

# **ETC3550/ETC5550**

## **Applied forecasting**

Week 7: Exponential smoothing



# Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a “level”, “trend” (slope) and “seasonal” component to describe a time series.
- The rate of change of the components are controlled by “smoothing parameters”:  $\alpha$ ,  $\beta$  and  $\gamma$  respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

# ETS models

**General notation**    E T S : ExponenTial Smoothing



Error   **T**rend   **S**eason

**Error:** Additive ("A") or multiplicative ("M")

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Diagram illustrating the components of the ETS notation: Error, Trend, and Season.

Error   **T**rend   **S**eason

**Error:** Additive ("A") or multiplicative ("M")

**Trend:** None ("N"), additive ("A"), multiplicative ("M"), or damped ("Ad" or "Md").

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Diagram illustrating the components of the ETS notation: Error, Trend, and Season.

Error   Trend   Season

**Error:** Additive ("A") or multiplicative ("M")

**Trend:** None ("N"), additive ("A"), multiplicative ("M"), or damped ("Ad" or "Md").

**Seasonality:** None ("N"), additive ("A") or multiplicative ("M")

# ETS models

## Additive Error

Trend Component		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
N	(None)	A,N,N	A,N,A	A,N,M
A	(Additive)	A,A,N	A,A,A	A,A,M
A <sub>d</sub>	(Additive damped)	A,A <sub>d</sub> ,N	A,A <sub>d</sub> ,A	A,A <sub>d</sub> ,M

## Multiplicative Error

Trend Component		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
N	(None)	M,N,N	M,N,A	M,N,M
A	(Additive)	M,A,N	M,A,A	M,A,M
A <sub>d</sub>	(Additive damped)	M,A <sub>d</sub> ,N	M,A <sub>d</sub> ,A	M,A <sub>d</sub> ,M

# ETS models

## Additive Error

		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
Trend Component	N (None)	A,N,N	A,N,A	<del>A,N,M</del>
	A (Additive)	A,A,N	A,A,A	<del>A,A,M</del>
	A <sub>d</sub> (Additive damped)	A,A <sub>d</sub> ,N	A,A <sub>d</sub> ,A	<del>A,A<sub>d</sub>,M</del>

## Multiplicative Error

		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
Trend Component	N (None)	M,N,N	M,N,A	M,N,M
	A (Additive)	M,A,N	M,A,A	M,A,M
	A <sub>d</sub> (Additive damped)	M,A <sub>d</sub> ,N	M,A <sub>d</sub> ,A	M,A <sub>d</sub> ,M

## AIC and cross-validation

Minimizing the AIC assuming Gaussian residuals is asymptotically equivalent to minimizing one-step time series cross validation MSE.



# Automatic forecasting

## From Hyndman et al. (IJF, 2002):

- Apply each model that is appropriate to the data. Optimize parameters and initial values using MLE (or some other criterion).
- Select best method using AICc:
- Produce forecasts using best method.
- Obtain forecast intervals using underlying state space model.

Method performed very well in M3 competition.

# Residuals

## Response residuals

$$\hat{e}_t = y_t - \hat{y}_{t|t-1}$$

## Innovation residuals

Additive error model:

$$\hat{\varepsilon}_t = y_t - \hat{y}_{t|t-1}$$

Multiplicative error model:

$$\hat{\varepsilon}_t = \frac{y_t - \hat{y}_{t|t-1}}{\hat{y}_{t|t-1}}$$

## Your turn

- 1 Try forecasting the Chinese GDP from the `global_economy` data set using an ETS model.

Experiment with the various options in the `ETS()` function to see how much the forecasts change with damped trend, or with a Box-Cox transformation. Try to develop an intuition of what each is doing to the forecasts.

[Hint: use `h=20` when forecasting, so you can clearly see the differences between the various options when plotting the forecasts.]

# Your turn

2

Find an ETS model for the Gas data from `aus_production` and forecast the next few years.

- ▶ Why is multiplicative seasonality necessary here?
- ▶ Experiment with making the trend damped. Does it improve the forecasts?