MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550
Applied forecasting

Week 4: Simple forecasting methods

Outline

1. Four benchmark methods
2. Time trends and dummy seasonality
3. Forecasting with transformations
4. Forecasting with decompositions

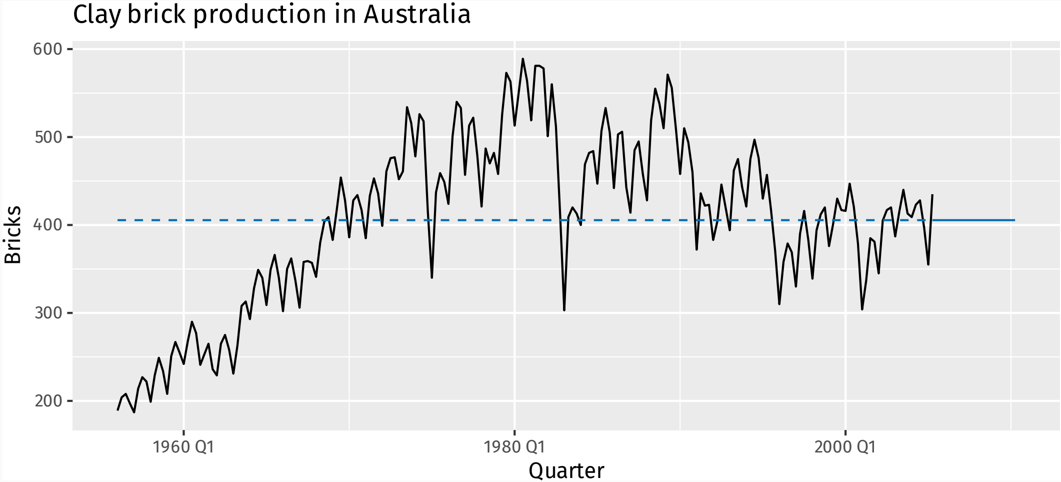
Outline

Mean method

```
MEAN(y)
```

- Forecast of all future values is equal to mean of historical data $\{y_1, \ldots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \cdots + y_T)/T$
- Implicit model: $y_t = c + \varepsilon_t$ where $\varepsilon_t \sim WN$.

Mean method



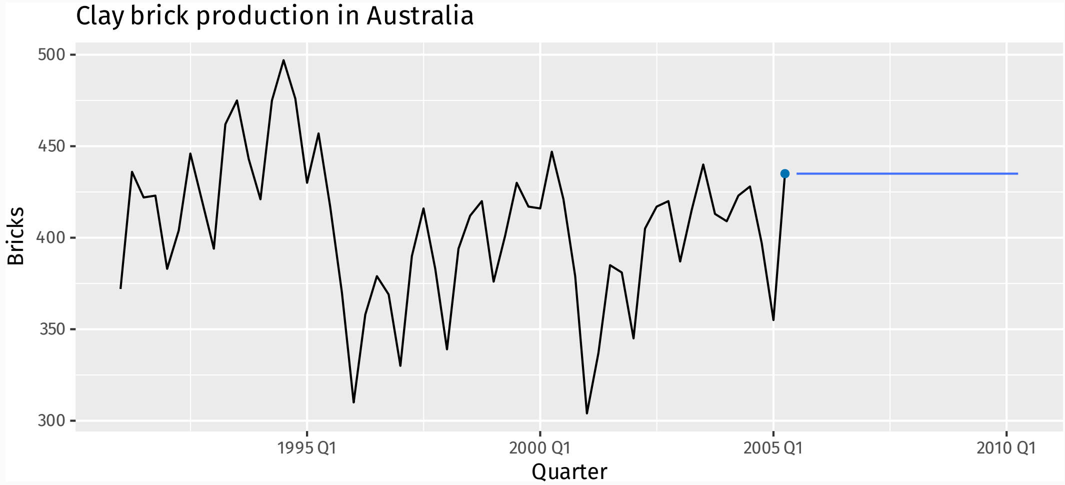
Clay brick production in Australia
600
500
400
300
200
Bricks
1960 Q1
1980 Q1
2000 Q1
Quarter

Naïve method

`NAIVE(y)`

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Implicit model is a random walk: $y_t = y_{t-1} + \varepsilon_t$
- Consequence of efficient market hypothesis.

Naïve method

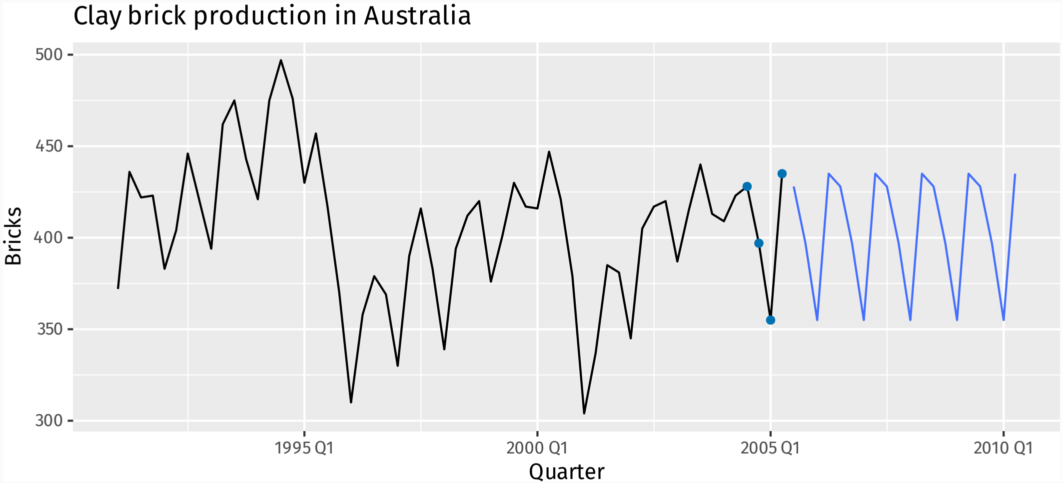


Seasonal Naïve method

```
SNAIVE(y ~ lag(m))
```

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.
- Implicit model: $y_t = y_{t-m} + \varepsilon_t$

Seasonal Naïve method



Drift method

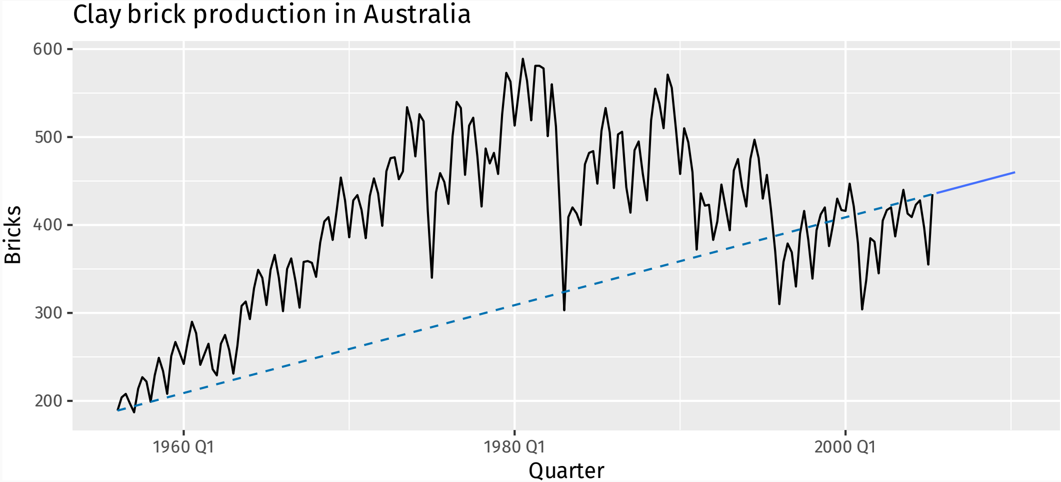
```
RW(y ~ drift())
```

- Forecasts equal to last value plus average change:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1}\sum_{t=2}^{T}(y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1}(y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.
- Implicit model: Random walk with drift: $y_t = c + y_{t-1} + \varepsilon_t$

Drift method



Outline

Linear time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \varepsilon_t$$

```
TSLM(y ~ trend())
```

- $x_{1,t} = t, \qquad t = 1, \dots, T$
- Forecasts: $\hat{y}_{T+h|T} = \beta_0 + \beta_1(T + h)$

Piecewise linear time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t$$

```
TSLM(y ~ trend(knots = c(tau1, tau2, ..., taup)))
```

- Bends at $\tau_1, \dots, \tau_p$
- $x_{1,t} = t$
- $x_{i,t} = (t - \tau_i)_+ = \begin{cases} 0 & t < \tau_i \\ (t - \tau_i) & t \geq \tau_i \end{cases}$
- Forecasts: $\beta_0 + \beta_1(T+h) + \beta_2(T+h-\tau_1)_+ + \cdots + \beta_{p+1}(T+h-\tau_p)_+$

Quadratic time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \varepsilon_t$$

- $x_{1,t} = t$
- $x_{2,t} = t^2$
- Forecasts: $\beta_0 + \beta_1(T + h) + \beta_2(T + h)^2$

NOT RECOMMENDED!

Daily dummy variables

```
TSLM(y ~ season())
```

- Using one dummy for each category gives too many dummy variables!
- The coefficients of the dummies are relative to the omitted category
- `season()` automatically generates the dummy variables for you.

Day	d1	d2	d3	d4
Monday	1	0	0	0
Tuesday	0	1	0	0
Wednesday	0	0	1	0
Thursday	0	0	0	1
Friday	0	0	0	0
Monday	1	0	0	0
Tuesday	0	1	0	0
Wednesday	0	0	1	0
Thursday	0	0	0	1
Friday	0	0	0	0

Outline

Forecasting with transformations

- Back-transformed point forecasts are medians.
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Let X have mean μ and variance σ^2.

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ:

$$Y = f(X) \approx f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2 f''(\mu).$$

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$$\text{E}[Y] = \text{E}[f(X)] \approx f(\mu) + \tfrac{1}{2}\sigma^2 f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

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$$\mathrm{E}[Y] \approx \begin{cases} e^{\mu}\left[1 + \frac{\sigma^2}{2}\right] & \lambda = 0; \\ (\lambda\mu + 1)^{1/\lambda}\left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda\mu+1)^2}\right] & \lambda \neq 0. \end{cases}$$

Outline

Forecasting with decompositions

$$y_t = \hat{S}_t + \hat{A}_t$$

- $\hat{A}_t$ is seasonally adjusted component
- $\hat{S}_t$ is seasonal component.

- Forecast $\hat{S}_t$ using a Seasonal Naïve method.
- Forecast $\hat{A}_t$ using a non-seasonal time series method.
- Combine forecasts of $\hat{S}_t$ and $\hat{A}_t$ to get forecasts of original data.

Decomposition models

`decomposition_model()` creates a decomposition model

- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naïve method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.