

## Extended Sampled Posterior P-value as a tool for Joint-Species Distribution Models checking

Ecologists use, among other things, Joint-Species Distribution Models (JSDMs) - which are types of Hierarchical Bayesian Models (which are types of Bayesian statistical parametric models defined in function of hyperparameters and unobservable parameters (latent variables)) - are used to model biodiversity hierarchically, i.e. at the level of species, ecological groups and communities of species. This modelling method makes it possible to share environmental information between species while taking into account the correlations between species. However, JSDMs make many mathematical assumptions due to their complex structure and the numerical limitations required to fit them. So doing appropriate Model Checking (which consists of testing the hypotheses made about a statistical parametric model) for these types of models is necessary to obtain greater confidence in the statistical results obtained from the predictions of these models.

Extended Internal Goodness-Of-Fit (GOF) p-values - approaches that mix frequentist principles with Bayesian inference, and are based on a method of using the same data used to estimate the model to critique the same model - are an important class of approaches for Hierarchical Bayesian Model checking. Extended Internal GOF p-values are used to quantify the degree of surprise in observed data given the specified data model and prior distribution and which are able to check the hypotheses made on the distributions of latent variables of Hierarchical Bayesian Models. In order to assess the quality of JSDMs it is important to use the Extended Internal GOF p-values to assess the goodness of fit between the observed data and the JSDM.

We show mathematically that a new Extended Internal GOF p-value, called the "extended sampled posterior p-value" - a type of External Internal GOF p-values, where observed data are compared with multiple replicated data sets where all replicated data are generated from a single parameter drawn from the posterior distribution -, has desirable mathematical properties that make it relevant in very different contexts and has a higher power (to detect lack of fit of the model to the data) than more Extended Internal GOF p-values which are known to be conservative (producing p-values biased toward 0.5) or numerically expensive.