

# ETC3550/ETC5550

## Applied forecasting

Week 8: ARIMA models



# ARIMA models

- AR:** autoregressive (lagged observations as inputs)
- I:** integrated (differencing to make series stationary)
- MA:** moving average (lagged errors as inputs)

An ARIMA model is rarely interpretable in terms of visible data structures like trend and seasonality. But it can capture a huge range of time series patterns.

# Stationarity

## Definition

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Transformations help to **stabilize the variance**.

For ARIMA modelling, we also need to **stabilize the mean**.

## Differencing

- Differencing helps to **stabilize the mean**.
- First differencing: *change* between consecutive observations:  $y'_t = y_t - y_{t-1}$ .
- Seasonal differencing: *change* between years:  $y'_t = y_t - y_{t-m}$ .

# Your turn

- 1 Does differencing make the Closing stock price series stationary for Amazon and Apple stocks?
- 2 What sorts of transformations and differencing are needed to make the Gas series from `aus_accommodation` stationary?

# Automatic differencing

Statistical tests to determine the required order of differencing.

- 1 Augmented Dickey Fuller test: null hypothesis is that the data are **non-stationary** and non-seasonal.
- 2 Kwiatkowski-Phillips-Schmidt-Shin (KPSS) test: null hypothesis is that the data are **stationary** and non-seasonal.

## Seasonal strength

STL decomposition:  $y_t = T_t + S_t + R_t$

Seasonal strength  $F_s = \max\left(0, 1 - \frac{\text{Var}(R_t)}{\text{Var}(S_t + R_t)}\right)$

If  $F_s > 0.64$ , do one seasonal difference.

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# Your turn

- 3 Do the unit root tests for the Gas series from `aus_accommodation`. Do they give the same numbers of difference as you chose?