



MONASH
University

MONASH
BUSINESS
SCHOOL

ETC3550/ETC5550

Applied forecasting

Week 4: Simple forecasting methods



Outline

- 1 Four benchmark methods
- 2 Time trends and dummy seasonality
- 3 Forecasting with transformations
- 4 Forecasting with decompositions

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Mean method

MEAN(y)

- Forecast of all future values is equal to mean of historical data $\{y_1, \dots, y_T\}$.
- Forecasts: $\hat{y}_{T+h|T} = \bar{y} = (y_1 + \dots + y_T)/T$
- Implicit model: $y_t = c + \varepsilon_t$ where $\varepsilon_t \sim WN$.

Mean method



Naïve method

NAIVE(y)

- Forecasts equal to last observed value.
- Forecasts: $\hat{y}_{T+h|T} = y_T$.
- Implicit model is a random walk: $y_t = y_{t-1} + \varepsilon_t$
- Consequence of efficient market hypothesis.

Naïve method



Seasonal Naïve method

SNAIVE($y \sim \text{lag}(m)$)

- Forecasts equal to last value from same season.
- Forecasts: $\hat{y}_{T+h|T} = y_{T+h-m(k+1)}$, where m = seasonal period and k is the integer part of $(h-1)/m$.
- Implicit model: $y_t = y_{t-m} + \varepsilon_t$

Seasonal Naïve method



Drift method

RW(y ~ drift())

- Forecasts equal to last value plus average change:

$$\begin{aligned}\hat{y}_{T+h|T} &= y_T + \frac{h}{T-1} \sum_{t=2}^T (y_t - y_{t-1}) \\ &= y_T + \frac{h}{T-1} (y_T - y_1).\end{aligned}$$

- Equivalent to extrapolating a line drawn between first and last observations.
- Implicit model: Random walk with drift: $y_t = c + y_{t-1} + \varepsilon_t$

Drift method



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Linear time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \varepsilon_t$$

TSLM($y \sim \text{trend}()$)

- $x_{1,t} = t, \quad t = 1, \dots, T$
- Forecasts: $\hat{y}_{T+h|T} = \beta_0 + \beta_1(T + h)$

Piecewise linear time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \cdots + \beta_k x_{k,t} + \varepsilon_t$$

TSLM($y \sim \text{trend}(\text{knots} = c(\text{tau1}, \text{tau2}, \dots, \text{taup}))$)

- Bends at τ_1, \dots, τ_p
- $x_{1,t} = t$
- $x_{i,t} = (t - \tau_i)_+ = \begin{cases} 0 & t < \tau_i \\ (t - \tau_i) & t \geq \tau_i \end{cases}$
- Forecasts: $\beta_0 + \beta_1(T+h) + \beta_2(T+h - \tau_1)_+ + \cdots + \beta_{p+1}(T+h - \tau_p)_+$

Quadratic time trends

$$y_t = \beta_0 + \beta_1 x_{1,t} + \beta_2 x_{2,t} + \varepsilon_t$$

- $x_{1,t} = t$
- $x_{2,t} = t^2$
- Forecasts: $\beta_0 + \beta_1(T + h) + \beta_2(T + h)^2$

NOT RECOMMENDED!

Daily dummy variables

TSLM($y \sim \text{season}()$)

- Using one dummy for each category gives too many dummy variables!
- The coefficients of the dummies are relative to the omitted category
- `season()` automatically generates the dummy variables for you.

| Day | d1 | d2 | d3 | d4 |
|-----------|----|----|----|----|
| Monday | 1 | 0 | 0 | 0 |
| Tuesday | 0 | 1 | 0 | 0 |
| Wednesday | 0 | 0 | 1 | 0 |
| Thursday | 0 | 0 | 0 | 1 |
| Friday | 0 | 0 | 0 | 0 |
| Monday | 1 | 0 | 0 | 0 |
| Tuesday | 0 | 1 | 0 | 0 |
| Wednesday | 0 | 0 | 1 | 0 |
| Thursday | 0 | 0 | 0 | 1 |
| Friday | 0 | 0 | 0 | 0 |

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- Back-transformed point forecasts are medians.
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Let X have mean μ and variance σ^2 .

Let $f(x)$ be back-transformation function, and $Y = f(X)$.

Taylor series expansion about μ :

$$Y = f(X) \approx f(\mu) + (X - \mu)f'(\mu) + \frac{1}{2}(X - \mu)^2f''(\mu).$$

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$$E[Y] = E[f(X)] \approx f(\mu) + \frac{1}{2}\sigma^2f''(\mu)$$

Bias adjustment

Box-Cox back-transformation:

$$y_t = \begin{cases} \exp(w_t) & \lambda = 0; \\ (\lambda W_t + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f(x) = \begin{cases} e^x & \lambda = 0; \\ (\lambda x + 1)^{1/\lambda} & \lambda \neq 0. \end{cases}$$

$$f''(x) = \begin{cases} e^x & \lambda = 0; \\ (1 - \lambda)(\lambda x + 1)^{1/\lambda - 2} & \lambda \neq 0. \end{cases}$$

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$$E[Y] \approx \begin{cases} e^\mu \left[1 + \frac{\sigma^2}{2} \right] & \lambda = 0; \\ (\lambda \mu + 1)^{1/\lambda} \left[1 + \frac{\sigma^2(1-\lambda)}{2(\lambda \mu + 1)^2} \right] & \lambda \neq 0. \end{cases}$$

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Forecasting with decompositions

$$y_t = \hat{S}_t + \hat{A}_t$$

- \hat{A}_t is seasonally adjusted component
 - \hat{S}_t is seasonal component.
-
- Forecast \hat{S}_t using a Seasonal Naïve method.
 - Forecast \hat{A}_t using a non-seasonal time series method.
 - Combine forecasts of \hat{S}_t and \hat{A}_t to get forecasts of original data.

Decomposition models

`decomposition_model()` creates a decomposition model

- You must provide a method for forecasting the `season_adjust` series.
- A seasonal naïve method is used by default for the `seasonal` components.
- The variances from both the seasonally adjusted and seasonal forecasts are combined.