ETC3550/ETC5550
Applied forecasting

Week 5: Accuracy evaluation

Outline

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Fitted values

- $\hat{y}_{t|t-1}$ is the forecast of y_t based on observations $y_1, \dots, y_{t-1}$.
- We call these "fitted values".
- Sometimes drop the subscript: $\hat{y}_t \equiv \hat{y}_{t|t-1}$.
- Often not true forecasts since parameters are estimated on all data.

For example:

- $\hat{y}_t = \bar{y}$ for average method.
- $\hat{y}_t = y_{t-1} + (y_T - y_1)/(T - 1)$ for drift method.

Forecasting residuals

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Assumptions

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2. $\{e_t\}$ have mean zero. If they don't, then forecasts are biased.

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Useful properties (for distributions & prediction intervals)

3. $\{e_t\}$ have constant variance.
4. $\{e_t\}$ are normally distributed.

ACF of residuals

- We assume that the residuals are white noise (uncorrelated, mean zero, constant variance). If they aren't, then there is information left in the residuals that should be used in computing forecasts.
- So a standard residual diagnostic is to check the ACF of the residuals of a forecasting method.
- We *expect* these to look like white noise.

Ljung-Box tests

r_k = autocorrelation of residual at lag k

Test *whole set* of r_k values simultaneously.

$$Q^* = T(T+2)\sum_{k=1}^{\ell}(T-k)^{-1}r_k^2$$

where ℓ is max lag being considered and T is number of observations.

- My preferences: $\ell = 10$ for non-seasonal data, $h = 2m$ for seasonal data (where m is seasonal period).
- If data are WN, Q^* has χ^2 distribution with ℓ degrees of freedom.

Outline

Forecast distributions

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution $y_{T+h} \mid y_1, \dots, y_T$.
- Most time series models produce normally distributed forecasts.
- The forecast distribution describes the probability of observing any future value.

Forecast distributions

Assuming residuals are normal, uncorrelated, sd = $\hat{\sigma}$:

Mean:	$y_{T+h\|T} \sim N(\bar{y}, (1 + 1/T)\hat{\sigma}^2)$
Naïve:	$y_{T+h\|T} \sim N(y_T, h\hat{\sigma}^2)$
Seasonal naïve:	$y_{T+h\|T} \sim N(y_{T+h-m(k+1)}, (k+1)\hat{\sigma}^2)$
Drift:	$y_{T+h\|T} \sim N(y_T + \frac{h}{T-1}(y_T - y_1), h\frac{T+h}{T}\hat{\sigma}^2)$

where k is the integer part of $(h-1)/m$.

Note that when $h = 1$ and T is large, these all give the same approximate forecast variance: $\hat{\sigma}^2$.

Prediction intervals

- A prediction interval gives a region within which we expect y_{T+h} to lie with a specified probability.
- Assuming forecast errors are normally distributed, then a 95% PI is

$$\hat{y}_{T+h|T} \pm 1.96\hat{\sigma}_h$$

where $\hat{\sigma}_h$ is the st dev of the h-step distribution.

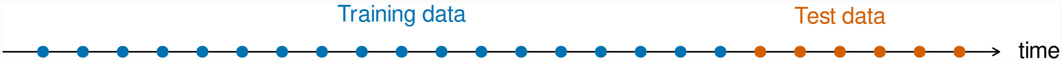
- When $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals.

Prediction intervals

- Point forecasts are often useless without a measure of uncertainty (such as prediction intervals).
- Prediction intervals require a stochastic model (with random errors, etc).
- For most models, prediction intervals get wider as the forecast horizon increases.
- Use `level` argument to control coverage.
- Check residual assumptions before believing them.
- Prediction intervals are usually too narrow due to unaccounted uncertainty.

Outline

Training and test sets



- A model which fits the training data well will not necessarily forecast well.
- A perfect fit can always be obtained by using a model with enough parameters.
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data.
- The test set must not be used for *any* aspect of model development or calculation of forecasts.
- Forecast accuracy is based only on the test set.

Forecast errors

Forecast “error”: the difference between an observed value and its forecast.

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h|T},$$

where the training data is given by $\{y_1, \dots, y_T\}$

- Unlike residuals, forecast errors on the test set involve multi-step forecasts.
- These are *true* forecast errors as the test data is not used in computing $\hat{y}_{T+h|T}$.

Measures of forecast accuracy

$$
\begin{aligned}
y_{T+h} &= (T+h)\text{th observation, } h = 1, \dots, H \\
\hat{y}_{T+h|T} &= \text{its forecast based on data up to time } T. \\
e_{T+h} &= y_{T+h} - \hat{y}_{T+h|T}
\end{aligned}
$$

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\begin{aligned}
\text{MAE} &= \text{mean}(|e_{T+h}|) \\
\text{MSE} &= \text{mean}(e_{T+h}^2) \qquad & \text{RMSE} &= \sqrt{\text{mean}(e_{T+h}^2)} \\
\text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|)
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\text{MAPE} &= 100\text{mean}(|e_{T+h}|/|y_{T+h}|)
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- MAE, MSE, RMSE are all scale dependent.
- MAPE is scale independent but is only sensible if $y_t \gg 0$ for all t, and y has a natural zero.

Scaled Errors

Proposed by Hyndman and Koehler (IJF, 2006).

- For non-seasonal time series, scale errors using naïve forecasts:

$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-1}\sum_{t=2}^{T}|y_t - y_{t-1}|}.$$

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$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-1}\sum_{t=2}^{T}|y_t - y_{t-1}|}.$$

- For seasonal time series, scale forecast errors using seasonal naïve forecasts:

$$q_{T+h} = \frac{e_{T+h}}{\frac{1}{T-m}\sum_{t=m+1}^{T}|y_t - y_{t-m}|}.$$

Scaled errors

Mean Absolute Scaled Error

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Root Mean Squared Scaled Error

$$\text{RMSSE} = \sqrt{\text{mean}(q^2_{T+h})}$$

where

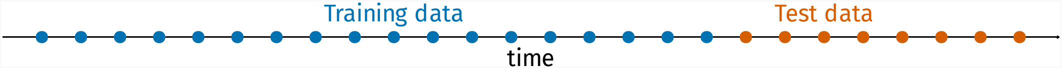
$$q^2_{T+h} = \frac{e^2_{T+h}}{\frac{1}{T-m}\sum_{t=m+1}^{T}(y_t - y_{t-m})^2},$$

and we set $m = 1$ for non-seasonal data.

Outline

Time series cross-validation

Traditional evaluation



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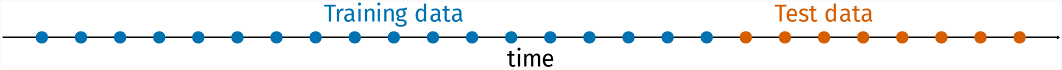

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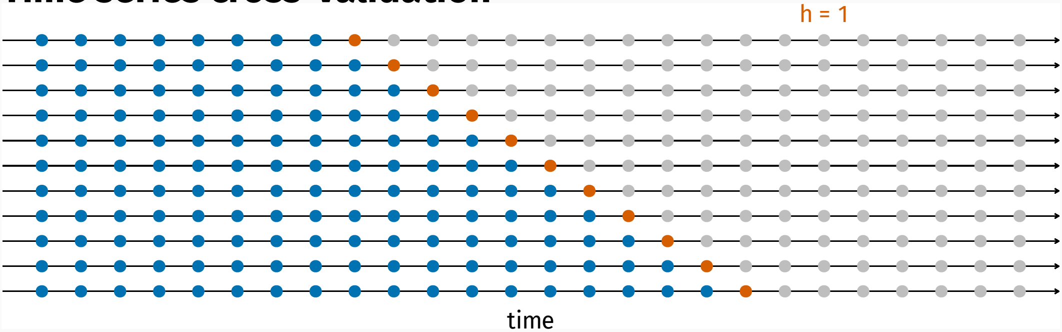

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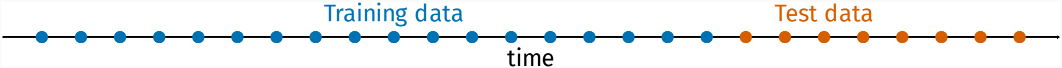

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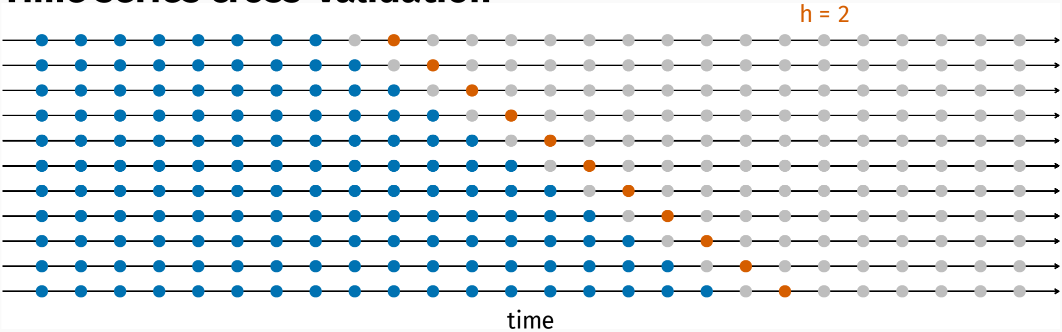

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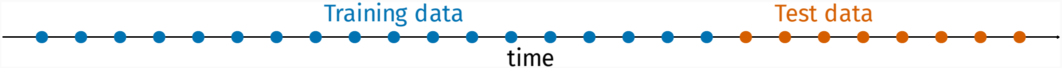

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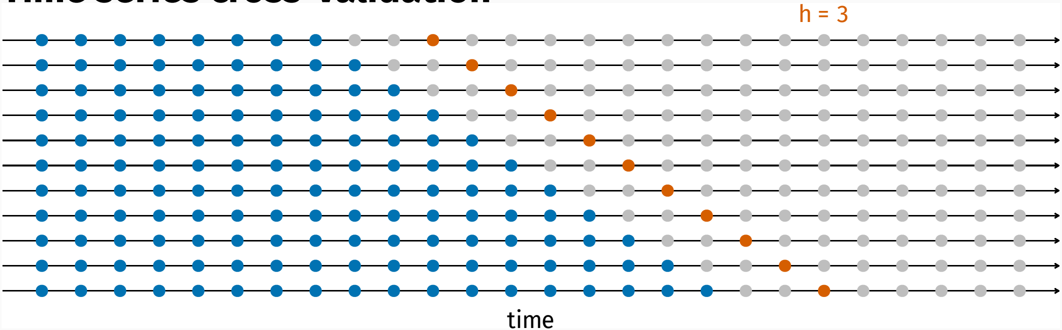

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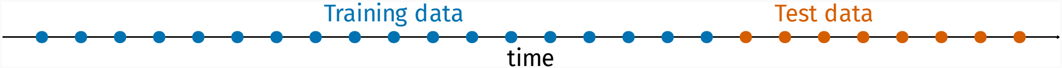

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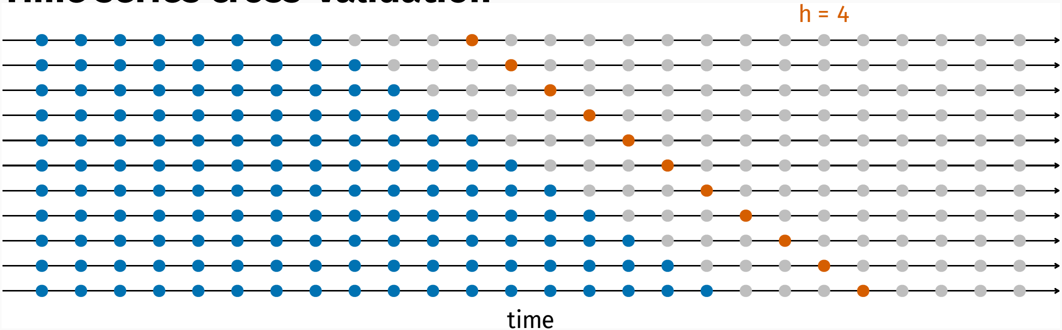

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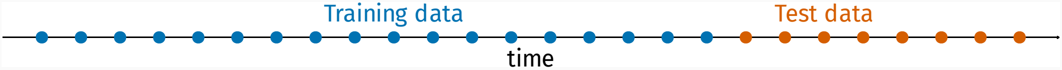

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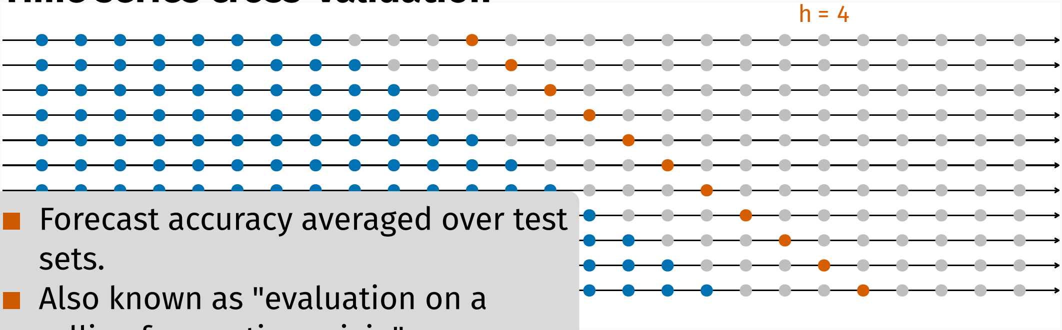

- Forecast accuracy averaged over test sets.
- Also known as "evaluation on a rolling forecasting origin"