

# ETC3550/ETC5550

## Applied forecasting

Week 7: Exponential smoothing



# Historical perspective

- Developed in the 1950s and 1960s as methods (algorithms) to produce point forecasts.
- Combine a “level”, “trend” (slope) and “seasonal” component to describe a time series.
- The rate of change of the components are controlled by “smoothing parameters”:  $\alpha$ ,  $\beta$  and  $\gamma$  respectively.
- Need to choose best values for the smoothing parameters (and initial states).
- Equivalent ETS state space models developed in the 1990s and 2000s.

# ETS models

**General notation**    E T S : ExponenTial S moothing



Error   T r e n d   S e a s o n

**Error:** Additive ("A") or multiplicative ("M")

# ETS models

**General notation**     E T S : ExponenTial Smoothing



Diagram illustrating the components of the ETS notation: Error, Trend, and Season.

Error   Trend   Season

**Error:** Additive ("A") or multiplicative ("M")

**Trend:** None ("N"), additive ("A"), multiplicative ("M"), or damped ("Ad" or "Md").

# ETS models

**General notation**     E T S : ExponenTial Smoothing



The diagram consists of three arrows pointing upwards from the words 'Error', 'Trend', and 'Season' to the letters 'E', 'T', and 'S' respectively in the 'ETS' part of the notation above.

Error Trend Season

**Error:** Additive ("A") or multiplicative ("M")

**Trend:** None ("N"), additive ("A"), multiplicative ("M"), or damped ("Ad" or "Md").

**Seasonality:** None ("N"), additive ("A") or multiplicative ("M")

# ETS models

## Additive Error

		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
	Trend Component			
	N (None)	A,N,N	A,N,A	A,N,M
	A (Additive)	A,A,N	A,A,A	A,A,M
	A <sub>d</sub> (Additive damped)	A,A <sub>d</sub> ,N	A,A <sub>d</sub> ,A	A,A <sub>d</sub> ,M

## Multiplicative Error

		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
	Trend Component			
	N (None)	M,N,N	M,N,A	M,N,M
	A (Additive)	M,A,N	M,A,A	M,A,M
	A <sub>d</sub> (Additive damped)	M,A <sub>d</sub> ,N	M,A <sub>d</sub> ,A	M,A <sub>d</sub> ,M

# ETS models

## Additive Error

Trend Component		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
N	(None)	A,N,N	A,N,A	<del>A,N,M</del>
A	(Additive)	A,A,N	A,A,A	<del>A,A,M</del>
A <sub>d</sub>	(Additive damped)	A,A <sub>d</sub> ,N	A,A <sub>d</sub> ,A	<del>A,A<sub>d</sub>,M</del>

## Multiplicative Error

Trend Component		Seasonal Component		
		N (None)	A (Additive)	M (Multiplicative)
N	(None)	M,N,N	M,N,A	M,N,M
A	(Additive)	M,A,N	M,A,A	M,A,M
A <sub>d</sub>	(Additive damped)	M,A <sub>d</sub> ,N	M,A <sub>d</sub> ,A	M,A <sub>d</sub> ,M

## AIC and cross-validation

Minimizing the AIC assuming Gaussian residuals is asymptotically equivalent to minimizing one-step time series cross validation MSE.



# Automatic forecasting

## **From Hyndman et al. (IJF, 2002):**

- Apply each model that is appropriate to the data. Optimize parameters and initial values using MLE (or some other criterion).
- Select best method using AICc:
- Produce forecasts using best method.
- Obtain forecast intervals using underlying state space model.

Method performed very well in M3 competition.

# Residuals

## Response residuals

$$\hat{e}_t = y_t - \hat{y}_{t|t-1}$$

## Innovation residuals

Additive error model:

$$\hat{\varepsilon}_t = y_t - \hat{y}_{t|t-1}$$

Multiplicative error model:

$$\hat{\varepsilon}_t = \frac{y_t - \hat{y}_{t|t-1}}{\hat{y}_{t|t-1}}$$