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# ETC3550/ETC5550

## Applied forecasting

Week 5: Accuracy evaluation



# Outline

- 1 Workshop tomorrow
- 2 Residuals
- 3 Forecast distributions
- 4 Forecast evaluation
- 5 Time series cross-validation

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# Workshop tomorrow

The workshop is online tomorrow.

Zoom link on Ed Discussion Forum

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# Fitted values

- $\hat{y}_{t|t-1}$  is the forecast of  $y_t$  based on observations  $y_1, \dots, y_{t-1}$ .
- We call these “fitted values”.
- Sometimes drop the subscript:  $\hat{y}_t \equiv \hat{y}_{t|t-1}$ .
- Often not true forecasts since parameters are estimated on all data.

## For example:

- $\hat{y}_t = \bar{y}$  for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$  for drift method.

# Forecasting residuals

**Residuals in forecasting:** difference between observed value and its fitted value:  $e_t = y_t - \hat{y}_{t|t-1}$ .

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## Assumptions

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- 2  $\{e_t\}$  have mean zero. If they don't, then forecasts are biased.



# Forecasting residuals

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## Useful properties (for distributions & prediction intervals)

- 3  $\{e_t\}$  have constant variance.
- 4  $\{e_t\}$  are normally distributed.

# ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

# Ljung-Box tests

$r_k$  = autocorrelation of residual at lag  $k$

Test *whole set* of  $r_k$  values simultaneously.

$$Q^* = T(T+2) \sum_{k=1}^{\ell} (T-k)^{-1} r_k^2$$

where  $\ell$  is max lag being considered and  $T$  is number of observations.

- My preferences:  $\ell = 10$  for non-seasonal data,  $\ell = 2m$  for seasonal data (where  $m$  is seasonal period).
- If data are WN,  $Q^*$  has  $\chi^2$  distribution with  $\ell$  degrees of freedom.

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# Forecast distributions

- A forecast  $\hat{y}_{T+h|T}$  is (usually) the mean of the conditional distribution  $y_{T+h} \mid y_1, \dots, y_T$ .
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

# Forecast distributions

Assuming residuals are normal, uncorrelated,  $\text{sd} = \hat{\sigma}$ :

**Mean:**  $y_{T+h|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$

**Naïve:**  $y_{T+h|T} \sim N(y_T, h\hat{\sigma}^2)$

**Seasonal naïve:**  $y_{T+h|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$

**Drift:**  $y_{T+h|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where  $k$  is the integer part of  $(h-1)/m$ .

Note that when  $h = 1$  and  $T$  is large, these all give the same approximate forecast variance:  $\hat{\sigma}^2$ .

# Prediction intervals

- A prediction interval gives a region within which we expect  $y_{T+h}$  to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where  $\hat{\sigma}_h$  is the st dev of the  $h$ -step distribution.

- When  $h = 1$ ,  $\hat{\sigma}_h$  can be estimated from the residuals.

# Prediction intervals

- Point forecasts are often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Prediction intervals are usually too narrow due to unaccounted uncertainty.



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# Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

# Forecast errors

Forecast “error”: the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by  $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing  $\hat{y}_{T+h|T}$ .

# Measures of forecast accuracy

$y_{T+h}$  =  $(T + h)$ th observation,  $h = 1, \dots, H$

$\hat{y}_{T+h|T}$  = its forecast based on data up to time  $T$ .

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$$

$$\text{MAE} = \text{mean}(|e_{T+h}|)$$

$$\text{MSE} = \text{mean}(e_{T+h}^2)$$

$$\text{MAPE} = 100\text{mean}(|e_{T+h}|/|y_{T+h}|)$$

$$\text{RMSE} = \sqrt{\text{mean}(e_{T+h}^2)}$$

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MAPE =  $100\text{mean}(|e_{T+h}|/|y_{T+h}|)$

- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if  $y_t \gg 0$  for all  $t$ , and  $y$  has a natural zero.

# Scaled Errors

Proposed by Hyndman and Koehler (IJF, 2006).

- For non-seasonal time series, scale errors using naïve forecasts:

$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-1} \sum_{t=2}^T |y_t - y_{t-1}|}.$$

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$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-1} \sum_{t=2}^T |y_t - y_{t-1}|}.$$

- For seasonal time series, scale forecast errors using seasonal naïve forecasts:

$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-m} \sum_{t=m+1}^T |y_t - y_{t-m}|}.$$

# Scaled errors

## Mean Absolute Scaled Error

$$\text{MASE} = \text{mean}(|q_{T+h}|)$$



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$$\text{MASE} = \text{mean}(|q_{T+h}|)$$

## Root Mean Squared Scaled Error

$$\text{RMSSE} = \sqrt{\text{mean}(q_{T+h}^2)}$$

where

$$q_{T+h}^2 = \frac{e_{T+h}^2}{\frac{1}{T-m} \sum_{t=m+1}^T (y_t - y_{t-m})^2},$$

and we set  $m = 1$  for non-seasonal data.

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# Time series cross-validation

## Traditional evaluation



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## Time series cross-validation



- Forecast accuracy averaged over test sets.
- Also known as "evaluation on a rolling forecasting origin"



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