Lab3 Report

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1 Problem1

1.1 Verify that the following three circuits represent the quantum oracles for a constant, a constant, and a balanced function, respectively.

From Table1, we verify the first and the second circuits are constant, for they only output 0 and 1, respectively, and that the third one is balanced, for it outputs 0 or 1 by 50%.

(x_0, x_1, x_2, x_3)	y_1	y_2	y_3
(1,1,1,1)	0	1	0
(1,1,1,0)	0	1	1
(1,1,0,1)	0	1	1
(1,1,0,0)	0	1	0
(1,0,1,1)	0	1	1
(1,0,1,0)	0	1	0
(1,0,0,1)	0	1	0
(1,0,0,0)	0	1	1
(0,1,1,1)	0	1	1
(0,1,1,0)	0	1	0
(0,1,0,1)	0	1	0
(0,1,0,0)	0	1	1
(0,0,1,1)	0	1	0
(0,0,1,0)	0	1	1
(0,0,0,1)	0	1	1
(0,0,0,0)	0	1	0

Table 1: Truth Table of the three circuits

1.2 Implement the Deutsch–Jozsa algorithm to decide whether the chosen oracle was constant or balanced.

If the output of the following circuit is 0, then it is constant; otherwise, it is balanced.

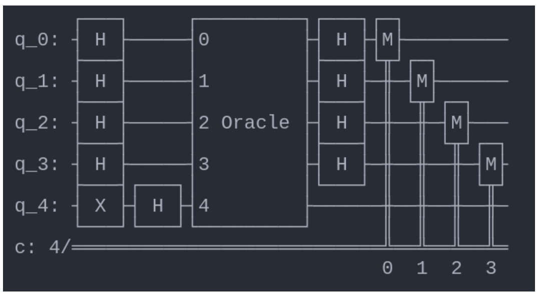


Figure 1: Implementation of Deutsch–Jozsa algorithm

2 Problem2

2.1 Pass $|\psi\rangle$ through the above mentioned circuit to see if it does the job: $I_{|x_0\rangle}|\psi\rangle = -a_{011}|011\rangle + \sum_{x \neq 011} a_x|x\rangle$

Original StateVector V_1:
0. +0.000000e+00j, 0. +0.000000e+00j, 0. +0.000000e+00j, 0. +0.000000e+00j,
0.5+0.000000e+00j, 0. +0.000000e+00j, 0.5+0.000000e+00j, 0. +0.000000e+00j,
0. +0.000000e+00j, 0. +0.000000e+00j, 0. +0.000000e+00j, 0. +0.000000e+00j,
-0.5-6.123234e-17j, 0. +0.000000e+00j, -0.5-6.123234e-17j, 0. +0.000000e+00j
Output StateVector V_1':
0. +0.000000e+00j, 0. +0.000000e+00j, 0. +0.000000e+00j, 0. +0.000000e+00j,
0.5+0.000000e+00j, 0. +0.000000e+00j, -0.5-6.123234e-17j, 0. +0.000000e+00j,
0. +0.000000e+00j, 0. +0.000000e+00j, 0. +0.000000e+00j, 0. +0.000000e+00j,
-0.5-6.123234e-17j, 0. +0.000000e+00j, 0.5+0.000000e+00j, 0. +0.000000e+00j
We can see that the red ones are flipped, whose index is 0110 and 1110, respectively.

2.2 Draw your circuit

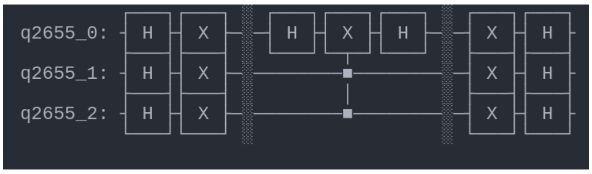
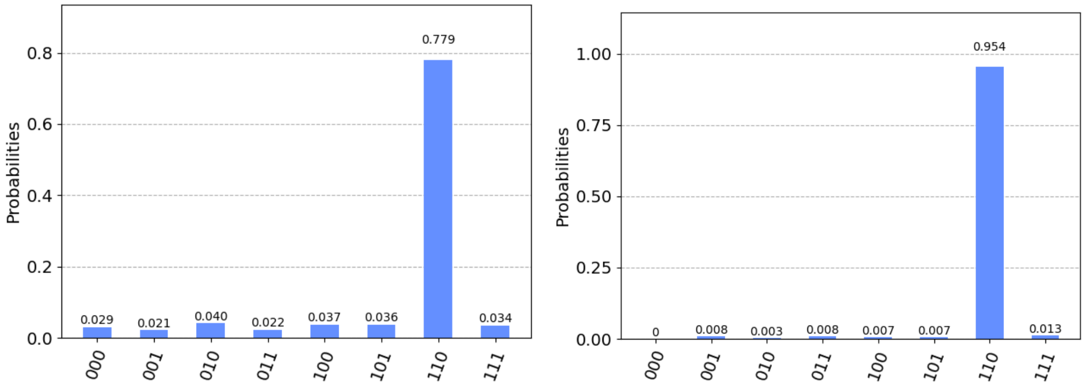


Figure 2: Circuit for amplification

2.3 Plot the probability of obtaining each state (including the probability of measuring the target x_0) along each stage



(a) The first iteration

(b) The second iteration

Figure 3: Probability of obtaining each state along each stage

2.4 After iterating the procedure by $\lfloor \sqrt{N} \rfloor$ times, do you always get the right solution '011' from the measurement outcome?

From Fig.3, Yes.

2.5 Plot the probability of getting the right solution versus the number of iterations, says 20.

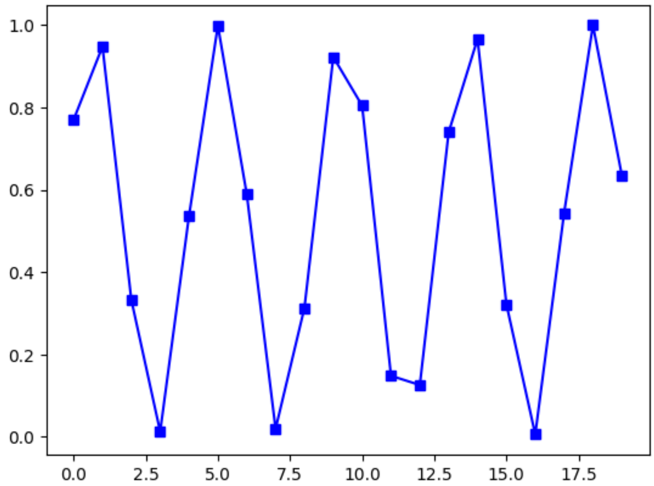


Figure 4: Probability of getting the right solution versus the number of iterations

2.6 Verify that the following circuit flips the sign of '011' and '101'

Original StateVector V_2:
0. +0.j, 0. +0.j, 0. +0.j, 0. +0.j,
0.70710678+0.j, 0.70710678+0.j, 0. +0.j, 0. +0.j
Output StateVector V_2':
0. +0.j, 0. +0.j, 0. +0.j, -0. +0.j,
0.70710678+0.j, -0.70710678+0.j, 0. +0.j, 0. +0.j
We can see that the red ones are flipped, whose index is 011 and 101, respectively.

2.7 Implement Grover's search algorithm for finding either '011' or '101'

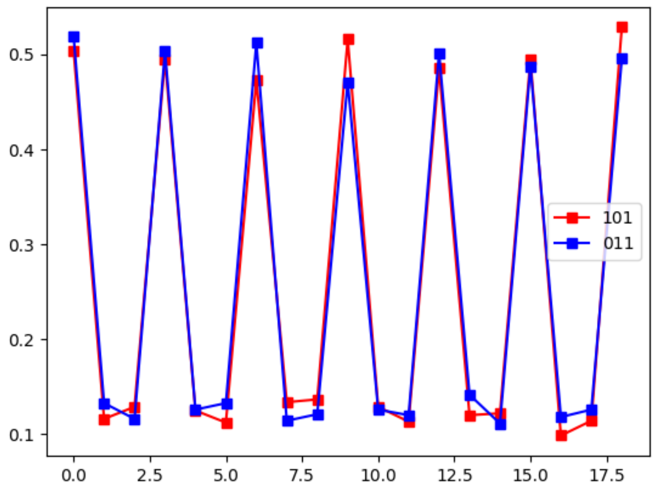


Figure 5: Run Grover's search algorithm for finding either '011' or '101' for 20 iterations

2.8 Consider another example where ‘011’, ‘101’, ‘110’, and ‘111’ are our targets. Use the following circuit as the such oracle, and run Grover’s algorithm on it. What do you get?

From Fig6, we can see that ’011’, ’101’, ’110’, and ’111’ have similar probabilities around 0.125, and this means that it is uniform.

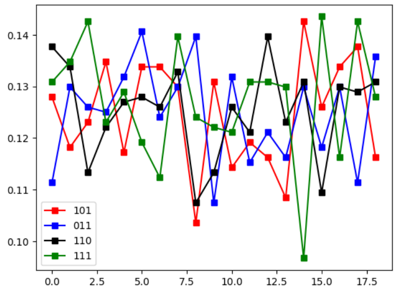
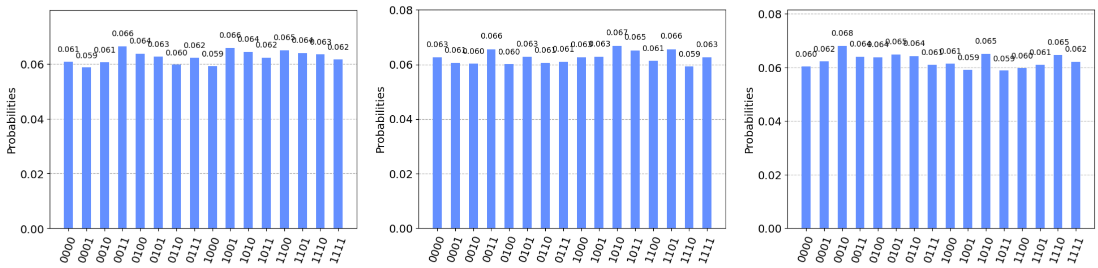


Figure 6: Run Grover’s algorithm on ‘011’, ‘101’, ‘110’, and ‘111’ as targets

3 Problem3

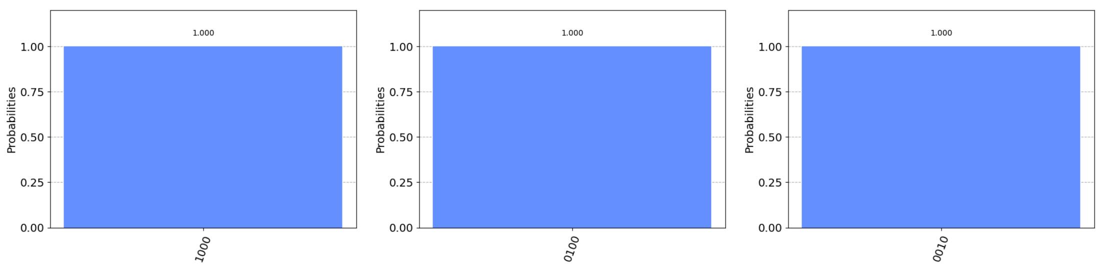
3.1 Measure each of them directly



(a) Measurement on circuit(1) (b) Measurement on circuit(2) (c) Measurement on circuit(3)

Figure 7: Result of directly measuring each circuit

3.2 Apply $\mathrm{QFT}_N^\dagger$ on them, what do you get? Why?



(a) Measurement on circuit(1) (b) Measurement on circuit(2) (c) Measurement on circuit(3)

Figure 8: Result of applying $\mathrm{QFT}_N^\dagger$ on each circuit

3.3 What information was retrieved by QFT?

By QFT, we can get the information of phase.

4 Problem4

4.1 Show that $7^x mod 15$ is periodic

From Fig.9, we can see the period of $7^x mod 15$ is 4.

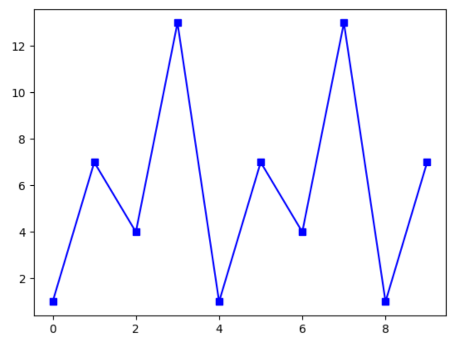


Figure 9: Period of $7^x mod 15$

4.2 Measure the second register. How likely are they? Why is that?

They are 4, 1, 13, and 7, and this means that f(x) will output these values uniformly. This result can be verified by Fig.9, which shows q2 is consistent with the value of f(x).

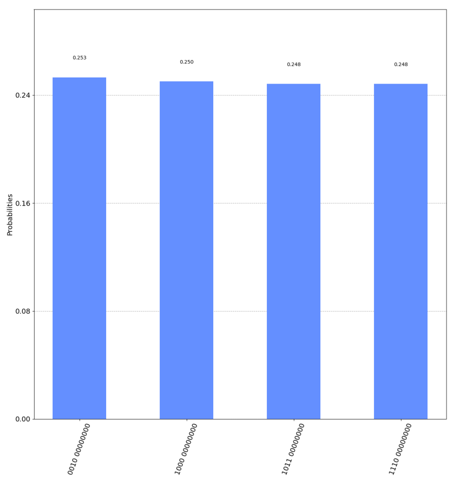
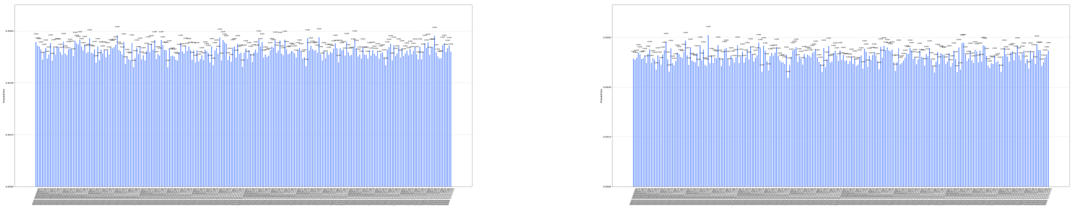


Figure 10: Measurement on the second register.

4.3 So overall we get a random number between 0 and N-1. Verify this by measuring both qr1 and qr2.



(a) Measurement on qr1

(b) Measurement on qr2

Figure 11: Result of measuring both qr1 and qr2

4.4 Apply $\mathrm{QFT}_N^\dagger$ on $|\mathrm{per}\rangle$. Show that all terms have zero amplitude except for the multiples of A. What did you really get?

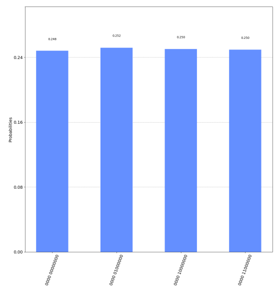


Figure 12: Result of applying $\mathrm{QFT}_N^\dagger$ on $|\mathrm{per}\rangle$

4.5 What is the probability of getting the right period in this case?

Phase	Fraction	Guess for r
0.50	1/2	2
0.75	3/4	4
0.25	1/4	4
0.00	0/1	1

Table 2: Guess for r

4.6 What is the chance that k_0 happens to be coprime to r?

From Table2, we can see the chance is 50%.

References

[1] https://qiskit.org/textbook/ch-algorithms/grover.html

[2] https://qiskit.org/textbook/ch-algorithms/shor.html