MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550 Applied forecasting

Week 12: Dynamic regression models

Regression with ARIMA errors

Regression models

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t,$$

- y_t modeled as function of k explanatory variables $x_{1,t}, \ldots, x_{k,t}$.
- In regression, we assume that ε_t is WN.
- Now we want to allow ε_t to be autocorrelated.

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Example: ARIMA(1,1,1) errors

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \eta_t,$$
$$(1 - \phi_1 B)(1 - B)\eta_t = (1 + \theta_1 B)\varepsilon_t,$$

where ε_t is white noise.

Estimation

If we minimize $\sum \eta_t^2$ (by using ordinary regression):

1. Estimated coefficients $\hat{\beta}_0, \ldots, \hat{\beta}_k$ are no longer optimal as some information ignored;
2. Statistical tests associated with the model (e.g., t-tests on the coefficients) are incorrect.
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- Minimizing $\sum \varepsilon_t^2$ avoids these problems.
- Maximizing likelihood similar to minimizing $\sum \varepsilon_t^2$.

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Original data

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$$\text{where} \quad \phi(B)(1-B)^d \eta_t = \theta(B)\varepsilon_t$$

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After differencing all variables

$$y'_t = \beta_1 x'_{1,t} + \cdots + \beta_k x'_{k,t} + \eta'_t.$$

where $\phi(B)\eta'_t = \theta(B)\varepsilon_t$,

$y'_t = (1-B)^d y_t, \quad x'_{i,t} = (1-B)^d x_{i,t}, \quad$ and $\eta'_t = (1-B)^d \eta_t$

Regression with ARIMA errors

- In R, we can specify an ARIMA(p, d, q) for the errors, then d levels of differencing will be applied to all variables $(y, x_{1,t}, \ldots, x_{k,t})$ during estimation.
- Check that ε_t series looks like white noise.
- AICc can be calculated for final model.
- Repeat procedure for all subsets of predictors to be considered, and select model with lowest AICc value.

Forecasting

- To forecast a regression model with ARIMA errors, we need to forecast the regression part of the model and the ARIMA part of the model and combine the results.
- Some predictors are known into the future (e.g., time, dummies).
- Separate forecasting models may be needed for other predictors.
- Forecast intervals ignore the uncertainty in forecasting the predictors.