MONASH University

MONASH BUSINESS SCHOOL

ETC3550/ETC5550
Applied forecasting

Week 5: Accuracy evaluation

Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations $y_1, \dots, y_{t-1}$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T-1)$ for drift method.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Forecasting residuals

Residuals in forecasting: difference between observed value and its fitted value: $e_t = y_t - \hat{y}_{t|t-1}$.

Assumptions

1. $\{e_t\}$ uncorrelated. If they aren't, then information left in residuals that should be used in computing forecasts.
2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

Useful properties (for distributions & prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Ljung-Box tests

r_k = autocorrelation of residual at lag k

Test *whole set* of r_k values simultaneously.

$$Q^* = T(T+2)\sum_{k=1}^{\ell}(T-k)^{-1}r_k^2$$

where ℓ is max lag being considered and T is number of observations.

- My preferences: $\ell = 10$ for non-seasonal data, $h = 2m$ for seasonal data (where m is seasonal period).
- If data are WN, Q^* has χ^2 distribution with ℓ degrees of freedom.

Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

Mean:	$y_{T+h\|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$
Naïve:	$y_{T+h\|T} \sim N(y_T, h\hat{\sigma}^2)$
Seasonal naïve:	$y_{T+h\|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$
Drift:	$y_{T+h\|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h-1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the h-step distribution.

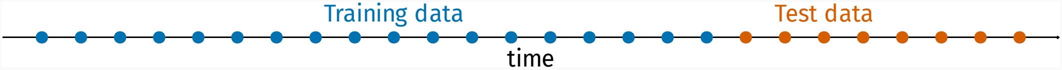
- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

- Point forecasts are often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Prediction intervals are usually too narrow due to unaccounted uncertainty.

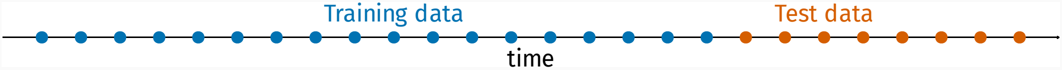
Time series cross-validation

Traditional evaluation

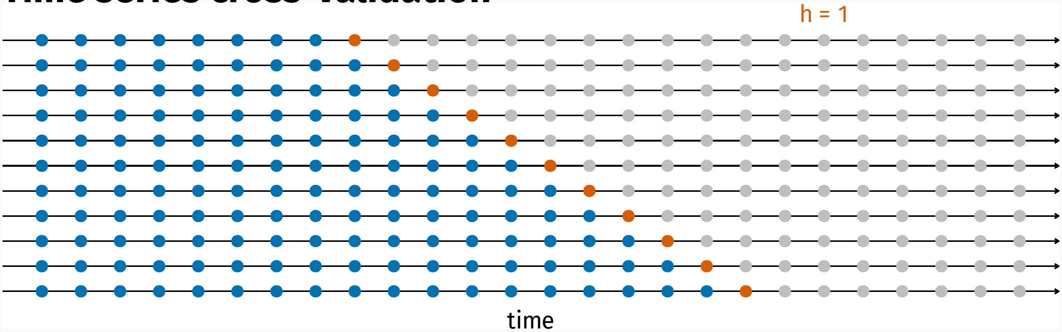


Time series cross-validation

Traditional evaluation

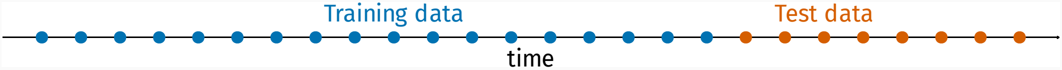


Time series cross-validation

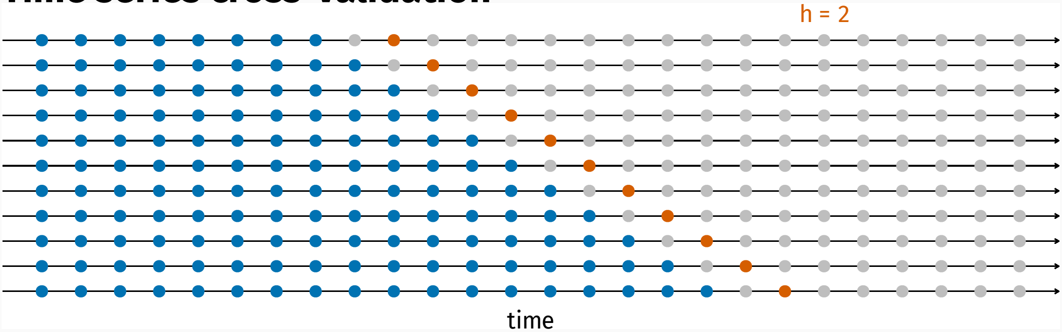


Time series cross-validation

Traditional evaluation

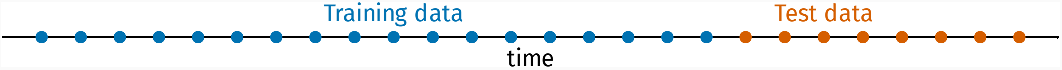


Time series cross-validation

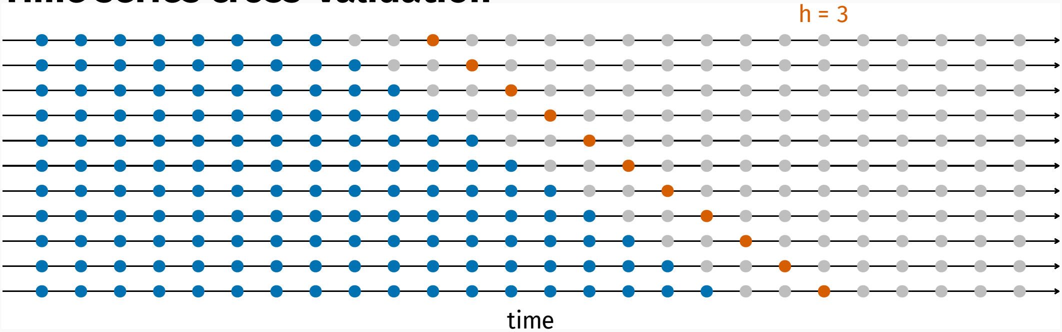


Time series cross-validation

Traditional evaluation

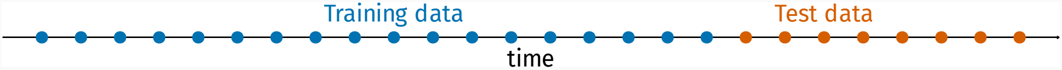


Time series cross-validation

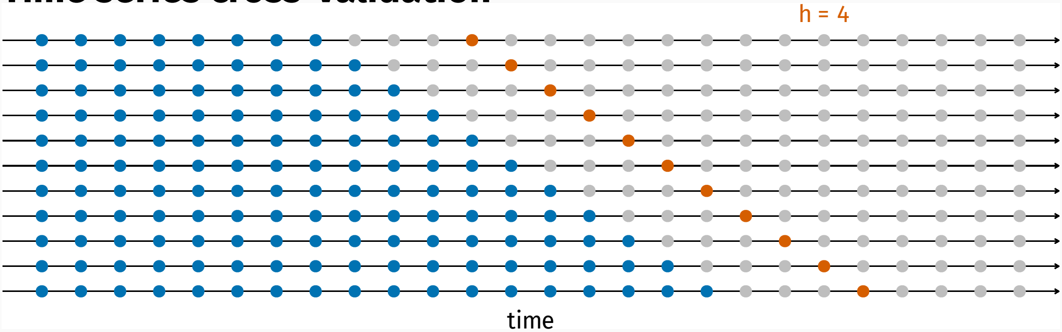


Time series cross-validation

Traditional evaluation

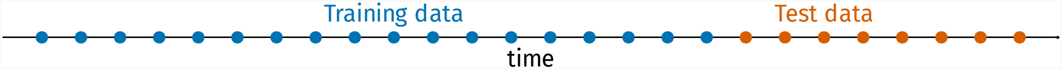


Time series cross-validation

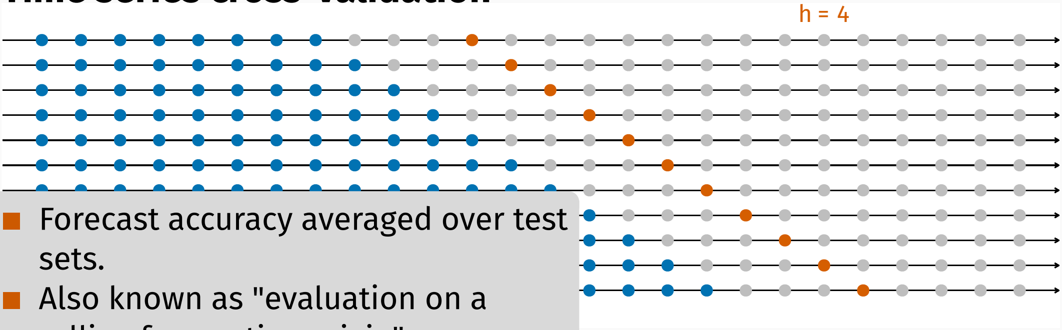


Time series cross-validation

Traditional evaluation



Time series cross-validation



- Forecast accuracy averaged over test sets.
- Also known as "evaluation on a rolling forecasting origin"

Your turn

1. Create a training set for household wealth (`hh_budget`) by withholding the last four years as a test set.
2. Fit all the appropriate benchmark methods to the training set and forecast the periods covered by the test set.
3. Compute the accuracy of your forecasts. Which method does best?
4. Do the residuals from the best method resemble white noise?