Számítógépes Modellezés3

Limesz, Derivált, Integrál

Direkt (normál) értékadás (=)
p legyen a 6. Chebysev polinom.

```
p = ChebyshevT[6, x]
```

$-1 + 18 x^2 - 48 x^4 + 32 x^6$

Helyettesítési érték meghatározásához a változó/határozatlan helyébe konkrét értéket teszünk

```
TableForm[Table[ChebyshevT[n, x], {n, 6}]]
```

x
$-1 + 2 x^2$
$-3 x + 4 x^3$
$1 - 8 x^2 + 8 x^4$
$5 x - 20 x^3 + 16 x^5$
$-1 + 18 x^2 - 48 x^4 + 32 x^6$

```
p /. x → 2
```

1351

(Formális) derivált

```
D[p, x]
```

$36 x - 192 x^3 + 192 x^5$

```
D[p, {x, 2}]
```

$36 - 576 x^2 + 960 x^4$

```
D[p, x, x]
```

$36 - 576 x^2 + 960 x^4$

Ugyanez, ha polinomfüggvényt definiálunk hozzárendelési szabállyal:

```
q[x_] := ChebyshevT[6, x];
```

```
q[1]
```

1

```
D[q[x], x]
```

$36 x - 192 x^3 + 192 x^5$

Határértékek

```
Limit[p, x → Infinity]
```

∞

```
Limit[(x^2 - 1) / (2 - x - 3 x^2), x → -Infinity]
```

$-\frac{1}{3}$

```
Limit[Sin[x] / x, x → 0]
```

1

Integrálok

$\int E^{-x^2} \, dx$

$\frac{1}{2} \sqrt{\pi} \, \text{Erf}[x]$

$\int_{-\infty}^{\infty} E^{-x^2} \, dx$

$\sqrt{\pi}$

$\text{exp1} = \int_{-1}^{1} E^{-x^2} \, dx$

$\sqrt{\pi} \, \text{Erf}[1]$

```
N[%]
```

1.49365

```
N[exp1]
```

1.49365

```
NIntegrate[E^(-x^2), {x, -1, 1}]
```

1.49365

```
Integrate[E^(-x^2), {x, -1, 1}]
```

$\sqrt{\pi} \, \text{Erf}[1]$

$\int_{-1}^{1} x^2 \, dx$

$\frac{2}{3}$

Sorozatok, Függvények

Sorozat: Hozzárendelési szabály vagy a képhalmaz egy véges szelete vagy grafikon
:= (SetDelayed, késleltetett értékadás)

$$a[n\_] := \frac{n}{n^2+5};$$

```
a[1]
```

$\frac{1}{6}$

```
a[2]
```

$\frac{2}{9}$

```
N[a[2]]
```

0.222222

Véges sorozatok generálása

```
Table[i^2, {i, 9, 12}]
```

{81, 100, 121, 144}

```
Table[i^3, {i, 1, 5, 2}]
```

{1, 27, 125}

Az a sorozat elsõ tíz eleme

```
Table[a[n], {n, 10}]
```

$$\left\{\frac{1}{6}, \frac{2}{9}, \frac{3}{14}, \frac{4}{21}, \frac{1}{6}, \frac{6}{41}, \frac{7}{54}, \frac{8}{69}, \frac{9}{86}, \frac{2}{21}\right\}$$

```
t = Table[{n, a[n]}, {n, 10}]
```

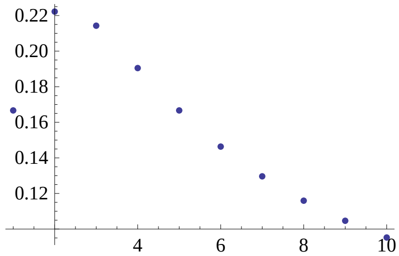
$$\left\{\left\{1, \frac{1}{6}\right\}, \left\{2, \frac{2}{9}\right\}, \left\{3, \frac{3}{14}\right\}, \left\{4, \frac{4}{21}\right\}, \left\{5, \frac{1}{6}\right\}, \left\{6, \frac{6}{41}\right\}, \left\{7, \frac{7}{54}\right\}, \left\{8, \frac{8}{69}\right\}, \left\{9, \frac{9}{86}\right\}, \left\{10, \frac{2}{21}\right\}\right\}$$

```
TableForm[t]
```

1	$\frac{1}{6}$
2	$\frac{2}{9}$
3	$\frac{3}{14}$
4	$\frac{4}{21}$
5	$\frac{1}{6}$
6	$\frac{6}{41}$
7	$\frac{7}{54}$
8	$\frac{8}{69}$
9	$\frac{9}{86}$
10	$\frac{2}{21}$

Ábrák

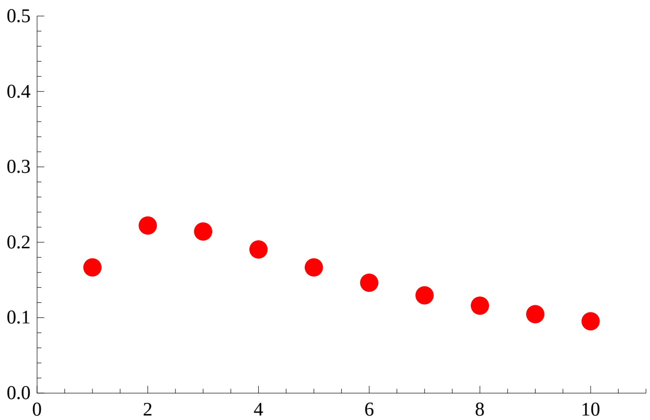
```
ListPlot[t]
```



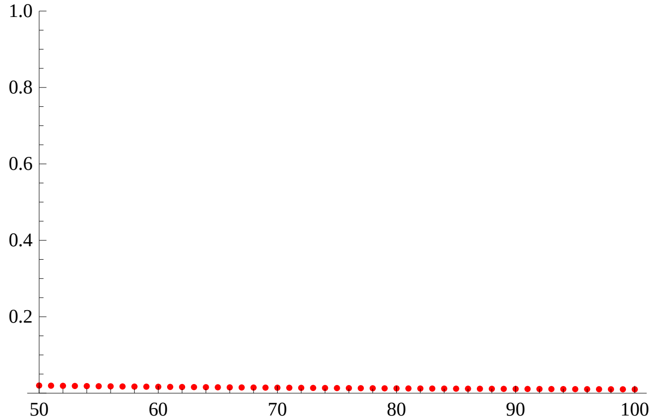

```
Options[ListPlot]
```

```
{AlignmentPoint → Center, AspectRatio → 1/GoldenRatio, Axes → True,
 AxesLabel → None, AxesOrigin → Automatic, AxesStyle → {}, Background → None,
 BaselinePosition → Automatic, BaseStyle → {}, ClippingStyle → None,
 ColorFunction → Automatic, ColorFunctionScaling → True, ColorOutput → Automatic,
 ContentSelectable → Automatic, CoordinatesToolOptions → Automatic, DataRange → Automatic,
 DisplayFunction :→ $DisplayFunction, Epilog → {}, Filling → None, FillingStyle → Automatic,
 FormatType :→ TraditionalForm, Frame → False, FrameLabel → None, FrameStyle → {},
 FrameTicks → Automatic, FrameTicksStyle → {}, GridLines → None, GridLinesStyle → {},
 ImageMargins → 0., ImagePadding → All, ImageSize → Automatic, ImageSizeRaw → Automatic,
 InterpolationOrder → None, Joined → False, LabelStyle → {}, MaxPlotPoints → ∞, Mesh → None,
 MeshFunctions → {#1 &}, MeshShading → None, MeshStyle → Automatic, Method → Automatic,
 PerformanceGoal :→ $PerformanceGoal, PlotLabel → None, PlotMarkers → None,
 PlotRange → Automatic, PlotRangeClipping → True, PlotRangePadding → Automatic,
 PlotRegion → Automatic, PlotStyle → Automatic, PreserveImageOptions → Automatic,
 Prolog → {}, RotateLabel → True, Ticks → Automatic, TicksStyle → {}}
```

```
ListPlot[t, PlotStyle → {Red, PointSize[.03]},
 AxesOrigin → {0, 0}, PlotRange → {{0, 11}, {0, .5}}]
```



```
ListPlot[Table[{n, a[n]}, {n, 50, 100}],
 PlotStyle → {RGBColor[1, 0, 0], PointSize[.01]}, PlotRange → {{49, 101}, {0, 1}}]
```



```
?? ListPlot
```

ListPlot[{y_1, y_2, …}] plots points corresponding to
a list of values, assumed to correspond to x coordinates 1, 2, … .
ListPlot[{{x_1, y_1}, {x_2, y_2}, …}] plots a list of points with specified x and y coordinates.
ListPlot[{$list_1$, $list_2$, …}] plots several lists of points. ≫

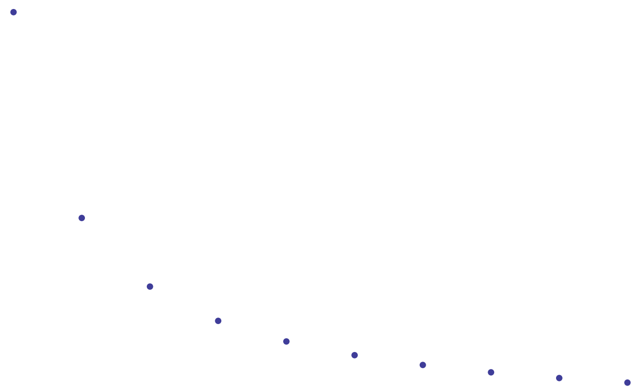
Attributes[ListPlot] = {Protected}

Options[ListPlot] = {AlignmentPoint → Center, AspectRatio → $\frac{1}{\text{GoldenRatio}}$, Axes → True, AxesLabel → None, AxesOrigin → Automatic, AxesStyle → {}, Background → None, BaselinePosition → Automatic, BaseStyle → {}, ClippingStyle → None, ColorFunction → Automatic, ColorFunctionScaling → True, ColorOutput → Automatic, ContentSelectable → Automatic, CoordinatesToolOptions → Automatic, DataRange → Automatic, DisplayFunction :→ $DisplayFunction, Epilog → {}, Filling → None, FillingStyle → Automatic, FormatType :→ TraditionalForm, Frame → False, FrameLabel → None, FrameStyle → {}, FrameTicks → Automatic, FrameTicksStyle → {}, GridLines → None, GridLinesStyle → {}, ImageMargins → 0., ImagePadding → All, ImageSize → Automatic, ImageSizeRaw → Automatic, InterpolationOrder → None, Joined → False, LabelStyle → {}, MaxPlotPoints → ∞, Mesh → None, MeshFunctions → {#1 &}, MeshShading → None, MeshStyle → Automatic, Method → Automatic, PerformanceGoal :→ $PerformanceGoal, PlotLabel → None, PlotMarkers → None, PlotRange → Automatic, PlotRangeClipping → True, PlotRangePadding → Automatic, PlotRegion → Automatic, PlotStyle → Automatic, PreserveImageOptions → Automatic, Prolog → {}, RotateLabel → True, Ticks → Automatic, TicksStyle → {}}

Ha az {{1,a1},{2,a2},{n,an}} pontokat akarjuk ábrázolni:

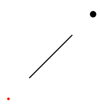
In[27]:=
```
ListPlot[Table[1 / n, {n, 10}], Axes → False]
```

Out[27]=

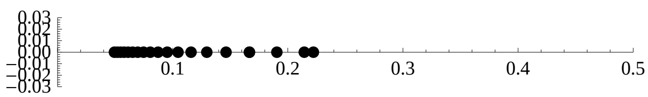


Grafikus objektumok ábrázolása

```
Show[Graphics[{PointSize[.02], Hue[.01], Point[{0, 0}]}],
 Graphics[Point[{1, 1}]], Graphics[Line[{{1 / 4, 1 / 4}, {3 / 4, 3 / 4}}]]]
```



```
Show[Table[Graphics[{PointSize[.02], Point[{a[n], 0}]}], {n, 20}],
 Axes → True, PlotRange → {{0, 1 / 2}, {-.03, .03}}]
```

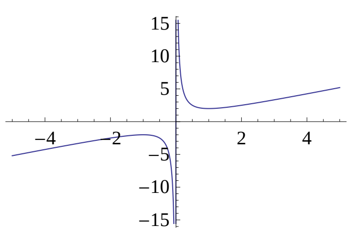



```
Show[Table[Graphics[{PointSize[.02], Point[{a[n], 0}]}], {n, 20}], Axes →
  False, PlotRange → {{0, 1 / 2}, {-.03, .03}}]
```

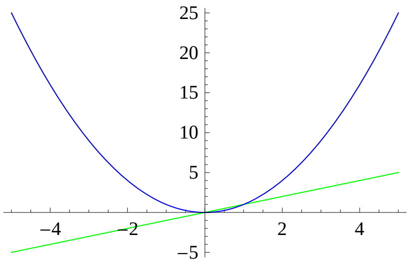
Függvény: Hozzárendelési szabály vagy grafikon

```
f[x_] := x + 1 / x;
```

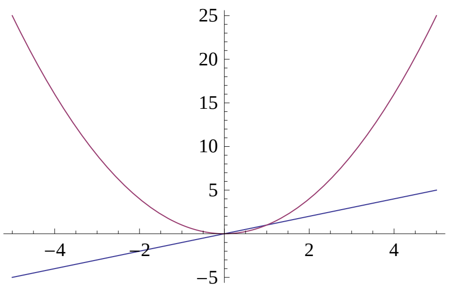
```
Plot[f[x], {x, -5, 5}]
```



```
Plot[{x, x^2}, {x, -5, 5}, PlotStyle → {{Green}, {Blue}}]
```



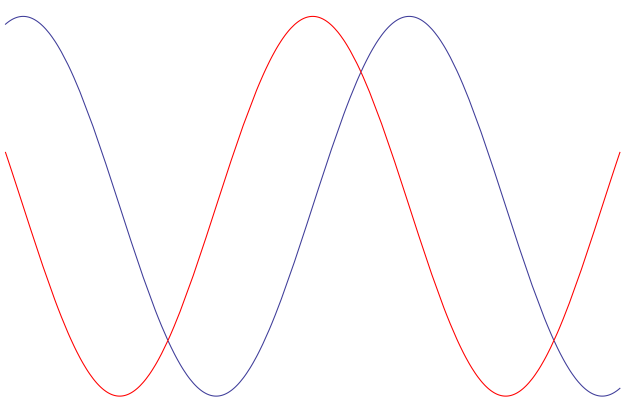

```
Plot[{x, x^2}, {x, -5, 5}]
```




```
MyFun[x_] := ArcTan[x];

MyDFun[x_] := D[MyFun[x], x];

MyDFun[x]
```

$$\frac{1}{1+x^2}$$

```
? Plot

?? Plot

Plot[{Sin[x], Cos[x]}, {x, -5, 5}, PlotStyle → {{}, {RGBColor[1, 0, 0]}}, Axes → False]
```



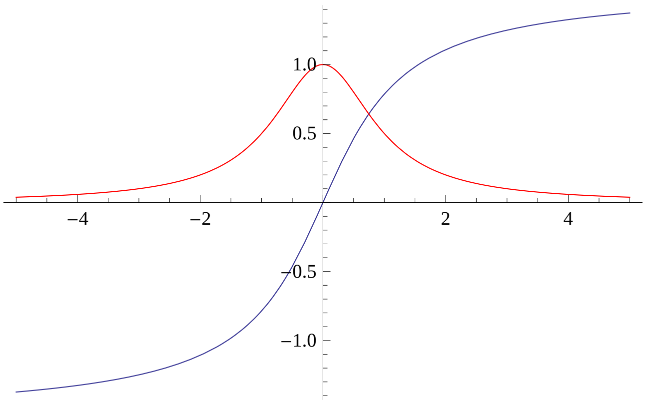
```
?? Plot
```

Plot[f, {x, x_{min}, x_{max}}] generates a plot of f as a function of x from x_{min} to x_{max}.
Plot[{f_1, f_2, …}, {x, x_{min}, x_{max}}] plots several functions f_i. ≫

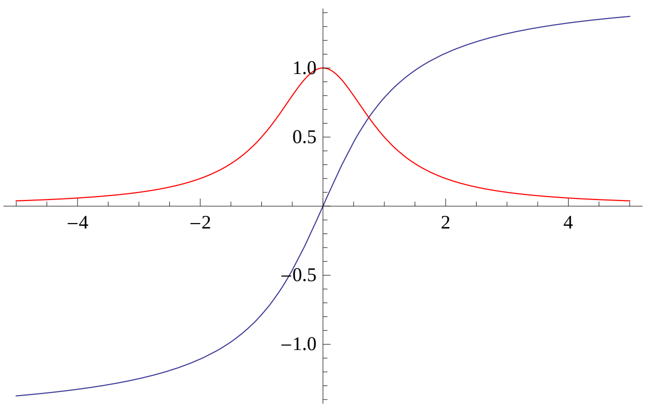
```
L = {MyFun[x], MyDFun[x]}
```

$\left\{\text{ArcTan}[x], \frac{1}{1+x^2}\right\}$

```
Plot[L, {x, -5, 5}, PlotStyle → {{}, {RGBColor[1, 0, 0]}}]
```




```
Plot[Evaluate[{MyFun[x], MyDFun[x]}], {x, -5, 5}, PlotStyle → {{}, {RGBColor[1, 0, 0]}}]
```



Gyakorló feladatok

Rajzoljunk le egy rac törtfgv (p/q) száml, nev + egy 3. konstans függvényt, ami piros⟺ ha p/q>0 és kék ha negatív!

Pl. f[x_]:=(x^2+x+6)/(x^2-2x-8)

Hint ?PlotStyle ?If ? ColorFunction

```
MyCF[x_, y_] := If[y > 0, Red, Blue]
```

```
MyCF[1, 2]
```

```
RGBColor[1, 0, 0]
```

```
MyCF[1, -3]
```

```
RGBColor[0, 0, 1]
```

```
Clear[p, q];
p[x_] = x^2 + x - 6; q[x_] = x^2 - 2 x - 8;
```

```
? ColorFunction
```

```
ColorFunction is an option for graphics functions which
   specifies a function to apply to determine colors of elements. ≫
```

```
36
```

Tiszta függvények (pure function)

```
(#^3 + 1) &[3]
```

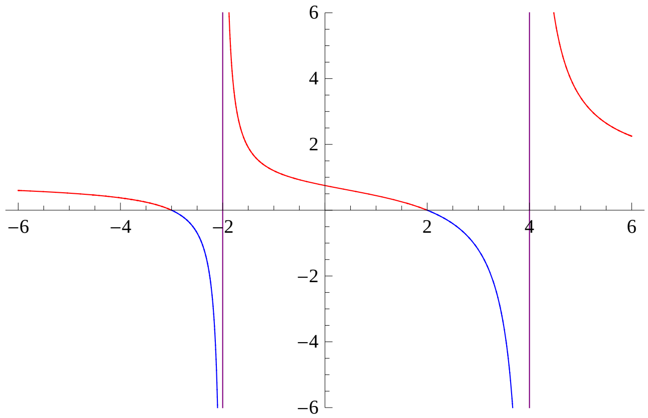
```
28
```

```
Sq[x_] := x^2
```

```
Sq[6]
```

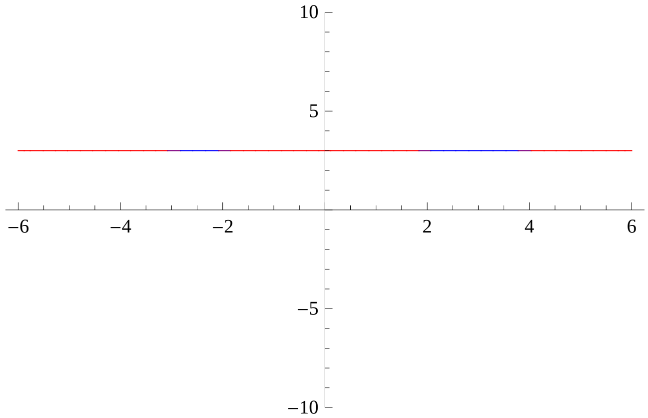
```
36
```

```
g2 = Plot[p[x] / q[x], {x, -6, 6}, ColorFunction → (MyCF[#1, #2] &)
  , ColorFunctionScaling → False, PlotRange → {-6, 6}]
```

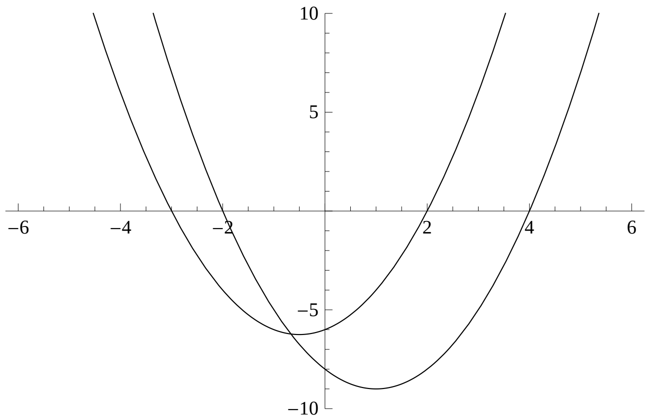



```
g2 = Plot[p[x] / q[x], {x, -6, 6}, ColorFunction →
   (If[#2 > 0, Red, Blue] &), ColorFunctionScaling → False, PlotRange → {-10, 10}]
```

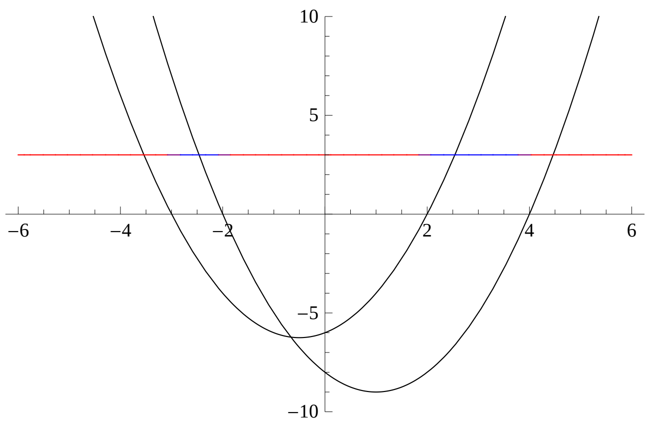
```
g2 = Plot[3, {x, -6, 6}, ColorFunction →
   (If[p[#1] / q[#1] > 0, Red, Blue] &), ColorFunctionScaling → False, PlotRange → {-10, 10}]
```



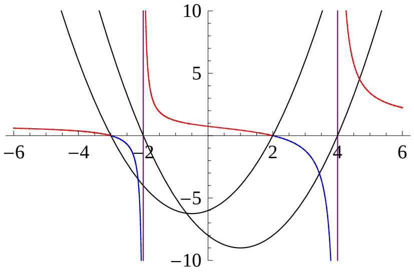

```
g1 = Plot[{p[x], q[x]}, {x, -6, 6}, PlotStyle -> Black, PlotRange → {-10, 10}]
```




```
Show[g1, g2]
```



```
Show[g1, g2]
```



Rajzoljuk le Sin[x] grafikonját 0 körüli Taylor polinomjaival

```
Tp[expr_, var_, n_] := Sum[(D[expr, {var, k}] /. var → 0) var^k / k!, {k, 0, n}]
```

```
Normal[Series[Sin[x], {x, 0, 3}]]
```

$$x - \frac{x^3}{6}$$

Listamüveletek

```
Prepend[{2, 3, 4}, 1]
```

```
{1, 2, 3, 4}
```

```
{1, {2, 3, 4}}
```

```
{1, {2, 3}}
```

```
{1, Sequence @@ {2, 3, 4}}
```

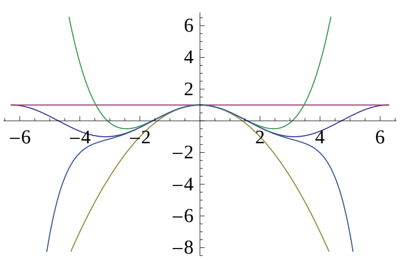
```
{1, 2, 3, 4}
```

```
Flatten[{1, {2, 3, 4}}]
```

```
{1, 2, 3, 4}
```

```
Plot[Evaluate[{Sin[x], Tp[Sin[x], x, 3]}], {x, -2 Pi, 2 Pi}]
```

```
Plot[Evaluate[{Cos[x], Sequence @@ Table[Tp[Cos[x], x, k], {k, 0, 6, 2}]}], {x, -2 Pi, 2 Pi}]
```



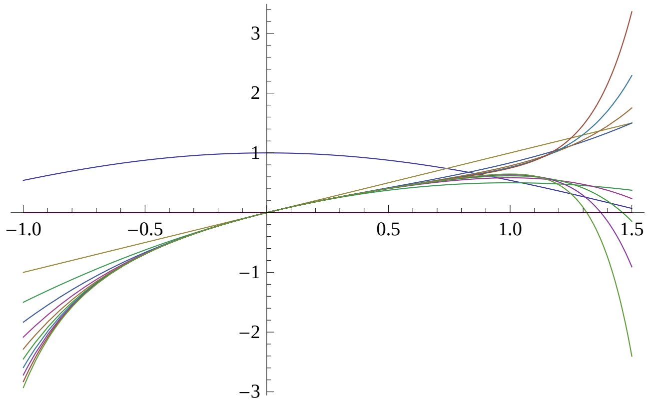

```
Tp[Sin[x], x, 3]
```

$$x - \frac{x^3}{6}$$

```
Table[Tp[Sin[x], x, n], {n, 5}]
```

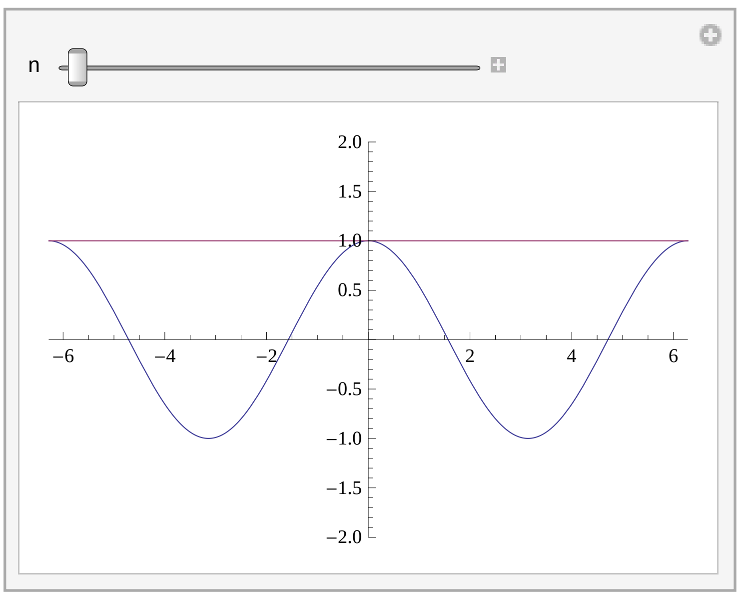
$$\left\{x,\ x,\ x-\frac{x^3}{6},\ x-\frac{x^3}{6},\ x-\frac{x^3}{6}+\frac{x^5}{120}\right\}$$

```
Plot[Evaluate[Prepend[Table[Tp[Log[1 + x], x, n], {n, 0, 10}], Cos[x]]], {x, -1, 3 / 2}]
```



Interaktív vizualizáció (Manipulate, Animate)

```
Manipulate[Plot[Evaluate[{Cos[x], Tp[Cos[x], x, n]}],
  {x, -2 Pi, 2 Pi}, PlotRange → {{-2 Pi, 2 Pi}, {-2, 2}}], {n, 0, 20, 2}]
```



```
Manipulate[Plot[Evaluate[{Log[1 + x], Tp[Log[1 + x], x, n]}],
  {x, -1, 2}, PlotRange → {{-1, 2}, {-5, 5}}], {n, 0, 20, 1}]
```