 1, -1, -2, -2, -1}
                 dy[ ] = {2, 1, -1, -1, -2, -1, 1, 2}
```

Step 2:

Take an array dp[N, N, steps + 1] which will store the probability of reaching (x,y) after (steps) number of moves.

Step 3:

Base case: if the number of steps is 0, then the probability that the Knight will remain inside the board is 1

Step 4:

Take the position (x, y) after s number of steps.

Step 5:

Take prob = 0.0 then check for each position reachable from (x, y) using the direction vectors and store it in a new position (nx, ny).

Step 6:

Check if this new position (nx, ny) is inside of the chessboard, if yes then add dp1[nx][ny][s - 1] / 8.0 to prob.

Step 7:

Store the prob in dp[x][y][s].

Step 8:

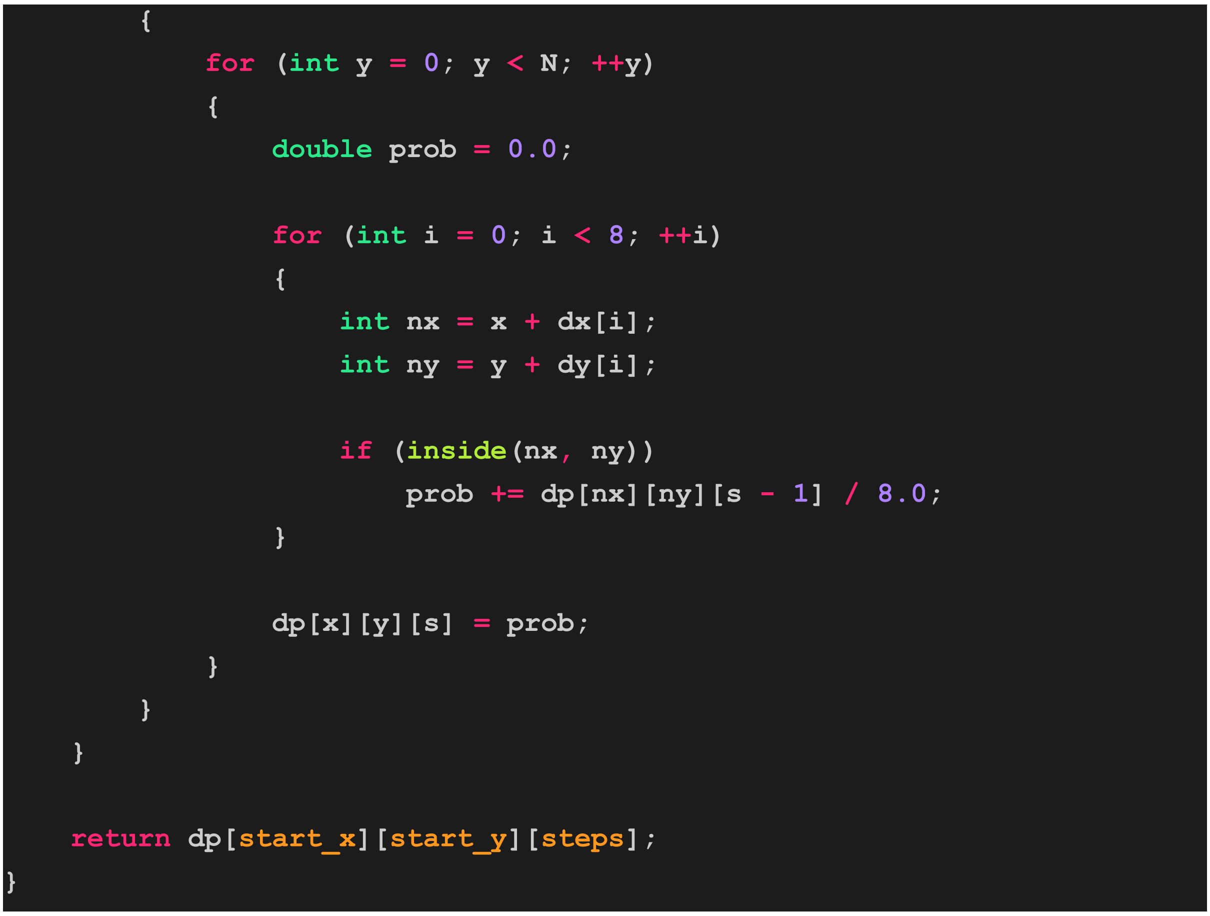
Keep repeating for the given number of steps.

Step 9:

The required probability will be stored in dp[start_x][start_y][k], where (start_x, start_y) are the given initial position of the knight and k is the number of steps.

Pseudo Code

```
int dx[] = {1, 2, 2, 1, -1, -2, -2, -1};
int dy[] = {2, 1, -1, -2, -2, -1, 1, 2};

bool inside(int x, int y)
{
    return (x >= 0 and x < N and y >= 0 and y < N);
}

double findProb(int start_x, int start_y, int steps)
{
    double dp[N][N][steps + 1];

    for (int i = 0; i < N; ++i)
        for (int j = 0; j < N; ++j)
            dp[i][j][0] = 1;

    for (int s = 1; s <= steps; ++s)
    {

        for (int x = 0; x < N; ++x)
```



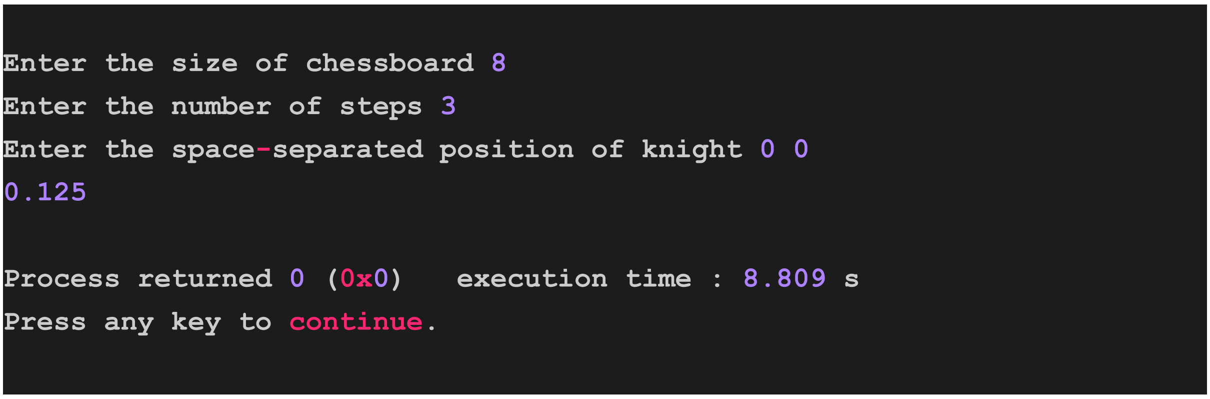
```
        {
            for (int y = 0; y < N; ++y)
            {
                double prob = 0.0;

                for (int i = 0; i < 8; ++i)
                {
                    int nx = x + dx[i];
                    int ny = y + dy[i];

                    if (inside(nx, ny))
                        prob += dp[nx][ny][s - 1] / 8.0;
                }

                dp[x][y][s] = prob;
            }
        }
    }

    return dp[start_x][start_y][steps];
}
```

Complexity analysis

Time Complexity: O(NxNxK), where N is the size of the board and K is the number of steps.

Space Complexity: O(NxNxK), where N is the size of the board and K is the number of steps.

Output



```
Enter the size of chessboard 8
Enter the number of steps 3
Enter the space-separated position of knight 0 0
0.125

Process returned 0 (0x0)   execution time : 8.809 s
Press any key to continue.
```

Conclusion

The method discussed above shows the mentioned problem can be solved using dynamic programming with a time and space complexity of O(NxNxK), where N is the size of the chessboard and K is the number of steps.

References

- https://www.geeksforgeeks.org/dynamic-programming/
- https://www.geeksforgeeks.org/understanding-time-complexity-simple-examples/