Tutorial & Homework #6 Due: 27 September 2018

1. Given a unity feedback system with open loop transfer function $G(s) = \frac{84}{s(s^7+5s^6+12s^5+25s^4+45s^3+50s^2+82s+60)}$ tell how many poles of the closed loop transfer function lie on the right half-plane, in the left half-plane and on the $j\omega$ axis.

2. Repeat problem (1) with $G(s) = \frac{8}{s(s^6-2s^5-s^4+2s^3+4s^2-8s-4)}$.

3. The closed loop transfer function for a system is $T(s) = \frac{s^2+K_1s+K_2}{s^4+K_1s^3+K_2s^2+4s+1}$. Determine the range of K_1 in order for the system to be stable. What is the relationship between K_1 and K_2 for stability.

4. For the transfer function $T(s) = \frac{K_1s+K_2}{s^4+K_1s^3+s^2+K_2s+1}$, find the constraints on K_1 and K_2 such that the function will have only two $j\omega$ poles.

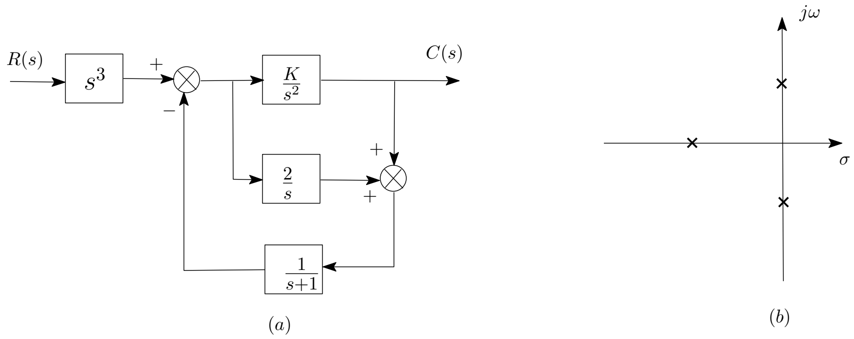
5. An interval polynomial is of the form $P(s) = a_0 + a_1s + a_2s^2 + a_3s^3 + a_4s^4 + a_5s^5 + a_6s^6 + \cdots$, with its coefficients belonging to intervals $x_i \leq a_i \leq y_i$, where x_i, y_i are prescribed constants. Kharitonov's theorem says that an interval polynomial has all its roots in the left half-plane if and each one of the following four extreme polynomials has its roots in the left half-plane

$$K_{ll}(s) = x_0 + x_1s + y_2s^2 + y_3s^3 + x_4s^4 + x_5s^5 + y_6s^6 + \cdots$$
$$K_{lu}(s) = x_0 + y_1s + y_2s^2 + x_3s^3 + x_4s^4 + y_5s^5 + y_6s^6 + \cdots$$
$$K_{ul}(s) = y_0 + x_1s + x_2s^2 + y_3s^3 + y_4s^4 + x_5s^5 + x_6s^6 + \cdots$$
$$K_{uu}(s) = y_0 + y_1s + x_2s^2 + x_3s^3 + y_4s^4 + y_5s^5 + x_6s^6 + \cdots$$

Use Kharitonov's theorem and the Routh-Hurwitz criterion to find if the following interval polynomial has any zeros in the right half-plane.

$$P(s) = a_0 + a_1s + a_2s^2 + a_3s^3, \quad 2 \leq a_0 \leq 3,\ 1 \leq a_1 \leq 2,\ 3 \leq a_2 \leq 5,\ a_3 = 1.$$

6. Find the value of K in the system shown in figure (a) that will place the closed loop poles as shown in figure (b)

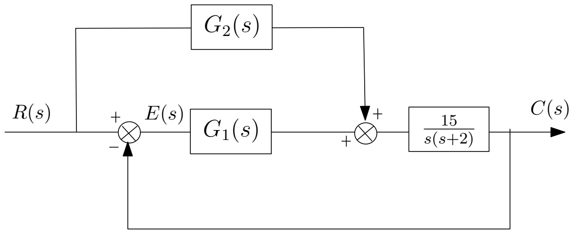


7. The unity feedback system where $G(s) = \frac{K(s^2+3s+30)}{s^n(s+5)}$ is to have 1/6000 error between an input of $10tu(t)$ and the output in the steady state. Find K and n to meet the specification. What are K_p, K_ν and K_a?

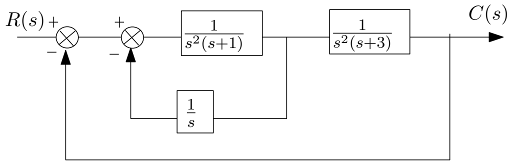
8. The unity feedback system where $G(s) = \frac{K(s+\alpha)}{(s+\beta)^2}$ is to be designed to meet the following specifications: steady-state error for unit step input = 0.1; damping ratio = 0.5; natural frequency = $\sqrt{10}$. Find K, α and β.

9. Given the unity feedback system where $G(s) = \frac{K}{s^n(s+a)}$, find the values of n, K, and a in order to meet specifications of 12% overshoot and $K_\nu = 110$.

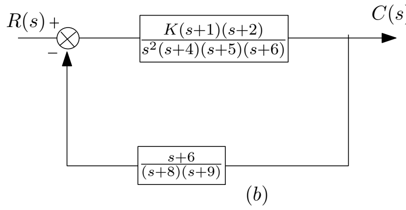
10. What are the restrictions on the feed-forward transfer function $G_2(s)$ in the system shown in figure below to obtain zero steady state error for step inputs if:
(a) $G_1(s)$ is a Type 0 transfer function (b) $G_1(s)$ is a Type 1 transfer function (c) $G_1(s)$ is a Type 2 transfer function



11. Given the systems in figure below, find the following: (a) The closed-loop transfer function (b) The system type (c) The steady-state error for an input $5u(t)$ (d) The steady-state error for an input $5tu(t)$

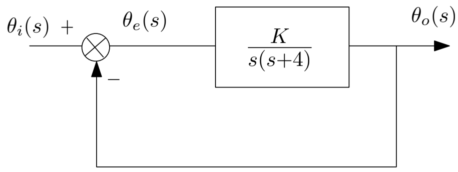


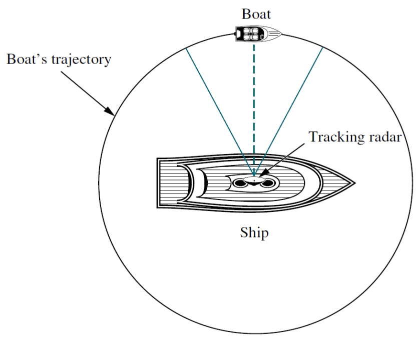
(a)



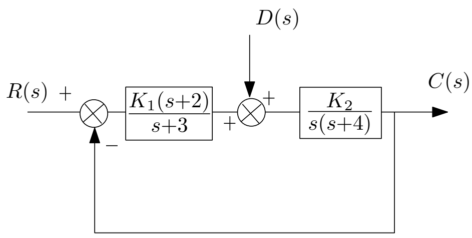
(b)

12. A boat is circling around a ship that is using a tracking radar. The speed of the boat is 20km/hr, and it is circling the ship at a distance of 1 kilometer, as shown in the figure below. A simplified model of the tracking system is provided as a block diagram in figure below. Find the value of K so that the boat is kept in the center of the radar beam with no more than 0.1 degree error.

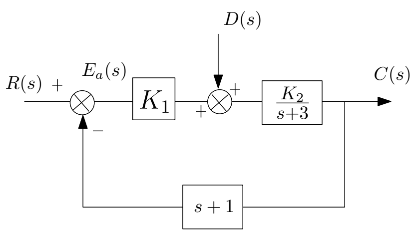




13. Design the values of K_1 and K_2 in the system shown in figure below to meet the following specifications: Steady-state error component due to a unit step disturbance is -0.000012; steady state error component due to a unit ramp input is 0.003.



14. For the system shown in figure below, find the sensitivity of the steady-state error for changes in K_1 and in K_2, when $K_1 = 100$ and $K_2 = 1$. Assume step inputs for both input and the disturbance.



15. For a closed loop system with $G(s) = \frac{K}{s(s+1)(s+3)}$ and $H(s) = s + a$, find the sensitivity of the steady-state error to parameter a. Assume a step input. Plot the sensitivity as a function of parameter a.