Computer Vision I

Heiko Neumann, Christian Jarvers, Daniel Schmid
Institute of Neural Information Processing
Ulm University

Assignment 2: Submission date (Moodle): June 6, 2019, 12:00.

1 Filter algebra (3 points)

This assignment deepens the understanding of filter operations. In Matlab, please use `imfilter(im, H, 'replicate', 'conv')` where the last parameters specifies the border handling and indexing mode (c.f. documentation for further details).

1. (*Handwritten assignment*) What is the minimum and maximum possible output value of the filter

$$H = \begin{pmatrix} -1 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}$$

when it is applied to an arbitrary single-channel image $I \in [0, 255]$?

2. Write a Matlab script which loads the image `lena.tif`, converts it to double and scales the grey values to the range $[0, 1]$. First, apply a Gaussian filter G_σ to the image I (you can create the filter with `fspecial('gaussian', hsize, sigma)`). Use $\sigma = 3$ and set the filter size according to the two-sigma rule[1]. On the blurred result image, apply the filter H from part 1, such that the transformation

$$R_1 = I * G_\sigma * H$$

is applied. What type of filter is H? What are image properties or features that such a filter is responsive to?

3. Now reverse the steps from the previous part, meaning that first the filter H and on this result the Gaussian filter G_σ should be applied, i.e.

$$R_2 = I * H * G_\sigma .$$

Calculate the absolute difference for each pixel position of the two results, i.e.

$$|R_1 - R_2| .$$

Do you notice any differences? Can you infer an additional property of the underlying convolution operation?

[1] In this case: $2 \cdot \lceil 2 \cdot \sigma \rceil + 1$, based on the empirical observation that 95 % of the data lies within two standard deviations from the mean (which is 0 here). This value $\cdot 2$ because of the symmetry of the Gaussian function and $+1$ due to the fact that the filter size should be odd for easier centering the kernel during convolution.

2 Discrete Fourier Transform (4 points)

1. Apply the Fourier transform to a short discrete signal to represent it as a set of sine and cosine waves. Given is a discrete signal with $N = 4$ values captured in time steps of $\Delta t = \frac{1}{4}s$ starting at $t_0 = 0$.

t_i	s_i
0	2
$\frac{1}{4}$	3
$\frac{1}{2}$	0
$\frac{3}{4}$	1

   a) Use the discrete Fourier transform to transfer this signal to the frequency domain:

$$S[u] = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} s[t] \cdot \exp[-i2\pi \frac{u}{N} \cdot t]$$

   where N is the number of sample points and u are the frequencies used to represent the sample. Apply this formula instead of using built-in Matlab functions (e.g., `fft`)

   b) The Fourier coefficients correspond to the amplitudes of component $\sin(\cdot)$ and $\cos(\cdot)$ functions of specific frequencies into which the signal is decomposed. To which frequency does each coefficient belong (i.e. name a Hz value for each coefficient)? Plot one of the frequencies as a sine wave in the range $[0, 1]$.

   c) Reconstruct the original signal by using the inverse Discrete Fourier Transform:

$$s_r[t] = \frac{1}{\sqrt{N}} \sum_{t=0}^{N-1} S[u] \cdot \exp[i2\pi \frac{u}{N} \cdot t]$$

   Pay attention to the sign in the transform kernel. Again, do not use built-in Matlab functions.

   d) Plot the original signal points, the frequency spectrum and your reconstructed signal.

3 Fourier Transform for Image Quality Assessment (3 points)

1. Load the images `flower01.png` and `flower02.png` into Matlab, convert them to double and scale the values to the range $[0, 1]$. Both images show the same object (a flower) but the first is in and the second out of focus. In this task we will decide with the help of the Fourier domain which image is sharp and which not. The central idea is to

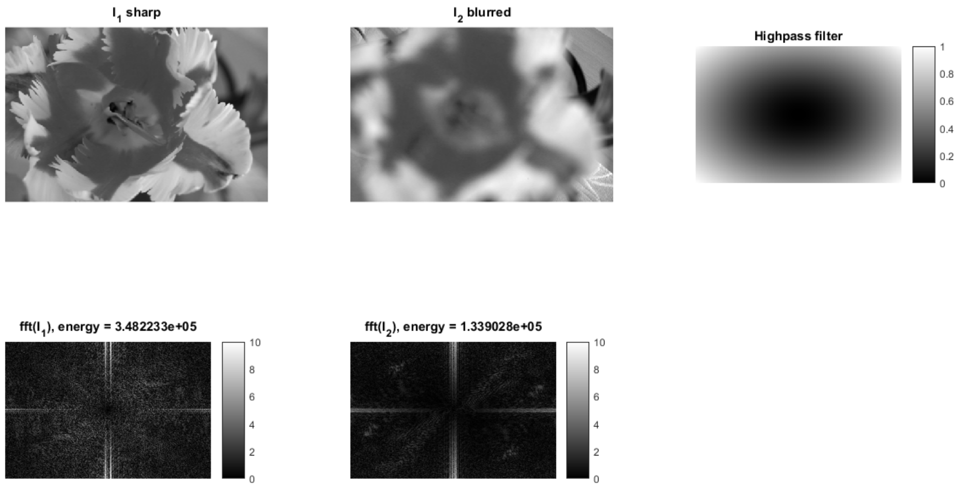

Figure 1: Possible solution for the 2^{nd} part of the Fourier transform assignment.

quantify the energy of high-frequency components in the image, as a sharp image will have more high frequency content. A possible solution for this task can be found in Figure 1.

a) Apply the 2D-Fourier transform (`fft2`) to both images, take the absolute value at each position and shift the frequency values so that the DC component is centered in the middle (hint: `fftshift`).

b) To decrease the effect of low frequencies, multiply both spectra with a high-pass filter. For this purpose, generate a Gaussian $G(x)$ with standard deviation equal to half of the height of the image (`fspecial`), scale this Gaussian such that its maximum value is one and generate a filter $f_h(x) = 1 - G(x)$.

c) In order to assess the energy content of the highpass filtered image, calculate the the quadratic sum of the amplitudes for all frequencies values. A sharp image should have a higher energy than a blurred one.

d) Plot the filtered amplitude spectra and annotate them with their energy.

Submission procedure

- Please work in groups of 2. Submission exclusively in Moodle (it is sufficient if one group member submits the solution in Moodle).
- Please name all Matlab main script files with the current assignment and exercise number, e.g. `sh01ex02.m` for the second assignment of the first assignment sheet.
- Please present your results in a pdf-document with

  - the names of all group members
  - and a brief description of your results and images.
- Please submit all files as a zip-document.

Have fun!