Reliability, Safety and Risk Analysis — Final Project

F. Circhetta, P. De Lucia

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Abstract

The aim of this project is to estimate the reliability and availability of a fairly simple system. Given the complexity involved with Continuous Time Discrete State Markov Processes - as it is impossible to derive an analytical solutions when the number of possible states exceeds 4 - time dependent solutions are obtained through series of indirect Monte Carlo simulations.

1 Introduction

The system under study is comprised of a 2-out-of-3 system chained in series with a fourth component. For each of them the failure and repair rates (λ_x and μ_x respectively, $x = A, B, C, D$) are known either by means of their real value or a specific distribution.

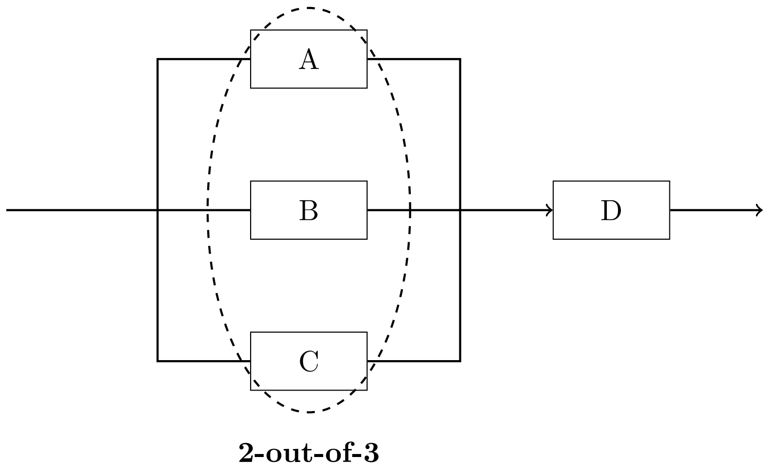

Figure 1: System under study taken from the project assignment.

2 Assumptions

- System is working at time $t = 0$;
- No common cause failures;
- A component under maintenance is failed;
- Every component has its own repair team;
- A component, taken individually, can fail even if the system is already failed;

- Components are continuously monitored and repairs start as soon as the component breaks;
- The repairs are always successful and bring the component to an "as good as new condition"

3 Algorithm

3.1 Input

Components' transition rates are recorded in a matrix. Then we define the system's failure logical function (anonymous function of the logical variable `state`, which is `0` if the corresponding component is working or `1` if it is failed).

3.2 Analytical solution (case study 1)

In order to validate the MC algorithm, the code provides an analytical solution for reliability and MTTF through the use of the symbolic package.

As for the reliability of the whole system, it is

$$\begin{aligned} R^{2oo3}(t) &= R_A R_B R_C + (1-R_A) R_B R_C + R_A(1-R_B)R_C + R_A R_B (1-R_C) \\ &= e^{-(\lambda_A+\lambda_B)t} + e^{-(\lambda_B+\lambda_C)t} + e^{-(\lambda_A+\lambda_C)t} - 2e^{-(\lambda_A+\lambda_B+\lambda_C)t} \\ R^{sys}(t) &= R^{2oo3}(t) \cdot e^{-\lambda_D t} \end{aligned}$$

While the equation for the MTTF has been derived as:

$$MTTF = \int_0^{+\infty} t f_T(t) dt = -R(t)t\Big|_0^{+\infty} + \int_0^{+\infty} R(t)dt = \int_0^{+\infty} R(t)dt \tag{1}$$

Once both reliability and MTTF are switched from symbolic to numerical values, the Monte Carlo simulation can be performed.

3.3 Reliability and availability

The simulation relies on the function `mc_sim`, which requires as inputs the components' transition rates matrix, the system's state, the analysis' time horizon, the number of simulations we want to perform and a logical indicator to assume (if `true`) the system definitively failed once it reaches this state. The function's output will be an array containing: the values on the time axis, the reliability and its variance, the availability and its variance.

The `mc_sim` function performs its computations through the following operations: in a `for` cycle iterating a number of times equal to that we want the simulation to be performed, while the time `t` is lower than the mission time, the code will be sampling transition times as

$$t_2 = t_1 - \frac{\ln(1-R_t)}{\lambda_{state}} \tag{2}$$

Where R_t is a random number extracted from a uniform distribution $U(0,1)$, while λ_{state} represents the transition rate of the system out from its current configuration and has been computed as the sum of all of the components' transition rates.

Now the affected component has to be found. To do so, given that the transition occurs at the sampled time `t`, the transition probabilities for each component will be

$$p_k = \frac{\lambda_{tr}^k}{\lambda_{state}}, k = 1, 2, \ldots, \#components \tag{3}$$

These probabilities will then be compared with a random number (again extracted from a uniform distribution $U(0,1)$) to determine which component will be subject to transition. Once the affected component is found its state has to be switched and will simply have to be switched to the opposite state since the components in the given system can only be in one state at the time and, when repair is a possibility, each of them is continuously monitored and repaired right after failure by a dedicated maintenance team.

The code then checks if the components' transition has led to system failure otherwise if the system previously was in a failed state. If the system has failed a lower time boundary is set at the first instant greater or equal to the transition time and thereon the system failure counters are increased, and so the simulation may start over again. If, on the other hand, maintenance is a possibility, the code will set an upper boundary for the down-time and account for the instantaneous unavailability through the use of counters. Reliability and availability (and their variance) can be calculated as the complementary functions of unreliability and unavailability, which have been found by applying the Monte Carlo integration method.

3.4 Average availability

The average availability is computed as the integral mean of the instantaneous availability over the mission time span

$$q_{T_p} = \frac{1}{T_p} \int_0^{T_p} q(t)dt \tag{4}$$

3.5 MTTF

We move to computing the MTTF through a Monte Carlo simulation. This calculation will be performed by a function similar to the previous employed for the reliability, but it will simply account for failure times in an array when the simulation results in a failed state. The MTTF is then computed following the Monte Carlo integration method together with its variance

$$MTTF = \int_0^{+\infty} t f_T(t)dt \tag{5}$$

To finish off, the code performs a last Monte Carlo simulation computing trials for the MTTF to provide information on the validity of the results obtained through validation at 1, 2 and 3-σ level of confidence.

3.6 Uncertainty on parameters

The last point we'll focus on is the Monte Carlo simulation of reliability and availability with a Bayesian approach when the real value of some parameters is unknown. In this case, we know the parameters are distributed according to a lognormal distribution. First of all, we'll need to move from logarithmic mean and variance to exact ones. To do so, given logarithmic mean values (m_i) and coefficient of variation (cv), we will perform the following calculations:

$$var_i = (cv \cdot m_i)^2, \mu_i = \ln\left(\frac{m_i^2}{\sqrt{var_i \cdot m_i^2}}\right), \sigma_i = \sqrt{\ln\left(\frac{var_i}{(m_i^2+1)}\right)} \tag{6}$$

These calculations give the correct parameters to employ in the log-normal random sampling of the two failure rates in the external loop, where they are iteratively substituted in the components' characteristics matrix to be employed by the Monte Carlo simulation function previously described to compute the reliabilities and availabilities together with their variances, in accordance with the following procedure:

$$R(t) = P(T > t) = \int_0^{+\infty} P(T > t|\lambda)p''(\lambda|E)d\lambda = \int_0^{+\infty} R(T|\lambda)p''(\lambda|E)d\lambda \tag{7}$$

Where p'' is the posterior distribution of the failure rate given the event E, which, while unknown, allows us to consider λ distributed as a lognormal.

4 Results

4.1 Case study 1: no repair

In the case of no available maintenance the system will most likely be failed at mission time ($\sim$ 21 days), with a value within the 95% confidence interval [0.032342; 0.034618], since the single components present MTTFs of 200, 250, 333 and 1000 hours, respectively, giving an overall MTTF of system in the 95% confidence interval [$181.50h$; $183.25h$].

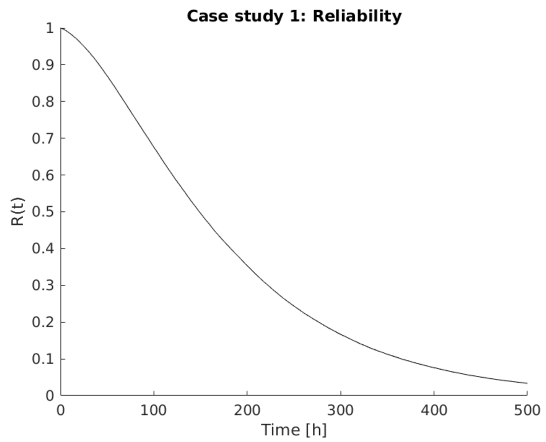

Figure 2: Time dependent reliability $R(t)$

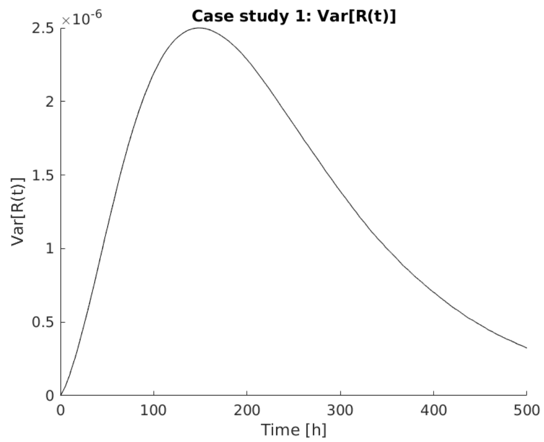

Figure 3: Time dependent $R(t)$ variance

For the first point of the project, we were able to validate the MTTF and reliability values over multiple trials given the analytical solution, the obtained results are satisfactory.

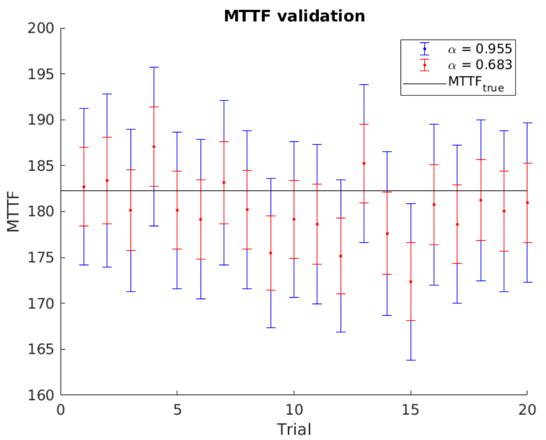

Figure 4: $MTTF$ validation

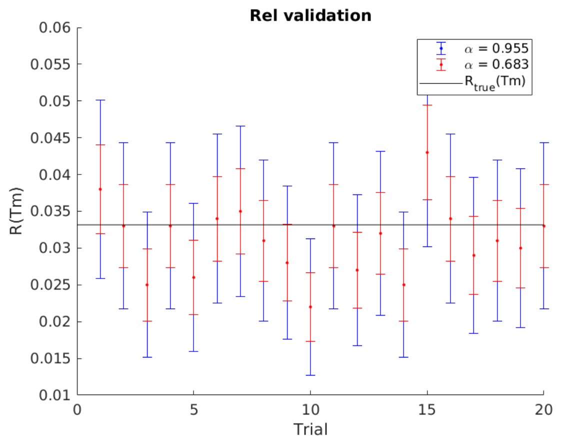

Figure 5: $R(500h)$ validation

4.2 Case study 2: repairs allowed

Introducing the possibility of maintenance, the reliability curve of the system will be decreasing more slowly, and with a value within the 95% confidence interval: [0.39168; 0.39787]. What actually is more relevant in this analysis is the trend shown by the availability, which, after a brief transition period, reaches an equilibrium around a stationary value. The mean availability in the mission time, in the 95% confidence interval, will be [0.99151; 0.99152], giving a mean downtime of $\sim$4.23 h. In this case the MTTF lies in the 95% confidence interval [$532.42h$; $539.14h$].

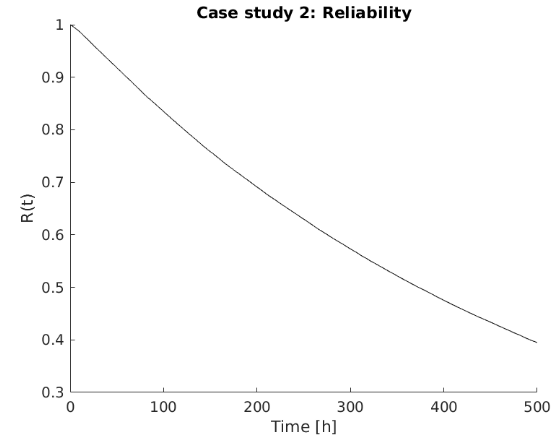

Figure 6: Time dependent reliability $R(t)$

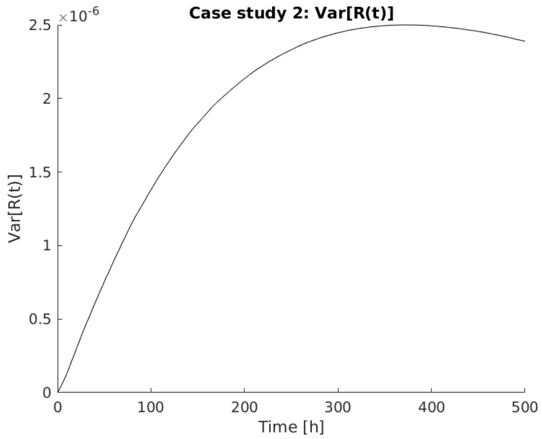

Figure 7: Time dependent $R(t)$ variance

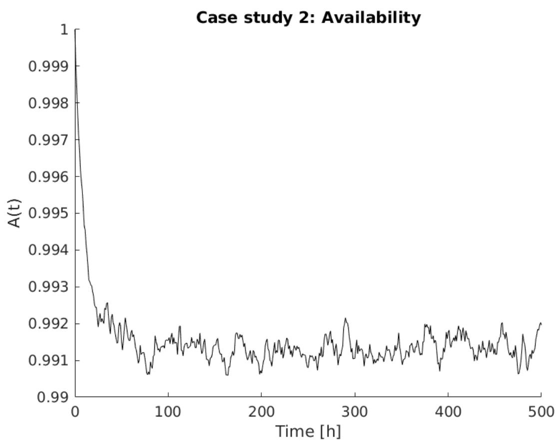

Figure 8: Time dependent availability $A(t)$

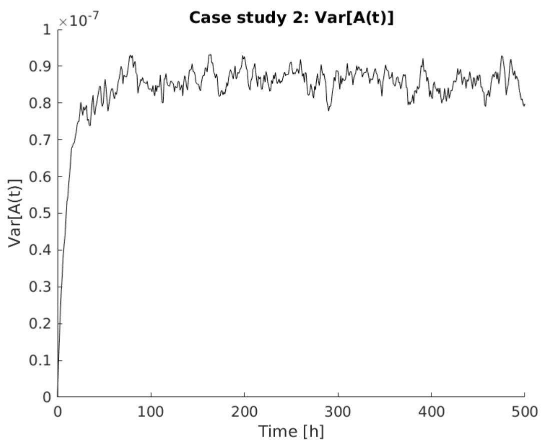

Figure 9: Time dependent $A(t)$ variance

4.3 Case study 3: Bayesian framework

As for point three of the project, the mean values assigned to the two failure rates were the same as the real ones for the previous analysis, so we didn't expect much change amongst the results. The Bayesian framework provides us with an analytic value for reliability and availability, even though the parameters presented are uncertain; this wouldn't have been the case if we were to employ the frequentist framework.

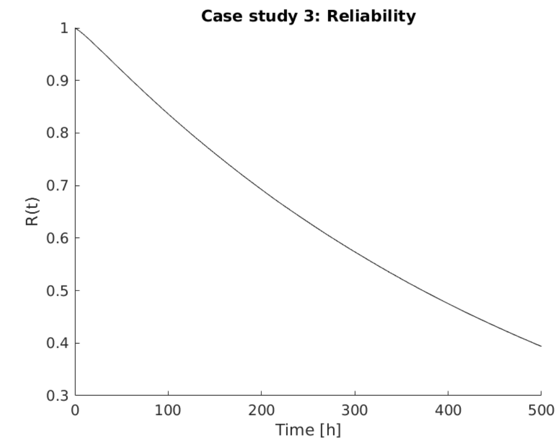

Figure 10: Time dependent reliability $R(t)$

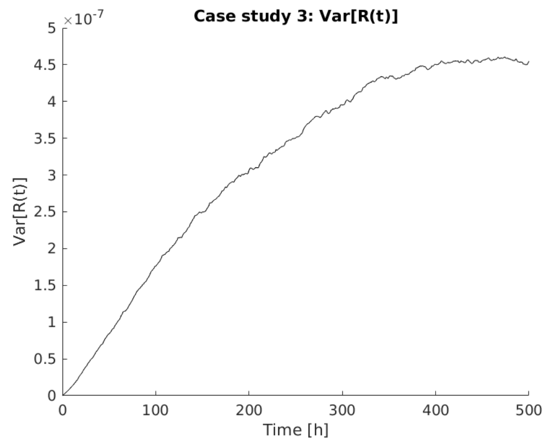

Figure 11: Time dependent $R(t)$ variance

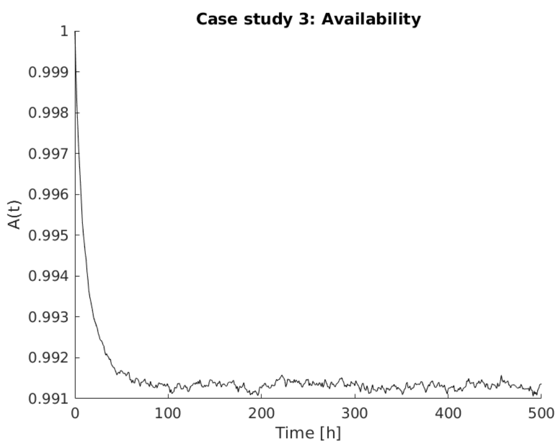

Figure 12: Time dependent availability $A(t)$

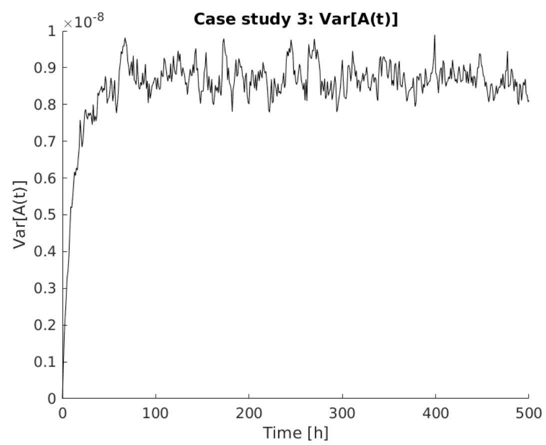

Figure 13: Time dependent $A(t)$ variance