

ETC3550/ETC5550

Applied forecasting

Week 7: Exponential smoothing



Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a “level”, “trend” (slope) and “seasonal” component to describe a time series.
- The rate of change of the components are controlled by “smoothing parameters”: α , β and γ respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

ETS models

General notation E T S : ExponenTial S moothing



Error T r e n d S e a s o n

Error: Additive ("A") or multiplicative ("M")

ETS models

General notation E T S : ExponenTial Smoothing



Diagram illustrating the components of the ETS notation: Error, Trend, and Season.

Error Trend Season

Error: Additive ("A") or multiplicative ("M")

Trend: None ("N"), additive ("A"), multiplicative ("M"), or damped ("Ad" or "Md").

ETS models

General notation E T S : ExponenTial Smoothing



The diagram shows three arrows pointing from the words 'Error', 'Trend', and 'Season' below to the letters 'E', 'T', and 'S' respectively in the 'ETS' part of the notation above.

Error Trend Season

Error: Additive ("A") or multiplicative ("M")

Trend: None ("N"), additive ("A"), multiplicative ("M"), or damped ("Ad" or "Md").

Seasonality: None ("N"), additive ("A") or multiplicative ("M")

ETS models

Additive Error

| | | Seasonal Component | | |
|--|----------------------------------|---------------------|---------------------|-----------------------|
| | | N (None) | A (Additive) | M (Multiplicative) |
| | Trend Component | | | |
| | N (None) | A,N,N | A,N,A | A,N,M |
| | A (Additive) | A,A,N | A,A,A | A,A,M |
| | A _d (Additive damped) | A,A _d ,N | A,A _d ,A | A,A _d ,M |

Multiplicative Error

| | | Seasonal Component | | |
|--|----------------------------------|---------------------|---------------------|-----------------------|
| | | N (None) | A (Additive) | M (Multiplicative) |
| | Trend Component | | | |
| | N (None) | M,N,N | M,N,A | M,N,M |
| | A (Additive) | M,A,N | M,A,A | M,A,M |
| | A _d (Additive damped) | M,A _d ,N | M,A _d ,A | M,A _d ,M |

ETS models

Additive Error

| | | Seasonal Component | | |
|-----------------|----------------------------------|---------------------|---------------------|------------------------------|
| | | N (None) | A (Additive) | M (Multiplicative) |
| Trend Component | N (None) | A,N,N | A,N,A | A,N,M |
| | A (Additive) | A,A,N | A,A,A | A,A,M |
| | A _d (Additive damped) | A,A _d ,N | A,A _d ,A | A,A_d,M |

Multiplicative Error

| | | Seasonal Component | | |
|-----------------|----------------------------------|---------------------|---------------------|-----------------------|
| | | N (None) | A (Additive) | M (Multiplicative) |
| Trend Component | N (None) | M,N,N | M,N,A | M,N,M |
| | A (Additive) | M,A,N | M,A,A | M,A,M |
| | A _d (Additive damped) | M,A _d ,N | M,A _d ,A | M,A _d ,M |

AIC and cross-validation

Minimizing the AIC assuming Gaussian residuals is asymptotically equivalent to minimizing one-step time series cross validation MSE.

Automatic forecasting

From Hyndman et al. (IJF, 2002):

- Apply each model that is appropriate to the data. Optimize parameters and initial values using MLE (or some other criterion).
- Select best method using AICc:
- Produce forecasts using best method.
- Obtain forecast intervals using underlying state space model.

Method performed very well in M3 competition.

Residuals

Response residuals

$$\hat{e}_t = y_t - \hat{y}_{t|t-1}$$

Innovation residuals

Additive error model:

$$\hat{\varepsilon}_t = y_t - \hat{y}_{t|t-1}$$

Multiplicative error model:

$$\hat{\varepsilon}_t = \frac{y_t - \hat{y}_{t|t-1}}{\hat{y}_{t|t-1}}$$

Your turn

- 1 Try forecasting the Chinese GDP from the `global_economy` data set using an ETS model.

Experiment with the various options in the `ETS()` function to see how much the forecasts change with damped trend, or with a Box-Cox transformation. Try to develop an intuition of what each is doing to the forecasts.

[Hint: use `h=20` when forecasting, so you can clearly see the differences between the various options when plotting the forecasts.]

Your turn

2

Find an ETS model for the Gas data from `aus_production` and forecast the next few years.

- ▶ Why is multiplicative seasonality necessary here?
- ▶ Experiment with making the trend damped. Does it improve the forecasts?