Classification and Regression Trees

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1. Growing trees
2. Pruning trees
3. Supernova

Trees

- Restrict to binary splits
- Computationally infeasible to build every possible tree
- Want a algorithm that builds a “good” tree.

Trees

Two components:

- Growing trees
- Pruning trees

Basic Idea

Growing trees:

- Choose the "best" possible binary split of the data
- Take each side of the split and find the next "best" split for each side.
- Continue until either each terminal node is "pure" or of some minimum size.

Will result in an over-fitted tree. Then need to prune tree:

- Using large tree prune back to find "best" tree of various sizes
- Use cross-validation to choose "best" size.

Growing trees

Need four things:

- A set of possible splits
- A measure of how good a split is
- A stop-splitting rule
- A rule for assigning a terminal node to a class.

Growing trees - Possible splits

Only split on one variable:

- Binary variables
  - one possible binary split
- Categorical variables
  - Splits of the form $x \in S$ where S could be any possible subset of the categorical levels.
  - $2^{L-1} - 1$ possible splits.
- Continuous or ordinal variables
  - Splits of the form $x \leq x_c$.
  - At most N possibilities - where x_c is halfway between to consecutive distinct values.

Need a measure to choose which spilt is best.

Growing trees - Best split

Introduce an impurity function f.
Let the impurity of a node t be,

$$I(t) = \sum_{i=1}^{C} f(p_{it}),$$

where p_{it} is the proportion of those in t that belong to class i in future samples (in practice use the proportions in the learning set possibly times a prior).

- Want $I(t)$ to be maximal when node contains equal amounts of each class.
- Want $I(t)$ to be minimal (i.e. 0) when node contains only one class.

Growing trees - Best split

Given a split, s, that sends a proportion p_R of the data to t_R and p_L to t_L the decrease in impurity from the split is,

$$\Delta I(s,t) = I(t) - p_R I(t_R) - p_L I(t_L)$$

Want to choose a split that maximises this decrease.
Some examples of f:

- Gini index $f(p) = p(1-p)$
- Information index $f(p) = -p\log p$

Growing trees - Stopping rule & assigning classes

Stopping rule

- Want a large tree - since we are going to prune later
- Keep growing until either terminal nodes are very small or are pure.

Assigning classes

- Assign to the class with the largest p_{it}

Pruning

- We now have a large tree which will have likely over-fitted the data. We want to prune back to a smaller tree.
- Basically, define a cost-complexity measure that is the misclassification cost of a tree penalized by its complexity.
  - cost-complexity of T = misclassification rate of $T + \alpha|T|$.
  - $|T|$ is the number of terminal nodes and measures the complexity of tree T.
- Find T that minimises the cost-complexity for various α.

Turns out this defines a sequence of nested subtrees of our original large tree.

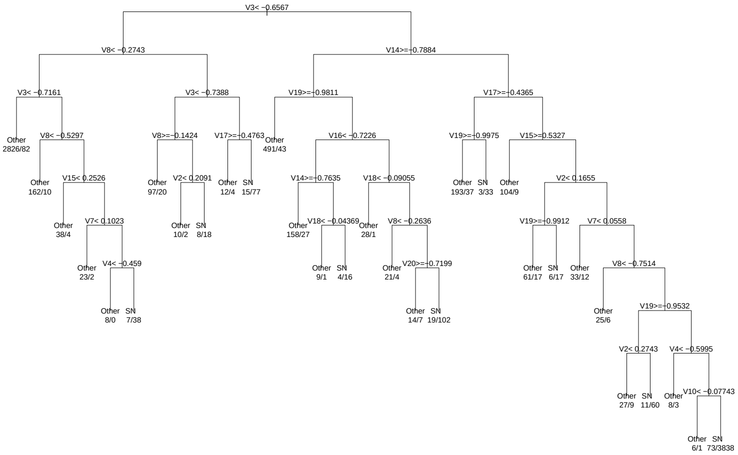
Choosing the level of pruning

- How do we choose the best sized tree from the sequence generated?
- Want to minimise the misclasification rate
- Misclassification rate on the training data will always decrease with increasing tree size
- Need an estimate if the misclassification rate for new data
- Cross validation!

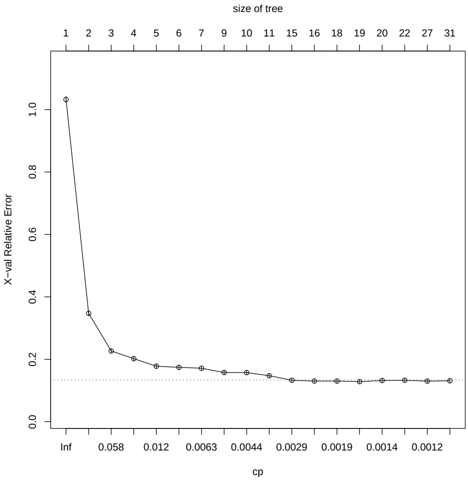
Supernova Data

- 5000 Supernova and 5000 other objects
- Split into two sets 9000 in training 1000 in test
- Build tree and prune based on training set using cross validation
- Test prediction on test set and compare to support vector machines.

Growing full length tree

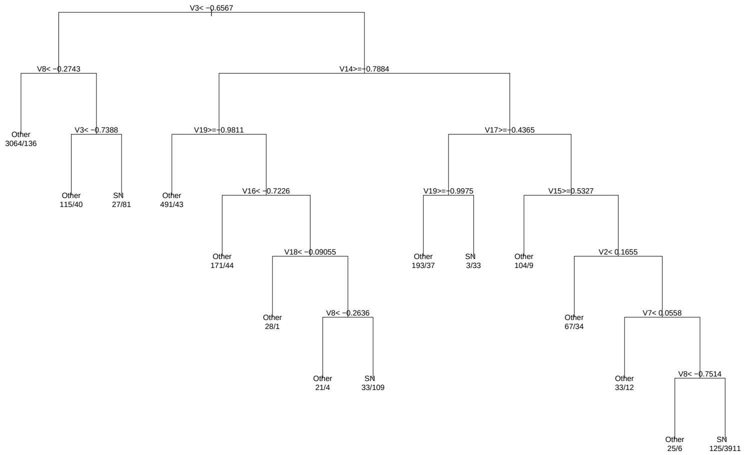


Pruning



- Look for smallest tree within 1sd of tree with minimum error.
- About 14 splits.

Pruned tree


V3< −0.6567
V8< −0.2743
V14>=−0.7884
Other
3064/136
V3< −0.7388
V19>=−0.9811
V17>=−0.4365
Other
115/40
SN
27/81
Other
491/43
V16< −0.7226
V19>=−0.9975
V15>=0.5327
Other
171/44
V18< −0.09055
Other
193/37
SN
3/33
Other
104/9
V2< 0.1655
Other
28/1
V8< −0.2636
Other
67/34
V7< 0.0558
Other
21/4
SN
33/109
Other
33/12
V8< −0.7514
Other
25/6
SN
125/3911

Left branch

Performance on test set

- Classification Tree

		Prediction: Other	Prediction: Supernova
Actual	Other	461	39
	Supernova	45	455

Error = 8.4%

- Best Support Vector Machine

		Prediction: Other	Prediction: Supernova
Actual	Other	484	16
	Supernova	32	468

Error = 4.8%

Other things to look into

- Try different impurity measures
- Using linear combination of variables as splits
- Priors?

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R package version 3.1-32.