129

Bayesian analysis of the historical functional linear model with application to air pollution forecasting

Historical functional linear models are used to analyse the relationship between a functional response and a functional predictor whereby only the past or recent past of the predictor process can affect the current outcome. In this work, we develop a Bayesian framework for the analysis of the historical functional linear model with multiple predictors.

Different from existing Bayesian approaches to historical functional linear models, our proposed methodology is able to handle multiple functional covariates with noise, measurement error and sparseness. The proposed model utilises the well-established connection between non-parametric smoothing and Bayesian methods to reduce sensitivity to the number of basis functions which are used to model the functional regression coefficients. We implement an efficient MCMC algorithm that uses a Choleski decomposition to sample from high-dimensional Gaussian distributions and takes advantage of the orthogonal properties of the functional principal components used to model the functional covariates. Our extensive simulation study shows substantial improvements in both the recovery of the functional regression surface and the true underlying functional response with higher coverage probabilities when compared to a classical model under which the measurement error is unaccounted for.

A major challenge with the collection of environmental data is that they are prone to measurement error, both random and systematic. Hence, our methodology provides a reliable functional data analytic framework for modelling environmental data. A study conducted in Cape Town, South Africa, has shown that there exists significant daily respiratory, cardiovascular and cerebrovascular mortality risk linked to ambient air pollution exposure, which is higher than reported in developed countries. Consequently, our focus is on the application of our method to forecast the level of daily atmospheric pollutants using meteorological information such as hourly records of temperature, humidity and wind speed from data collected by the City of Cape Town, South Africa. The forecasts provided by the proposed method are highly competitive against the functional autoregressive model with exogenous variables of order 1 which is traditionally used for functional time series.