Assignment 5 - TDT4136

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November 01, 2022

1 Models and entailment in propositional logic

1.1 Modelling

a)

A	B	$\neg A$	$\neg B$	$\neg A \land \neg B$	$\neg A \land \neg B \Rightarrow \neg B$
false	false	true	true	true	true
false	true	true	false	false	true
true	false	false	true	false	true
true	true	false	false	false	true

The statement is *true*

b)

A	B	$\neg A$	$\neg B$	$\neg A \lor \neg B$	$\neg A \lor \neg B \Rightarrow \neg B$
false	false	true	true	true	true
false	true	true	false	true	false
true	false	false	true	true	true
true	true	false	false	false	true

The statement is $false$

c)

A	B	$\neg A$	$\neg A \land B$	A $\lor B$	$\neg A \land B \Rightarrow A \lor B$
false	false	true	false	false	true
false	true	true	true	true	true
true	false	false	false	true	true
true	true	false	false	true	true

The statement is *true*

d)

A	B	A $\Rightarrow B$	A $\iff B$	(A $\Rightarrow B) \Rightarrow (A \iff B)$
false	false	true	true	true
false	true	true	false	false
true	false	false	false	true
true	true	true	true	true

The statement is $false$

e)

A	B	C	$\neg B$	(A $\Rightarrow B) \iff C$	A $\vee \neg B \vee C$	$\models$
true	true	true	false	true	true	true
true	true	false	false	false	true	true
true	false	true	true	false	true	true
true	false	false	true	true	true	true
false	true	true	false	true	true	true
false	true	false	false	false	false	true
false	false	true	true	true	true	true
false	false	false	true	false	true	true

The statement is *true*

f)

A	B	$\neg A$	$\neg B$	$\neg A \Rightarrow \neg B$	A $\wedge \neg B$	$(\neg A \Rightarrow \neg B) \wedge (A \wedge \neg B)$
false	false	true	true	true	false	false
false	true	true	false	false	false	false
true	false	false	true	true	true	true
true	true	false	false	true	false	false

Satisfiable: *true*

g)

A	B	$\neg A$	$\neg B$	$\neg A \iff \neg B$	A $\wedge \neg B$	$(\neg A \iff \neg B) \wedge (A \wedge \neg B)$
false	false	true	true	true	false	false
false	true	true	false	false	false	false
true	false	false	true	false	true	false
true	true	false	false	true	false	false

Satisfiable: $false$

1.2 Trouble in the lab

a)

- S_1 = Is tank occupied
- S_2 = Is tank toxicity level high
- S_3 = Is tank electrical charge high

b)

- $C_1 = \neg S_2 \wedge \neg S_3$
- $C_2 = \neg S_1 \wedge S_2$
- $C_3 = S_3$ (Assumes that the electrical charge level is "dangerous" if it's "high")

c)

S_1	S_2	S_3	C_1	C_2	C_3	$C_1 \vee C_2 \vee C_3$
true	true	true	false	false	true	true
true	true	false	false	false	false	false
true	false	true	false	false	true	true
true	false	false	true	false	false	true
false	true	true	false	true	true	true
false	true	false	false	true	false	true
false	false	true	false	false	true	true
false	false	false	true	false	false	true

d) `11001010`: Occupied tank with high toxicity level and low electrical charge. Binary id of `01010` (10). Tank gate should be open.

`01110110`: Unoccupied tank with high toxicity level and high electrical charge. Binary id of `10110` (22). Tank gate should be closed as both C_2 and C_3 is true.

2 Resolution in propositional logic

2.1 Conjunctive Normal Form

a) $A \vee (B \wedge C \wedge \neg D)$ is already in CNF

b) $\neg(\mathrm{A} \Rightarrow \neg B) \wedge \neg(C \Rightarrow \neg D)$ becomes $A \wedge B \wedge C \wedge D$

c) $\neg((\mathrm{A} \Rightarrow B) \wedge (C \Rightarrow \neg D))$ becomes $(A \wedge \neg B) \vee (C \wedge D)$

d) $(A \wedge B) \vee (C \Rightarrow D)$ becomes $(A \wedge B) \vee \neg C \vee D$

e) $A \iff (B \Rightarrow \neg C)$ becomes $A \iff (\neg B \vee \neg C)$ becomes $(A \Rightarrow (\neg B \vee \neg C)) \wedge ((\neg B \vee \neg C) \Rightarrow A)$ becomes $(\neg A \vee \neg B \vee \neg C) \wedge ((B \wedge C) \vee A)$

2.2 Inference in propositional logic

S_W: is true if it's warm	A_E: is true if I enjoy
S_R: is true if it's raining	A_B: is true if I pick up berries
S_S: is true if it's sunny	A_W: is true if I will get wet

R_1: $(S_W \wedge S_S) \Rightarrow A_E$. CNF: $\neg S_W \vee \neg S_S \vee A_E$ (If the weather is both Sunny and Warm, then I Enjoy)

R_2: $(S_W \wedge \neg S_R) \Rightarrow A_B$. CNF: $\neg S_W \vee S_R \vee A_B$ (If the weather is both Warm and Nice (not Raining), then I pick up Berries.)

R_3: $S_R \Rightarrow \neg A_B$. CNF: $\neg S_R \vee \neg A_B$ (If it is Raining, then I won't pick up Berries.)

R_4: $S_R \Rightarrow A_W$. CNF: $\neg S_R \vee A_W$ (If it is Raining, then I will get wet.)

R_5: S_W

R_6: S_R

R_7: S_S

The literal S_W in R_5 and S_S in R_7 resolves with the literal $\neg S_W$ and $\neg S_S$ in R_1 to give the resolvent:

R_8: A_E ($Q_2 \Rightarrow$ proved)

The literal S_R in R_6 resolves with the literal $\neg S_R$ in R_3 and $\neg S_R$ in R_4 to give the resolvents:

R_9: A_B ($Q_1 \Rightarrow$ proved)

R_{10}: A_W ($Q_3 \Rightarrow$ proved)

3 Representation in First-Order Logic (FOL)

3.1 Predicates

a) Emily is either a surgeon or a lawyer: $Occupation(Emily, Surgeon) \vee Occupation(Emily, Lawyer)$

b) Joe is an actor, but he also holds another job: $\exists x\ Occupation(Joe, Actor) \wedge Occupation(Joe, x)$

c) All surgeons are doctors: $\forall x\ Occupation(x, Surgeon) \Rightarrow Occupation(x, Doctor)$

d) Joe does not have a lawyer: $\nexists x\ Customer(Joe, x) \wedge Occupation(x, Lawyer)$

e) Emily has a boss who is a lawyer: $\exists x\ Boss(x, Emily) \wedge Occupation(x, Lawyer)$

f) There exists a lawyer all of whose customers are doctors: $\exists x\ Occupation(x, Lawyer) \wedge (\forall y\ \ Customer(y, x) \Rightarrow Occupation(y, Doctor))$

g) Every surgeon has a lawyer: $\forall x\ Occupation(x, Surgeon) \Rightarrow (\exists y\ \ Customer(x, y) \wedge Occupation(y, Lawyer))$

3.2 Functions as predicates

a) Divisible(x, y): $\exists z\ \ z < x \wedge x = z \times y$

b) Even(x): $Divisible(x, 2)$

c) Odd(x): $\neg Divisible(x, 2)$

d) Odd(x): $Even(x + 1)$

e) Prime(x): $Divisible(x, x) \wedge (\nexists y\ \ y < x \wedge Divisible(x, y))$

f) $\exists ! x\ \ Prime(x)$

g) Every integer number is equal to a product of prime numbers

4 Resolution in FOL

a)
- $R_1 : \forall x\ \ (\neg GG(x) \wedge \neg Sone(x)) \vee (GG(x) \wedge Sone(x))$
- $R_2 : \forall x\ \ (\neg RV(x) \wedge \neg Reveluv(x)) \vee (RV(x) \wedge Reveluv(x))$
- $R_3 : \forall x\ \ (\neg BP(x) \wedge \neg Blink(x)) \vee (BP(x) \wedge Blink(x))$
- $R_4 : \forall x\ \ \neg Reveluv(x) \vee Ballads(x)$
- $R_5 : \forall x\ \ \neg Blink(x) \vee Dance(x)$
- $R_6 : \forall x\ \ \neg Dance(x) \vee \neg Ballads(x) \vee CH(x)$
- $R_7 : \forall x\ \ \neg Drama(x) \vee \neg Ballads(x) \vee HE(x)$
- $R_8 : \forall x\ \ \neg Sone(x) \vee \neg Electro(x) \vee DJH(x)$
- $R_9 : \forall x\ \ \neg Sone(x) \vee \neg Drama(x) \vee SEO(x)$
- $R_{10} : \forall x\ \ \neg Sone(x) \vee \neg Ballads(x) \vee TAE(x)$

b) *If a new user u_1 is a fan of GG and identifies as Reveluv, then TAE will be a good recommendation*

- $Q_1: \ (GG(u_1) \wedge Reveluv(u_1)) \Rightarrow TAE(u_1)$.
- $R_2, R_3, R_5, R_6, R_7, R_8$ *and* R_9 are not relevant because all of them resolves to true and therefore not a part of the KB for this resolution.
- We use resolution to check $KB \wedge \neg TAE(u_1)$ which equals $R_4 \wedge R_{10} \wedge \neg TAE(u_1)$
- The literal $GG(u_1)$ in Q_1 resolves with the literal $GG(u_1)$ in R_1 to give the resolvent: $Sone(u_1)$
- The literal $Reveluv(u_1)$ in Q_1 resolves with the literal $Reveluv(u_1)$ in R_4 to give the resolvent: $Ballads(u_1)$ which is true
- R_{10} resolves to $TAE(u_1)$ with $Sone(u_1)$ and $Ballads(u_1)$
- The final clause is then $TAE(u_1) \wedge \neg TAE(u_1)$. This is a contradiction and we've therefore proved that TAE will be a good recommendation.

c) *If a new user u_1 is a fan of GG and identifies as Reveluv, then HE will be a good recommendation*

- $Q_2: \ (GG(u_1) \wedge Reveluv(u_1)) \Rightarrow HE(u_1)$.
- $R_1, R_2, R_3, R_5, R_6, R_8, R_9$ *and* R_{10} are not relevant because all of them resolves to true and therefore not a part of the KB for this resolution.
- We use resolution to check $KB \wedge \neg HE(u_1)$ which equals $R_4 \wedge R_7 \wedge \neg HE(u_1)$
- The literal $Reveluv(u_1)$ in Q_1 resolves with the literal $Reveluv(u_1)$ in R_4 to give the resolvent: $Ballads(u_1)$ which is true
- The final clause is $(\neg Drama(u_1) \vee HE(u_1)) \wedge \neg HE(u_1)$. This is not a contradiction since $Drama(u_1)$ is false and the statement therefore can be satisfied. Since there is not contradiction, we've disproved that HE will be a good recommendation.

d) *Given what you know, if another user u_2 claims to be a Sone, a Reveluv, a Blink, and likes Drama; what are the possible artists and genre recommendations the system will provide?*

- $Sone(u_2)$ resolves with R_1 which gives $GG(u_2)$
- $Reveluv(u_2)$ resolves with R_2 which gives $RV(u_2)$
- $Blink(u_2)$ resolves with R_3 which gives $BP(u_2)$
- $Reveluv(u_2)$ resolves with R_4 which gives $Ballads(u_2)$
- $Blink(u_2)$ resolves with R_5 which gives $Dance(u_2)$
- $Ballads(u_2) \wedge Dance(u_2)$ resolves with R_6 which gives $CH(u_2)$
- $Ballads(u_2) \wedge Drama(u_2)$ resolves with R_7 which gives $HE(u_2)$
- $Sone(u_2) \wedge Ballads(u_2)$ resolves with R_9 which gives $SEO(u_2)$
- $Sone(u_2) \wedge Ballads(u_2)$ resolves with R_{10} which gives $TAE(u_2)$
- The system will recommend the artists GG, RV, BP, CH, HE, SEO and TAE.
- The system will recommend the genres Ballads and Dance.