Baryon Imbalance

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October 18, 2017

The Big Bang should have created equal amounts of matter and antimatter and yet barely any antimatter is observed. Neither general relativity or the standard model offer explanations for this imbalance. This problem is referred to as the baryon asymmetry problem. One possible explanation is that matter and antimatter dominate different large scale regions of the universe. At astronomical distances matter and antimatter are indistinguishable but at the boundaries between these regions annihilation would occur offering an observational test of this theory.

1. Estimate the number density of the IGM.

   We can approximate the density of the IGM by calculating the cosmological mean baryon density. We have by the Friedmann equation:

$$\rho_{\mathrm{IGM}} \sim \rho_{\mathrm{b}} = \Omega_{\mathrm{b}}\rho_{\mathrm{crit}} = \Omega_{\mathrm{b}}\frac{3H^2}{8\pi G} \tag{1}$$

   Approximating the IGM as all neutral hydrogen we have the number density:

$$n_{\mathrm{IGM}} = \frac{\Omega_{\mathrm{b}}}{m_{\mathrm{H}}}\frac{3H^2}{8\pi G} \tag{2}$$

   Evaluating at order of magnitude, $\Omega_{\mathrm{b}} = 0.04$, $H = 70$ km/s/Mpc:

$$n_{\mathrm{IGM}} \sim 2 \times 10^{-7}\ \mathrm{cm}^{-3} \tag{3}$$

2. Estimate the particle collision rate in the IGM, as well as the matter-antimatter collision rate assuming equal amounts of each.

   The mean free path is given by:

$$l_{\mathrm{IGM}} = \frac{1}{n_{\mathrm{IGM}}\sigma} = \frac{1}{n_{\mathrm{IGM}}\pi a^2} \sim 6 \times 10^{22}\ \mathrm{cm} \sim 20\ \mathrm{kpc} \tag{4}$$

   Where we approximate neutral hydrogen as a sphere of the Bohr radius, a. We need the mean particle velocity in the IGM to calculate the collision rate. Assuming the IGM is in equilibrium with the CMB (applying the equipartition theorem for a monoatomic gas) we have:

$$v = \sqrt{\frac{3k_{\mathrm{b}}T_{\mathrm{CMB}}}{m_{\mathrm{H}}}} \sim 260\ \mathrm{m/s} \tag{5}$$

   The particle collision rate is then:

$$r = \frac{v}{l_{\mathrm{IGM}}} \sim 4 \times 10^{-19}\ \mathrm{s}^{-1} \tag{6}$$

We need to include a multiplicative factor of $2f(1-f)$, where f is the antimatter fraction, to get the matter-antimatter collision rate:

$$r \sim 2 \times 10^{-19}\ \mathrm{s}^{-1} \tag{7}$$

A more useful quantity is the volumetric collision rate:

$$\Gamma = n_{\mathrm{IGM}} \times r \sim 4 \times 10^{-26}\ \mathrm{cm}^{-3}\ \mathrm{s}^{-1} \tag{8}$$

3. Assume that we are at the center of a spherical region of space dominated by matter. Write an expression for the gamma-ray surface brightness due to matter-antimatter annihilations (assuming equal matter and antimatter).

   The total number of photons produced in a infinitesimal volume per time is given by:

$$\frac{dN}{dt} = \Gamma \times 4\pi r^2 dr \tag{9}$$

So the intensity:

$$dI_\nu = \frac{dN}{dA\ dt\ d\Omega\ d\nu} = \frac{4\pi\Gamma r^2 dr}{4\pi r^2\ dt\ 4\pi\ d\nu} \sim \frac{\Gamma}{4\pi} dr \tag{10}$$

The final equality is made because our source is (basically monochromatic). Integrating for I:

$$I = \int_r^{r+\delta r} \frac{\Gamma}{4\pi} dr' = \frac{\Gamma}{4\pi}\delta r \tag{11}$$

In this case the mean free path for collisions seems like a reasonable estimate for δr, and additionally since the sources are not isotropic but instead output two photons in opposite directions I *believe* we need an extra factor of $2/4\pi$:

$$I = \frac{\Gamma}{8\pi^2} l_{IGM} \sim 2.4 \times 10^{-5}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1} \tag{12}$$

4. If we observed a radiation background of energy 3×10^5 keV of brightness, $I = 10^{-4}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}$ that we could be pretty sure is from annihilation at a matter-antimatter boundary, what would this imply about the value of Ω_{b} and the scale of our matter dominated region (again assume equal matter and antimatter)?

   The energy of the photons:

$$\epsilon = \frac{1}{2} m_h c^2 (1+z)^{-1} \tag{13}$$

$$z = \frac{m_h c^2}{2\epsilon} - 1 \sim 0.5 \tag{14}$$

Applying equations 2, 4, 8, and 13 we have:

$$I = \frac{n_{IGM} v}{8\pi^2} = 2.4 \times 10^{-5} \left(\frac{\Omega_b}{0.04}\right)\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1} \tag{15}$$

So if we observed $I = 10^{-4}\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}$ that would imply:

$$\Omega_b \sim 0.16 \tag{16}$$

5. Based on the data given write an expression relating Ω_b, and the redshift the hypothesized boundary. If we take $\Omega_\mathrm{b} = 0.04$ what limiting distance to the hypothesized boundary does that require.

   Based on the attached plot the gamma-ray background spectrum appears to be consistent with a power-law, at least to order of magnitude. I estimate the power-law index to be about, $\gamma = -\frac{11}{5} \sim 2$, and taking the convenient value of $I(\epsilon = 1\ \mathrm{keV}) = 10$ we can solve for the normalization factor:

$$I(\epsilon) = (10\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}) \left(\frac{\epsilon}{\mathrm{keV}}\right)^{-2} \tag{17}$$

   With this relation we can at least put an upper bound on the gamma ray surface brightness due to matter-antimatter annihilations from our hypothesized large scale region boundary:

$$2.4 \times 10^{-5} \left(\frac{\Omega_b}{0.04}\right)\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1} \leq (10\ \mathrm{cm}^{-2}\ \mathrm{s}^{-1}\ \mathrm{sr}^{-1}) \left(\frac{\epsilon}{\mathrm{keV}}\right)^{-2} \tag{18}$$

   Taking $\frac{1}{2} m_h c^2 \sim 5 \times 10^5$ keV we have:

$$10^{-5} \geq \frac{1}{(1+z)^2} \left(\frac{\Omega_b}{0.04}\right) \tag{19}$$

   Taking $\Omega_b = 0.04$ we get a very harsh constraint:

$$z \geq 10^{5/2} \sim 300 \tag{20}$$

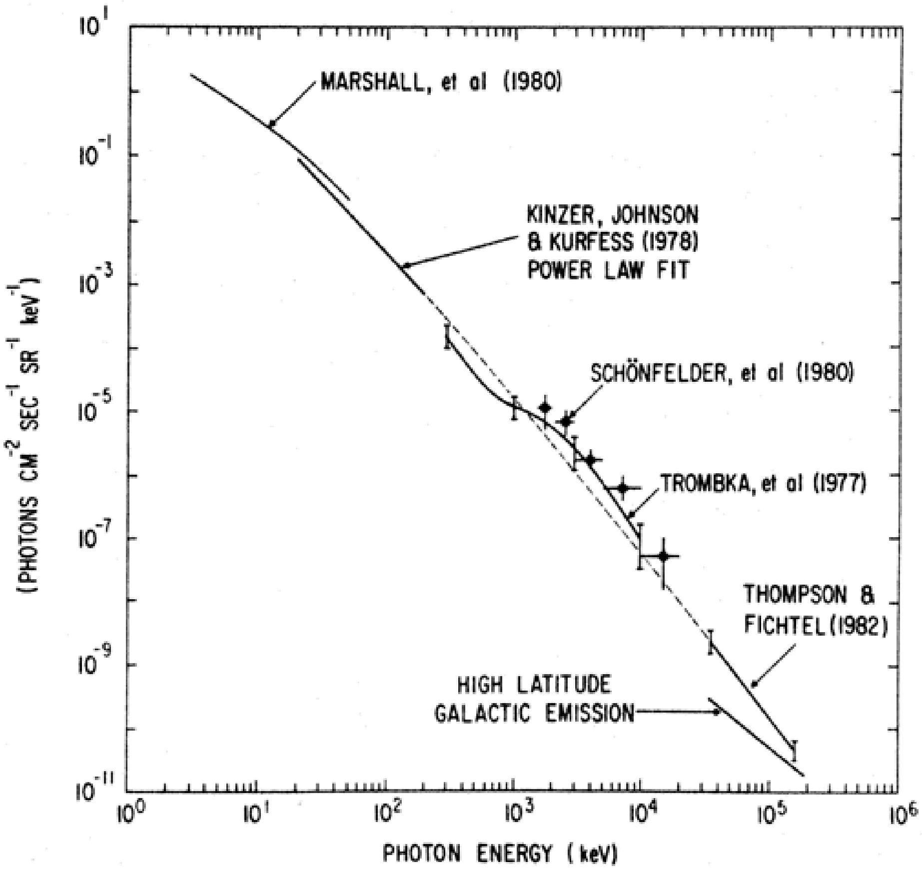

Figure 1: Gamma-ray background surface brightness spectrum (From Trombka and Fichtel 1983).