Optimization-based restoration of damaged audio signals

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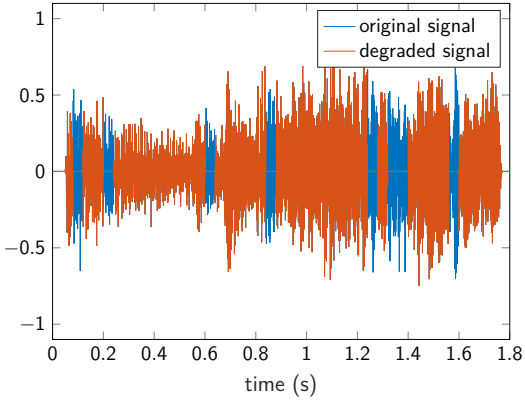
GitHub repository:

https://github.com/ondrejmokry/InpaintingLecture

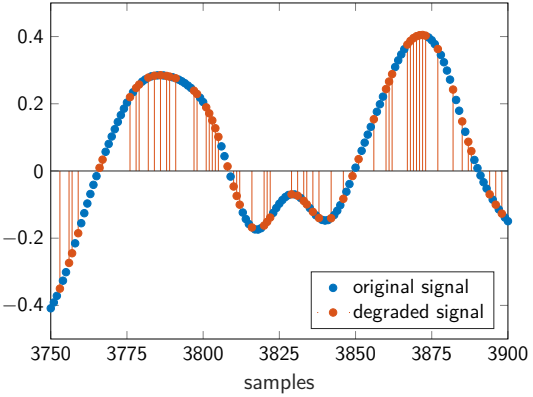
What do we mean by damaged audio signals

Sample loss

Time domain



(a) missing signal blocks



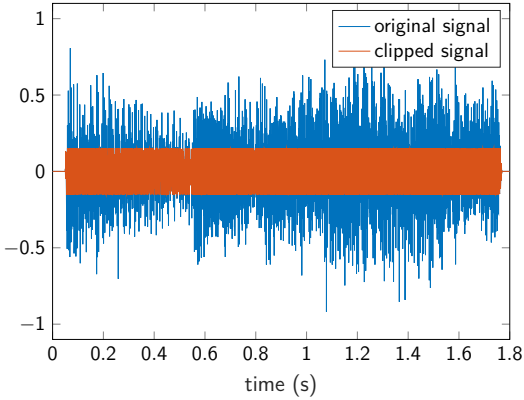
(b) missing random signal samples

Figure: Signal degraded by lost samples.

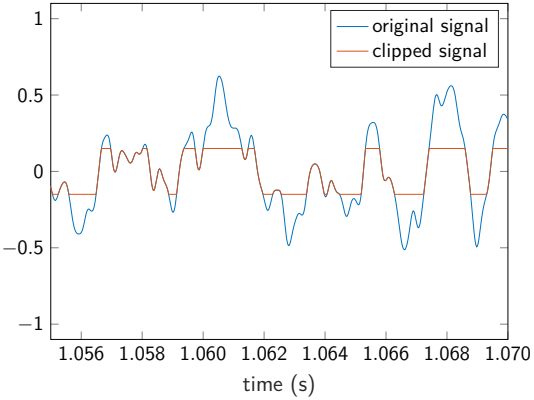
In case (a), 18 % of all samples are lost in blocks of length 40 ms. In case (b), 60 % of randomly selected samples are lost.

Clipping

Time domain



(a) the whole signal



(b) selection

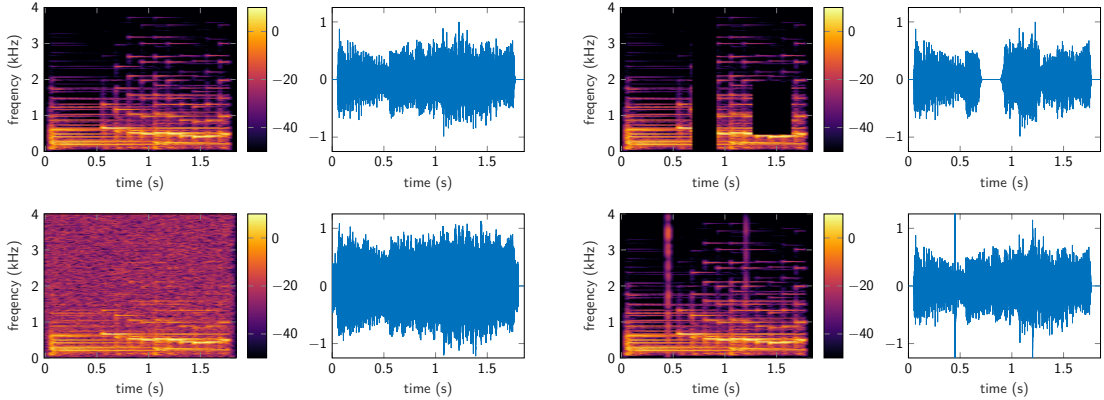
Figure: Signal degraded by clipping.

55 % of all samples are clipped

Other examples

that would fit in our framework

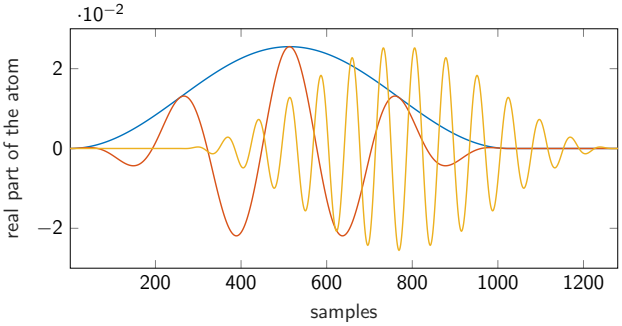
- additive noise
- impulsive noise
- missing values in a transformed domain (occluded spectrogram)



Short-time Fourier transform

A very brief and slightly mathematical overview

- Correlation with translated (τ) + modulated (ω) versions of the window function ($\mathbf{g}$)



- Fourier transform in translated "snip-outs" of the signal ($\mathbf{x}$)

$$\mathbf{x}(t) \mapsto \mathbf{C}(\tau, \omega) = \langle \mathbf{x}, \mathbf{g}_{\tau,\omega} \rangle, \quad \mathbf{g}_{\tau,\omega}(t) = \mathbf{w}(t-\tau)\mathbf{e}^{\mathrm{i}\omega t}$$

Missing signal blocks

Time-frequency domain

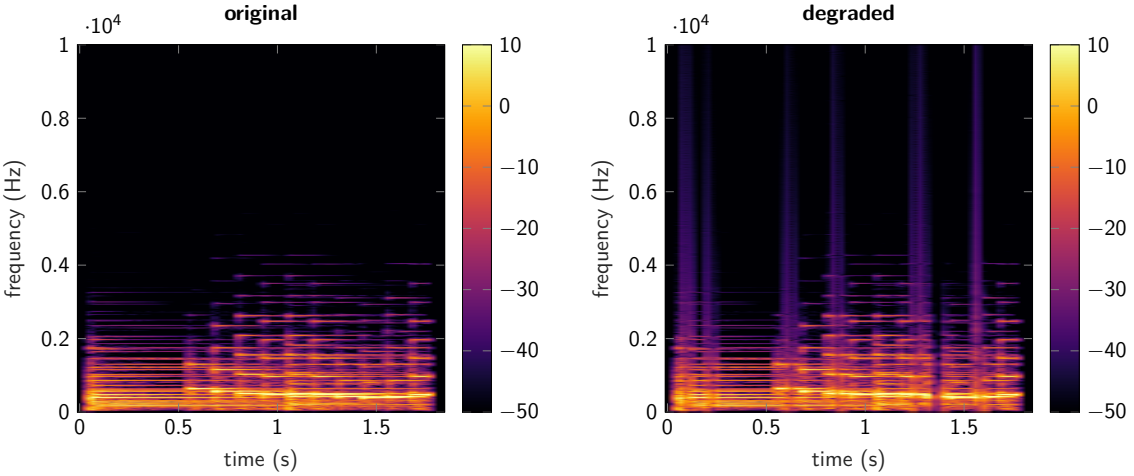
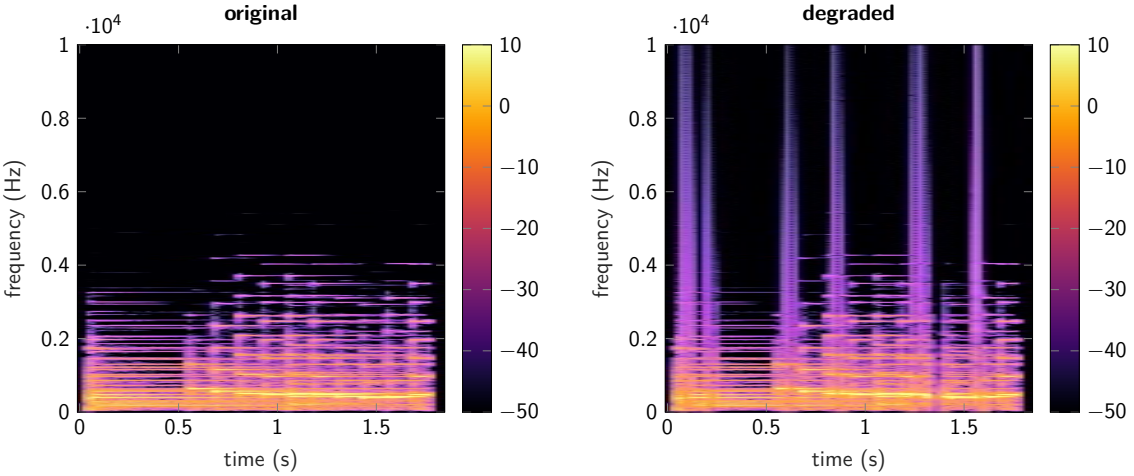


Figure: 18 % of all samples are lost in blocks of length 40 ms

Audio links: original, degraded

Missing signal blocks

Time-frequency domain, hopefully better visibility



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Missing random samples

Time-frequency domain

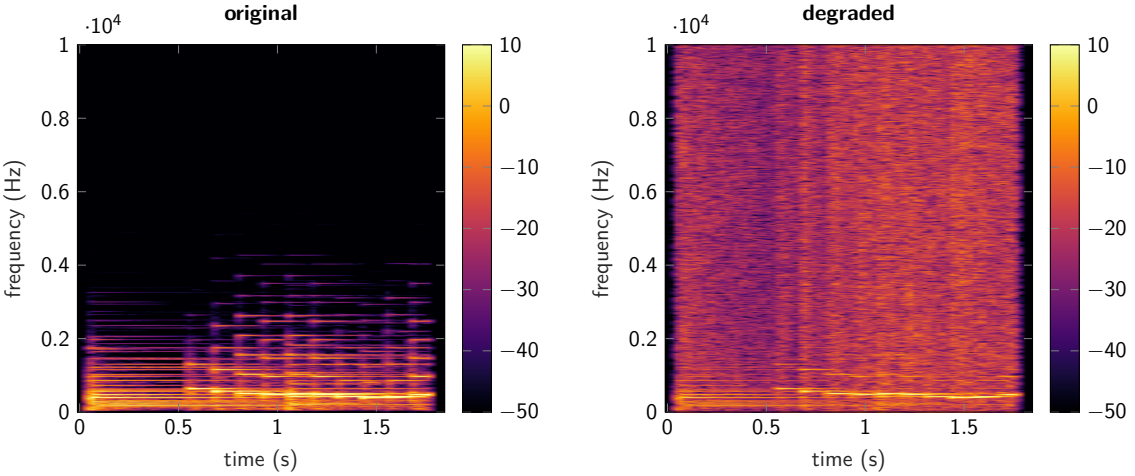


Figure: 60 % of randomly selected samples are lost

Audio links: original, degraded

Clipping

Time-frequency domain

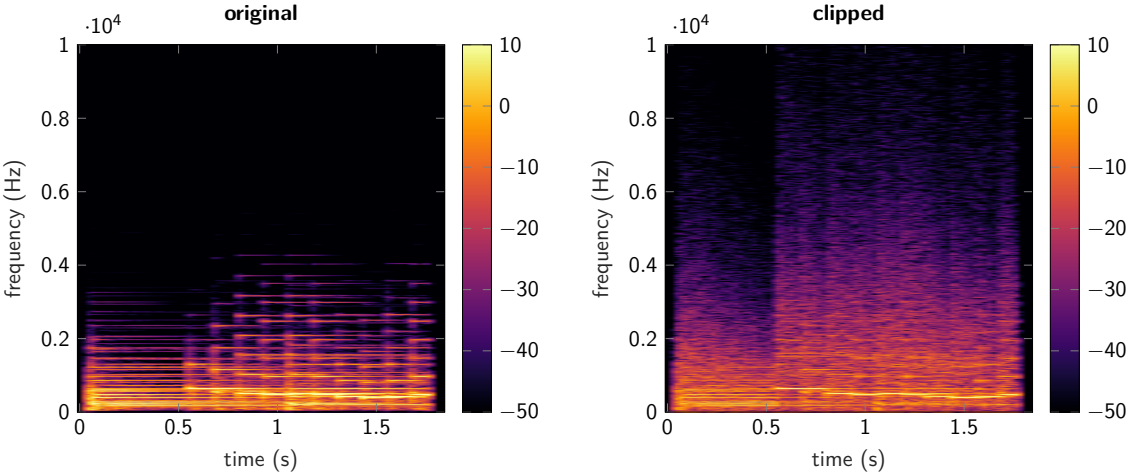


Figure: 55 % of all samples are clipped

Audio links: original, clipped

What do we mean by damaged audio signals

Typical situations

- Sample loss → *inpainting*
  - Damaged medium (LP, CD, wax phonograph cylinders and any other historical media)
  - Transmission error → *packet loss concealment*
  - Impulsive noise → *declicking*
  - Low sampling rate → *interpolation*
- Clipping → *declipping*
  - Overflow of the dynamic range of the microphone / AD converter
- (Usually additive) noise → *denoising*

Audio examples

- clean signal

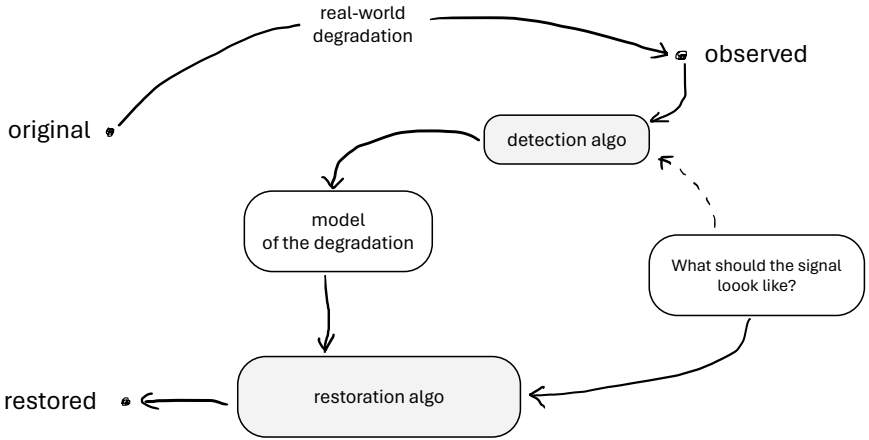
- different solutions to different problems
- (for declipping, the clipped samples are considered lost)

clipping	**long gaps**	**random loss**
55% clipped	40 ms per gap	60% lost
method 1	method 1	method 1
method 2	method 2	method 2
method 3	method 3	method 3
method 4	method 4	method 4

Optimization-based restoration

Optimization-based restoration

Generic scheme, part I



Optimization-based restoration

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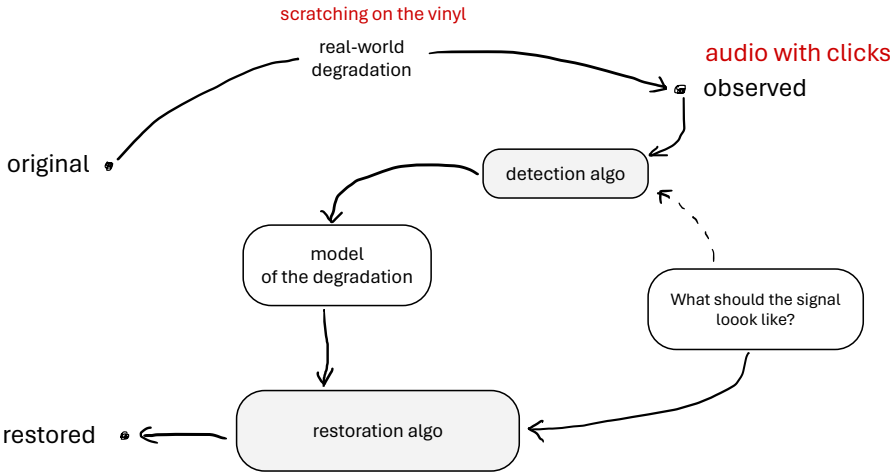

Inverse problems more formally

What is the inversion?

- We want to invert the degradation/observation process

- Clean signal $\xrightarrow{\text{deg}}$ Degraded signal
- Clean signal $\xleftarrow{\text{deg}^{-1}}$ Degraded signal

- Inversion is not possible in general due to loss of information
- We seek a signal that fits the observation and is regularized by a function R

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- We seek a signal that fits the observation and is regularized by a function R

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \{E(\mathbf{x}, \mathbf{y}) + R(\mathbf{x})\} \qquad \text{(regularized problem)}$$

- $\mathbf{y}$ is the (degraded) observation
- $\mathbf{x}$ is the variable of the problem
- $\hat{\mathbf{x}}$ is the solution
- $E(\mathbf{x}, \mathbf{y})$ ensures that the solution $\hat{\mathbf{x}}$ fits the observed signal $\mathbf{y}$, i.e. the relation $\hat{\mathbf{x}} \xrightarrow{\text{deg}} \mathbf{y}$ holds

Inverse problems more formally

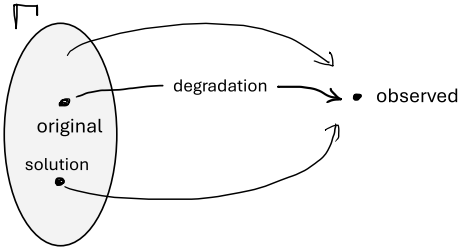
The feasible set

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \{E(\mathbf{x}, \mathbf{y}) + R(\mathbf{x})\} \qquad \text{(regularized problem)}$$

- Set of signals consistent with the observation:

$$\mathbf{x} \in \Gamma = \{\mathbf{u} \mid M_R \mathbf{u} = M_R \mathbf{y}\} \qquad \text{(inpainting consistency in time domain)}$$

  - $M_R\mathbf{y}$ is the observed signal
  - M_R selects the reliable samples



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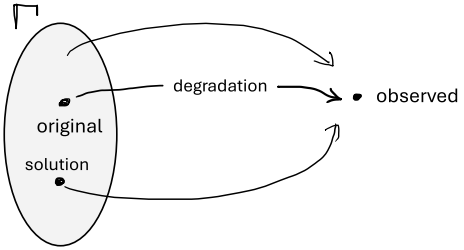
- (De)clipping with symmetric clipping level θ:

  $M_R \mathbf{u} = M_R \mathbf{y}$, $M_H \mathbf{u} > \theta$, $M_L \mathbf{u} < -\theta$



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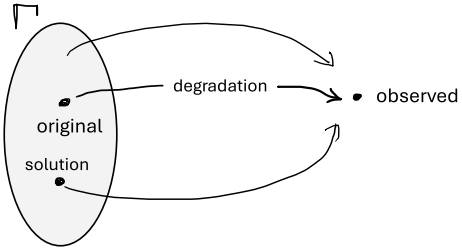
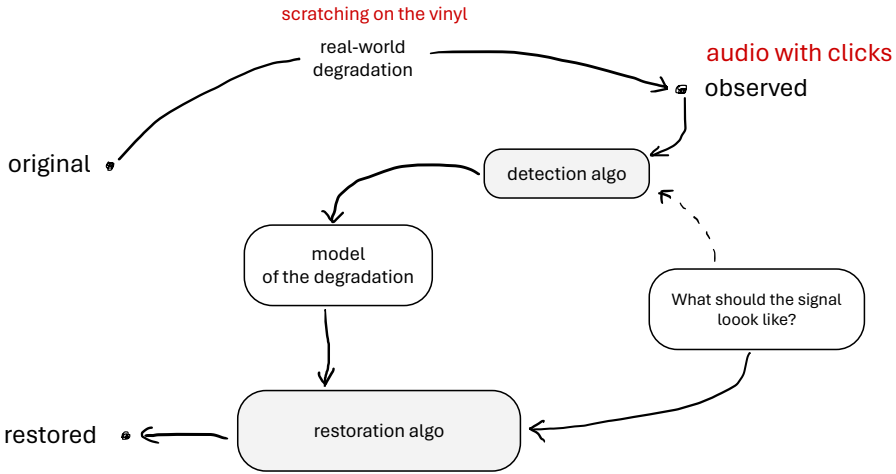
- Examples of the the error term:
  - $E(\mathbf{x}, \mathbf{y}) = 0$ if $\mathbf{x} \in \Gamma$, ∞ if $\mathbf{x} \notin \Gamma$ (noiseless case)
  - Distance of $\mathbf{x}$ from the set Γ



Optimization-based restoration

Generic scheme, part II



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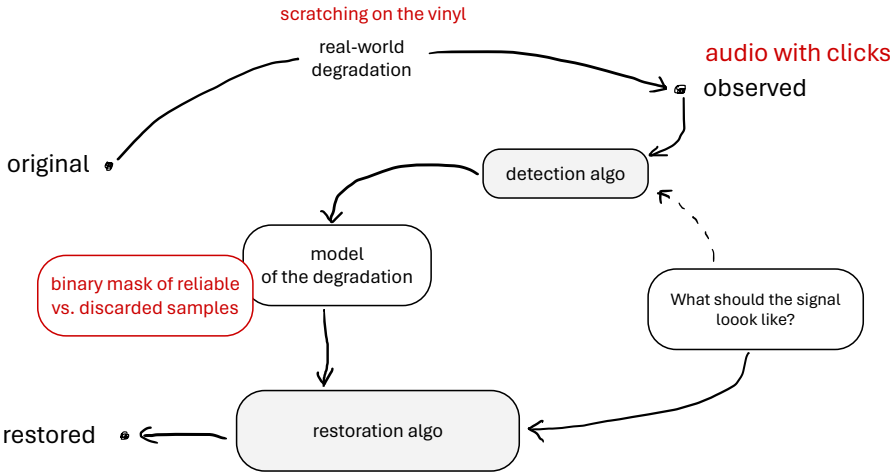

Overview of current approaches

to inpainting and related audio restoration problems

- Janssen (1986) assumes that the signal is governed by an **AR (autoregressive) model**
- Abel (1991) utilizes **limited bandwidth** – possible non-smooth solutions contain high frequencies; therefore he forces bandlimitedness; but oversampling is needed, which is not practical
- Fong (2001) also **autoregressive** assumption, but the waveform is found by Monte-Carlo particle filtering
- Selesnick (2013) proposes to penalize **high values of signal derivatives** and finds his solution via least squares, explicitly
- Bilen (2015) uses **non-negative matrix factorization** to decompose audio to "notes" and their activation patterns (analog to MIDI)
- Takahashi (2015) uses autoregressive assumption, but through **low-rank properties** of certain matrices generated from the signal
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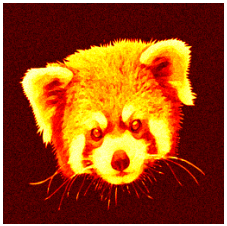
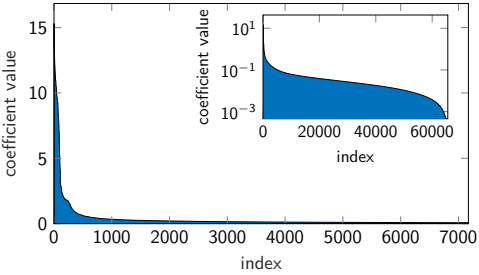
Most recently developed methods are based on **signal sparsity**.

Sparse representations

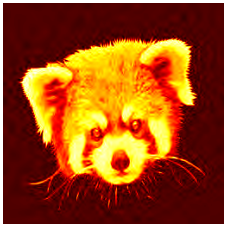
original



all coefficients



5 % largest coefficients



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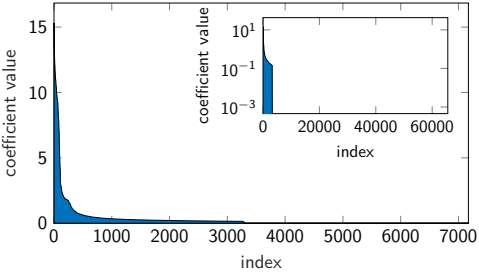
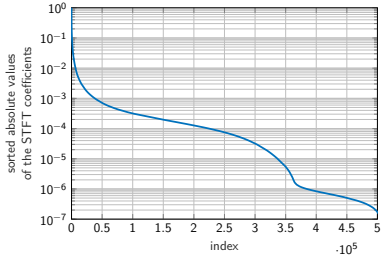


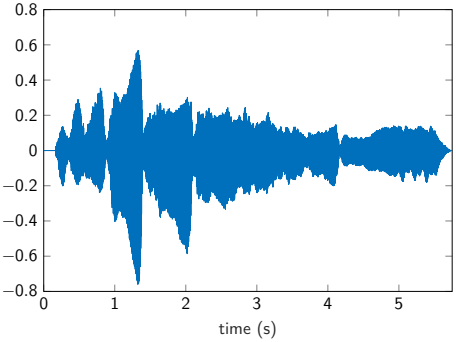
Figure: Images are likely to be sparse under wavelet transform.

Sparse representations

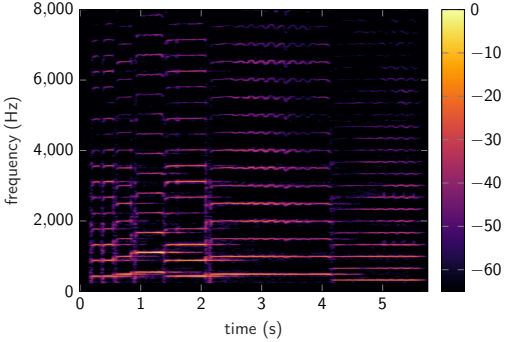
In audio

- Crucial role of a transform
- Audio: STFT, Constant-Q, ...
- Image processing: DCT (used in JPEG)
- What is actually meant by *sparse*?





STFT



- Time-domain signal (vector) $\mathbf{y}$ is modeled as a sum of basis signals $\mathbf{d}_n$ with proper weights c_n (coefficients of the linear combination):

$$\mathbf{y} = \sum_n c_n \mathbf{d}_n + \boldsymbol{\eta} = \mathbf{D}\mathbf{c} + \boldsymbol{\eta}$$

- We **synthesize** the signal
- Sparse $\mathbf{c}$ means most c_n are zero

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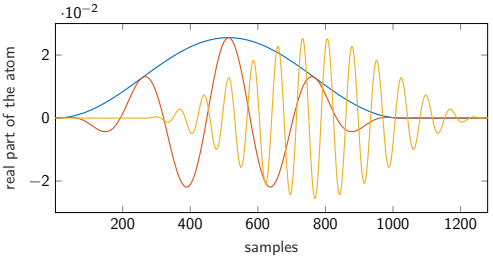
- Example – inverse STFT:
  $\mathbf{d}_n$ are the shifted and modulated windows



Sparse representations

Synthesis signal model

- **Synthesis** model $\mathbf{y} = \mathbf{D}\mathbf{c}$
- For transforms like DCT and DFT, there is a *unique* representation $\mathbf{c} = \mathbf{D}^{-1}\mathbf{y}$
  $\mathbf{D}^{-1}$ is called **analysis**, also denoted $\mathbf{A}$
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- Sparsity (in suitable domain) can be the regularizer for an ill-posed inverse problem (i.e. non-sparse solutions will not be preferred during the search)

- Notation: $\|\mathbf{c}\|_0 =$ number of non-zero elements of $\mathbf{c}$

- Truly sparse solutions not achievable (and not necessary) in practical problems
- Moreover, optimizing true sparsity has combinatorial complexity, it is *NP-hard*
- Different approaches to approximate sparsity available:
  - ℓ_1-norm-based: $\|\mathbf{c}\|_1 = \sum_n |c_n|$

    taxicab, Manhattan metric

    compare with Euclidean $\|\mathbf{c}\|_2 = \sqrt{\sum_n |c_n|^2}$
  - greedy algorithms

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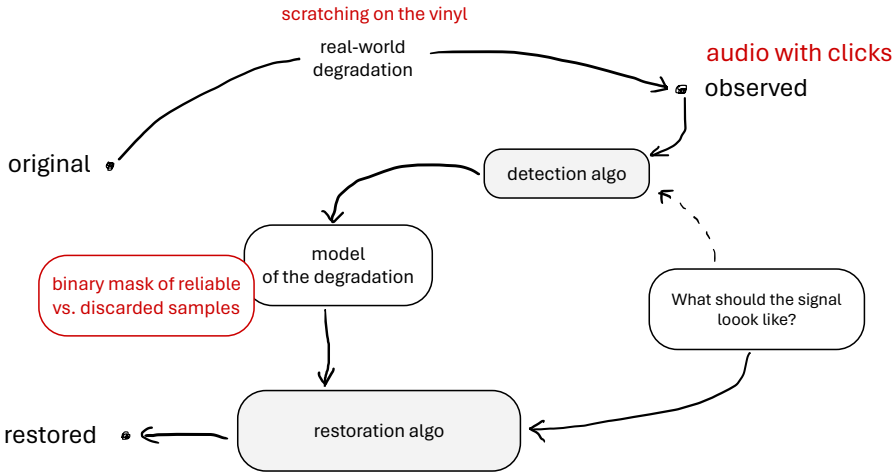
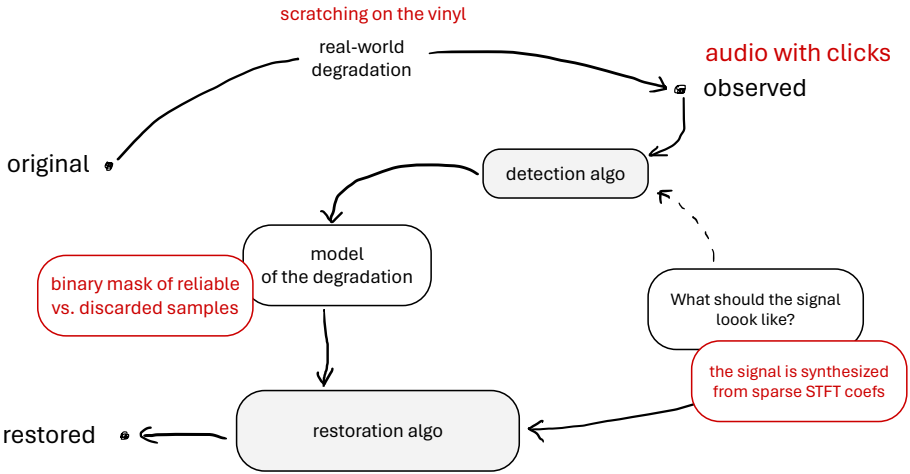
Optimization-based restoration

Generic scheme, part III



Optimization-based restoration

Generic scheme, part III



Example: convex method

ℓ_1-norm-based

$$\arg\min_{\mathbf{c}} \|\mathbf{c}\|_1 \quad \text{subject to} \quad M_\mathrm{R}\mathbf{D}\mathbf{c} = M_\mathrm{R}\mathbf{y}$$

$\mathbf{c}$	time-frequency coefficients of the restored signal
$\mathbf{y}$	observed time-domain signal
M_R	selection of the observed (reliable) samples
$\mathbf{D}$	synthesis operator: coefficients $\mapsto$ signal

- objective is convex
- constraints are linear

- practically solvable using splitting algorithms

Example: convex method

ℓ_1-norm-based

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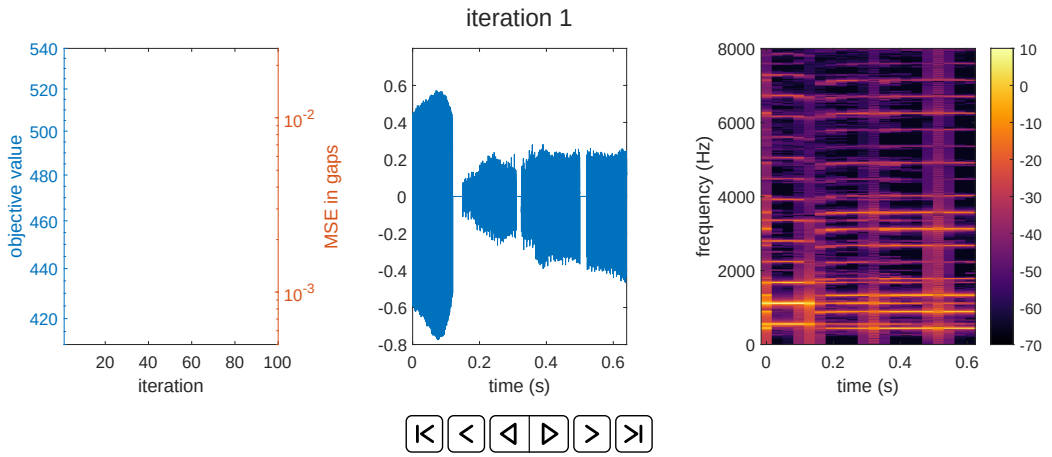

left – sparsity $\|\mathbf{c}\|_1$, MSE in time domain, **center** – synthesized solution $\mathbf{D}\mathbf{c}$, **right** – iterates (coefficients) $\mathbf{c}$

2nd example: convex method

ℓ_1-norm-based

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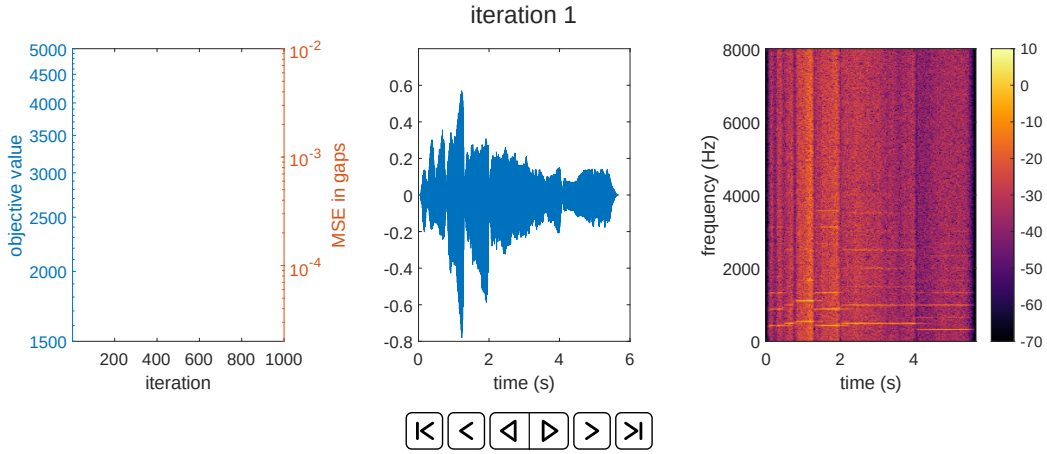
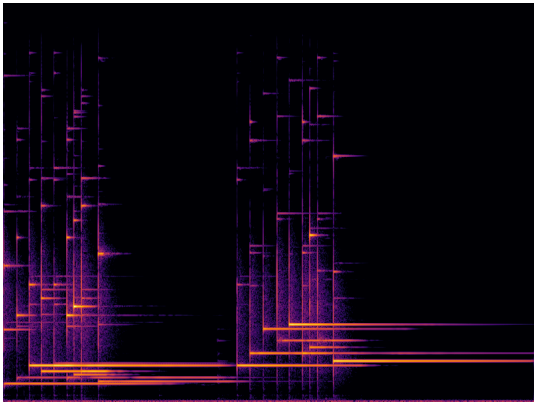


left – sparsity $\|\mathbf{c}\|_1$, MSE in time domain, **center** – synthesized solution $\mathbf{D}\mathbf{c}$, **right** – iterates (coefficients) $\mathbf{c}$, **audio link**

Other approaches and signal models

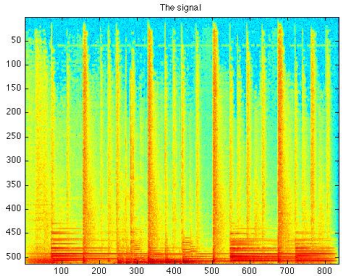
Social sparsity

- Music contains harmonic and transient components
- Horizontal and vertical structures in spectrogram
- Creating groups of coefficients (*group* or *social* sparsity)
- Using this as regularizers instead of plain sparsity



Social sparsity

Audio decomposition into harmonic and transient components

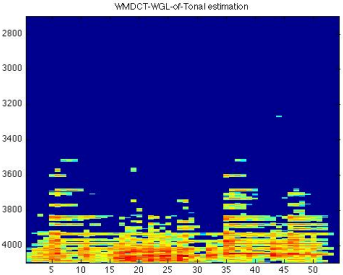


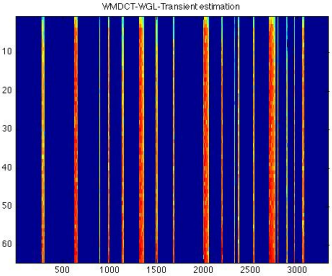
Audio links:

original

tonal

transient





Autoregressive modeling

Autoregressive (AR) model

Signal samples are linear combinations of previous samples + white noise.

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. . . from other perspective

AR process is an output of an all-pole IIR filter whose input is white noise.

Autoregressive modeling

Autoregressive (AR) model

Signal samples are linear combinations of previous samples + white noise.

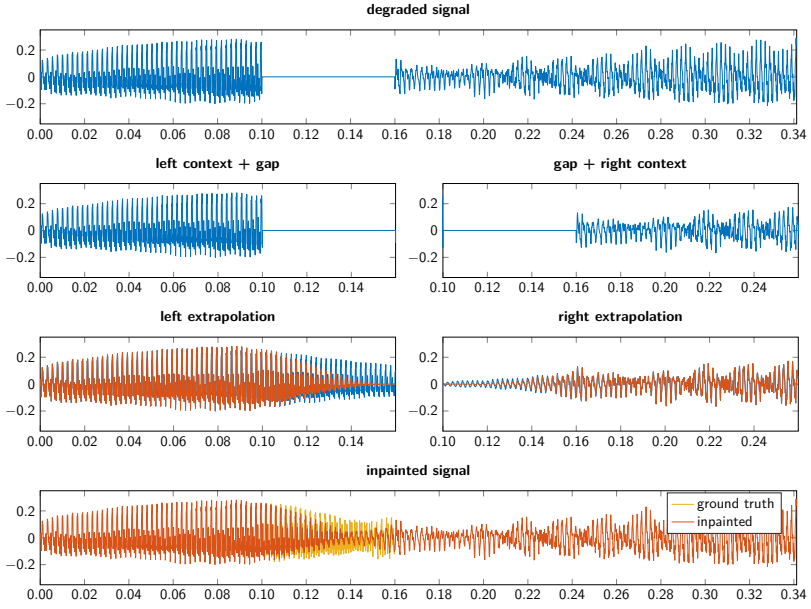
. . . from other perspective

AR process is an output of an all-pole IIR filter whose input is white noise.

- Mostly in line with the source-filter model of speech
- Applicable also on music in practice (with high model orders)

- Useful model for analysis (and extrapolation!) of time series, including audio signals

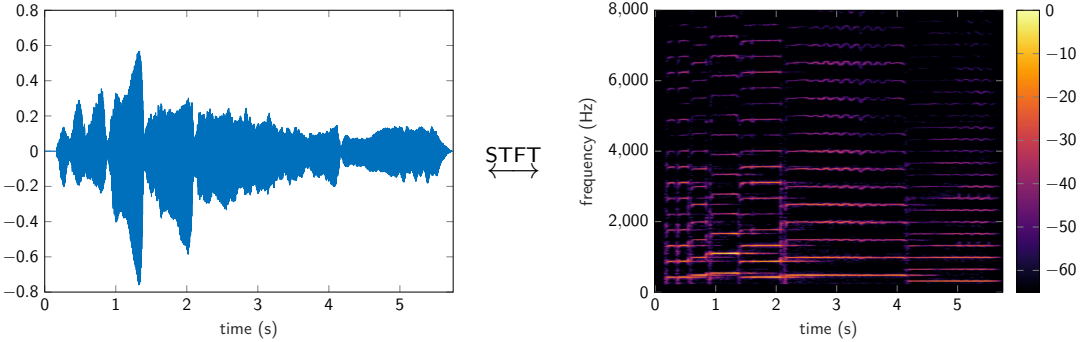
- Simple approach for long gaps – extrapolation of the reliable sections
- More demanding approach – Janssen's iteration



time axis in seconds, note the time-stretching of the right context

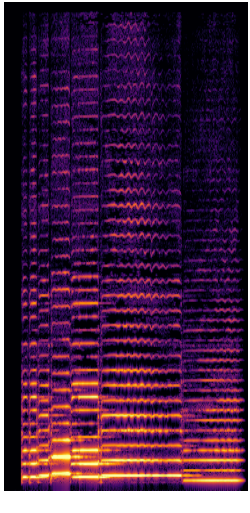
Matrix factorization

- Musical and speech signals consist of repeating spectral patterns (notes, syllables)
- The these patterns are (simultaneously) activated
- The mix differs in time, but the set of the patterns stays constant

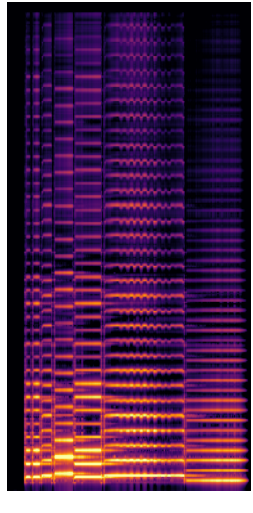


- This can be modeled as a non-negative matrix factorization (NMF) of the power spectrogram **P**

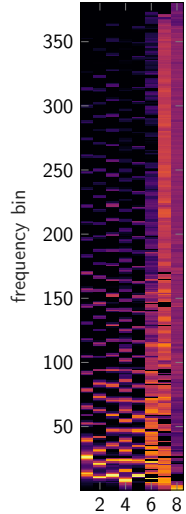
P



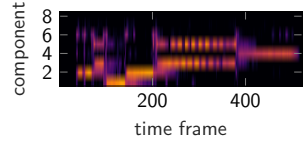
V = WH ≈ P



W



H



Audio links:

whole signal
component 1
component 2
component 3
component 4
component 5
component 6
component 7
component 8

On the computational complexity and evaluation

On the computational complexity

- Work focused on restoration quality, usually **far from real-time** processing
- Implementations in **Matlab** (LTFAT has backend in C) and **PyTorch**
- Sparsity-based methods
  - dominated by signal synthesis and analysis (STFT)
- AR-based methods
  - inversion of very large matrices
  - extrapolation-based method is much faster but with limited usability (only long gaps)
- NMF-based methods
  - even worse matrix manipulations

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- Computational complexity depends on the number of missing samples for some algorithms
- Usually there is a trade-off between time and restoration quality

Evaluation of the reconstruction quality

- SDR (signal-to-distortion ratio)
  - easiest and simplest method
  - compares only the physical similarity of waveforms
  - does not correspond to human hearing
- PEMO-Q
  - free Matlab implementation for research
- PEAQ, PESQ
  - ITU-R standard for evaluating audio (speech) quality
  - used very often, but with limited use for audio inpainting (not very sensitive)
  - free unofficial implementations available
    - C and Matlab
    - Python
- ViSQOL
  - relatively new method
  - C++ implementation on GitHub
  - good results in general
  - not tested for audio inpainting

ODG	Impairment description
0.0	Imperceptible
−1.0	Perceptible, but not annoying
−2.0	Slightly annoying
−3.0	Annoying
−4.0	Very annoying

Future research and possible cooperation

Future research and possible cooperation

- Ideas on future research
  - (model-based) deep learning
  - making current methods computationally feasible (possibly real-time)
- Bachelor & diploma & dissertation theses
- Collaboration
  - Jiří Schimmel
  - IRIT (Toulouse), ARI (Vienna)
- Research funding
  - MŠMT projects (2013–2016)
  - FWF–GAČR project (2017–2019)
  - GAČR projects (2020–2022, 2023–2025)

This is the end...

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Thank you for your attention!

Bonus:
Non-convex inpainting method

Non-convex inpainting method

Sparse audio inpainter (SPAIN)

$$\arg\min_{\mathbf{x},\mathbf{c}} \|\mathbf{c}\|_0 \quad \text{subject to} \quad M_R\mathbf{x} = M_R\mathbf{y},\ \|\mathbf{A}\mathbf{x} - \mathbf{c}\| < \varepsilon$$

$\mathbf{c}$	time-frequency coefficients of the restored signal
$\mathbf{x}$	the restored signal
$\mathbf{y}$	observed time-domain signal
M_R	selection of the observed (reliable) samples
$\mathbf{A}$	analysis operator: signal $\mapsto$ coefficients
ε	chosen tolerance

- constraints are linear and convex
- objective is non-convex

- theoretically not solvable in polynomial time
- need for a heuristic algorithm

Non-convex inpainting method

Sparse audio inpainter (SPAIN)

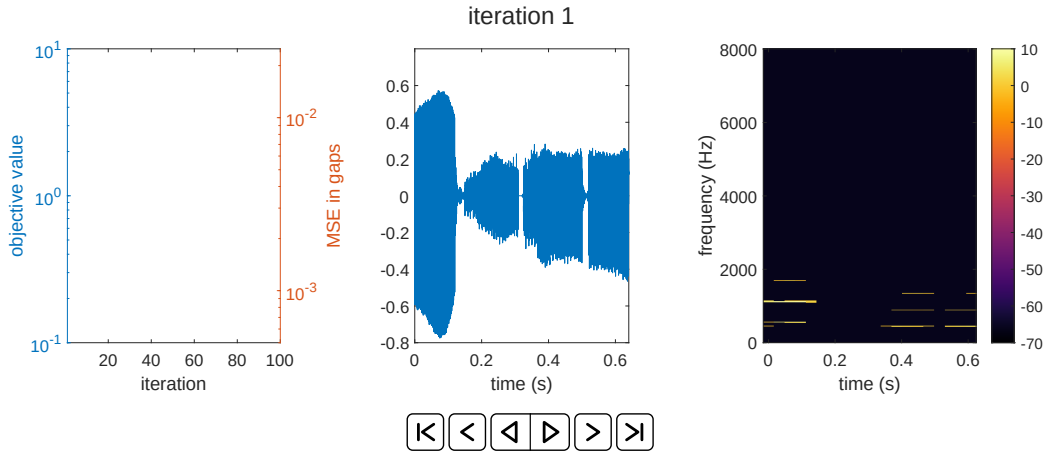
$$\arg\min_{\mathbf{x},\mathbf{c}} \|\mathbf{c}\|_0 \quad \text{subject to} \quad M_R\mathbf{x} = M_R\mathbf{y},\ \|\mathbf{A}\mathbf{x} - \mathbf{c}\| < \varepsilon$$

- it is almost solvable for known sparsity k of the solution
- first idea:
  - start with low k
  - search for the best approximation of $\mathbf{A}\mathbf{x}$ with k-sparse $\mathbf{c}$
  - increase k and repeat
- second idea:
  - merge it with a convex method (add some intermediate steps) such that it works in practice

Non-convex inpainting method

Sparse audio inpainter (SPAIN)

$$\arg\min_{\mathbf{x},\mathbf{c}} \|\mathbf{c}\|_0 \quad \text{subject to} \quad M_R\mathbf{x} = M_R\mathbf{y},\ \|\mathbf{A}\mathbf{x} - \mathbf{c}\| < \varepsilon$$



left – error $\|\mathbf{A}\mathbf{x} - \mathbf{c}\|$, MSE in time domain, **center** – the solution $\mathbf{x}$, **right** – the sparse coefficients $\mathbf{c}$

Non-convex inpainting method

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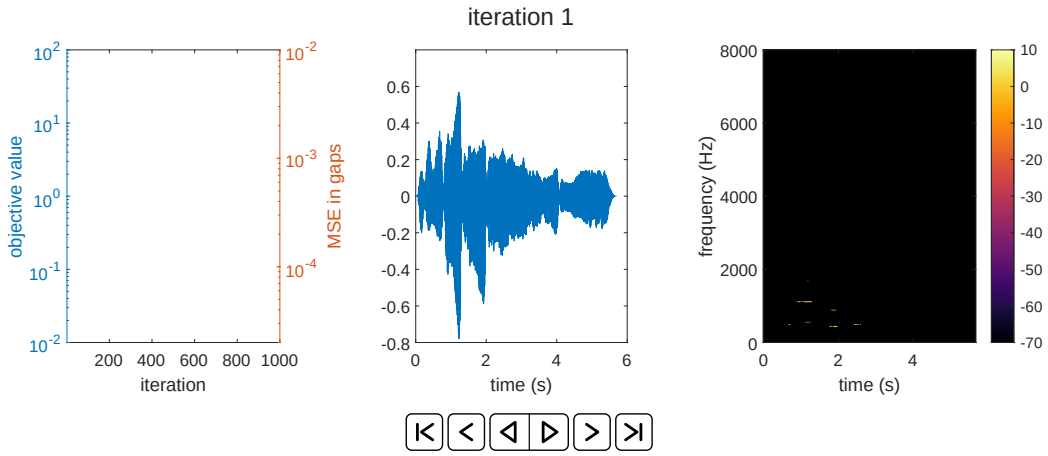


left – error $\|\mathbf{A}\mathbf{x} - \mathbf{c}\|$, MSE in time domain, **center** – the solution $\mathbf{x}$, **right** – the sparse coefficients $\mathbf{c}$, **audio link**

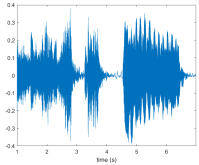
Bonus:

Audio database

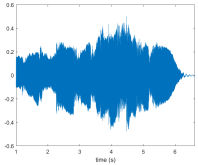
Evaluation of the reconstruction quality

Audio Database

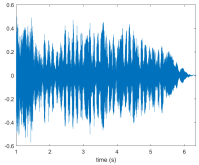
- 10 musical excerpts with approximate length 7 seconds
- Different degrees of signal sparsity w.r.t. STFT
- 44.1 kHz sampling rate, 16 bps



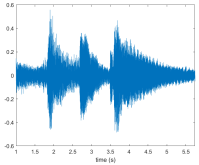
(a) violin



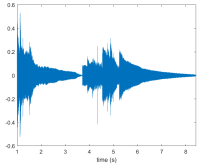
(b) clarinet



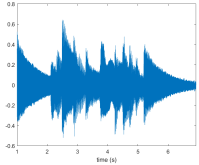
(c) bassoon



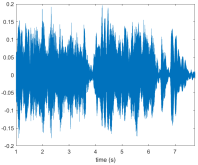
(d) harp



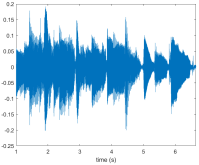
(e) glockenspiel



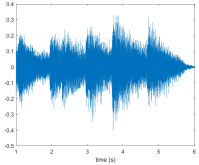
(f) celesta



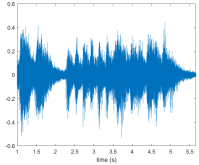
(g) accordion



(h) guitar



(i) piano



(j) wind ensemble

Evaluation of the reconstruction quality

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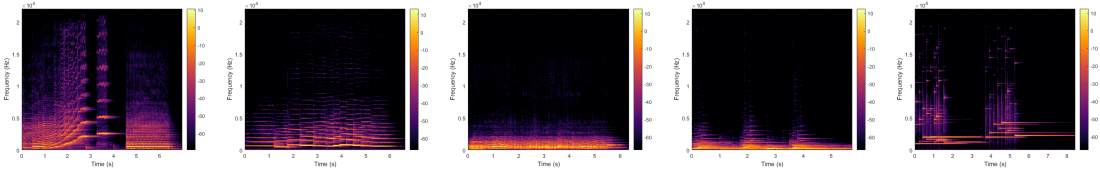

(a) violin (b) clarinet (c) bassoon (d) harp (e) glockenspiel

(f) celesta (g) accordion (h) guitar (i) piano (j) wind ensemble