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# ETC3550/ETC5550

## Applied forecasting

Week 4: Simple forecasting methods



# Outline

- 1 Four benchmark methods
- 2 Time trends and dummy seasonality
- 3 Forecasting with transformations
- 4 Forecasting with decompositions

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# Mean method

MEAN( $y$ )

- Forecast of all future values is equal to mean of historical data  $\{y_1, \dots, y_T\}$ .
- Forecasts:  $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T$
- Implicit model:  $y_t = c + \varepsilon_t$  where  $\varepsilon_t \sim WN$ .

# Mean method



# Naïve method

NAIVE( $y$ )

- Forecasts equal to last observed value.
- Forecasts:  $\hat{y}_{T+h|T} = y_T$ .
- Implicit model is a random walk:  $y_t = y_{t-1} + \varepsilon_t$
- Consequence of efficient market hypothesis.

# Naïve method



# Seasonal Naïve method

SNAIVE( $y \sim \text{lag}(m)$ )

- Forecasts equal to last value from same season.
- Forecasts:  $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$ , where  $m$  = seasonal period and  $k$  is the integer part of  $(h-1)/m$ .
- Implicit model:  $y_t = y_{t-m} + \varepsilon_t$



# Seasonal Naïve method



# Drift method

RW(y ~ drift())

- Forecasts equal to last value plus average change:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1} (y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.
- Implicit model: Random walk with drift:  $y_t = c + y_{t-1} + \varepsilon_t$

# Drift method



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# Linear time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \varepsilon_t$$

TSLM( $y \sim \text{trend}()$ )

- $x_{1,t} = t, \quad t = 1, \dots, T$
- Forecasts:  $\hat{y}_{T+h|T} = \beta_0 + \beta_1(T + h)$

# Piecewise linear time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t$$

`TSLM(y ~ trend(knots = c(tau1, tau2, ..., taup)))`

- Bends at  $\tau_1, \dots, \tau_p$
- $x_{1,t} = t$
- $x_{i,t} = (t - \tau_i)_+ = \begin{cases} 0 & t < \tau_i \\ (t - \tau_i) & t \geq \tau_i \end{cases}$
- Forecasts:  $\beta_0 + \beta_1(T+h) + \beta_2(T+h - \tau_1)_+ + \cdots + \beta_{p+1}(T+h - \tau_p)_+$

# Quadratic time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \varepsilon_t$$

- $x_{1,t} = t$
- $x_{2,t} = t^2$
- Forecasts:  $\beta_0 + \beta_1(T + h) + \beta_2(T + h)^2$

**NOT RECOMMENDED!**

# Daily dummy variables

TSLM( $y \sim \text{season}()$ )

- Using one dummy for each category gives too many dummy variables!
- The coefficients of the dummies are relative to the omitted category
- `season()` automatically generates the dummy variables for you.

| Day       | d1 | d2 | d3 | d4 |
|-----------|----|----|----|----|
| Monday    | 1  | 0  | 0  | 0  |
| Tuesday   | 0  | 1  | 0  | 0  |
| Wednesday | 0  | 0  | 1  | 0  |
| Thursday  | 0  | 0  | 0  | 1  |
| Friday    | 0  | 0  | 0  | 0  |
| Monday    | 1  | 0  | 0  | 0  |
| Tuesday   | 0  | 1  | 0  | 0  |
| Wednesday | 0  | 0  | 1  | 0  |
| Thursday  | 0  | 0  | 0  | 1  |
| Friday    | 0  | 0  | 0  | 0  |



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Let  $X$  have mean  $\mu$  and variance  $\sigma^2$ .

Let  $f(x)$  be back-transformation function, and  $Y = f(X)$ .

Taylor series expansion about  $\mu$ :

$$Y = f(X) \approx f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2f''(\mu).$$

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$$E[Y] = E[f(X)] \approx f(\mu) + \frac{1}{2}\sigma^2f''(\mu)$$

# Bias adjustment

## Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

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$$E[Y] \approx \begin{cases} e^\mu \left[ 1 + \frac{\sigma^2}{2} \right] & \lambda = 0; \\ (\lambda \mu + 1)^{1/\lambda} \left[ 1 + \frac{\sigma^2(1-\lambda)}{2(\lambda \mu + 1)^2} \right] & \lambda \neq 0. \end{cases}$$

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# Forecasting with decompositions

$$y_t = \hat{S}_t + \hat{A}_t$$

- $\hat{A}_t$  is seasonally adjusted component
  - $\hat{S}_t$  is seasonal component.
- 
- Forecast  $\hat{S}_t$  using a Seasonal Naïve method.
  - Forecast  $\hat{A}_t$  using a non-seasonal time series method.
  - Combine forecasts of  $\hat{S}_t$  and  $\hat{A}_t$  to get forecasts of original data.



# Decomposition models

`decomposition_model()` creates a decomposition model

- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naïve method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.