One possible solution area for a means of generating thrust for our rocket lander was through the use of cold gas thrusters. Cold gas thrusters are commonplace in aerospace applications today but rarely used as a primary source of propulsion on heavy objects. They are much more commonly used as an implement for small corrections in orientation. This is due to their lack of combustion. Instead of using combustible gasses and materials like traditional mono-propellant and bi-propellant combustion rocket engines, cold-gas thrust relies solely on the acceleration of inert gasses to produce thrust. As a result, they produce less thrust per unit weight than combustible gasses since chemical energy is not being utilized.

Cold-gas thrusters utilize inert gasses that are stored in pressurized tanks. The gas is discharged at high pressure and low velocity from the tank into a converging-diverging nozzle. In this nozzle, subsonic flow is ideally accelerated to Mach 1 at the nozzle 'throat' (the thinnest portion of the nozzle). From here the sonic flow is accelerated through the diverging portion of the nozzle to supersonic velocities and the pressure drops accordingly. Exit velocity is determined by properties of the chosen gas and the ratio of exit area and throat area. Thrust (denoted by F) is then produced at the exit in accordance with *Equation 1*.

$$\textit{Eqn 1:}\ \ F = \dot{m} \cdot V_e + (p_e - p_o) \cdot A_e$$

In equation 1, thrust is calculated by multiplying mass flow rate through the nozzle and the average exit velocity. A correction factor is then used to compensate for the difference between exit pressure and free stream pressure. The various components of this equation can be solved by hand using a set of equations known as the 'Rocket Thrust Equations'. These equations are seen below (*equations 2-6*). In the following order, the equation for mass flow rate, the equation for exit mach, the equation exit temperature, the equation for exit pressure and the equation for exit velocity.

$$\textit{Eqn 2:}\ \ \dot{m} = \frac{A^{*} p_c}{\sqrt{T_c}} \sqrt{\frac{\gamma}{R}} \left(\frac{\gamma+1}{2}\right)^{-\left(\frac{\gamma+1}{2(\gamma-1)}\right)}$$

$$\textit{Eqn 3:}\ \ \frac{A_e}{A^{*}} = \left(\frac{\gamma+1}{2}\right)^{-\left(\frac{\gamma+1}{2(\gamma-1)}\right)} \frac{\left(1+\frac{\gamma-1}{2}M_e^2\right)^{\frac{\gamma+1}{2(\gamma-1)}}}{M_e}$$

$$\textit{Eqn 4:}\ \ \frac{T_e}{T_c} = \left(1 + \frac{\gamma-1}{2}M_e^2\right)^{-1}$$

$$Eqn\ 5: \quad \frac{p_e}{p_c} = \left(1 + \frac{\gamma-1}{2}M_e^2\right)^{\frac{-\gamma}{\gamma-1}}$$

$$Eqn\ 6: \quad V_e = M_e\sqrt{\gamma R T_e}$$

In these equations, A^* is the designation for the area of the throat, p_c and T_c are chamber pressure and temperature respectively. Gamma (γ) is the specific heat ratio of the gas being used, and R is the gas constant for the gas being used. These 6 equations hold for almost all converging-diverging nozzle flows.

Using these equations and known nozzle geometries, the various parameters associated with nozzle flow can be determined. For our team's application, we are estimating a need for approximately 100 Newtons (this includes a 45% margin of error) of static thrust for a time period of 30 seconds to possibly 2 minutes. In order to determine the maximum static thrust available in a cold-gas set-up, the team used the dimensions of a previous NUSTARS, competition used, converging-diverging nozzle, acquired from the manufacturer engineering drawing (Figure A1), as well as the required storage conditions for Carbon Dioxide (one of the most commonly used cold-gas thruster gasses). Carbon Dioxide in these applications is stored as a liquid and requires a tank pressure of 4.5 MPa at ambient temperature. Ambient temperature for this application was taken as standard atmospheric temperature at sea level (288.15 Kelvin). Using these conditions, equations 1-6 were used to calculate the maximum static thrust of our selected nozzle. These calculations can be seen in (Figure A2).

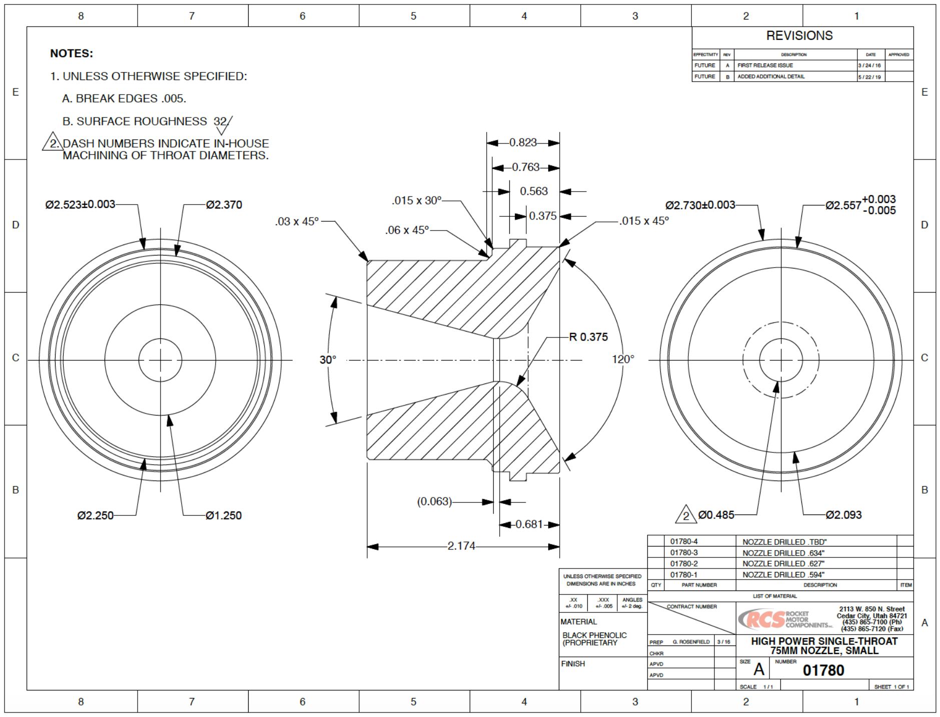

Figure A1: Nozzle Dimensions

CO_2 Cold-gas

$\gamma = 1.289$ $\quad A_e = 7.917 \times 10^{-4}\ m^2$

$R = 188.9\ J/kgK$ $\quad A^* = 1.1919 \times 10^{-4}\ m^2$

$P_c = 4.5\ MPa$ $\quad T_c = 288.15\ K$

$$② \quad \dot{m} = \frac{(1.1919 \times 10^{-4}\ m^2)(4.5 \times 10^6\ N/m^2)}{\sqrt{288.15\ K}} \sqrt{\frac{1.289}{188.9\ J/kg\,K}} (1.1445)^{-3.96}$$

$$\boxed{\dot{m} = 1.5294\ kg/s}$$

$$③ \quad \frac{7.917 \times 10^{-4}\ m^2}{1.1919 \times 10^{-4}\ m^2} = (1.1445)^{-3.96} \frac{(1 + .1445\ M_e^2)^{3.96}}{M_e}$$

$$\boxed{M_e = 3.1956}$$

$$④ \quad T_e = T_c (1 + .1445\ M_e^2)^{-1}$$

$$\boxed{T_e = 116.395\ K}$$

$$⑤ \quad P_e = P_c (1 + .1445\ M_e^2)^{-4.46}$$

$$\boxed{P_e = 78956.32\ Pa}$$

$$⑥ \quad V_e = M_e \sqrt{(1.289)(188.9)(116.395)}$$

$$\boxed{V_e = 537.975\ m/s}$$

$$① \quad F = (1.5294\ \tfrac{kg}{s})(537.975\ m/s) + (78956 - 101325) A_e$$

$$F = 822.779\ N + (-17.71\ N)$$

$$\boxed{F = 805.069\ N}$$

Maximum Static Thrust

Figure A2: Maximum Static Thrust Calculations

From these initial calculations, it can be seen that the nozzle set-up is able to produce our required amount of thrust using compressed Carbon Dioxide. From here, we transferred our computing needs to ANSYS Fluent. By creating and meshing an identical geometry in ANSYS Design Modeler, we were able to set up a flow simulation in Fluent to verify our results and allow us to more easily compute future values of interest. In Fluent, a density-based solver was used and we enabled energy conservation equations as well as a k-epsilon turbulence model. The fluid material was set to carbon-dioxide and the density was changed to 'ideal-gas' to allow for compressibility in the supersonic flow. We set the inlet boundary conditions to a pressure of 4.5 MPa and a temperature of 288.15 K. From here, we enabled a monitor of the average outlet velocity in order to monitor the simulation convergence. We then initialized the simulation and ran it. The results of the simulation were gathered and compiled alongside our hand calculation values in Table A1. Along with these values, contour plots for the velocity (Figure A3), and pressure (Figure A4) were plotted to get a better visualization of the flow inside the nozzle.

	Hand Calculations	FLUENT Values	Error
Average Outlet Velocity	537.975 m/s	519.187 m/s	3.49%
Mass Flow Rate	1.529 kg/s	1.596 kg/s	4.38%
Average Outlet Pressure	78,956.32 Pa	90,228.8 Pa	14.27%
Thrust	805.069 N	820.512 N	1.92%

Table A1: Hand Calculations vs Fluent Values (4.5MPa chamber pressure)

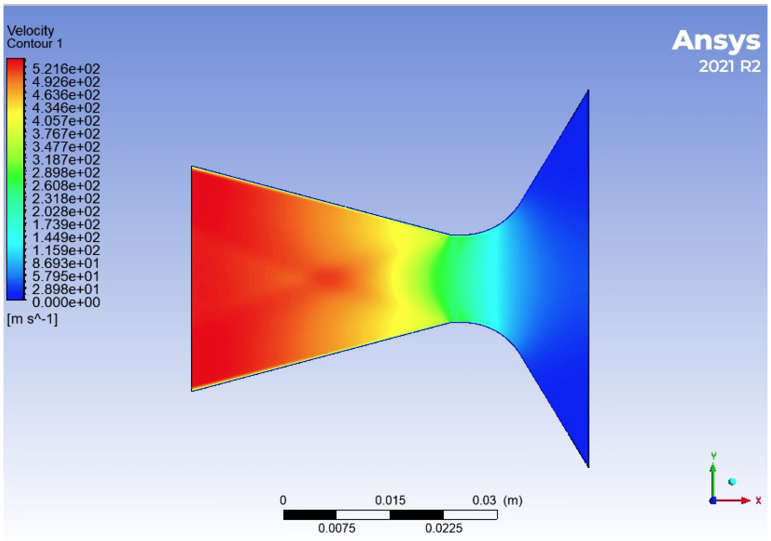

Figure A3: Velocity Magnitude Contour Plot (4.5MPa chamber pressure)

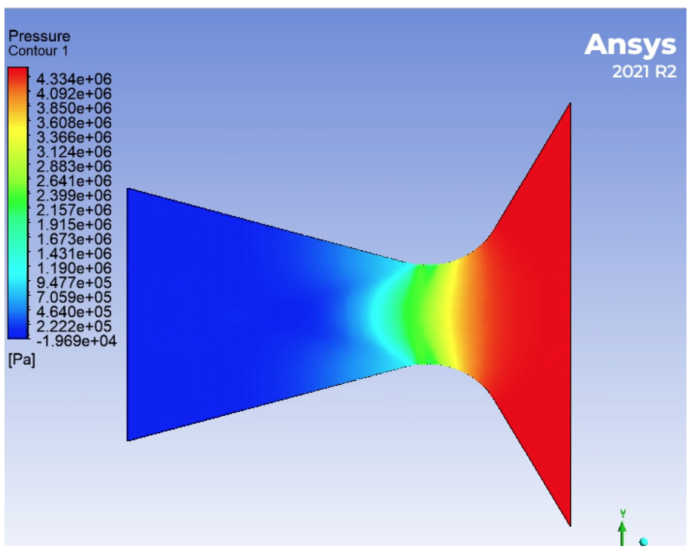

Figure A4: Pressure Contour Plot (4.5 MPa chamber pressure)

While these initial results do indicate that the required thrust can be met, further calculations had to be done in order to determine the parameters for our estimated required thrust. After verifying the validity of our ANSYS Fluent set-up with our hand calculations, we were confident in using solely Fluent for the remainder of the calculations. In order to determine the parameters required to achieve 100 N of thrust, the chamber pressure was varied, simulating the use of a pressure regulator between the storage tank and the inlet to the nozzle. After some trial and error, it was found that an inlet pressure of 1.0 MPa would result in an outlet velocity of 414.8505 m/s, a mass flow rate of 0.3868 kg/s and an average exit pressure of 46,047.04 Pa. These result in a static thrust of 116.7 Newtons, 16.7% above our estimated required maximum thrust. While these results do verify the capability of the cold gas thruster to produce the required thrust, it comes at a great cost in the form of mass flow rate. In order to achieve roughly our estimated required maximum static thrust, the nozzle will discharge 0.3868 kg of Carbon Dioxide per second. For even just a 30 second flight time, this equates to an extra 11.604 kilograms or 25.6 pounds of liquid carbon dioxide, as well as a pressure tank and regulator to be factored into our payload which would then increase our maximum required thrust. This increase in maximum thrust would then result in an increase in mass flow rate. Aside from the obvious mass problem encountered in this situation, the contour plots for this chamber pressure reveal concerning information. The pressure contour plot (Figure A5) looks fairly normal, but the velocity magnitude contour plot (Figure A6) indicates the formation of an oblique shock inside the nozzle.

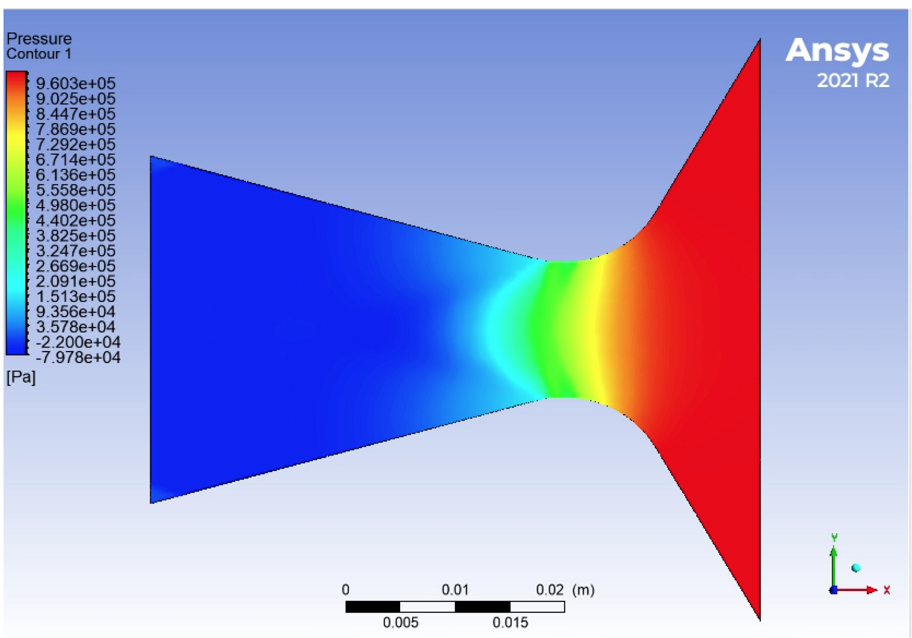

Figure A5: Pressure Contour Plot (1 MPa chamber pressure)

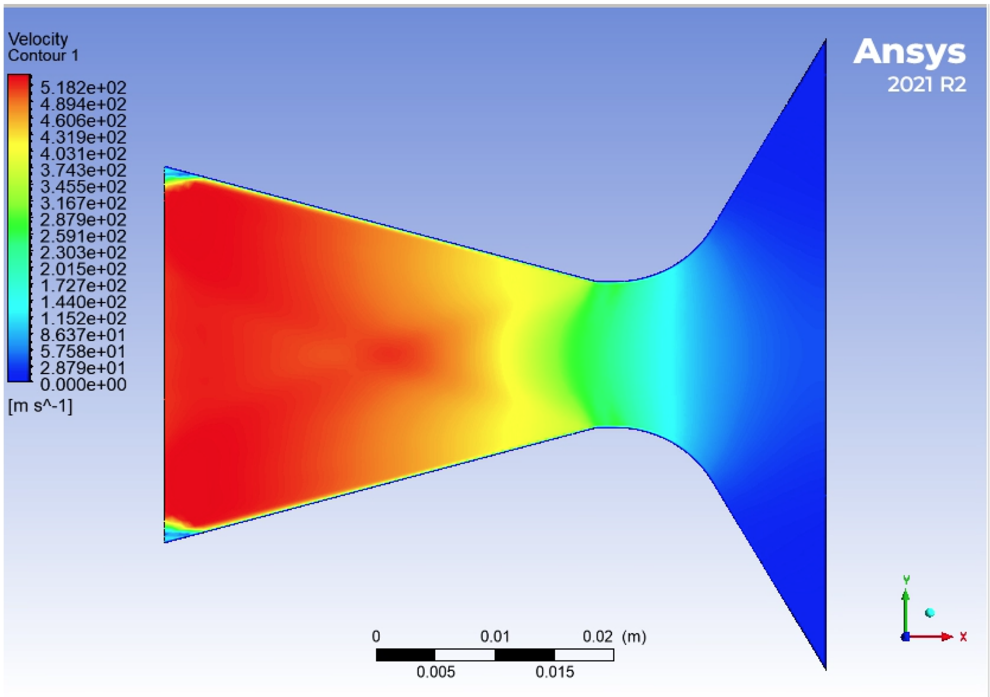

Figure A6: Velocity Magnitude contour plot (1 MPa chamber pressure)

The flow for this condition appears normal until just before the exit. Along the boundary layer at the exit of the nozzle, boundary layer expansion can be seen taking place. This is a strong indicator of flow over-expansion which will lead to an internal oblique shock. A likely cause of this is low chamber pressure. The flow still accelerates to supersonic velocities but the pressure drops so much that the boundary layer begins to expand into it. This will reduce overall thrust produced as well as cause shocks inside the nozzle which can cause structural problems if not accounted for. Adding to the worry of this situation. The flow is over expanding due to low chamber pressure but this is at our estimated maximum thrust. In order to control the lander and land softly this thrust will have to be able to be throttled down to below the weight of the lander in order to descend. This will require even lower chamber pressure which will likely cause more problems with overexpansion and thrust loss. To verify these predictions, chamber pressure was set to 500,000 Pascals and 300,000 Pascals and the velocity contours were once again plotted to visualize the flow. The 500,000 Pascal velocity plot can be seen below as Figure A7 and the 300,000 Pascal plot can be seen as Figure A8.

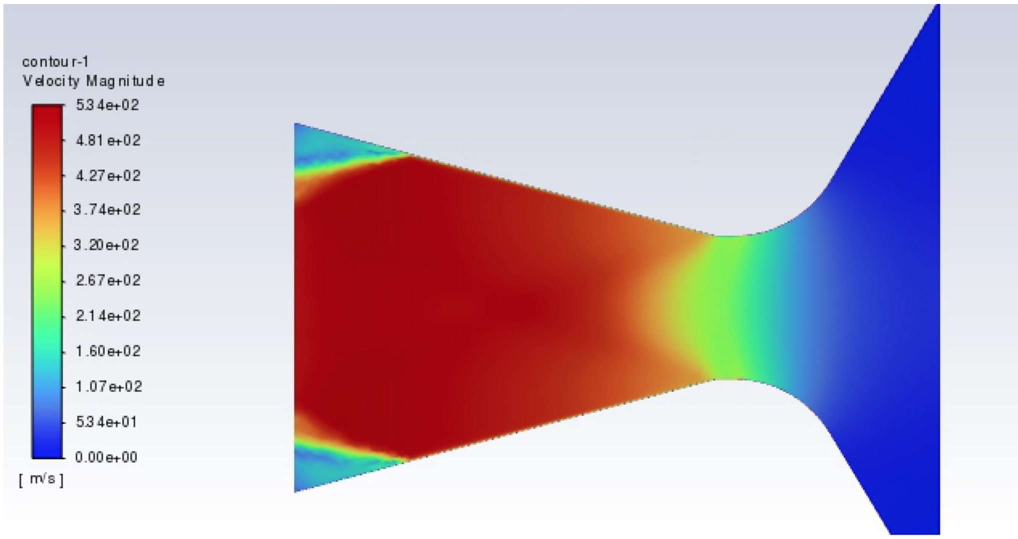

Figure A7: Velocity Contour Plot (500,000 Pa chamber pressure)

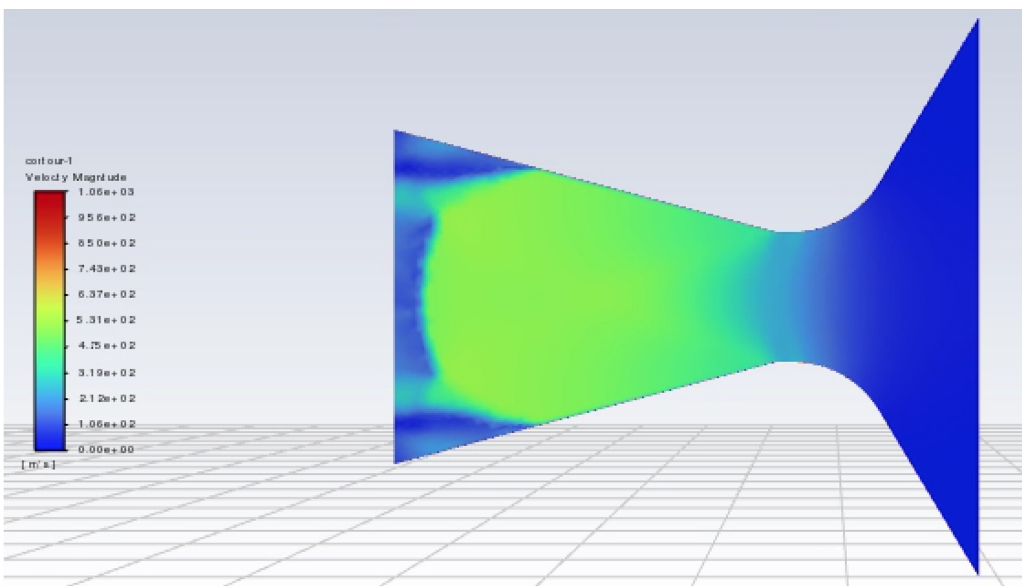

Figure A8: Velocity contour plot (300,000 Pa chamber pressure)

The 500,000 Pa chamber pressure results in a thrust of just over 21.57 Newtons but the oblique shock formation inside the nozzle is much more drastic (Figure A7). The 300,000 Pa chamber pressure results in almost no thrust at all as a result of the oblique shocks completely disrupting the flow in the diverging portion of the C-D nozzle (Figure A8).

As a result of these simulations and the numeric solutions they produced, the team has confidently determined that cold-gas thrust is not a viable option as a primary propulsion source. This method requires an incredible amount of mass to be taken on board during the flight in order to produce a thrust capable of supporting our mission. Furthermore, using the nozzle already used by NUSTARS, achieving low enough thrust capable of allowing the lander to descend would be very challenging to accomplish without the formation of oblique shocks and a dramatic performance drop off.