Számítógépes Modellezés—4

Gyakorló feladatok

▪ 1

Rajzoljunk le egy rac törtfgv (p/q) száml, nev + egy 3. konstans függvényt, ami piros⟺ ha p/q>0 és kék ha negatív!

Pl. f[x_]:=(x^2+x+6)/(x^2-2x-8)

Hint ?PlotStyle ?If ? ColorFunction

```
Clear[p, q];
p[x_] = x^2 + x - 6; q[x_] = x^2 - 2 x - 8;
```

```
p[x] / q[x]
```

$$\frac{-6 + x + x^2}{-8 - 2 x + x^2}$$

```
((x + 1) / (1 - x^2))[[2, 1]]
```

$1 - x^2$

```
Numerator[((x + 1) / (1 - x^2))]
```

$1 + x$

```
Solve[Numerator[((x + 1) / (1 - x^2))] == 0, x]
```

{{x → -1}}

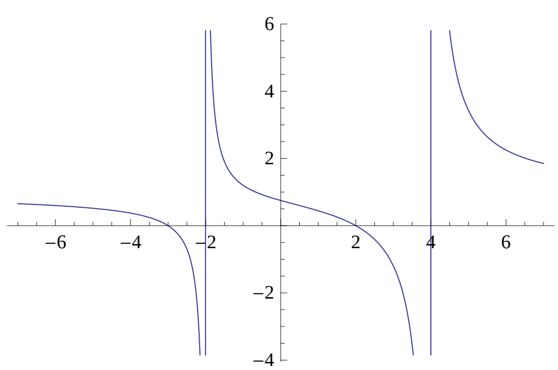
```
Lz = x /. Solve[p[x] == 0, x]
```

{-3, 2}

```
Ls = x /. Solve[q[x] == 0, x]
```

{-2, 4}

```
Plot[p[x] / q[x], {x, -7, 7}]
```



```
D[p[x] / q[x], x] // Together
```

$$\frac{-20-4x-3x^2}{\left(-8-2x+x^2\right)^2}$$

```
f1[x_] := Module[{z}, D[p[z] / q[z], z] /. z → x]
```

```
Together[f1[x]]
```

$$\frac{-20-4x-3x^2}{\left(-8-2x+x^2\right)^2}$$

```
NSolve[f1[x] == 0, x]
```

```
{{x → -0.666667 + 2.49444 ⅈ}, {x → -0.666667 - 2.49444 ⅈ}}
```

```
Denominator[((x + 1) / (1 - x^2))]
```

$1 - x^2$

```
? ColorFunction
```

```
ColorFunction is an option for graphics functions which
   specifies a function to apply to determine colors of elements.  >>
```

```
MyCF[x_, y_] := If[y > 0, Red, Blue]
```

```
MyCF2[x_, y_] := If[x > 0, Red, Blue]
```

```
MyCF[1, -1]
```

```
RGBColor[0, 0, 1]
```

```
3^2
```

```
9
```

```
g[x_] := x^3
```

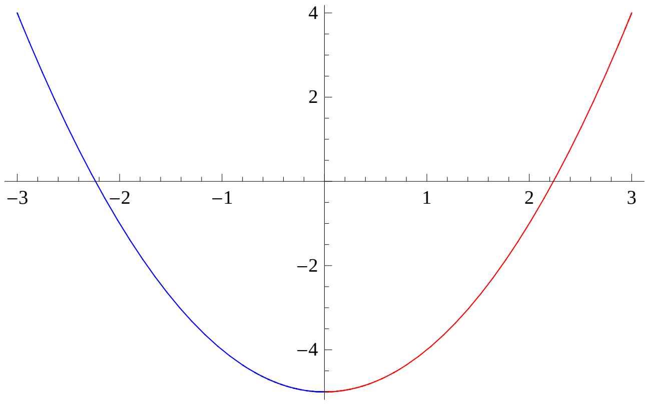
```
g[5]
```

```
125
```

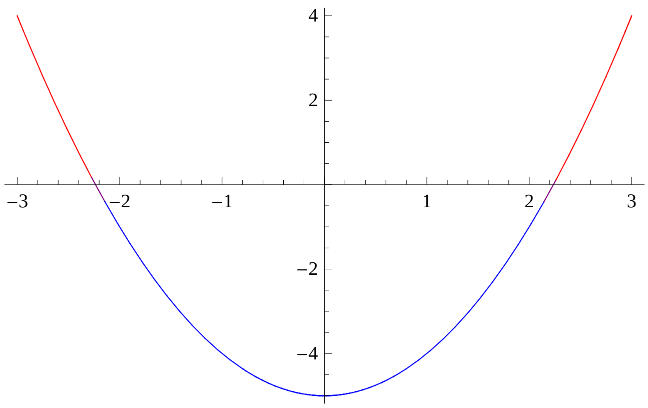
```
(#^3) &[5]
```

```
125
```

```
Plot[x^2 - 5, {x, -3, 3}, ColorFunction → (MyCF2[#1, #2] &), ColorFunctionScaling → False]
```




```
Plot[x^2 - 5, {x, -3, 3}, ColorFunction → (MyCF[#1, #2] &), ColorFunctionScaling → False]
```



Tiszta függvények (pure function)

```
pp[x_] = x^3 - x;
```

```
pp[1]
```

```
0
```

```
If[((D[pp[z], z] /. (z → 2)) > 0), Red, Blue]
```

```
RGBColor[1, 0, 0]
```

```
z
```

```
z
```

```
D[pp[z], z]
```

$-1 + 3\, z^2$

```
(D[pp[z], z] /. (z → x))
```

$-1 + 3\, x^2$

```
h[x_, y_] = x^2 + y^2
```

$x^2 + y^2$

```
h[{x_, y_}] := x + y
```

```
h[1, 2]
```

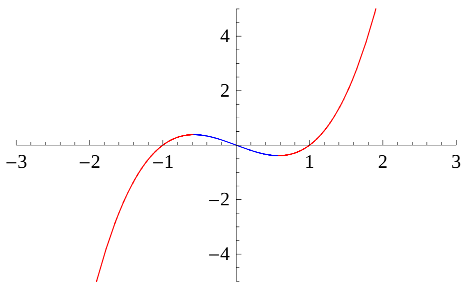
5

```
h[{1, 2}]
```

3

```
MyCF3[x_, y_] := If[(Evaluate[D[pp[z], z]] /. z -> x) > 0, Red, Blue]
```

```
Plot[pp[x], {x, -3, 3}, ColorFunction → (MyCF3[#1, #2] &),
 ColorFunctionScaling → False, PlotRange → {{-3, 3}, {-5, 5}}]
```



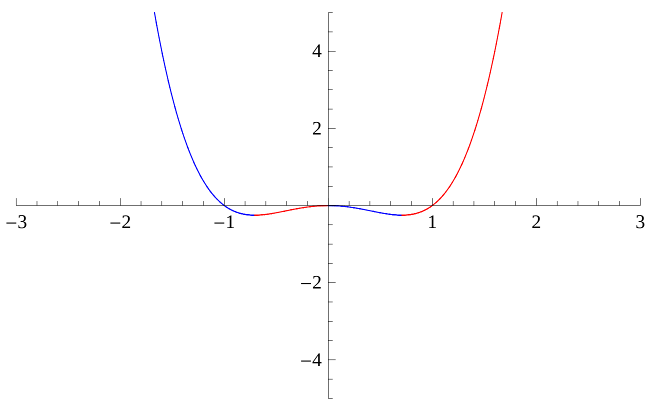

```
pq[x_] := x^4 - x^2
```

```
MyCF4[x_, y_] := If[(Evaluate[D[pq[z], z]] /. z -> x) > 0, Red, Blue]
```

```
pq[2]
```

12

```
Plot[pq[x], {x, -3, 3}, ColorFunction → (MyCF4[#1, #2] &),
 ColorFunctionScaling → False, PlotRange → {{-3, 3}, {-5, 5}}]
```



```
Options[Plot]
```

```
{AlignmentPoint → Center, AspectRatio → 1/GoldenRatio, Axes → True, AxesLabel → None,
 AxesOrigin → Automatic, AxesStyle → {}, Background → None, BaselinePosition → Automatic,
 BaseStyle → {}, ClippingStyle → None, ColorFunction → Automatic, ColorFunctionScaling → True,
 ColorOutput → Automatic, ContentSelectable → Automatic, CoordinatesToolOptions → Automatic,
 DisplayFunction :→ $DisplayFunction, Epilog → {}, Evaluated → Automatic,
 EvaluationMonitor → None, Exclusions → Automatic, ExclusionsStyle → None,
 Filling → None, FillingStyle → Automatic, FormatType :→ TraditionalForm, Frame → False,
 FrameLabel → None, FrameStyle → {}, FrameTicks → Automatic, FrameTicksStyle → {},
 GridLines → None, GridLinesStyle → {}, ImageMargins → 0., ImagePadding → All,
 ImageSize → Automatic, ImageSizeRaw → Automatic, LabelStyle → {}, MaxRecursion → Automatic,
 Mesh → None, MeshFunctions → {#1 &}, MeshShading → None, MeshStyle → Automatic,
 Method → Automatic, PerformanceGoal :→ $PerformanceGoal, PlotLabel → None,
 PlotPoints → Automatic, PlotRange → {Full, Automatic}, PlotRangeClipping → True,
 PlotRangePadding → Automatic, PlotRegion → Automatic, PlotStyle → Automatic,
 PreserveImageOptions → Automatic, Prolog → {}, RegionFunction → (True &),
 RotateLabel → True, Ticks → Automatic, TicksStyle → {}, WorkingPrecision → MachinePrecision}
```

■ 2

Rajzoljuk le Sin[x] grafikonját 0 körüli Taylor polinomjaival

In[141]:=
```
TP[expr_, var_, x0_, n_] := Sum[(D[expr, {var, k}] /. var → x0) / k! (var - x0)^k, {k, 0, n}]
```

In[142]:=
```
TP[Sin[x], x, 0, 3]
```

Out[142]= $x - \frac{x^3}{6}$

In[143]:=
```
Table[TP[Sin[z], z, 0, k], {k, 0, 9, 2}] // TableForm
```

Out[143]//TableForm=

0

z

$z - \frac{z^3}{6}$

$z - \frac{z^3}{6} + \frac{z^5}{120}$

$z - \frac{z^3}{6} + \frac{z^5}{120} - \frac{z^7}{5040}$

```
Table[TP[Sin[z], z, 0, k], {k, 0, 9}] // TableForm
```

0

z

z

$z - \frac{z^3}{6}$

$z - \frac{z^3}{6}$

$z - \frac{z^3}{6} + \frac{z^5}{120}$

$z - \frac{z^3}{6} + \frac{z^5}{120}$

$z - \frac{z^3}{6} + \frac{z^5}{120} - \frac{z^7}{5040}$

$z - \frac{z^3}{6} + \frac{z^5}{120} - \frac{z^7}{5040}$

$z - \frac{z^3}{6} + \frac{z^5}{120} - \frac{z^7}{5040} + \frac{z^9}{362\,880}$

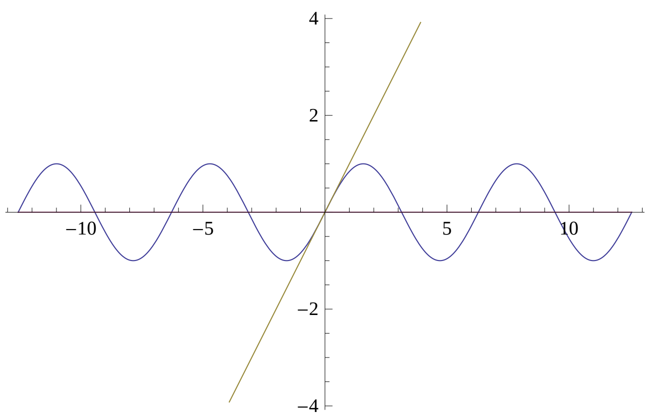
```
Prepend[{1, 2}, 3]
```

{3, 1, 2}

```
Join[{3}, {1, 2}]
```

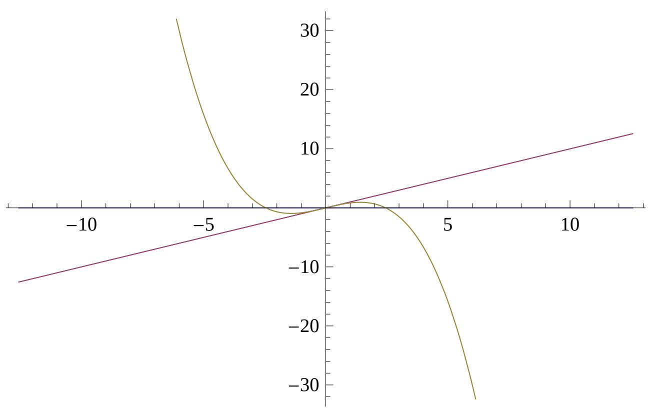
{3, 1, 2}

```
Plot[Evaluate[Prepend[Table[TP[Sin[x], x, 0, k], {k, 0, 2, 2}], Sin[x]]], {x, -4 Pi, 4 Pi}]
```



In[144]:= `Plot[Evaluate[Table[TP[Sin[x], x, 0, k], {k, 0, 4, 2}]], {x, -4 Pi, 4 Pi}]`

Out[144]=



```
Series[Sin[x], {x, 0, 5}]
```

$$x - \frac{x^3}{6} + \frac{x^5}{120} + O[x]^6$$

```
Normal[Series[Sin[x], {x, 0, 5}]]
```

$$x - \frac{x^3}{6} + \frac{x^5}{120}$$

```
TP[Sin[x], x, 0, 5]
```

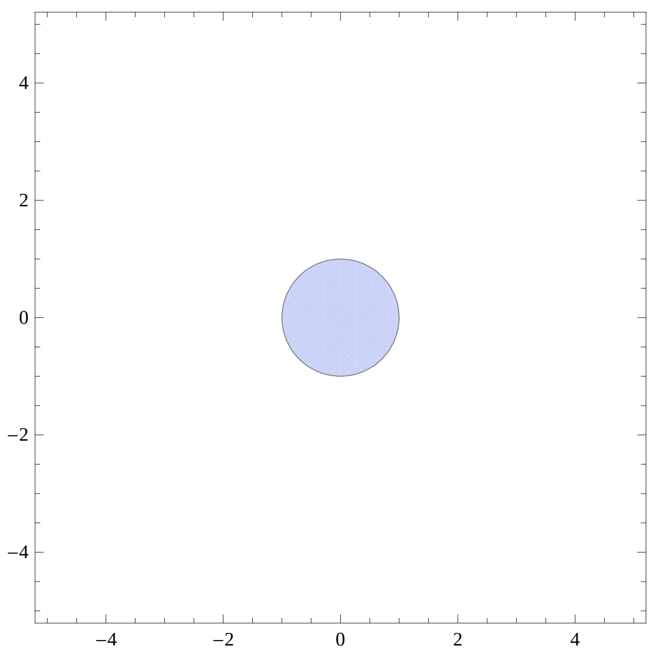
$$x - \frac{x^3}{6} + \frac{x^5}{120}$$

Listamüveletek

```
Solve[{x^2 + y^2 == 5, x - y == 1}, {x, y}]
```

```
ContourPlot[{x^2 + y^2 == 1, x - y == 1}, {x, -3, 3}, {y, -3, 3}]
```

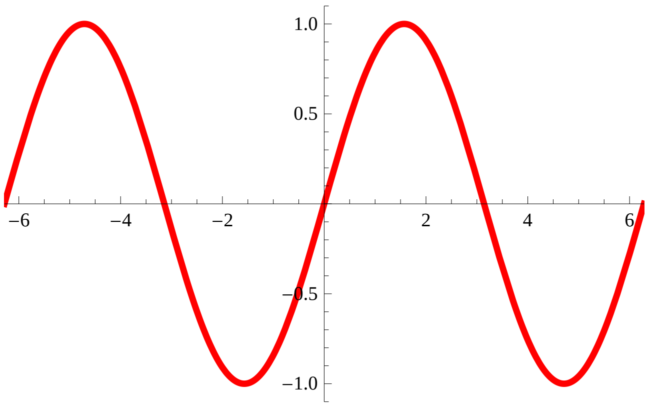
```
RegionPlot[x^2 + y^2 < 1, {x, -5, 5}, {y, -5, 5}]
```



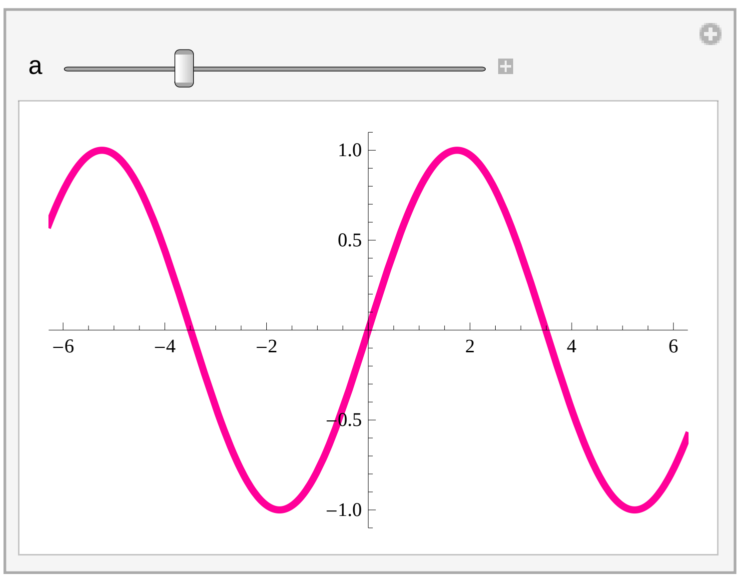

```
? *Plot*
```

Interaktív vizualizáció (Manipulate, Animate)

```
Plot[Sin[1 x], {x, -2 π, 2 π}, PlotRange → {{-2 π, 2 π}, {-1.1, 1.1}},
 PlotStyle → {Thickness[.01 (1 / 1)], Hue[1]}]
```




```
Manipulate[Plot[Sin[a x], {x, -2 π, 2 π}, PlotRange → {{-2 π, 2 π}, {-1.1, 1.1}},
  PlotStyle → {Thickness[.01 (1 / a)], Hue[a]}], {a, .5, 2, .1}]
```




```
Manipulate[TableForm[{{{(a + b) ^n, "=", Expand[(a + b) ^n]}}}], {n, 2, 5, 1}]
```

n

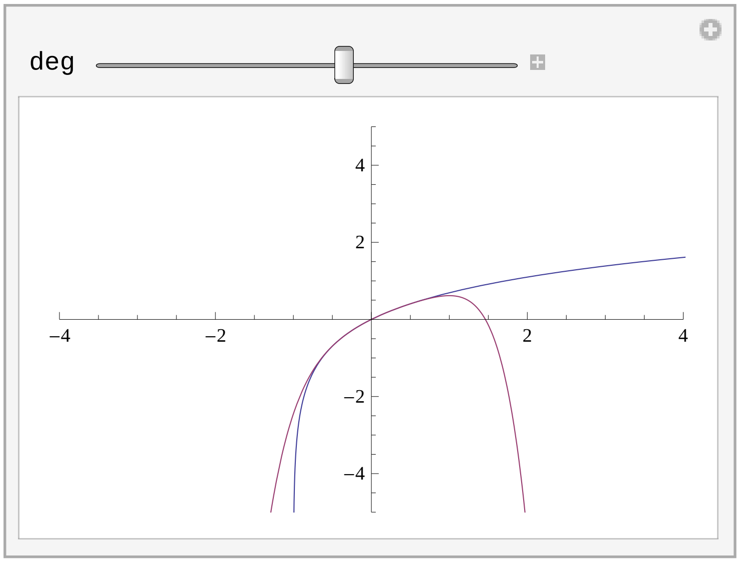
$(a+b)^4$

=

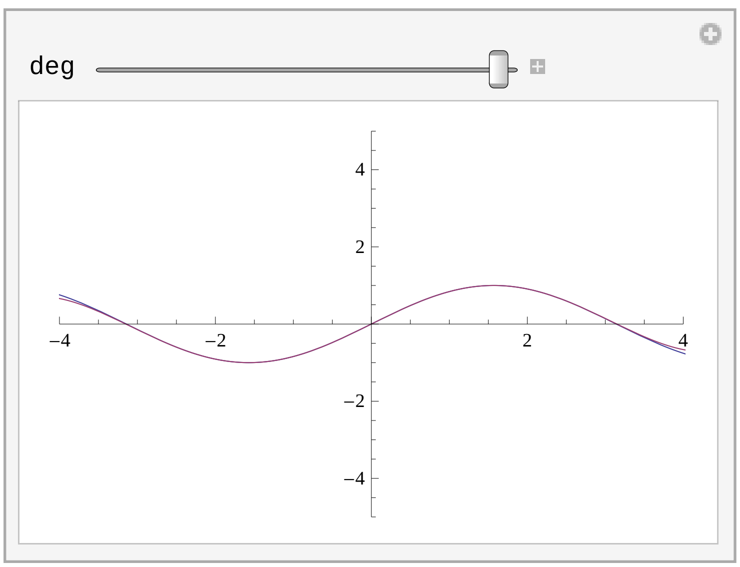
$a^4 + 4 a^3 b + 6 a^2 b^2 + 4 a b^3 + b^4$

Feladat: Interaktív Taylor polinom-fgv, érintôegyenes demo

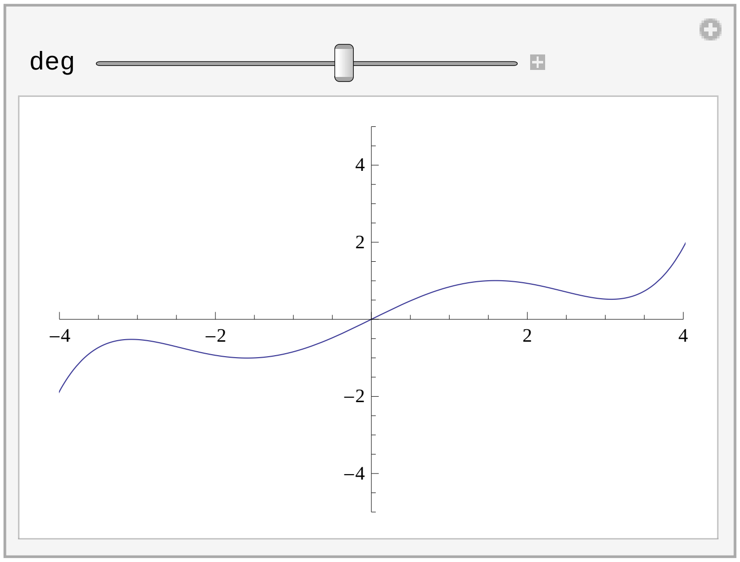
```
Manipulate[Plot[Evaluate[{Log[1 + x], TP[Log[1 + x], x, 0, deg]}],
  {x, -2 π, 2 π}, PlotRange → {{-4, 4}, {-5, 5}}], {deg, 0, 10, 1}]
```



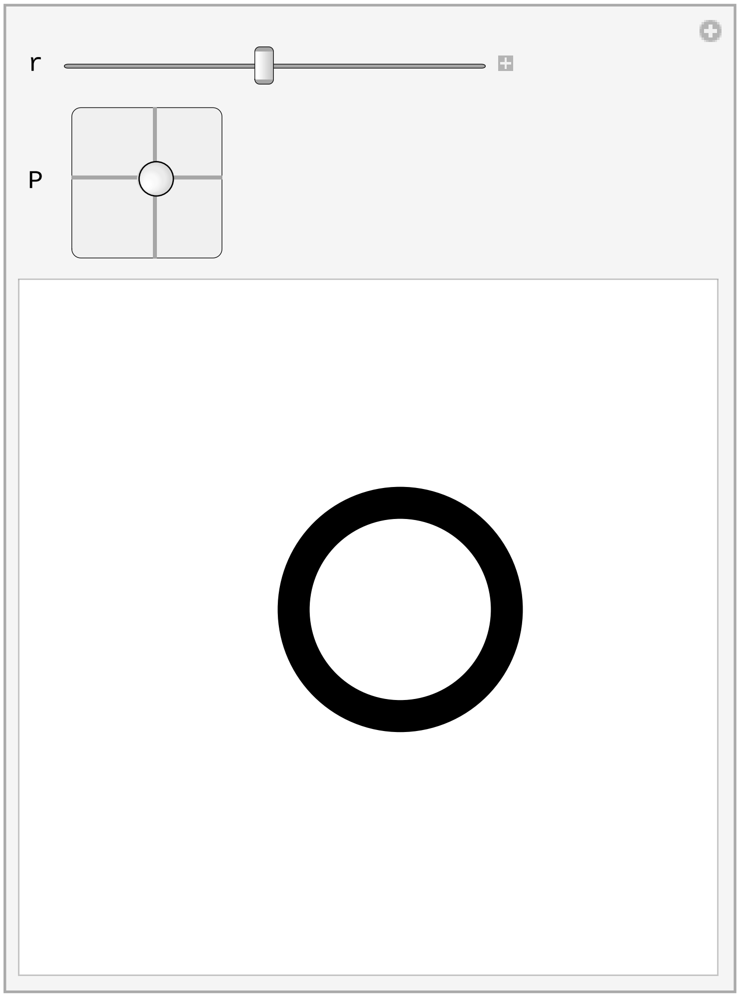

```
Manipulate[Plot[Evaluate[{Sin[x], TP[Sin[x], x, 0, deg]}],
  {x, -2 π, 2 π}, PlotRange → {{-4, 4}, {-5, 5}}], {deg, 0, 10, 1}]
```



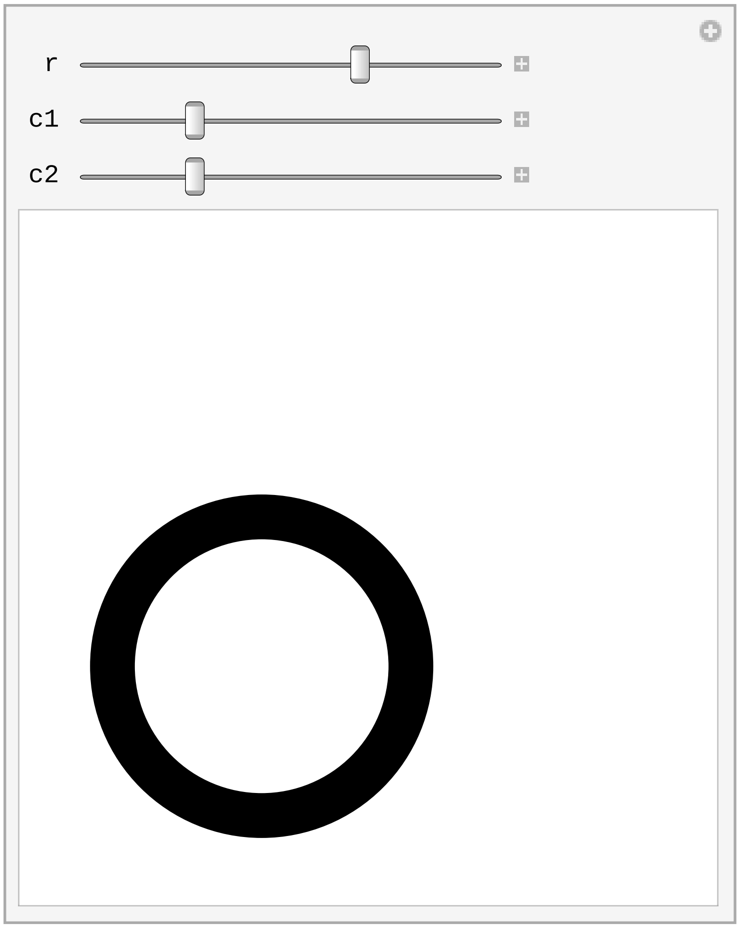
```
Manipulate[Plot[Evaluate[TP[Sin[x], x, 0, deg]],
  {x, -2 π, 2 π}, PlotRange → {{-4, 4}, {-5, 5}}], {deg, 0, 10, 1}]
```



```
Manipulate[Show[Graphics[{Thickness[r / 20], Circle[P, r]}], PlotRange -> {{-3, 3}, {-3, 3}}],
 {{r, 1}, .1, 2, .1}, {P, {-2, -2}, {2, 2}, Slider2D}]
```



```
Manipulate[Show[Graphics[{Thickness[r / 20], Circle[{c1, c2}, r]}],
  PlotRange -> {{-3, 3}, {-3, 3}}], {r, .1, 2, .1}, {c1, -2, 2, .1}, {c2, -2, 2, .1}]
```



Egyenletek

Solve[{eqns},{vars}]

```
Solve[x^2 - 2 x - 7 == 0, x]
```

```
x /. Solve[x^2 - 2 x - 7 == 0, x]
```

$\left\{1-2\sqrt{2},\ 1+2\sqrt{2}\right\}$

```
Solve[{x^2 + y^2 == 5, x - y == 2}, {x, y}]
```

$\left\{\left\{x\to\frac{1}{2}\left(2-\sqrt{6}\right),\ y\to\frac{1}{2}\left(-2-\sqrt{6}\right)\right\},\ \left\{x\to\frac{1}{2}\left(2+\sqrt{6}\right),\ y\to\frac{1}{2}\left(-2+\sqrt{6}\right)\right\}\right\}$

```
{x, y} /. Solve[{x^2 + y^2 == 5, x - y == 2}, {x, y}]
```

$\left\{\left\{\frac{1}{2}\left(2-\sqrt{6}\right),\ \frac{1}{2}\left(-2-\sqrt{6}\right)\right\},\ \left\{\frac{1}{2}\left(2+\sqrt{6}\right),\ \frac{1}{2}\left(-2+\sqrt{6}\right)\right\}\right\}$

Ellenôrzés

```
{x^2 + y^2 == 5, x - y == 2} /. Solve[{x^2 + y^2 == 5, x - y == 2}, {x, y}]
```

```
{{True, True}, {True, True}}
```

```
NSolve[x^2 - 2 x - 7 == 0, x]
```

```
{{x → -1.82843}, {x → 3.82843}}
```

```
NSolve[Cos[x] == x, x]
```

```
Solve::tdep :
 The equations appear to involve the variables to be solved for in an essentially
     non-algebraic way. >>
```

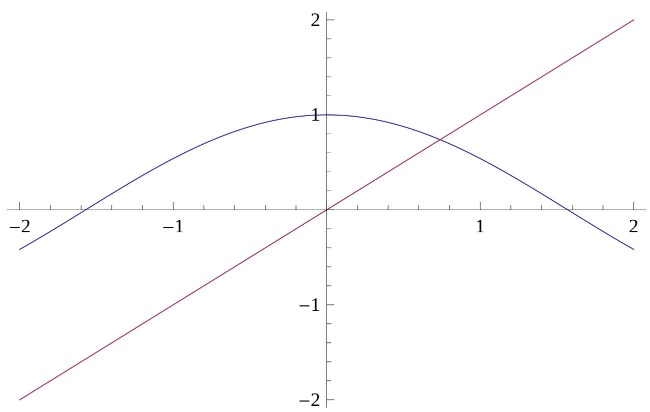
```
NSolve[Cos[x] == x, x]
```

Megoldás: iterációs módszerek, késôbb.

```
FindRoot[Cos[x] == x, {x, .9}]
```

```
{x → 0.739085}
```

```
Plot[{Cos[x], x}, {x, -2, 2}]
```



Lineáris egyeneletrendszer

```
A = {{1, 2}, {3, 4}}; b = {3, 4};
```

```
LinearSolve[A, b]
```

$\left\{-2, \frac{5}{2}\right\}$

```
Solve[{x + 2 y == 3, 3 x + 4 y == 4}, {x, y}]
```

$\left\{\left\{x \to -2, y \to \frac{5}{2}\right\}\right\}$

Diiferenciaegyenlet, egyenletrendszer

```
RSolve[a[n] == 2 a[n - 1], a[n], n]
```

$\{\{a[n] \to 2^{-1+n}\, C[1]\}\}$

```
RSolve[{a[n] == 2 a[n - 1], a[1] == 2}, a[n], n]
```

$\{\{a[n] \to 2^{n}\}\}$

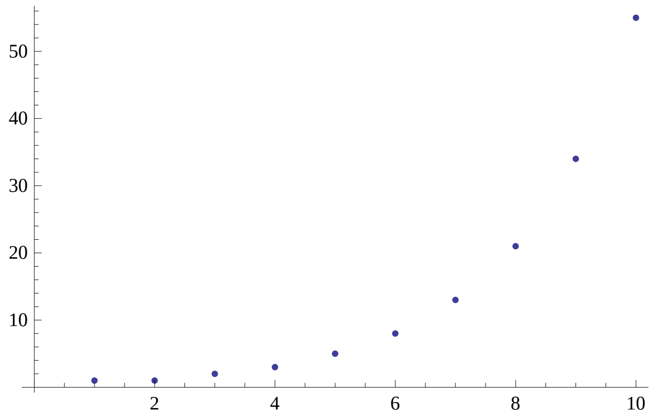
```
RSolve[{a[n] == a[n - 1] + a[n - 2], a[0] == 0, a[1] == 1}, a[n], n]
```

{{a[n] → Fibonacci[n]}}

```
a[n] /. RSolve[{a[n] == a[n - 1] + a[n - 2], a[0] == 0, a[1] == 1}, a[n], n][[1]]
```

Fibonacci[n]

```
ListPlot[Table[Evaluate[
  a[n] /. RSolve[{a[n] == a[n - 1] + a[n - 2], a[0] == 0, a[1] == 1}, a[n], n][[1]]], {n, 10}]]
```




```
RSolve[{a[n] == a[n - 1] + b[n - 1], b[n] == a[n - 1] - b[n - 1]}, {a[n], b[n]}, n]
```

$$\left\{\left\{a[n] \to 2^{-\frac{3}{2}+\frac{n}{2}} \left(1 - (-1)^n + \sqrt{2} + (-1)^n \sqrt{2}\right) C[1] - 2^{-\frac{3}{2}+\frac{n}{2}} (-1 + (-1)^n)\, C[2],\right.\right.$$
$$\left.\left. b[n] \to -2^{-\frac{3}{2}+\frac{n}{2}} (-1 + (-1)^n)\, C[1] + 2^{-\frac{3}{2}+\frac{n}{2}} \left(-1 + (-1)^n + \sqrt{2} + (-1)^n \sqrt{2}\right) C[2]\right\}\right\}$$

Diiferenciálegyenlet, egyenletrendszer, késleltetés

```
DSolve[y'[x] == y[x], y[x], x]
```

$\{\{y[x] \to e^{x}\, C[1]\}\}$

```
DSolve[{y'[x] == y[x], y[0] == 1}, y[x], x]
```

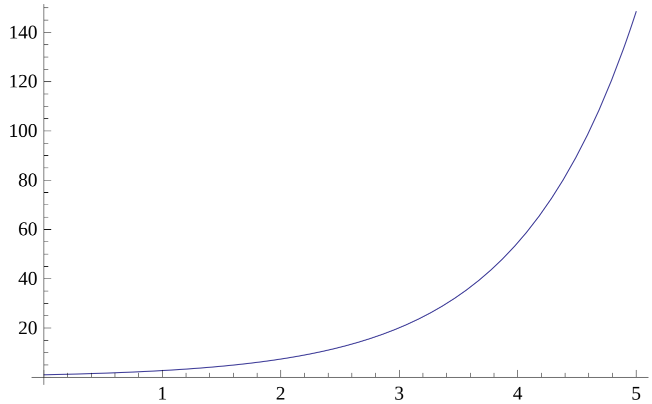
$\{\{y[x] \to e^{x}\}\}$

Numerikus, modellezés, alkalmazások szempontjából fontos

```
s1 = NDSolve[{y'[x] == y[x], y[0] == 1}, y, {x, 0, 5}]
```

```
{{y → InterpolatingFunction[{{0., 5.}}, <>]}}
```

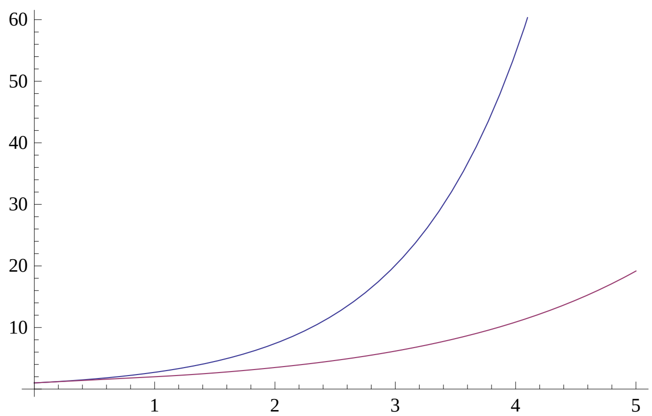
```
Plot[Evaluate[y[x] /. s1[[1]]], {x, 0, 5}]
```




```
s2 = NDSolve[{y'[x] == y[x - 1], y[x /; x ≤ 0] == 1}, y, {x, 0, 5}]
```

```
{{y → InterpolatingFunction[{{0., 5.}}, <>]}}
```

```
Plot[Evaluate[y[x] /. {s1[[1]], s2[[1]]}], {x, 0, 5}]
```



Alkalmazások késôbb