Lecture 12: Multiagent-Systems and Game Theory

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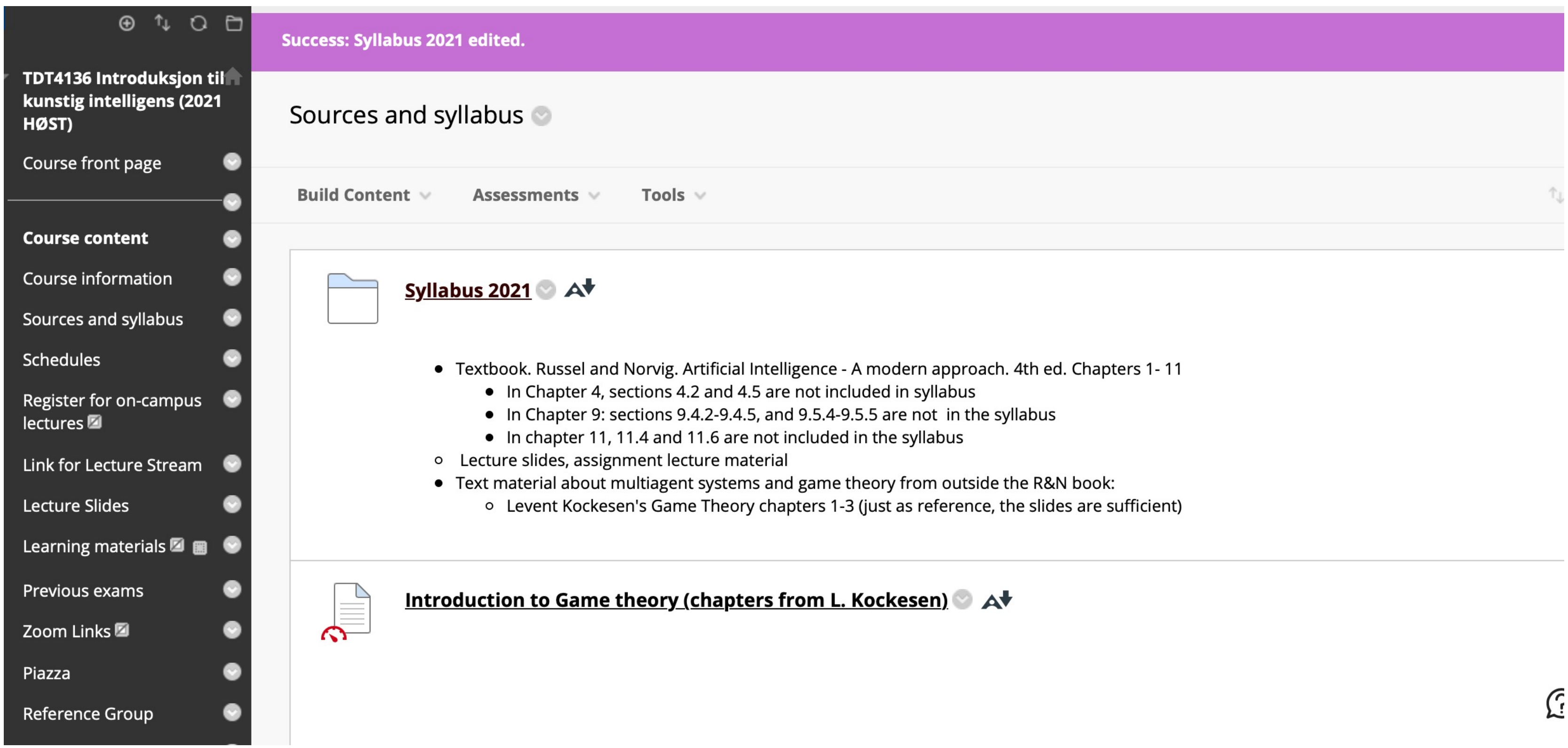
Outline

- Micro vs macro aspects
- Strategical thinking
- Strategical normal games
- Game theory solution concepts
  - Dominant (strongly and weakly) strategy equilibrium
  - Iterated elimination of dominated strategies
  - Nash equilibrium
- Pareto optimal and Social Welfare properties

Reading material



Agent Design (micro) vs. Society Design (macro)

Micro-questions:

- How do we build agents that are capable of independent, autonomous actions in order to successfully carry out the tasks that we delegate to them?

Macro-questions:

- Types and characteristics of interactions between agents, interaction protocols
- Behaviour in societies of self-interested agents?
  - Cooperative
  - Competitive

Game theory

- Studies the decisions of agents in a multiagent environment where
  – Actions of each agent have an effect in the environment
  – So, outcome of an action of one agent may depend on other agents' actions
  – Hence, "strategical decision" making

Strategical Reasoning

- An agent’s decision depends on what it reasons/thinks the other agent(s) will do.
  - The agent is **uncertain** about the decisions (i.e, actions) of the other participating agents
  - Tries to predict other’s next decisions
  - Computes how these affect him/her/them
  - Makes own decisions accordingly

Let us play

- Pretend we are playing a game. Here's how it works. You must choose a number, x between [0, 100].

- Assume we would then calculate the average of all the numbers. Then the person whose number is closest to 2/3 of the average will win.

- Assume that eveybody is rational.

- You choose x = ??

Different types of games in Classical Game Theory

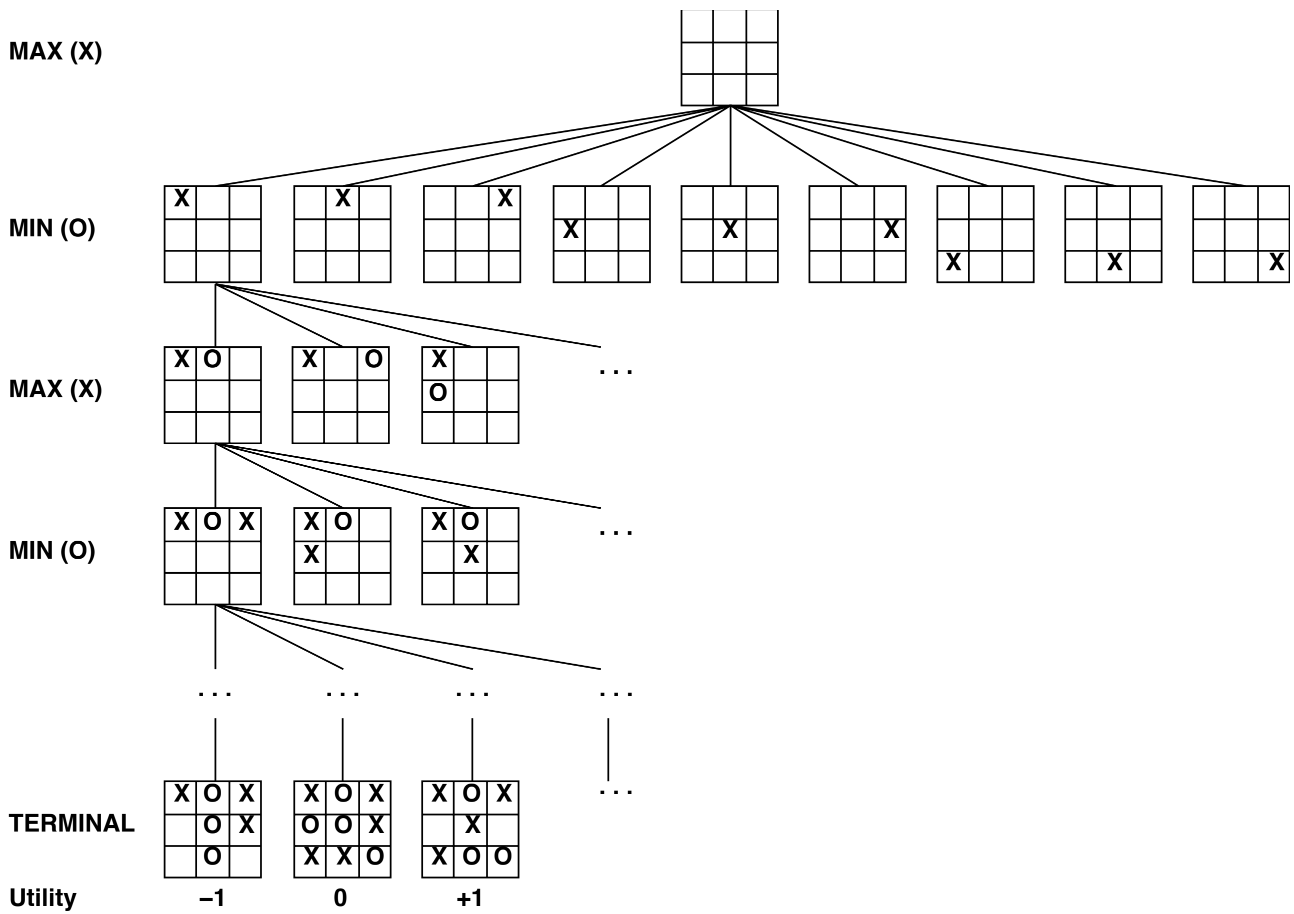
- Sequential games – chapter 6 in R&N
- Repeated games – very interesting
- Strategic/simultaneous normal form games - today

Sequential (extensive form) games

- Turn taking
- At each point the agents may decide/change their strategy
- Representation: tree
- Example: tik tak toe, chess, go

Example : tic-tac-toe

Sequential games, represented as a tree



Strategic normal form games

- One-shot
- The agent chooses its strategy only once at the beginning of the game, and all agents take their actions simultaneously.
- Main issue: predict what the other agent(s) will play.
- Represented: payoff matrix (will see soon)
- Example: Prisoner’s dilemma, battle of sexes, stag hunt

Example 1: Prisoner's Dilemma

- Two men are collectively charged with a crime and held in separate cells.
  - If one confesses and the other does not, the confessor will be freed, and the other will be jailed for three years.
  - If both confesses, then each will be jailed for two years.
  - If neither confesses, then they will each be jailed for one year.

- They are both rational and both know the other is rational
- No communication - no way of agreement.
- What kind of behavior would you expect them to display?


THE PRISONER'S DILEMMA
THE CRASH IS PSYCHOLOGICAL. IF EVERYONE MAKES A LEAP OF FAITH AND STARTS SPENDING AGAIN, WE'LL BE FINE!
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SOMEONE HAS TO SPEND FIRST. I'M GOING TO LEAD BY EXAMPLE!
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SURE IS A LOT OF LITTER OUT HERE


Example 2: Stag Hunt



- Describes a conflict between safety and social cooperation, “trust dilemma”
- Story is due to Jean-Jacques Rousseau: two individuals go out on a hunt. Each can individually choose to hunt a stag or hunt a hare.
- Each player must choose an action without knowing the choice of the other.
- If an individual hunts a stag, they must have the cooperation of their partner in order to succeed. An individual can get a hare by themselves, but a hare is worth much less than a stag.

Example 3: Battle of Sexes

- Jonathan and Sofie are married and they are in their offices on a Friday evening trying to figure out what they should do after work. They cannot get in touch with each other but would like to meet and spend the evening going to a **movie or an opera**.

- Jonathan likes movies better while Sofie would rather go to an opera.

- However, being in love, the most important thing for them is to do something together; both view the night "wasted" unless they spend it together.

- If they cannot communicate, where should they go?

Assumptions of Classical Game theory

- Agents are *rational*
  - They have well-defined objectives/preferences over a set of outcomes and
  - They choose the actions that lead them to their preferred outcomes

- Agents have as common knowledge
  - The rules of the game
  - Know that other agents are rational and the others know that they know they are rational…

Defining Strategical environments

- Main components
  1. Agents
  2. Strategies/actions
  3. Outcomes
  4. Payoffs (utilities)

Strategic Normal Games

A game in strategic normal form is defined by:

1. Agents: (N>1)
2. Actions:
   - Each agent *i* chooses an action a_i from its own action set A_i
   - The vector a=$(a_1, \ldots., a_n)$ of individual actions is called a **joint action** (or **action profile** or strategy profile). The set *A* is the set of all joint actions.

agent j

agent i		action1	action2
	action 1	1 1	4 1
	action 2	1 4	4 4

Right now don't think about the numbers in the "squares"

Outcomes

3. Outcomes: Suppose we have the case where two agents have Action={a,b}, i.e., each agent has just two possible actions that it can perform. Four possible different outcomes can be produced by the system:

$$\tau(a,a) = \omega_1 \qquad \tau(a,b) = \omega_2$$
$$\tau(b,a) = \omega_3 \qquad \tau(b,b) = \omega_4$$

Utility Function

4. Utility function:
   - Each agent *i* has its own utility function $u_i(a)$ that measures the goodness of the outcomes, i.e., joint actions.
   - Each agent may give different preferences to different joint actions.
   - A **payoff matrix** representation shows the utilities of **joint actions** for each agent (*coming soon*).

A Simple Interaction Environment

- Assume we have just two agents, *Agent = {i, j}*
- All actions an agent *i* can perform are $A_i = \{a_1, a_2, ..\}$
- All possible outcomes (i.e. outcome space) of the system are $\Omega = \{\omega_1, \omega_2, ..\}$ where each ω represents an outcome corresponding to a collection of actions, one for each agent
- Hence, actual outcome ω depends on the combination of actions taken by *i* and *j*.
- So, environment behavior is given by the state transformer function:
  $\tau : A_i \times A_j \rightarrow \Omega$

State Transformer Function

- Assume each agent has just two possible actions that it can perform, *C* and *D*.

- Here is an environment controlled by agent *j only*:

$\tau(D,D) = \omega_1$ $\tau(D,C) = \omega_2$ $\tau(C,D) = \omega_1$ $\tau(C,C) = \omega_2$

- Now, a state transformer function of an environment which is sensitive to actions of both agents:

$\tau(D,D) = \omega_1$ $\tau(D,C) = \omega_2$ $\tau(C,D) = \omega_3$ $\tau(C,C) = \omega_4$

- And here is another environment where none of the agents has any influence:

$\tau(D,D) = \omega_1$ $\tau(D,C) = \omega_1$ $\tau(C,D) = \omega_1$ $\tau(C,C) = \omega_1$

Preferences

- Utility functions lead to **preference orderings** over outcomes:

  $\omega >_{\mathbf{i}} \omega'$ iff $u_{\mathbf{i}}(\omega) > u_{\mathbf{i}}(\omega')$ where u_i is the utility fn of agent i.

- Preferences determines the selection of actions

Utilities and Preferences - example

- Suppose the agents have utility functions as follows:

$u_i(\omega_1) = 1 \quad u_i(\omega_2) = 1 \quad u_i(\omega_3) = 4 \quad u_i(\omega_4) = 4$

$u_j(\omega_1) = 1 \quad u_j(\omega_2) = 4 \quad u_j(\omega_3) = 1 \quad u_j(\omega_4) = 4$

where $\omega_1=\{D,D\}$, $\omega_2=\{D,C\}$, $\omega_3=\{C,D\}$, $\omega_4=\{C,C\}$,

- We say agent i and j's preferences (over outcomes) are as follows:

i's: $\omega_4 \geq_i \omega_3 >_i \omega_2 \geq_i \omega_1$ ($(C,C) \geq_i (C,D) >_i (D,C) \geq_i (D,D)$)

j's: $\omega_4 \geq_j \omega_2 >_j \omega_3 \geq_j \omega_1$ ($(C,C) \geq_j (D,C) >_j (C,D) \geq_j (D,D)$)

- If you were agent i, what would you prefer to do, C or D? Why?

Payoff Matrix

- Assume agent i has Ai={a,b} and agent j has Aj={c,d}
- Agent's utilities are shown as a payoff matrix:

agent j

agent i	action c	action d
action a	u_2 / u_1	u_4 / u_3
action b	u_6 / u_5	u_8 / u_7

OR

a_j

a_i	action c	action d
action a	u_1, u_2	u_3, u_4
action b	u_5, u_6	u_7, u_8

- The payoff u_1 is the utility for agent *i* when agent *i* chooses action *a* and agent *j* chooses action *c*.
- The payoff u_2 is the utility for agent *j* when agent *i* chooses action *a* and agent *j* chooses action *c*.

Rational action

- We can characterize the example in (slide) as this payoff matrix:

agent j

agent i	D	C
D	1 1	4 1
C	1 4	4 4

payoff for agent *j*

- "C" is the rational choice for *i*, because *i* prefers all outcomes that arise through C over all outcomes that arise through D, no matter what j does. Indeed both agents prefer "C" independently from each other!

- In this situation agents do not need to worry about what the other agent will do - **no "strategic thinking".**

Example : Prisoner's Dilemma (PD)

Two men are collectively charged with a crime and held in separate cells.

- Common knowledge, the rules of the game - they are told that:
  - If one confesses and the other does not, the confessor will be freed, and the other will be jailed for 3 years.
  - If both confesses, then each will be jailed for 2 years.
  - If neither confesses, then they will each be jailed for 1 year.
  - They are both rational and both know the other is rational

- No communication - no way of agreement.
- What kind of behavior would you expect them to display?

PD - II

Confess = defect
Stay silent= cooperate

	a_j	
a_i	cooperate	defect
cooperate	3, 3	1, 4
defect	4, 1	2, 2

- Note that the numbers of the payoff matrix do not refer to years in prison but to how good an outcome is for the agent, the shorter time in jail the better.
- Top left: Reward for mutual cooperation (1 year each).
- Top right: If *i* cooperates and *j* defects, *i* gets sucker's payoff of 1, while *j* gets 4 (3 vs zero years)
- Bottom left: If *j* cooperates and *i* defects, *j* gets sucker's payoff of 1, while *i* gets 4 (e.g., 3 vs zero years).
- Bottom right: If both defect, then both get punishment for mutual defection (e.g., 2 years each).

Canonical PD payoff matrix

	Coop	Defect
Coop	R,R	S,T
Defect	T,S	P,P

Applies when this condition holds:

$T > R > P > S$

Solution Concepts

- A solution to a game is a prediction of the outcome of the game using the assumption that all agents are rational and strategic.

- How does the game theory predict?
  1. Strictly(Strongly) *dominant* equilibriums.
  2. Weakly *dominant* equilibriums.
  3. Iterated elimination of **dominated** actions.
  4. Nash equilibrium.

Solution concept 1: Strictly Dominant Strategy Equilibrium

- An action is strictly dominant if the agent prefers it over all of its other actions, regardless of what action(s) the other agent(s) prefer to choose.
- A dominant action must be unique, and when it exists, a rational agent will choose it.
- The strictly dominant strategy equilibrium is the joint strategy where all the agents choose the strictly dominant action – if there exists.

Strictly Dominant Strategy equilibrium - example

agent j

		defect	cooperate
agent i	defect	2, 2	4, 1
	cooperate	1, 4	3, 3

- Preferences:
  - i's: **D**,D > C,D and **D**,C > C,C (compare rows)
  - j's: D,**D** > D,C and C,**D** > C,C (compare columns)

- "Defect" is the **strictly dominant strategy** for both agents.

- (D,D) is the strictly (also called **strongly)** dominant strategy equilibrium.

Strictly Dominant Strategy (SDS) - formal

- A_i is the set of all actions of agent ***i***.
- a_{-i} is a joint action taken by all agents other than ***i*.**
- A_{-i} is the set of joint actions except the action of agent ***i*** *, i.e.,* $a_{-i} \in A_{-i}$
- Given a game in strategic normal form, an action $a_i \in A_i$ for agent ***i*** <u>strictly dominates</u> action $b_i \in A_i$ if

  $U_i(a_i, a_{-i}) > U_i(b_i, a_{-i}) \qquad \forall a_{-i} \in A_{-i}$

From the classroom – about the definition of strongly dominant strategies

$A_1 : \{C, D\}$

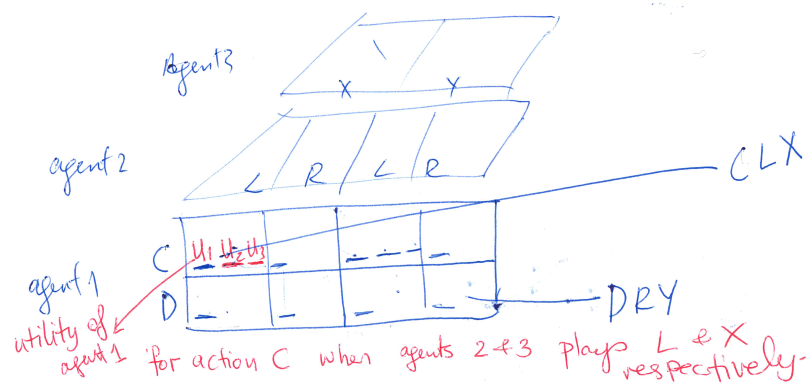
$A_2 : \{L, R\}$

$A_3 : \{X, Y\}$

$A : \{CLX, CLY, CRX, CRY, DLX, DLY, \ldots\}$

$A_{-1} : \{LX, LY, RX \ldots\}$

$u_i(a_i, a_{-i}) > u(b_i, a_{-i}) \ \forall_x \ a_{-i} \in A_{-i}$



SDS equilibrium - another example

- Does this game have a dominant strategy for agent i?
- For agent j ?

agent j

agent i		action a	action b
	action a	1, 1	1, 4
	action b	4, 1	4, 4

SDS equilibrium Example - II

		agent j	
		action a	action b
agent i	action a	1, 1	1, 4
	action b	4, 1	4, 4

- Preferences:
  - i's: **b**,b = **b**,a > **a**,b = **a**,a (compare rows)
  - j's: b,**b** = a,**b** > b,**a** = a,**a** (compare columns)
- Action b is a strictly dominant action for agent i.
- Action b is a strictly dominant action for agent j.
- Thus (b,b) is a strictly dominant strategy equilibrium.

Solution concept 2: Weakly Dominant Strategy Equilibrium

- For an agent i, a_i weakly **dominates** b_i if
  - $u_i(a_i, a_{-i}) \geq u_i(b_i, a_{-i}) \quad \forall a_{-i} \in A_{-i}$ and
  - $u_i(a_i, a_{-i}) > u_i(b_i, a_{-i})$ for some $a_{-i} \in A_{-I}$
- a_i is called weakly dominant if it weakly dominates every action in Ai.

WDS equilibrium

agent j

agent i

	action a	action b
action a	2, 1	0, 2
action b	2, 3	4, 3

- Neither agent has a strongly dominant strategy.
- Action *b* is a weakly dominant strategy for both agents *i* and *j*.
- Therefore *(b,b)* is a weakly dominant strategy equilibrium.
- Rational agents will usually play this strategy.

Example: No dominant strategy at all

agent j

agent i

	L	M	R
U	1,0	1,2	0,1
D	0,3	0,1	2,0

- Are there/What are the dominant strategies?
- The game has no dominant strategy equilibrium.

Solution Concept 3: Iterated elimination of Dominated strategies

- This solution concept may be used in games where there is no dominant strategy equilibrium.
- There may be dominated action strategies
- Key idea: A rational agent will never choose a suboptimal action, i.e. a dominated action.

Dominated action

- Take a game in normal strategic form and consider any two actions $a_i, b_i \in A_i$ for any player $i \in N$.

- We say that a_i is strictly dominated by b_i if

  $$u_i(a_i, a_{-i}) < u_i(b_i, a_{-i}) \qquad \forall a_{-i} \in A_{-i}$$

- We say that a_i is weakly dominated by b_i if

  $$u_i(a_i, a_{-i}) \leq u_i(b_i, a_{-i}) \qquad \forall a_{-i} \in A_{-i}$$

  $$u_i(a_i, a_{-i}) < u_i(b_i, a_{-i}) \qquad \text{for some} \quad a_{-i} \in A_{-i}$$

Iterated Elimination of Strongly Dominated Strategies - procedure

- **A rational agent will never choose a suboptimal action, i.e. a dominated action.**

- IESDS is a solution technique that iteratively eliminates strictly dominated actions from all agents until no more actions are strictly dominated

Iterated elimination of strictly dominated actions - Example

agent j

agent i

	L	M	R
U	1,0	1,2	0,1
D	0,3	0,1	2,0

agent j

agent i

	L	M	R
U	1,0	1,2	0,1
D	0,3	0,1	2,0

agent j

agent i

	L	M	R
U	1,0	1,2	0,1
D	0,3	0,1	2,0

- R is strictly dominated for agent *j* (by action M).
- We eliminate R because agent *i*, being rational, knows that agent *j* will not play R.
- Now agent *j* notices that D is strictly dominated for agent *i*. Agent *j* eliminated in his head the action for agent *i*.
- Eliminate L for agent *j*.
- The order of elimination does not matter in IESD

Iterated elimination of weakly dominated actions - Example

- If no strongly dominated actions, then use weakly dominated ones to eliminate

	L	R
U	$3, 1$	$2, 0$
M	$4, 0$	$1, 1$
D	$4, 4$	$2, 4$

- Does the order matter?
- Start with eliminating U
- Start with M

Let us play

- Pretend we are playing a game. Here's how it works. You must choose a number, x between [0, 100].

- Assume we would then calculate the average of all the numbers. Then the person whose number is closest to 2/3 of the average will win.

- Assume that eveybody is rational.

- You choose x = ??

IESDS/IEWDS - problems

- Different outcomes may arise as outcomes that survive, depending on the order of elimination in IEWDS.
- There may be left more than one solution after elimination.
- Hence, need a stronger solution concept.
  - Nash equilibrium

Solution concept 4: Nash Equilibrium

Definitions: Pareto Optimality and Social Welfare

- Two important notions for comparison and selection of solutions; How good is a solution?

  1. Pareto optimality
  2. Social welfare maximization

Pareto Optimality(efficiency) – defn.

- A solution (i.e., a strategy profile, e.g., $(a_1, a_2, \ldots, a_n)$) S^P is Pareto optimal, if there is **no** solution S' with:

  $\exists$ agent i : $U_i(S') > U_i(S^P)$ and

  $\forall$ agent j: $U_j(S') \geqq U_j(S^P)$ i.e., none of the others are worse off

- In plain english: there is no other outcome where some agents can increase their payoffs without decreasing the payoffs of other agents.

Example on pareto optimality

Agent1

	A	B
A	9,9	7, 10
B	10, 7	8, 8

- Which joint action(s) is/are pareto optimal?
  - (A,A) : optimal because no other outcome makes an agent better without making the other worse
  - (A,B): optimal, because payoff "7" of agent1 can be improved but then agent2 is worse off.
  - (B,A): optimal
  - (B,B): **not** optimal. Both agents are better off when they shift to (A,A)

Social Welfare – defn.

- Social welfare value (of a solution)
  - Sum of utilities of all agents for this strategy profile.
  - If a solution maximizes social welfare, i.e., social optimum, then the available utilities are not wasted.

- if a solution is a social optimum, then it is Pareto efficient.
  - The converse does not hold

Solution concept: Nash Equilibrium - Definition

- **Nash equilibrium** is a strategy profile (i.e., a collection of strategies, one for each player) such that each strategy is a **best response** (maximizes payoff) to all the other agents' strategies

- **Best response:** The best response of agent *i* is given by $BR_i(a_{-i}) = \{a_i \in A_i : u_i(a_i, a_{-i}) \geq u_i(b_i, a_{-i}) \text{ for all } b_i \in A_i\}$.

How to find the Nash equilibrium(s)

- One way of finding Nash eq. in two-person strategic form games is to utilize the best response correspondences in the payoff matrix.

- Mark the best response(s) of each agent *given the action choice of the other agent*. A strategy profile where best responses of all agents intersect is a Nash equilibrium (not necessarily a singleton set)

agent j

agent i

	x	y
x	<u>2</u>, <u>1</u>	0, 0
y	0, 0	<u>1</u>, <u>2</u>

- The Nash equilibriums are (x,x) and (y,y).

Finding NE -example

Agent j

Agent i

	X	Y	Z
A	2,1	2,5	2,7
B	0,1	4,6	6,0
C	7,3	5,1	0,1

Nash Equilibrium?

(C,X) is NE

Finding NE – another example

Agent j

Agent i

	X	Y	Z
A	4, 2	5, 2	1, 1
B	1, 3	4, 0	0, 2
C	3, 1	2, 3	1, 2

NE: (A,X) and (A,Y)

Example: Prisoner's Dilemma

agent j

agent i

	defect	cooperate
defect	2, 2	4, 1
cooperate	1, 4	3, 3

- The strategy profile (defect, defect) is a Nash Equilibrium.
  - If agent *i* changes its strategy(defect) to cooperate - the new situation is (cooperate, defect) - agent i will be worse off (payoff from 2 to 1).
  - If *j* changes its strategy (defect→ cooperate) it will be worse off (payoff, again, from 2 to 1).

- So defection is the best response to all possible strategies.

Example: Prisoner's Dilemma- cont.

- Pareto optimal: (C,D), (D,C) and (C,C)
- Socially welfare maximizing: (C,C)
- The solution is (D,D)
  - Unique Nash (also strongly dominant strategy profile) is not Pareto optimal
  - Not socially (welfare) maximizing

- Intuition says this is not the best outcome – not social optimum. Surely they should both cooperate and each get payoff of 3!
- This apparent paradox is the fundamental problem of multi-agent interactions.

Multiple Nash equilibria

- Not every interaction scenario has a single Nash equilibrium

		agent j	
		l	r
agent i	l	1, 1	0, 0
	r	0, 0	2, 2

- Consider two agents each of whom chooses either l (left) or r (right). If their choices do not match, they receive a payoff of zero. See the other payoff values in the figure.
- There are two Nash equilibria, (l,l) and (r,r).
- The two Nash equilibria are not equal. Would the agents possibly ”agree” (coordinate) upon one of the solutions?
  – (r,r) is *social optimum*.
- This type of game is known as “pure coordination game”

Multiple Nash equilibria Example : Battle of sexes

husband

wife

	movie	opera
movie	2, 1	0, 0
opera	0, 0	1, 2

- The story: A couple is arguing about what to do for entertainment in the evening.

- The couple is in love, so the most important thing for the players is to do something together; both view the night ”wasted” unless they spend it together.

Battle of Sexes- cont.

agent j

agent i

	movie	opera
movie	<u>2</u>, <u>1</u>	0, 0
opera	0, 0	<u>1</u>, <u>2</u>

- Given agent i plays m, the best response for agent j is to play m, which is expressed by underscoring player j's payoff at (m,m), and its best response to "o" is "o".
- Similar for agent i, and the Nash equilibria are
  - (m,m) and (o,o). No strict equilibrium.
- Hence, they agree that cooperation is better but "disagree" about the best outcome

Multiple Nash equilibria Example : Stag Hunt

Two individuals go out on a hunt. Each can individually choose to hunt a stag or hunt a hare. If an individual hunts a stag, they must have the cooperation of his partner in order to succeed. An individual can get a hare by themself, but a hare is worth less than a stag

agent j

agent i

	Hare	Stag
Hare	1, 1	2, 0
Stag	0, 2	3, 3

- Each agent wants to do what the other does - which may be different than what they *say* they'll do.
- Coordination game

Stag Hunt- cont.

agent j

agent i

	Hare	Stag
Hare	1, 1	2, 0
Stag	0, 2	3, 3

- Two pure Nash equilibria: (H,H)and (S,S).
- (S,S) is pareto optimal/payoff dominant (more payoff) while (H, H) is risk dominant (less risky)
- Higher payoff vs safer?
- If one agent trusts the other (that they will cooperate to hunt stag) then the other can do no better than to cooperate. But if ... So, hunting stags is most beneficial for society but requires a lot of trust among its members.

Stag Hunt – solution selection

- Conflicting opinions
- Harsanyi and Selten (1988) propose that the payoff dominant equilibrium is the **rational choice** in the stag hunt game,
- Harsanyi (1995) changes this conclusion to take risk dominance as the relevant selection criterion.

Changing the environment/norms in Stag hunt game

- If both agents hunt a hare the hares will be given to others
- If only one catches a hare (while the other tries Stag) then the two agents will share the hare

- How would you hunt?

No Nash Equilibrium

- There is no Nash in Zero-sum interactions

- Zero-sum games: preferences of agents are diametrically opposed, we have strictly competitive scenarios.

- Zero-sum encounters are those where utilities sum to zero.

  $u_i(\boldsymbol{\omega}) + u_j(\boldsymbol{\omega}) = 0 \quad \forall \boldsymbol{\omega} \in \Omega$, where $\boldsymbol{\omega}$ is an outcome

Zero-sum – Example

agent j

agent i		H	T
	H	1, -1	-1, 1
	T	-1, 1	1, -1

- Matching pennies game

- Two agents. Each chooses either Head or Tail simultaneously. If the choice differ, agent *i* pays agent *j* (let's say 100 NOK). If they choose the same, agent *j* pays agent *i*.

- No possibility for cooperation or coordination or anything.

- What may happen?

Mixed Strategies

agent j

agent i

	H	T
H	-1 +1	+ 1, -1
T	+1, -1	-1, +1

- No Nash: Because in Nash, the players don't change their strategies even when they know what the other agent will play
- Here for every strategy pair, there is an agent that may want to change their strategy to get more payoff.
- Hence, no **pure** strategy equilibrium?
- What now? Randomized behaviour : Mixed strategies

Why inefficient outcomes becomes solution?

- Need for new theories

- Need for modification in the modelled environment?
  - E.g., mafia norms, “nobody talks to authorities”
  - Legislations/Regulations?
  - Incentives/subsidization

Summary

END of GT part of the Syllabus

- The rest of the slides are for those curious about mixed strategies.

Mixed Strategies

- Principle idea: agents don't choose a strategy but a strategy distribution

- Choose probability of choosing each strategy
  - In mixed strategies, a pure strategy H is an instance of mixed strategies of choosing H with probabilities between 0 and 1.

- If an agent chooses H with probability q (a number between 0 and 1), then the problem is transformed to finding the mixed strategies (e.g., probability of selecting H) that are best response to each other.

How to find the Mixed Strategies

- No “pure” (strict choice of either H or T) strategy equilibrium

		agent j	
		H	T
agent i	H	-1 +1	+ 1, -1
	T	+1, -1	-1, +1

- **Assume** agent-i plays pure strategy H, and agent-j plays H with prob q. Expected payoff of agent-i when playing H:

  $(- 1) * q + 1* (1-q) = 1-2q$

- If agent-i plays pure strategy T when agent-j plays H with probability q, then expected payoff of agent-i would be:

  $q – (1-q)= 2q-1$

Equilibrium with the mixed strategies

- If 1-2q ≠ 2q-1 then it makes a difference for agent i to choose either pure H or pure T.
- Why try to be (and make the other agent) indifferent in expected payoffs of alternative actions ?
  - It is like minimizing the ability of the opponent to discover your play-pattern.
- We know that there is no pure strategy equilibrium in Matching pennies. If agent-i responds with a mixed strategy, agent-j would also make itself unpredictable - indifferent between playing H and T.
- So in any Nash equilibrium it is required that

  1-2q= 2q-1 i.e., q=1/2
- The situation is symmetric, therefore agent i also plays mixed strategy, i.e., H with probability p=1/2

agent j

agent i		H	T
	H	-1 +1	+ 1, -1
	T	+1, -1	-1, +1

Mixed Strategy in Battle of Sexes (BoS)

agent i

	movie	opera
movie	2, 1	0, 0
opera	0, 0	1, 2

- **Assume** agent-j plays M with prob q. Expected payoff of agent-i:
  - For agent-i, $U_i(M)=U_i(O)$ (see the preceding slide)
    => $2q = 1(1-q)$ => $q= 1/3$
- Similarly agent-i will randomize to make agent-j indifferent between M and O: $U_j(M)=U_j(O)$
  => $p= 2(1-p)$ => $p=2/3$
- So (2/3, 1/3) is a mixed strategy
  - Interpretation: the wife may prefer 2/3 of times to go to movie and the husband 1/3 of the times.

Meaning of Mixed Strategies

- Why/when are the mixed used?
- Various interpretations:
  - to make your opponent more uncertain/confused
  - randomize when you are uncertain about the other agent’s actions

- Suggests that the play is repeated

Why inefficient outcomes becomes solution?

- Need for new theories

- Need for modification in the modelled environment?
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Different types of games

- Simultaneous/strategic normal form games
- Sequential (extensive form) games
- Repeated games

Repeated/Evolutionary Games

Repeated games

– Focus: remembering past behavior of others, strategies based on past behavior of other agents.
– Main issue: evolution of cooperation, learning, opponent modeling
– Example: prisoner's dilemma repeatedly played

Iterated Prisoner's Dilemma

- How to evolve cooperation?
  - One answer: play the game more than once.
- If you know you will be meeting your opponent again, then the incentive to defect appears to evaporate.

Axelrod’s Tournament

- Suppose you play iterated prisoner’s dilemma against a range of opponents. What strategy should you choose, so as to maximize your overall payoff?
- Axelrod (1984) investigated this problem, with a computer tournament for programs playing the prisoner’s dilemma.
- In the tournament, programs played games against each other and themselves repeatedly.

Strategies in Axelrod’s Tournament

- ALLD:
  - Always defect - the hawk strategy.
- RANDOM
  - Selects either cooperate or defect on random.
- TIT-FOR-TAT
  - On round $t = 0$, cooperate
  - On round $t > 0$, do what your opponent did on round $t-1$.
- TESTER
  - On 1st round defect. If the opponent ever retaliated with defection, then play TIT-FOR-TAT. Otherwise play a repeated sequence of cooperating for two rounds, then defecting.
- JOSS
  - As TIT-FOR-TAT, except periodically defect.

Recipes for Success in Axelrod's Tournament

Axelrod suggested the following rules for succeeding in his tournament:

– Don't be envious:

  Don't Play as if it were zero sum!

– Be nice:

  Start by cooperating, and reciprocate cooperation.

– Retaliate appropriately:

  Always punish defection immediately, but use "measured" force - don't overdo it.

– Don't hold grudges:

  Always reciprocate cooperation immediately.