

# ETC3550/ETC5550

## Applied forecasting

Week 5: Simple Exponential smoothing



# Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a “level”, “trend” (slope) and “seasonal” component to describe a time series.
- The rate of change of the components are controlled by “smoothing parameters”:  $\alpha$ ,  $\beta$  and  $\gamma$  respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

# Big idea: control the rate of change

$\alpha$  controls the flexibility of the **level**

- If  $\alpha = 0$ , the level never updates (mean)
- If  $\alpha = 1$ , the level updates completely (naive)

$\beta$  controls the flexibility of the **trend**

- If  $\beta = 0$ , the trend is linear
- If  $\beta = 1$ , the trend changes suddenly every observation

$\gamma$  controls the flexibility of the **seasonality**

- If  $\gamma = 0$ , the seasonality is fixed (seasonal means)
- If  $\gamma = 1$ , the seasonality updates completely (seasonal naive)

# Models and methods

## Methods

- Algorithms that return point forecasts.

## Models

- Generate same point forecasts but can also generate forecast distributions.
- A stochastic (or random) data generating process that can generate an entire forecast distribution.
- Allow for “proper” model selection.

# Simple Exponential Smoothing

## Iterative form

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## Component form

Forecast equation  
Smoothing equation

$$\begin{aligned}\hat{y}_{t+h|t} &= \ell_t \\ \ell_t &= \alpha y_t + (1 - \alpha) \ell_{t-1}\end{aligned}$$

# Simple Exponential Smoothing

## Component form

Forecast equation

$$\hat{y}_{t+h|t} = l_t$$

Smoothing equation

$$l_t = \alpha y_t + (1 - \alpha)l_{t-1}$$



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Forecast error:  $e_t = y_t - \hat{y}_{t|t-1} = y_t - l_{t-1}$ .

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## Error correction form

$$y_t = l_{t-1} + e_t$$

$$l_t = l_{t-1} + \alpha(y_t - l_{t-1})$$

$$= l_{t-1} + \alpha e_t$$

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Specify probability distribution:  $e_t = \varepsilon_t \sim \text{NID}(0, \sigma^2)$ .

# ETS(A,N,N): SES with additive errors

## ETS(A,N,N) model

Observation equation

$$y_t = l_{t-1} + \varepsilon_t$$

State equation

$$l_t = l_{t-1} + \alpha \varepsilon_t$$

where  $\varepsilon_t \sim \text{NID}(0, \sigma^2)$ .

- “innovations” or “single source of error” because equations have the same error process,  $\varepsilon_t$ .
- Observation equation: relationship between observations and states.
- State equation(s): evolution of the state(s) through time.

# ETS(M,N,N): SES with multiplicative errors.

- Specify relative errors  $\varepsilon_t = \frac{y_t - \hat{y}_{t|t-1}}{\hat{y}_{t|t-1}} \sim \text{NID}(0, \sigma^2)$
- Substituting  $\hat{y}_{t|t-1} = l_{t-1}$  gives:
  - ▶  $y_t = l_{t-1} + l_{t-1}\varepsilon_t$
  - ▶  $e_t = y_t - \hat{y}_{t|t-1} = l_{t-1}\varepsilon_t$

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## ETS(M,N,N) model

Observation equation

$$y_t = l_{t-1}(1 + \varepsilon_t)$$

State equation

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- Models with additive and multiplicative errors with the same parameters generate the same point forecasts but different prediction intervals.