

Computer Vision I - Sheet 2

Group 2

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1 Filter algebra

1.1

Given the filter $H = \begin{pmatrix} -1 & -2 & 0 \\ -2 & 0 & 2 \\ 0 & 2 & 1 \end{pmatrix}$ and an Image $I \in [0, 255]$ the exercise arises the question what minimum and maximum output values this filter could create. Because all values in the image are positive, the maximum of 1275 is reached for a pixel region like $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 255 \\ 0 & 255 & 255 \end{pmatrix}$. Analogous the minimum value of -1275 is for instance reached in a pixel region like $\begin{pmatrix} 255 & 255 & 0 \\ 255 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$.

1.2

The source code used for this exercise is located in sh02ex01.m. The resulting images for task 2 are shown in the figures 2 and 3. For comparison purposes the original image is displayed in figure 1. The filter H emphasises large contrasts along the diagonal from the bottom left to the top right corner. Because the filter only emphasises high contrasts, it belongs to the class of high-pass filters.



Figure 1: Original image

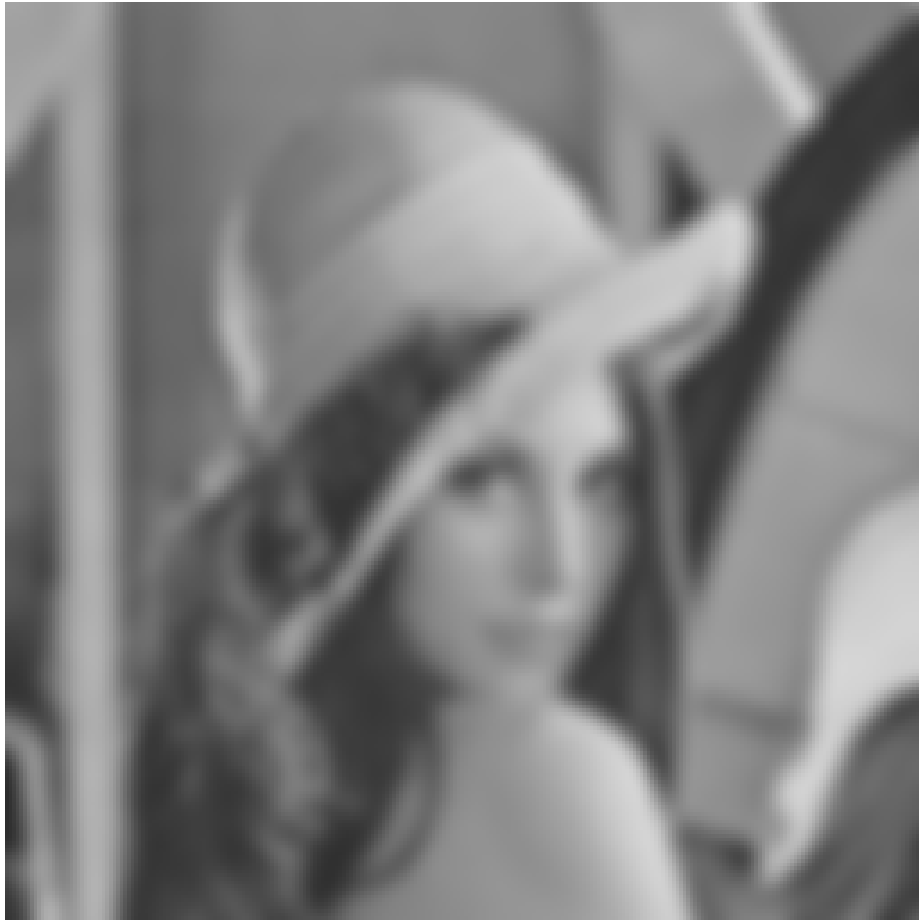


Figure 2: Gaussian kernel applied to lena.tiff



Figure 3: Resulting image for $R_1 = I * G_\sigma * H$

1.3

The figures 4 to 6 show the results to task 3. Although the image shown in figure 6 looks quite dark, the sum over all pixels in the image is approximately 31. Compared to the size of the image ($256\text{px} \times 256\text{px}$) this number is so small, that one could argue that the error is coming from the small inaccuracies related to calculating with floating point numbers. Therefore the example shows the commutative property the convolution owns quite well.

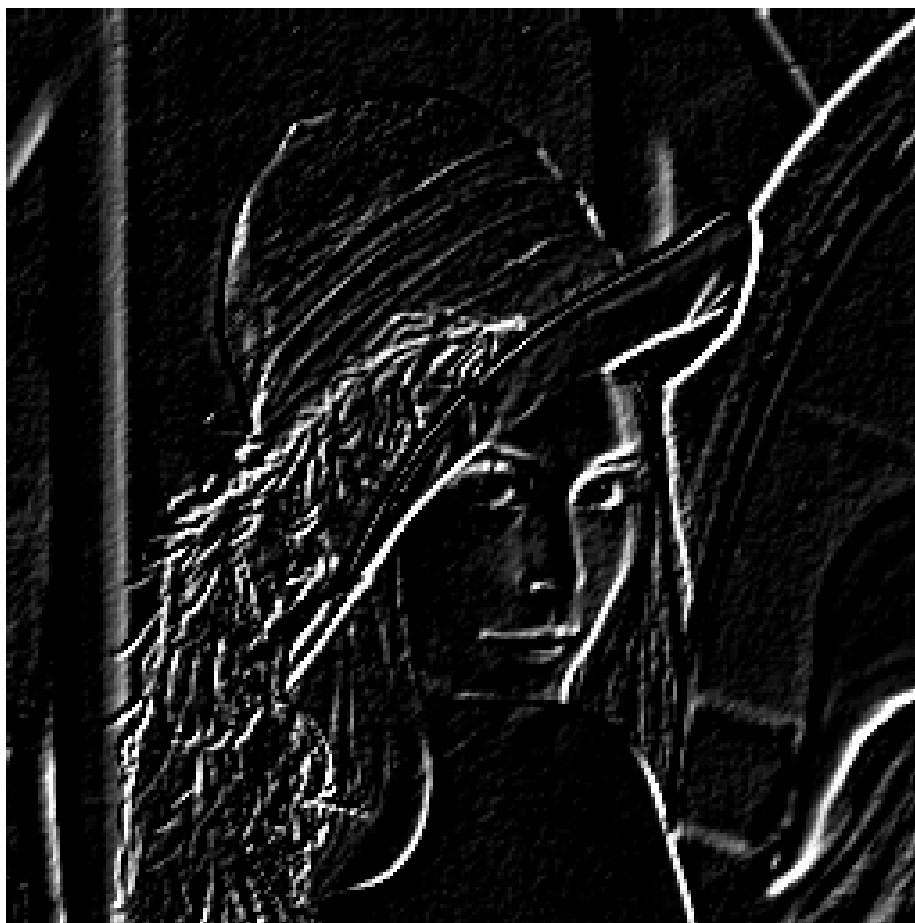


Figure 4: Filter H applied to lena.tiff



Figure 5: Resulting image for $R_2 = I * H * G_\sigma$

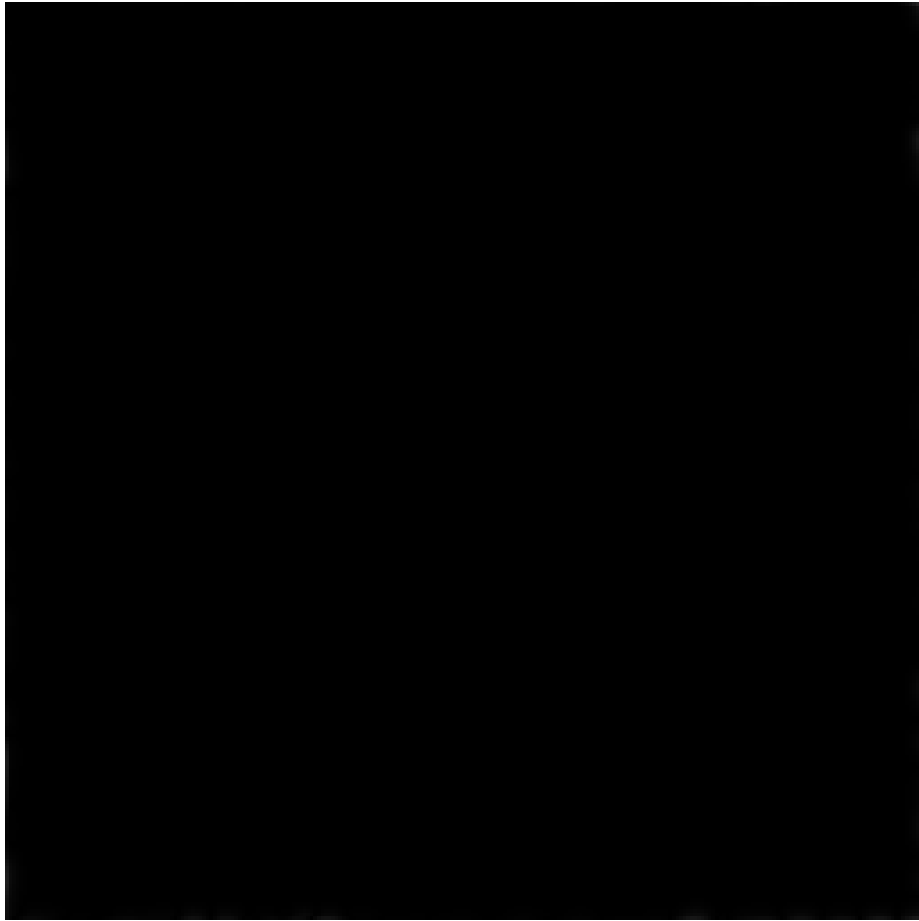


Figure 6: Resulting image for $|R_1 - R_2|$

2 Discrete Fourier Transform

2.1

Our implementation of the Discrete Fourier Transform can be found in the file `fourier.m`. Figure 7 shows the results of applying our DFT to the given signal.

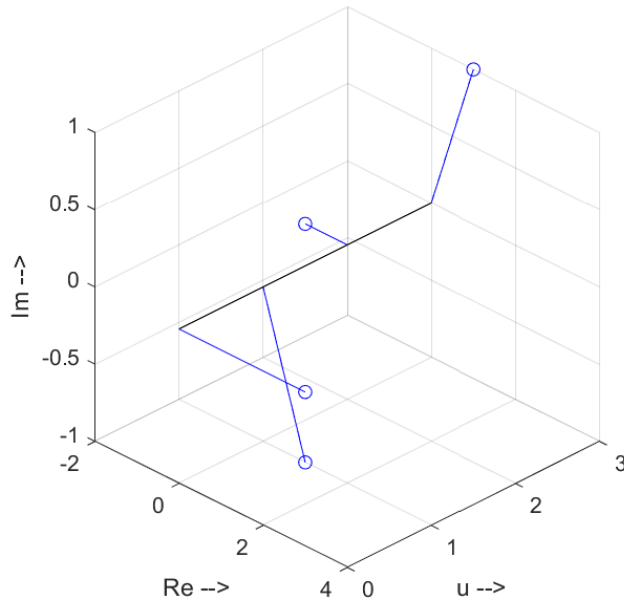


Figure 7: Complex visualization of the results of the DFT

2.2

Figure 8 shows the sine wave which corresponds to the vector at $u = 1$ in Figure 7. Because this vector has both a real and an imaginary component not equal to zero it would also create a cosine wave. Both waves together would then represent this part in the original signal.

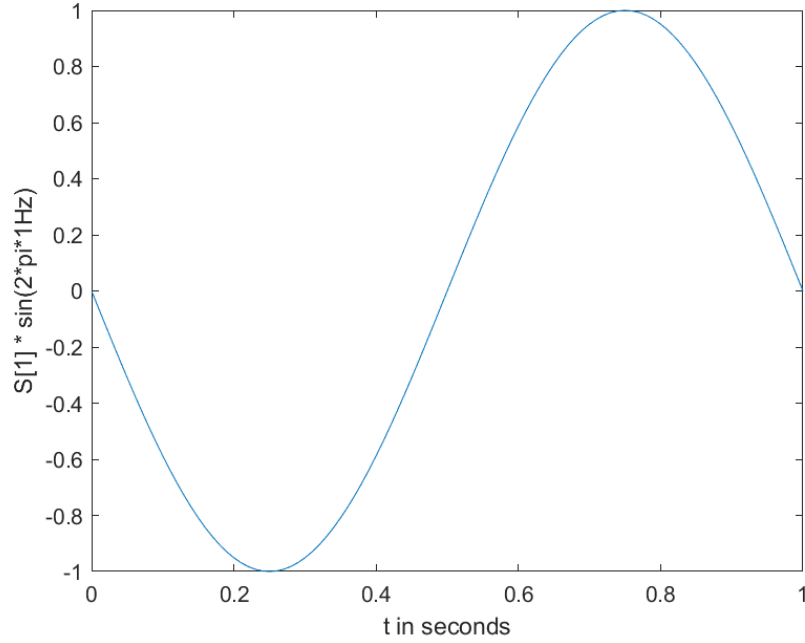


Figure 8: Sinus component of the complex value representing 1Hz

2.3

The source code used for this task can be found in the file ifourier.m.

2.4

Figure 9 shows the results gained from task a) to c). Because the third value of the reconstructed signal is not exact zero but of the magnitude 10^{-16} Matlab expands the vertical axis in the third plot a bit into the negative direction. Besides this small difference, the reconstructed signal looks the same compared to the original signal. Because our signal was sampled with a sampling frequency of 4Hz and the sampled signal has 4 points, the frequency resolution of the frequency spectrum is $\frac{4\text{Hz}}{4} = 1\text{Hz}$.

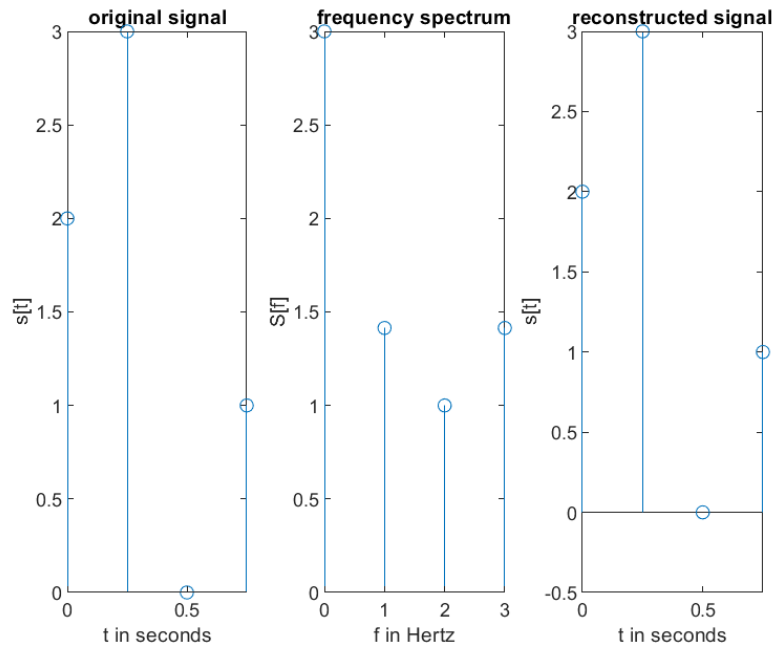


Figure 9: Plots showing results from task a) to c)

3 Fourier Transform for Image Quality Assessment

The source used to create the figures shown in this section can be found in the Matlab script sh03ex03.m. Figure 10 shows the original images together with their already high-pass filtered frequency spectra calculated with `fft2`. The spectra have been shifted using `fftshift`. Figure 3 shows the high-pass filter which was used. According to the exercise we set σ to the half of the image height. In a first try we used the rule for the filter size given in exercise 1. But this lead to a filter which was much bigger than the original image. Therefore we scaled it so that it fits to the image and can be directly applied in the frequency domain by multiplication. Above the images one can find the energy calculated for this image. The left image is the sharp one, thus we expected the energy of it's filtered spectrum being higher than in the filtered spectrum of the other image. Our calculation proves what we expected, the energy of the left spectrum is about twice the energy of the right spectrum.

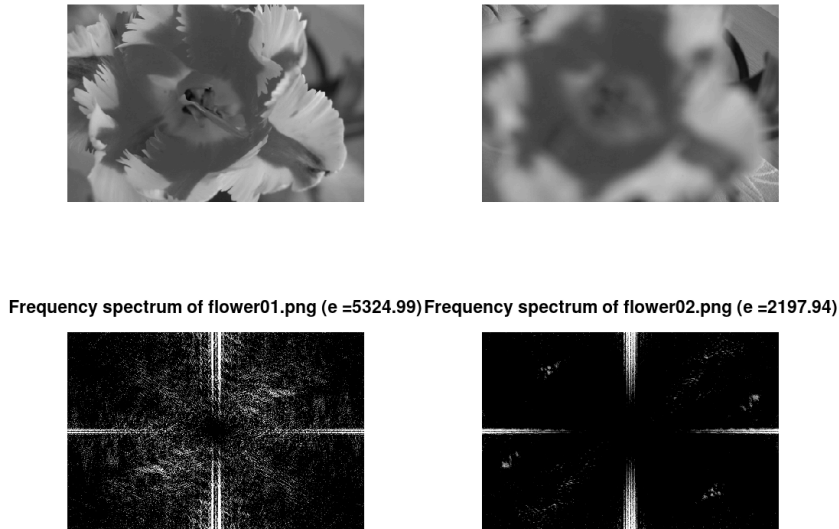


Figure 10: Original images and their frequency spectra

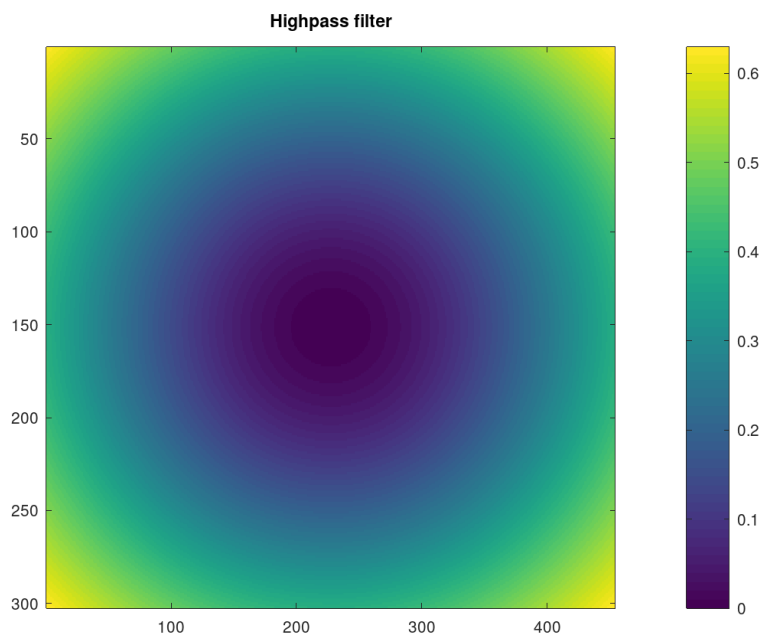


Figure 11: Highpass filter