(8) Difference-in-Differences

Causal Data Science for Business Analytics

Christoph Ihl
Hamburg University of Technology
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Basic Model

Focus: ATT After Treatment

- Unconfoundedness assumption $\{Y(0), Y(1)\} \perp\!\!\!\perp T$ helps to identify the ATE: $\tau_{\text{ATE}} = \mathbb{E}[Y_i|T_i = 1] - \mathbb{E}[Y_i|T_i = 0]$.
- Average Treatment Effect on the Treated (ATT): $\tau_{\text{ATT}} = \mathbb{E}[Y_i(1) - Y_i(0)|T_i = 1]$
  - Weaker identification assumption suffices: $Y(0) \perp\!\!\!\perp T|X$:

$$
\begin{aligned}
\tau_{\text{ATT}} = \mathbb{E}[Y_i(1) - Y_i(0)|T_i = 1] &= \mathbb{E}[Y_i(1)|T_i = 1] - \mathbb{E}[Y_i(0)|T_i = 1] \\
&= \mathbb{E}[Y_i|T_i = 1] - \mathbb{E}[Y_i(0)|T_i = 1] \\
&= \mathbb{E}[Y_i|T_i = 1] - \mathbb{E}[Y_i(0)|T_i = 0] \\
&= \mathbb{E}[Y_i|T_i = 1] - \mathbb{E}[Y_i|T_i = 0]
\end{aligned}
$$

- Introducing time periods before and after treatment $t = 0, 1$:
  - $\tau_{\text{DiD}} = \mathbb{E}[Y_{i,t=1}(1) - Y_{i,t=1}(0)|T_i = 1]$

Assumptions and Definition

- In addition to SUTVA (consistency & no interference), two new assumptions:

Assumption (A.pt) "Parallel Trends"

$$\mathbb{E}[Y_{i,t=1}(0) - Y_{i,t=0}(0)|T_i = 1] = \mathbb{E}[Y_{i,t=1}(0) - Y_{i,t=0}(0)|T_i = 0].$$

- Equivalent to unconfoundedness of the change (rather than potential outcomes themselves): $(Y_1(0) - Y_0(0)) \perp\!\!\!\perp T$.

Assumption (A.na) "No Anticipation"

$\mathbb{E}[Y_{i,t=0}(1) - Y_{i,t=0}(0)|T_i = 1] = 0$ or $Y_{i,t=0}(1) = Y_{i,t=0}(0)$

- Treatment has no effect on the treatment group before it is administered.

Definition "Difference-in-Differences ATT"

- Given consistency, parallel trends, and no anticipation, the ATT is given by the difference between change in the treated group and change in the control group:

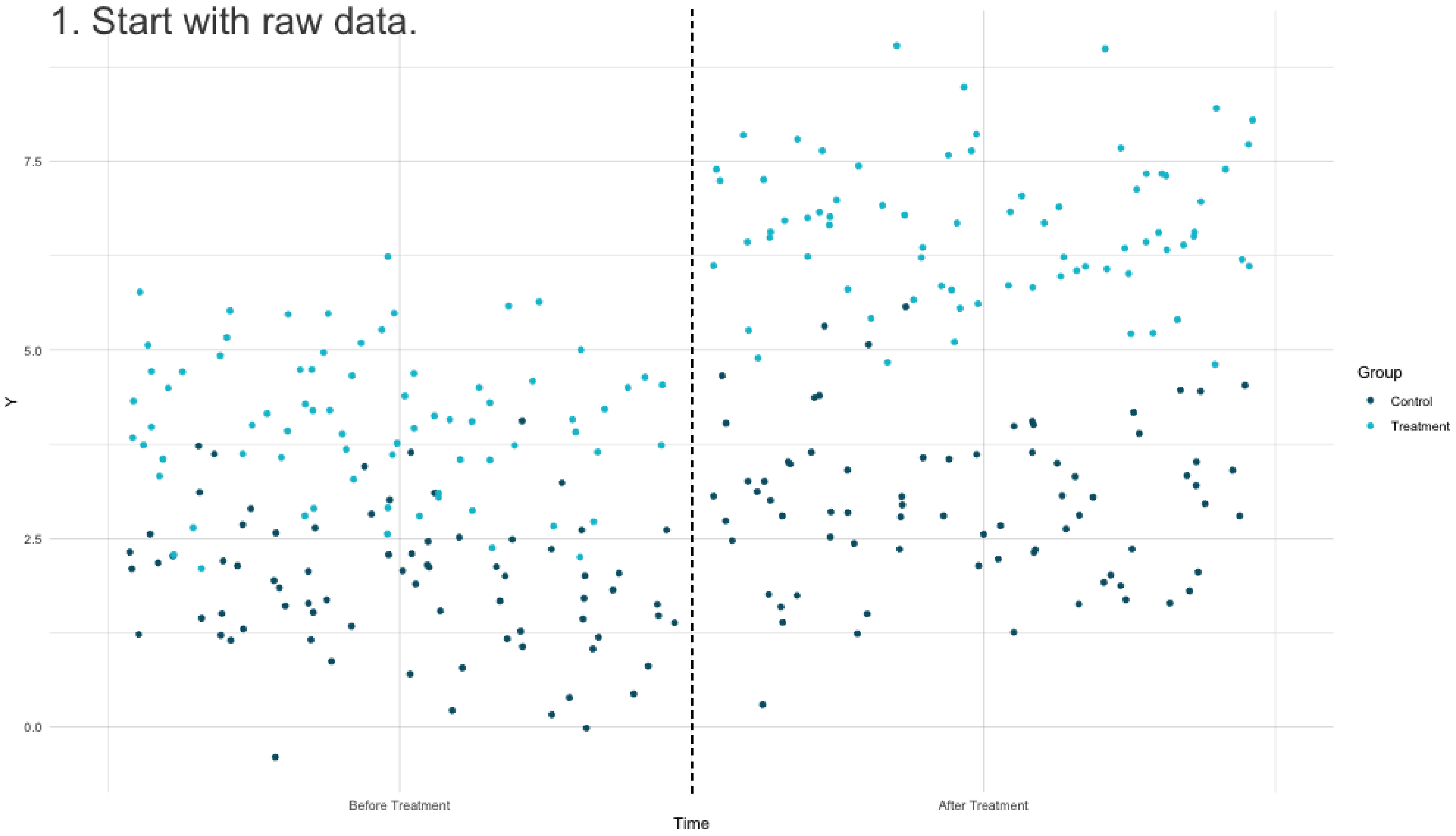
$$\tau_{\text{DiD}} = \mathbb{E}[Y_{i,t=1}(1) - Y_{i,t=1}(0)|T_i = 1] = (\mathbb{E}[Y_{i,t=1}|T_i = 1] - \mathbb{E}[Y_{i,t=0}|T_i = 1]) - (\mathbb{E}[Y_{i,t=1}|T_i = 0] - \mathbb{E}[Y_{i,t=0}|T_i = 0]).$$

Identification

- Proof:

$$
\begin{aligned}
\tau_{\text{DiD}} = \mathbb{E}[Y_{i,1}(1) - Y_{i,1}(0)|T_i = 1] &\overset{\text{LIE}}{=} \mathbb{E}[Y_{i,1}(1)|T_i = 1] - \mathbb{E}[Y_{i,1}(0)|T_i = 1] \\
&= \mathbb{E}[Y_{i,1}(1)|T_i = 1] - \mathbb{E}[Y_{i,0}(0)|T_i = 1] - \mathbb{E}[Y_{i,1}(0)|T_i = 1] + \mathbb{E}[Y_{i,0}(0)|T_i = 1] \\
&\overset{\text{(A.na)}}{=} \mathbb{E}[Y_{i,1}(1)|T_i = 1] - \mathbb{E}[Y_{i,0}(1)|T_i = 1] - \mathbb{E}[Y_{i,1}(0)|T_i = 1] + \mathbb{E}[Y_{i,0}(0)|T_i = 1] \\
&\overset{\text{(SUTVA)}}{=} \mathbb{E}[Y_{i,1}|T_i = 1] - \mathbb{E}[Y_{i,0}|T_i = 1] - (\mathbb{E}[Y_{i,1}(0) - Y_{i,0}(0)|T_i = 1]) \\
&\overset{\text{(A.pt)}}{=} \mathbb{E}[Y_{i,1}|T_i = 1] - \mathbb{E}[Y_{i,0}|T_i = 1] - (\mathbb{E}[Y_{i,1}(0) - Y_{i,0}(0)|T_i = 0]) \\
&\overset{\text{LIE}}{=} \mathbb{E}[Y_{i,1}|T_i = 1] - \mathbb{E}[Y_{i,0}|T_i = 1] - (\mathbb{E}[Y_{i,1}(0)|T_i = 0] - \mathbb{E}[Y_{i,0}(0)|T_i = 0]) \\
&\overset{\text{SUTVA}}{=} \underbrace{\mathbb{E}[Y_{i,1}|T_i = 1] - \mathbb{E}[Y_{i,0}|T_i = 1]}_{\text{change in the treated group}} - \underbrace{(\mathbb{E}[Y_{i,1}|T_i = 0] - \mathbb{E}[Y_{i,0}|T_i = 0])}_{\text{change in the control group}}
\end{aligned}
$$

How DiD Estimation Works



Diff-in-Diffs Regression

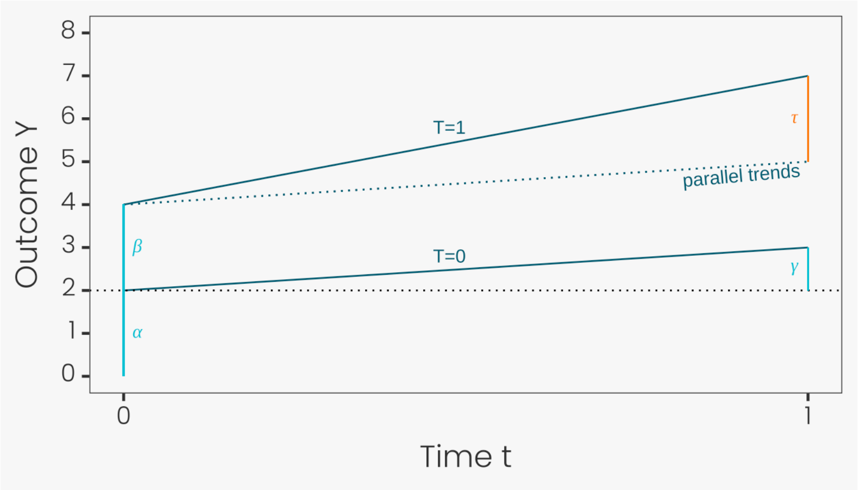
- DiD estimator τ_{DiD} can be obtained by two types of regressions:

1. Two-way fixed effects (TWFE) regression:
    - $Y_{i,t} = \alpha_i + \gamma_t + \tau_{\text{DiD}}(T_i \times t) + \epsilon_{i,t}$
    - Individual fixed effects α_i: capture time-invariant characteristics of individuals.
    - Time fixed effects γ_t: capture time-specific effects common to all individuals.
    - Interaction between the treatment T_i and a time dummy t: $T_i \times t$.
    - Works for panel data: i.e. same observations before and after treatment.

2. Regressing Y_i on the treatment T_i, a time dummy t and their interaction $T_i \times t$:
    - $Y_{i,t} = \alpha + \beta T_i + \gamma t + \tau_{\text{DiD}}(T_i \times t) + \epsilon_{i,t}$
    - Works for both panel data and repeated cross-sectionional data: i.e. different observations before and after treatment.

Diff-in-Diffs Regression

- Graphical interpretation of $Y_{i,t} = \alpha + \beta T_i + \gamma t + \tau_{\text{DiD}}(T_i \times t) + \epsilon_{i,t}$:
  - $\alpha = \mathbb{E}[Y_{i,0}|T_i = 0]$ is the mean outcome of the nontreated at $t = 0$.
  - $\beta = \mathbb{E}[Y_{i,0}|T_i = 1] - \mathbb{E}[Y_{i,0}|T_i = 0]$ is the mean difference in outcomes across treatment groups at $t = 0$.
    - This selection bias should remain constant in $t = 1$.
  - $\gamma = \mathbb{E}[Y_{i,1}|T_i = 0] - \mathbb{E}[Y_{i,0}|T_i = 0]$ is the time trend in mean outcomes among the non-treated.
    - This trend should be the same (parallel) for the treated group.



- Alternative interpretation:

$$\tau_{\text{DiD}} = \underbrace{\mathbb{E}[Y_{i,1}|T_i = 1] - \mathbb{E}[Y_{i,1}|T_i = 0]}_{\text{difference in post-treatment}} \underbrace{-(\mathbb{E}[Y_{i,0}|T_i = 1] - \mathbb{E}[Y_{i,0}|T_i = 0])}_{\text{difference in pre-treatment}}$$

Basic DiD: Example

- Repeated cross-sectional data: Assess house prices before and after a new highway is built. Repeated cross-section data of 179 houses in 1978 and 142 houses in 1981 in Kiel, Germany. Not the same houses over time.

```r
library(wooldridge)  # load wooldridge package for data
library(fixest) # load fixest package for FE regression
data(kielmc)  # load kielmc data
attach(kielmc)   # attach data
kielmc$Y = kielmc$rprice  # define outcome
kielmc$T = kielmc$nearinc # define treatment group
kielmc$t = kielmc$y81     # define period dummy

feols(Y ~ T + t + T:t,    # eqivalent to lm(Y ~ T*t)
      data = kielmc,
      vcov = vcov_cluster("cbd") # cluster st.error w.r.t. distance to center
)
```

```
OLS estimation, Dep. Var.: Y
Observations: 321
Standard-errors: Clustered (cbd)
            Estimate Std. Error  t value   Pr(>|t|)
(Intercept)  82517.2    3221.92 25.61117  < 2.2e-16 ***
T           -18824.4    7796.29 -2.41453 0.02146099 *
t            18790.3    5154.86  3.64516 0.00090981 ***
T:t         -11863.9    6621.82 -1.79164 0.08236582 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 30,053.9   Adj. R2: 0.166131
```

Basic DiD: Example 2

- Panel data: Assess the impact of participating in the U.S. National Supported Work (NSW) training program targeted to individuals with social and economic problems on their real earnings. Experimental vs. non-experimental control group.

```
library(tidyverse)
library(fixest)
library(haven) # to read Stata files
# 1. Experimental data
data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-
# ---- Difference-in-means - Averages
with(data, {y11 = mean(re[year == 78 & ever_treated == 1])
              y01 = mean(re[year == 78 & ever_treated == 0])
              dim = y11 - y01
              dim})
```

```
[1] 1794.342
```

```
# 2. Non-Experimental data
data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-
# ---- Difference-in-means - Averages
with(data, {y11 = mean(re[year == 78 & ever_treated == 1])
              y01 = mean(re[year == 78 & ever_treated == 0])
              dim = y11 - y01
              dim})
```

```
[1] -8497.516
```

```
# ---- Difference-in-Differences - Averages
with(data, {y00 = mean(re[year == 75 & ever_treated == 0])
              y01 = mean(re[year == 78 & ever_treated == 0])
              y10 = mean(re[year == 75 & ever_treated == 1])
              y11 = mean(re[year == 78 & ever_treated == 1])
              did = (y11 - y10) - (y01 - y00)
              did})
```

```
[1] 3621.232
```

```
data$post_treat = data$ever_treated * (data$year == 78)
# ---- Difference-in-Differences - Two-Way Fixed Effects Regression
feols(re ~ post_treat | id + year,
        data = data |> filter(year %in% c(75, 78)),
        vcov = vcov_cluster(c("id", "year")))
# ---- Difference-in-Differences - Interactive Regression
feols(re ~ ever_treated + I(year == 78) + post_treat,
        data = data |> filter(year %in% c(75, 78)),
        vcov = vcov_cluster(c("id", "year")))
```

```
OLS estimation, Dep. Var.: re
Observations: 32,354
Fixed-effects: id: 16,177,  year: 2
Standard-errors: Clustered (id & year)
           Estimate Std. Error t value Pr(>|t|)
post_treat  3621.23     609.84 5.93801  0.10621
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 3,859.0     Adj. R2: 0.670904
                Within R2: 0.002483

OLS estimation, Dep. Var.: re
Observations: 32,354
Standard-errors: Clustered (id & year)
               Estimate Std. Error   t value  Pr(>|t|)
(Intercept)    13650.80    51.4092  265.5324 0.0023975 **
ever_treated  -12118.75    99.1118 -122.2736 0.0052064 **
I(year == 78)   1195.86    21.2944   56.1583 0.0113350 *
post_treat      3621.23    41.0534   88.2078 0.0072170 **
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 9,428.0   Adj. R2: 0.017819
```

Parallel Trends Conditional on Covariates

Parallel Trends Violations

1. Functional form misspecification:
   - Sensitive to (even monotonic) transformations of the outcome (e.g. logarithm) unless the treatment is randomly assigned (Roth and Sant'Anna, 2021).

2. Compositional change with repeated cross-sections:
   - Sample composition may have changed between the pre and post period in ways that are correlated with treatment (Sant'Anna and Xu, 2023).

3. Time-varying Confounding:
   - **Confounding in diff-in-diffs**: covariate with time-varying effect on the outcome or a time-varying difference between groups.
   - **Time-invariant confounding**: covariate differences between groups that are invariant over time, or covariate changes that are invariant across groups - cancel out due to differencing.
   - **Cross-sectional confounding** in comparison: covariate associated with treatment & outcome.

Types of Confounding Covariates

- Time-invariant covariate: X_i
  - Does **not** change over time for an individual.
  - **Confounder** if the means of the covariate are different in the two groups **and** it has a time-varying effect on the outcome.
- Time-varying covariate: $X_{i,t}$
  - Does change over time for an individual.
  - **Confounder** if the covariate means evolve differently between the two groups **or** the covariate means start at different levels and evolve in parallel, and the covariate has a time-varying effect on the outcome.

Handling of Confounding Covariates

- Most estimation approaches treat **all covariates as time-invariant**:
  - Time-varying covariates are fixed at their pre-treatment value across all periods.
  - Advantages: avoids time-varying confounders affected by treatment (mediators are "bad controls") and helps to reduce the dimensionality of the problem.
  - Disadvantage: no control for time trends in X independently of the treatment and thus PT violation remains.
- Newer estimation approaches can handle time-varying covariates in more flexible ways (e.g. Caetano and Callaway, 2023):
  - All covariates values from each period in each period (highest dimensionality).
  - Covariates values from the current period and base period.
  - Changes in covariate values from period to period.
  - Average covariate values across time periods.

Conditional Parallel Trends

- Parallel trend assumption often appear plausible only after controlling for observed covariates $\mathbf{X_i}$:

Assumption (A.cpt) "Conditional Parallel Trends"

$\mathbb{E}[Y_{i,t=1}(0) - Y_{i,t=0}(0)|T_i = 1, \mathbf{X_i}] = \mathbb{E}[Y_{i,t=1}(0) - Y_{i,t=0}(0)|T_i = 0, \mathbf{X_i}]$ and

$\mathbf{X_i}$ is not affected by T_i: $\mathbf{X_i}(1) = \mathbf{X_i}(0) = \mathbf{X_i}$

- Conditional unconfoundedness of the change (rather than potential outcomes themselves): $(Y_1(0) - Y_0(0)) \perp\!\!\!\perp T \mid \mathbf{X_i}$.

Assumption (A.cna) "Conditionally No Anticipation"

$\mathbb{E}[Y_{i,t=0}(1) - Y_{i,t=0}(0)|T_i = 1, \mathbf{X_i}] = 0$

- Treatment has no effect on the treatment group before it is administered within the same strata of $\mathbf{X_i}$.

Assumption (A.pos) "Positivity / Common Support / Overlap"

$\Pr(T_i = 1 \mid \mathbf{X_i}) < 1 \text{ and } \Pr(T_i = 1) > 0.$

- For each treated unit with covariates $\mathbf{X_i}$, there are at least some untreated units in the population with the same $\mathbf{X_i}$.

ATT Conditional on Covariates: Identification

- Given the conditional parallel trends assumption, no anticipation assumption, and overlap condition, the ATT conditional on $\mathbf{X_i} = \mathbf{x}$, $\tau_{\text{DiD}}(\mathbf{x})$, can be identified for all $\mathbf{x}$ with $\Pr(T_i = 1 \mid \mathbf{X_i} = \mathbf{x}) > 0$ as:

$$
\begin{aligned}
\tau_{\text{DiD}}(\mathbf{x}) &= \mathbb{E}[Y_{i,1}(1) - Y_{i,1}(0) | T_i = 1, \mathbf{X_i} = \mathbf{x}] \\
&\overset{\text{LIE}}{=} \mathbb{E}[Y_{i,1}(1)|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,1}(0)|T_i = 1, \mathbf{x}] \\
&= \mathbb{E}[Y_{i,1}(1)|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,0}(0)|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,1}(0)|T_i = 1, \mathbf{x}] + \mathbb{E}[Y_{i,0}(0)|T_i = 1, \mathbf{x}] \\
&\overset{\text{(A.cna)}}{=} \mathbb{E}[Y_{i,1}(1)|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,0}(1)|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,1}(0)|T_i = 1, \mathbf{x}] + \mathbb{E}[Y_{i,0}(0)|T_i = 1, \mathbf{x}] \\
&\overset{\text{(SUTVA)}}{=} \mathbb{E}[Y_{i,1}|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,0}|T_i = 1, \mathbf{x}] - (\mathbb{E}[Y_{i,1}(0) - Y_{i,0}(0)|T_i = 1, \mathbf{x}]) \\
&\overset{\text{(A.cpt)}}{=} \mathbb{E}[Y_{i,1}|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,0}|T_i = 1, \mathbf{x}] - (\mathbb{E}[Y_{i,1}(0) - Y_{i,0}(0)|T_i = 0, \mathbf{x}]) \\
&\overset{\text{LIE}}{=} \mathbb{E}[Y_{i,1}|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,0}|T_i = 1, \mathbf{x}] - (\mathbb{E}[Y_{i,1}(0)|T_i = 0, \mathbf{x}] - \mathbb{E}[Y_{i,0}(0)|T_i = 0, \mathbf{x}]) \\
&\overset{\text{SUTVA}}{=} \underbrace{\mathbb{E}[Y_{i,1}|T_i = 1, \mathbf{x}] - \mathbb{E}[Y_{i,0}|T_i = 1, \mathbf{x}]}_{\text{change for T=1 and X=x}} - \underbrace{(\mathbb{E}[Y_{i,1}|T_i = 0, \mathbf{x}] - \mathbb{E}[Y_{i,0}|T_i = 0, \mathbf{x}])}_{\text{change for T=0 and X=x}}
\end{aligned}
$$

ATT Conditional on Covariates: Estimation

- The unconditional ATT can then be identified by averaging $\tau_{\text{DiD}}(\mathbf{x})$ over the distribution of X_i in the treated population.
- Using the law of iterated expectations, we have:

$$\tau_{\text{DiD}} = \mathbb{E}[Y_{i,1}(1) - Y_{i,1}(0)|T_i = 1] = \mathbb{E}_{\mathbf{X_i}}\left[\underbrace{\mathbb{E}[Y_{i,1}(1) - Y_{i,1}(0)|T_i = 1, \mathbf{X_i}]}_{\tau_{\text{DiD}}(\mathbf{X_i})} \mid T_i = 1\right]$$

Two-Way Fixed Effects (TWFE) Regression

- Augment TWFE specification with covariates (e.g. Zeldow and Hatfield, 2021):
  - Time-invariant covariate with time-variant effect on outcome:
    - $Y_{i,t} = \alpha_i + \gamma_t + \tau_{\text{DiD}}(T_i \times t) + \delta(X_i \times t) + \epsilon_{i,t}$
  - Time-variant covariate with time-invariant effect on outcome:
    - $Y_{i,t} = \alpha_i + \gamma_t + \tau_{\text{DiD}}(T_i \times t) + \delta X_{it} + \epsilon_{i,t}$
  - Time-variant covariate with time-variant effect on outcome:
    - $Y_{i,t} = \alpha_i + \gamma_t + \tau_{\text{DiD}}(T_i \times t) + \delta(X_{it} \times t) + \epsilon_{i,t}$

- Rather strong additional assumptions needed (e.g. Caetano and Callaway, 2023):
  - Treatment effect is homogeneous across different values of X.
  - Outcome is linear in X.
  - Only controlling for covariate changes, not for levels.

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TWFE Regression: Example

```r
library(tidyverse)
library(fixest)
library(haven) # to read Stata files

data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-Inference-2/master/Lab/Lalonde/lalonde_nonexp_panel.dta")

data$post_treat = data$ever_treated * (data$year == 78)
data$post = as.integer(data$year == 78)

# ---- Difference-in-Differences - Two-Way Fixed Effects Regression
feols(
  re ~ post_treat + age:post + agesq:post + agecube:post + educ:post + educsq:post + marr:post + nodegree:post + black:post + hisp:post | id + year,
  data = data |> filter(year %in% c(75, 78)),
  vcov = vcov_cluster(c("id", "year"))
)
```

```
OLS estimation, Dep. Var.: re
Observations: 32,354
Fixed-effects: id: 16,177,  year: 2
Standard-errors: Clustered (id & year)
                Estimate Std. Error   t value Pr(>|t|)
post_treat     2450.964333 645.332739  3.797985 0.163900
age:post      -1392.968721 188.627206 -7.384771 0.085686 .
post:agesq       32.005450   5.576397  5.739450 0.109818
post:agecube     -0.254779   0.052340 -4.867790 0.128988
post:educ      -132.810162  95.337273 -1.393056 0.396361
post:educsq      10.228897   3.957951  2.584392 0.235037
post:marr      -578.337803 163.583509 -3.535429 0.175485
post:nodegree   417.398327 193.694598  2.154930 0.276597
post:black     -281.607046 205.559277 -1.369955 0.401418
post:hisp      -126.167682 234.813629 -0.537310 0.686116
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 3,737.9     Adj. R2: 0.691052
                Within R2: 0.064074
```

Outcome Regression Adjustment

- Regression Adjustment exploits the fact that under conditional parallel trends, strong overlap, and no anticipation the ATT can be written as (Heckman, Ichimura and Todd, 1997):

$$\begin{aligned}
\tau_{\text{DiD}} &= \mathbb{E}_{\mathbf{X_i}}\left[\underbrace{\mathbb{E}[Y_{i,1}-Y_{i,0}|T_i=1,\mathbf{X_i}]-\mathbb{E}[Y_{i,1}-Y_{i,0}|T_i=0,\mathbf{X_i}]}_{\tau_{\text{DiD}}(\mathbf{X_i})}\mid T_i=1\right] \\
&\overset{\text{LIE}}{=} \mathbb{E}[Y_{i,1}-Y_{i,0}|T_i=1]-\mathbb{E}_{\mathbf{X_i}}\left[\,\mathbb{E}[Y_{i,1}-Y_{i,0}|T_i=0,\mathbf{X_i}]\mid T_i=1\right] \\
&\overset{\text{LIE}}{=} \mathbb{E}[Y_{i,1}-Y_{i,0}|T_i=1]-\mathbb{E}_{\mathbf{X_i}}\left[\,\mathbb{E}[Y_{i,1}|T_i=0,\mathbf{X_i}]-\mathbb{E}[Y_{i,0}|T_i=0,\mathbf{X_i}]\mid T_i=1\right] \\
&:= \mathbb{E}[Y_{i,1}-Y_{i,0}|T_i=1]-\mathbb{E}_{\mathbf{X_i}}\left[\,\mu_1(0,\mathbf{X_i})-\mu_0(0,\mathbf{X_i})\mid T_i=1\right]
\end{aligned}$$

- Potential outcome evolution for the treatment group is imputed with a regression based only on X of the control group.
- Sample version: $\hat{\tau}_{\text{DiD}}=\frac{1}{N_1}\sum_{i:T_i=1}\left((Y_{i,1}-Y_{i,0})-(\hat{\mu}_1(0,\mathbf{X_i})-\hat{\mu}_0(0,\mathbf{X_i}))\right)$
- 1. Estimate the conditional expectation of the outcome at time t, $\hat{\mu}_t(T_i=0,\mathbf{X_i})$ among untreated units.
- 2. Create prediction for each treated unit using the covarite values X_i among the treated units.
- 3. Calculate difference between observed and predicted difference for each treated unit and average.

Outcome Regression Adjustment: Example

```r
library(tidyverse)
library(DRDID)
library(haven) # to read Stata files

data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-Inference-2/master/Lab/Lalonde/lalonde_nonexp_panel.dta")

# ---- Difference-in-Differences - Double-robust
ordid(
  yname = "re", tname = "year", idname = "id", dname = "ever_treated",
  xformla = ~ age + agesq + agecube + educ + educsq + marr + nodegree + black + hisp + re74 + u74,
  data = data |> filter(year == 75 | year == 78)
)
```

```
 Call:
ordid(yname = "re", tname = "year", idname = "id", dname = "ever_treated",
    xformla = ~age + agesq + agecube + educ + educsq + marr +
        nodegree + black + hisp + re74 + u74, data = filter(data,
        year == 75 | year == 78))
------------------------------------------------------------------
 Outcome-Regression DID estimator for the ATT:

   ATT     Std. Error  t value    Pr(>|t|)  [95% Conf. Interval]
1769.9984   643.0996    2.7523     0.0059    509.5233  3030.4736
------------------------------------------------------------------
 Estimator based on panel data.
 Outcome regression est. method: OLS.
 Analytical standard error.
------------------------------------------------------------------
 See Sant'Anna and Zhao (2020) for details.
```

Outcome Regression with ML: Example

```r
library(tidyverse)
library(mlr3)
library(mlr3learners)
library(haven) # to read Stata files

data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-Inference-2/master/Lab/Lalonde/lalonde_nonexp_panel.dta")
data <- data |> filter(year == 75 | year == 78) |>
  select(-treat, -data_id) |>
  pivot_wider(names_from = year, values_from = re, names_prefix = "re_") |>
  mutate(re_diff = re_78 - re_75)
data_pred <- data |> select(re_diff, ever_treated, age, agesq, agecube, educ, educsq,
                              marr, nodegree, black, hisp, re74, u74)

# Define prediction model for the outcome difference delta_mu
task_mu <- as_task_regr(data_pred |> select(-ever_treated), target = "re_diff")
lrnr_mu <- lrn("regr.ranger", predict_type = "response")
# learn outcome difference delta_mu among untreated
lrnr_mu$train(task_mu, row_ids = which(data$ever_treated == 0))
# predict outcome difference delta_mu among treated
delta_mu <- lrnr_mu$predict(task_mu, row_ids = which(data$ever_treated == 1))$response

Y1 <- data$re_78[data$ever_treated == 1]
Y0 <- data$re_75[data$ever_treated == 1]

mean( (Y1 - Y0) - delta_mu )
```

```
[1] 2201.86
```

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Inverse Probability Weighting (IPW)

- The IPW approach proposed by Abadie (2005):
  - $\tau_{\text{DiD}} = \mathbb{E}\left(\frac{Y_{i,1}-Y_{i,0}}{P(T_i=1)} \frac{T_i-e(\mathbf{X_i})}{1-e(\mathbf{X_i})}\right)$
  - $e(\mathbf{X_i}) = P[T_i = 1|\mathbf{X_i}]$
- Sample version:
  - $\hat{\tau}_{\text{DiD}} = \frac{1}{N}\sum_i \left(\frac{Y_{i,1}-Y_{i,0}}{P(T_i=1)} \frac{T_i-\hat{e}(\mathbf{X_i})}{1-\hat{e}(\mathbf{X_i})}\right)$
- Intuition: what happens when $T_i = 1$ and $T_i = 0$?
  - Weighting with the propensity only happens to the control group's first differences – not the treatment groups!
  - Why? Because it's the $Y_1(0)$ that is missing, not the $Y_1(1)$.

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IWP: Example

```r
library(tidyverse)
library(DRDID)
library(haven) # to read Stata files

data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-Inference-2/master/Lab/Lalonde/lalonde_nonexp_panel.dta")

# ---- Difference-in-Differences - Double-robust
ipwdid(
  yname = "re", tname = "year", idname = "id", dname = "ever_treated",
  xformla = ~ age + agesq + agecube + educ + educsq + marr + nodegree + black + hisp + re74 + u74,
  data = data |> filter(year == 75 | year == 78)
)
```

```
 Call:
ipwdid(yname = "re", tname = "year", idname = "id", dname = "ever_treated",
    xformla = ~age + agesq + agecube + educ + educsq + marr +
        nodegree + black + hisp + re74 + u74, data = filter(data,
        year == 75 | year == 78))
------------------------------------------------------------------
 IPW DID estimator for the ATT:

   ATT     Std. Error  t value    Pr(>|t|)  [95% Conf. Interval]
2048.1972   724.1233    2.8285     0.0047    628.9156  3467.4788
------------------------------------------------------------------
 Estimator based on panel data.
 Hajek-type IPW estimator (weights sum up to 1).
 Propensity score est. method: maximum likelihood.
 Analytical standard error.
------------------------------------------------------------------
 See Sant'Anna and Zhao (2020) for details.
```

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IWP with ML: Example

```r
library(tidyverse)
library(mlr3)
library(mlr3learners)
library(haven) # to read Stata files
data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-Inference-2/master/Lab/Lalonde/lalonde_nonexp_panel.dta")
data <- data |> filter(year == 75 | year == 78) |>
  select(-treat, -data_id) |>
  pivot_wider(names_from = year, values_from = re, names_prefix = "re_")
data_pred <- data |> select(re_78, re_75, ever_treated, age, agesq, agecube, educ, educsq,
                            marr, nodegree, black, hisp, re74, u74)

# Define prediction model for the propensity score e
task_e <- as_task_classif(data_pred |> select(-re_78, -re_75), target = "ever_treated")
lrnr_e <- lrn("classif.ranger", predict_type = "prob")
# Learn propensity score e among all observations
lrnr_e$train(task_e)
# Predict propensity score ehat
ehat <- lrnr_e$predict(task_e)$prob[, 2]

# Calculate the ATT
T <- data$ever_treated
P <- mean(data$ever_treated)
Y1 <- data$re_78
Y0 <- data$re_75

mean( (Y1-Y0)/P * (T - ehat)/(1-ehat) )
```

```
[1] 2925.12
```

Doubly Robust Estimation

- Outcome regression and IPW approaches can also be combined in the context of Diff-in-Diffs to form “doubly-robust” (DR) methods that are valid if either the outcome model or the propensity score model is correctly specified (Sant’Anna and Zhao, 2020):
  - $\tau_{\text{DiD}} = \mathbb{E}\left(\left(Y_{i,1} - Y_{i,0} - (\mu_1(0, \mathbf{X_i}) - \mu_0(0, \mathbf{X_i}))\right)\left(\frac{T_i - e(\mathbf{X_i})}{P(T_i)(1 - e(\mathbf{X_i}))}\right)\right)$
- Sample version:
  - $\hat{\tau}_{\text{DiD}} = \frac{1}{N}\sum_i \left(\left(Y_{i,1} - Y_{i,0} - (\hat{\mu}_1(0, \mathbf{X_i}) - \hat{\mu}_0(0, \mathbf{X_i}))\right)\left(\frac{T_i - \hat{e}(\mathbf{X_i})}{P(T_i)(1 - \hat{e}(\mathbf{X_i}))}\right)\right)$
- Double machine learning for difference-in-differences models (Chang, 2020).

Doubly Robust Estimation: Example

```r
library(tidyverse)
library(DRDID)
library(haven) # to read Stata files

data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-Inference-2/master/Lab/Lalonde/lalonde_nonexp_panel.dta")

# ---- Difference-in-Differences - Double-robust
drdid(
  yname = "re", tname = "year", idname = "id", dname = "ever_treated",
  xformla = ~ age + agesq + agecube + educ + educsq + marr + nodegree + black + hisp + re74 + u74,
  data = data |> filter(year == 75 | year == 78)
)
```

```
 Call:
drdid(yname = "re", tname = "year", idname = "id", dname = "ever_treated",
    xformla = ~age + agesq + agecube + educ + educsq + marr +
        nodegree + black + hisp + re74 + u74, data = filter(data,
        year == 75 | year == 78))
------------------------------------------------------------------
 Further improved locally efficient DR DID estimator for the ATT:

   ATT     Std. Error  t value    Pr(>|t|)  [95% Conf. Interval]
2032.9217   707.4779    2.8735     0.0041    646.265   3419.5784
------------------------------------------------------------------
 Estimator based on panel data.
 Outcome regression est. method: weighted least squares.
 Propensity score est. method: inverse prob. tilting.
 Analytical standard error.
------------------------------------------------------------------
 See Sant'Anna and Zhao (2020) for details.
```

Doubly Robust Estimation with ML: Example

```r
library(tidyverse)
library(mlr3)
library(mlr3learners)
library(haven) # to read Stata files

data <- haven::read_dta("https://raw.github.com/Mixtape-Sessions/Causal-Inference-2/master/Lab/Lalonde/lalonde_nonexp_panel.dta")
data <- data |> filter(year == 75 | year == 78) |>
  select(-treat, -data_id) |>
  pivot_wider(names_from = year, values_from = re, names_prefix = "re_") |>
  mutate(re_diff = re_78 - re_75)
data_pred <- data |> select(re_diff, ever_treated, age, agesq, agecube, educ, educsq,
                            marr, nodegree, black, hisp, re74, u74)

# Define prediction model for the propensity score e
task_e <- as_task_classif(data_pred |> select(-re_diff), target = "ever_treated")
lrnr_e <- lrn("classif.ranger", predict_type = "prob")
# Learn propensity score e among all observations
lrnr_e$train(task_e)
# Predict propensity score ehat
ehat <- lrnr_e$predict(task_e)$prob[, 2]

# Define prediction model for the outcome difference delta_mu
task_mu <- as_task_regr(data_pred |> select(-ever_treated), target = "re_diff")
lrnr_mu <- lrn("regr.ranger", predict_type = "response")
# learn outcome difference delta_mu among untreated
lrnr_mu$train(task_mu, row_ids = which(data$ever_treated == 0))
```

```
[1] 2308.23
```

Staggered treatment Timing

Staggered Timing

- Remember basic DiD model:
  - Two periods and a common treatment date.
  - Identification from parallel trends and no anticipation.
- Active recent literature has focused on relaxing the first assumption:
  - What if there are multiple periods and units adopt treatment at different times?
  - Maintaining parallel trends and no anticipation assumptions.

- Notation:
  - Panel of observations i and time periods $t = 1 \ldots T_t$.
  - Units adopt a binary treatment at different dates $G_i \in (1, \ldots, T_t) \cup \infty$.
    - where $G_i = \infty$ means **never-treated**.
  - Potential outcomes $Y_{it}(g)$ depend on time (t) and time you were first treated (g).

- Literature is now starting to consider cases with continuous treatment & treatments that turn on/off.
  - still developing; for a review see de Chaisemartin and D'Haultfœuille (2023).

Extending the Identifying Assumptions

- Key identifying assumptions from the canonical model are extended in a natural way:

Assumption (A.stpt) "Parallel Trends"

$$\mathbb{E}[Y_{i,t}(\infty) - Y_{i,t-1}(\infty)|G_i = g] = \mathbb{E}[Y_{i,t-1}(\infty) - Y_{i,t}(\infty)|G_i = g'] \quad \forall g, g', t.$$

- Intuitively, says that if treatment hadn't happened, all "adoption cohorts" would have parallel average outcomes in all periods. (Note: could impose slightly weaker versions, e.g. only require PT post-treatment).

Assumption (A.stna) "No Anticipation"

$$\mathbb{E}[Y_{i,t}(g) - Y_{i,t}(\infty)|T_i = 1] = 0 \quad \text{or} \quad Y_{i,t}(g) = Y_{i,t}(\infty) \quad \forall t < g$$

- Treatment has no effect on the treatment group before it is administered.

TWFE Regression with Staggered Timing

- Suppose we extend Two-way fixed effects (TWFE) regression to staggered treatment timing:
  - $Y_{i,t} = \alpha_i + \gamma_t + \beta D_{it} + \epsilon_{i,t}$
  - where $D_{it} = 1[t \geq G_i]$ is an indicator for whether the unit has been treated by time t.
- Given no anticipation and parallel trends across all adoption cohorts:
  - if treatment effect is constant across time and units, $Y_{it}(g) - Y_{it}(\infty) \equiv \tau$, identification possible: $\tau = \beta$.
  - if treatment effect is heterogeneous, i.e. depends on time since treatement, $Y_{it}(t-r) - Y_{it}(\infty) \equiv \tau_r$, then identification fails, because some τ_r's may get negative weights.
- **Reason**:
  - Clean comparisons: DiD's between treated and not-yet-treated units.
  - Forbidden comparisons: DiD's between already-treated units (who began treatment at different times).
    - can lead to negative weights, if treatment effects in the already treated "control group" change over time.

Forbidden Comparisons in TWFE: Intuition

- 1. Suppose two period model with two groups: always treated (in both periods) & switchers (treated only in period 2).
  - With two periods, $Y_{i,t} = \alpha_i + \gamma_t + \beta D_{it} + \epsilon_{i,t}$ is the same as $\Delta Y_i = \alpha + \beta \Delta D_i + u_i$ (by first-differencing).
  - $\Delta D_i = 1$ for switchers and 0 for the control group of always treated, thus:
  - $\hat{\beta} = \left(\overline{Y}_{\text{switchers},2} - \overline{Y}_{\text{switchers},1}\right) - \left(\overline{Y}_{\text{AT},2} - \overline{Y}_{\text{AT},1}\right)$
  - Problem: if treatment effect for always-treated grows over time, $\hat{\beta}$ can get negative weights.
- 2. Frisch-Waugh-Lovell theorem says that we can obtain β in $Y_{i,t} = \alpha_i + \gamma_t + \beta D_{it} + \epsilon_{i,t}$ in two steps:
  - 1. Regress $D_{i,t}$ on fixed effects (in a linear probability model - LPM): $D_{i,t} = \tilde{\alpha}_i + \tilde{\gamma}_t + \tilde{\epsilon}_{i,t}$.
  - 2. Regress $Y_{i,t}$ on $D_{i,t} - \hat{D}_{i,t}$, thus: $\beta = \frac{\mathbb{E}(Y_{i,t}(D_{i,t} - \hat{D}_{i,t}))}{Var(D_{i,t} - \hat{D}_{i,t})}$.
  - However, LPMs can predict $\hat{D}_{i,t} > 1$, and Y_{it} can get negative weight.
- 3. Even if weights are non-negative (i.e. individual τ_i's are constant - no dynamics), β might still be biased:
  - $\beta = \sum_{i=1}^{N} \sum_{t=1}^{T_t} w_{it} \beta_{it}$: w_{it} is inversely proportional to the variance of β_{it}.
    - Proportional to available information for i, i.e. the number of observations pre and post treatment.
  - But are individuals in the middle of the panel also the most representative of the population?

Dynamic TWFE Regression with Staggered Timing

- Sun and Abraham (2021) show that similar issues arise with dynamic TWFE ("event study") specifications:
  - $Y_{i,t} = \alpha_i + \gamma_t + \sum_{k \neq 0} \beta_k D_{it}^k + \epsilon_{i,t}$
  - where $D_{it}^k = 1[t - G_i = k]$ are leading and lagging "event time" dummies.

- This dynamic specification yields a sensible causal estimand when there is heterogeneity only in time since treatment.
- However, if there is heterogeneity in dynamic treatment effects also across adoption cohorts, then:
  - Like for static TWFE, β_k may put negative weight on treatment effects after k periods for some units.
  - Furthermore, β_k may be **"contaminated"** by treatment effects at different leads and lags $k' \neq k$.

- Thus, interpreting β_k as estimates ...
  - of the dynamic effects of treatment $(k > 0)$ may be misleading.
  - for pre-trends tests $(k < 0)$ may also be misleading.
    - We will return to pre-trends tests later.

New DiD-Estimators for Staggered Timing

- Estimators based on **clean aggregated comparisons**:
  - Callaway and Sant'Anna (2021): R package 'did' (**Focus**)
  - Sun and Abraham (2021): R package 'fixest'
- Estimators based on **imputation**:
  - Gardner, Thakral, Tô, and Yap (2024): R package 'did2s' (**Focus**)
  - Borusyak, Jaravel, Spiess (2024): R package 'did_imputation'
  - Wooldridge (2021): R package 'etwfe'
- Estimators that can handle **non-absorbing and/or non-binary treatments**:
  - de Chaisemartin and D'Haultfoeuille (2020): R package 'DIDmultiplegt'
- Estimators based on **stacking**:
  - Cengiz, Dube, Lindner, and Zipperer (2019)
  - Dube, Girardi, Jorda, and Taylor (2023)

Estimator by Callaway & Sant'Anna (2021)

- Callaway & Sant'Anna (2021) define the group-time-specific treatment effect on the treated:
  - $\tau(g,t) = \mathbb{E}[Y_{i,t}(g) - Y_{i,t}(\infty) | G_i = g]$, with $t \geq g$.
  - ATT in period t for units first treated in period g.

- Under PT and No Anticipation, it can be identified as:
  - $\tau(g,t) = \underbrace{\mathbb{E}[Y_{i,t} - Y_{i,g-1} | G_i = g]}_{\text{change for cohort g}} - \underbrace{\mathbb{E}[Y_{i,t} - Y_{i,g-1} | G_i = \infty]}_{\text{change for never-treated}}$
  - Similar to the basic model, this is a two-group two-period comparison.
    - Similar identification proof (next).
  - Differences:
    - 1. Period $g-1$ is pre-treatment period (right before cohort g becomes treated).
    - 2. More flexibility in terms of comparison group: (a) never-treated, (b) not-yet-treated, (c) not-yet-but-eventually-treated, (d) last-to-be-treated.

- Sample version of $\tau(g,t)$:
  - $\hat{\tau}(g,t) = \frac{1}{N_g} \sum_{i=1}^{N_g} (Y_{i,t} - Y_{i,g-1}) 1[G_i = g] - \frac{1}{N_\infty} \sum_{i=1}^{N_\infty} (Y_{i,t} - Y_{i,g-1}) 1[G_i = \infty]$

Callaway & Sant'Anna (2021): Proof

- Start with identification result and work backwards:
  - $\mathbb{E}[Y_{i,t} - Y_{i,g-1} | G_i = g] - \mathbb{E}[Y_{i,t} - Y_{i,g-1} | G_i = \infty]$
- Apply definition of Potential Outcomes:
  - $\mathbb{E}[Y_{i,t}(g) - Y_{i,g-1}(g) | G_i = g] - \mathbb{E}[Y_{i,t}(\infty) - Y_{i,g-1}(\infty) | G_i = \infty]$
- Use No Anticipation to substitute $Y_{i,g-1}(\infty)$ for $Y_{i,g-1}(g)$:
  - $\mathbb{E}[Y_{i,t}(g) - Y_{i,g-1}(\infty) | G_i = g] - \mathbb{E}[Y_{i,t}(\infty) - Y_{i,g-1}(\infty) | G_i = \infty]$
- Add and subtract $\mathbb{E}[Y_{i,t}(\infty) | G_i = g]$:
  - $\mathbb{E}[Y_{i,t}(g) - Y_{i,g-1}(\infty) | G_i = g] - \mathbb{E}[Y_{i,t}(\infty) - Y_{i,g-1}(\infty) | G_i = \infty] + \mathbb{E}[Y_{i,t}(\infty) | G_i = g] - \mathbb{E}[Y_{i,t}(\infty) | G_i = g]$
- Rearrange terms:
  - $\mathbb{E}[Y_{i,t}(g) - Y_{i,t}(\infty) | G_i = g] + \underbrace{\mathbb{E}[Y_{i,t}(\infty) - Y_{i,g-1}(\infty) | G_i = g] - \mathbb{E}[Y_{i,t}(\infty) - Y_{i,g-1}(\infty) | G_i = \infty]}_{=0}$
- QED:
  - $\tau(g,t) = \mathbb{E}[Y_{i,t}(g) - Y_{i,t}(\infty) | G_i = g]$

Callaway & Sant'Anna (2021): Aggregation

- If have a large number of observations and relatively few groups/periods, can report $\hat{\tau}(g,t)$'s directly.
- If there are many groups/periods, the $\hat{\tau}(g,t)$'s may be very imprecisely estimated and/or too numerous to report.
- In these cases, it is often desirable to report meaningful averages of the $\hat{\tau}(g,t)$'s.
- Four aggregation schemes:
  - Simple:
    - Computes a single weighted average of all group-time average treatment effects with weights proportional to group size.
  - Dynamic:
    - Computes event-study parameters which average the $\hat{\tau}(g,t)$'s at a particular lag since (or lengths of exposure to) the treatment.
    - Can also be constructed for k < 0 to estimate "pre-trends".
  - Group:
    - Computes group averages which average the $\hat{\tau}(g,t)$'s for a particular cohort treated a g.
  - Calendar:
    - Computes "calendar averages" which average the $\hat{\tau}(g,t)$'s for a particular calendar time (year).

Callaway & Sant'Anna (2021): Further Variants

- Anticipation:
  - In many applications, units may observe that an intervention is about to occur, so that they change their behaviors before the intervention is actually implemented.
  - Straightforward adaptation: if there is one period of anticipation, set the base period to $g-2$ rather than $g-1$, so that:
    - $\tau(g,t) = \mathbb{E}[Y_{i,t} - Y_{i,g-2} | G_i = g] - \mathbb{E}[Y_{i,t} - Y_{i,g-2} | G_i = \infty]$

- Covariates:
  - Staggered timing estimator can also be extended to include covariates:
    - 1. Conditional outcome regression.
    - 2. Inverse Probability Weighting.
    - 3. Doubly Robust Estimation.

Callaway & Sant'Anna (2021): Example

- Assess the impact of job displacement (i.e. losing job w/o own fault, e.g. mass layoff) on income of 1,298 individuals.

```r
library(did) # Load the 'did' package
library(fixest) # Load the 'fixest' package
temp_file <- tempfile(fileext = ".RData") # Define a temporary file path
# Download the file from Dropbox
download.file("https://www.dropbox.com/scl/fi/wnp1hrkz00izr72h6ua1t/job_
load(temp_file) # Load the RData file into the R session
rm(temp_file) # Optionally, remove the temporary file


# Check the structure of the loaded data
head(job_displacement_data)

# run TWFE
fixest::feols(income ~ i(year >= group) | id + year,
              data=job_displacement_data,
              cluster=c("id", "year"))
```

```
       id year group income female white occ_score
1 7900002 1984     0  31130      1     1         4
2 7900002 1985     0  32200      1     1         3
3 7900002 1986     0  35520      1     1         4
4 7900002 1987     0  43600      1     1         4
5 7900002 1988     0  39900      1     1         4
6 7900002 1990     0  38200      1     1         4

OLS estimation, Dep. Var.: income
Observations: 11,682
Fixed-effects: id: 1,298,  year: 9
Standard-errors: Clustered (id & year)
                    Estimate Std. Error  t value Pr(>|t|)
year >= group::TRUE -6455.36    2041.49 -3.16208 0.013353 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
RMSE: 14,824.0     Adj. R2: 0.674268
                 Within R2: 0.002425
```

Callaway & Sant'Anna (2021): Example cont'd

- Assess the impact of job displacement (i.e. losing job w/o own fault, e.g. mass layoff) on income of 1,298 individuals.

Imputation-based Estimation: Procedure

- Recall the TWFE specification (with covariates) with the (heterogeneous) τ_{it} if PT assumption holds:
  - $Y_{i,t} = \alpha_i + \gamma_t + \tau_{it} D_{it} + \delta X_{it} + \epsilon_{i,t}$
  - where $D_{it} = T_i \times t$ and $\mathbb{E}[\epsilon_{i,t} | \{D_{it}, X_{it}\}_{t=1}^{T_t}] = 0$
- If we have some t where $D_{it} = 0$ for all i (e.g. pre-treatment period observations), two-stage procedure:
  - 1. Using all observations with $D_{it} = 0$, regress $Y_{i,t}$ on the fixed effect α_i and γ_t as well as on the covariates X_{it}:
    - $Y_{i,t}(0) = \alpha_i + \gamma_t + \delta X_{it} + \epsilon_{i,t}$.
    - Obtain $\hat{Y}_{i,t}(0) = \hat{\alpha}_i + \hat{\gamma}_t + \hat{\delta} X_{it}$.
  - 2. Regress adjusted outcomes $Y_{i,t} - \hat{Y}_{i,t}(0)$ on D_{it} to obtain $\hat{\tau}_{it}$:
    - $(Y_{i,t} - \hat{Y}_{i,t}(0)) = \alpha_0 + \tau_{it} D_{it} + \epsilon_{i,t}$.
- With events studies of the form $Y_{i,t} = \alpha_i + \gamma_t + \sum_{k \neq 0} \tau_{it}^k D_{it}^k + \delta X_{it} + \epsilon_{i,t}$:
  - 1st stage remains the same.
  - 2nd stage: Regress adjusted outcomes $Y_{i,t} - \hat{Y}_{i,t}(0)$ on event dummies D_{it}^k to obtain $\hat{\tau}_{it}^k$:
    - $(Y_{i,t} - \hat{Y}_{i,t}(0)) = \alpha_0 + \sum_{k \neq 0} \tau_{it}^k D_{it}^k + \epsilon_{i,t}$.

Imputation-based Estimation: Comparison

- Approaches of Gardner, Thakral, Tô, and Yap (2024) and Borusyak, Jaravel, Spiess (2024) are very similar:
  - Same point estimates, but differ in deriving standard errors.
- Key difference to C&S (2021) is the trade-off between efficiency and strength of identifying assumption:
  - Plus: averaging over multiple pre-treatment periods (instead of one) can increase precision.
  - Minus: parallel trends need to hold for all groups and time periods (instead of only post-treatment parallel trends).

Imputation-based Estimation: Example

- Assess the impact of job displacement (i.e. losing job w/o own fault, e.g. mass layoff) on income of 1,298 individuals.

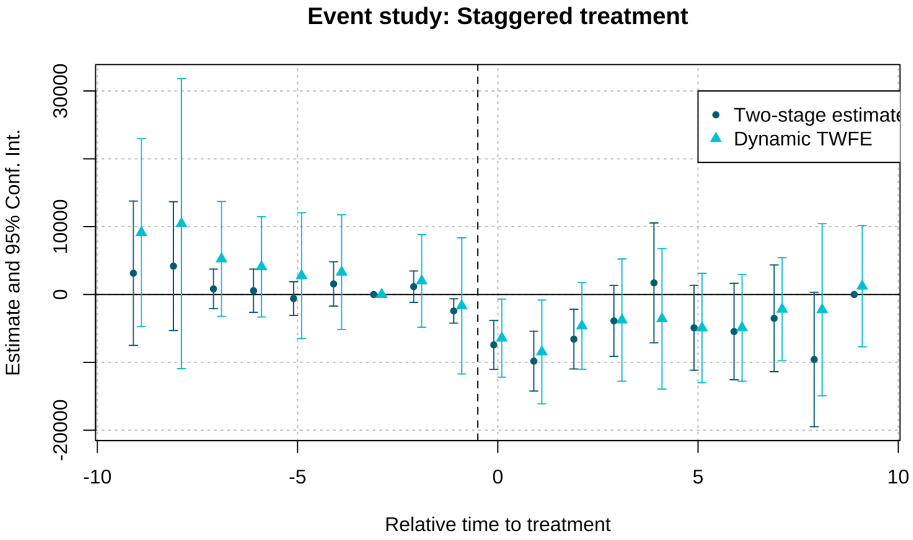
```r
library(did2s) # Load the 'did' package
library(tidyverse) # Load the 'tidyverse' package
temp_file <- tempfile(fileext = ".RData") # Define a temporary file path
# Download the file from Dropbox
download.file("https://www.dropbox.com/scl/fi/wnp1hrkz00izr72h6ua1t/job_
load(temp_file) # Load the RData file into the R session
rm(temp_file) # Optionally, remove the temporary file

job_displacement_data <- job_displacement_data |>
  # create time-variant treatment indicator D_it
  mutate(d_it = case_when(
    group == 0 ~ 0,
    year >= group ~ 1,
    TRUE ~ 0)
  ) |>
  # create releative event-time indicator D_it_k
  mutate(d_it_k = case_when(
    group == 0 ~ Inf,
    TRUE ~ year - group)
  )


# Static model
result <- did2s(
  job_displacement_data,
  yname = "income",
```

```
                                  result
Dependent Var.:                   income

d_it               -5,900.8** (2,151.9)
_______________    ____________________
S.E. type                         Custom
Observations                      11,448
R2                               0.00689
Adj. R2                          0.00689
---
Signif. codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```



Testing for Parallel Trends

Testing for Pre-existing Trends

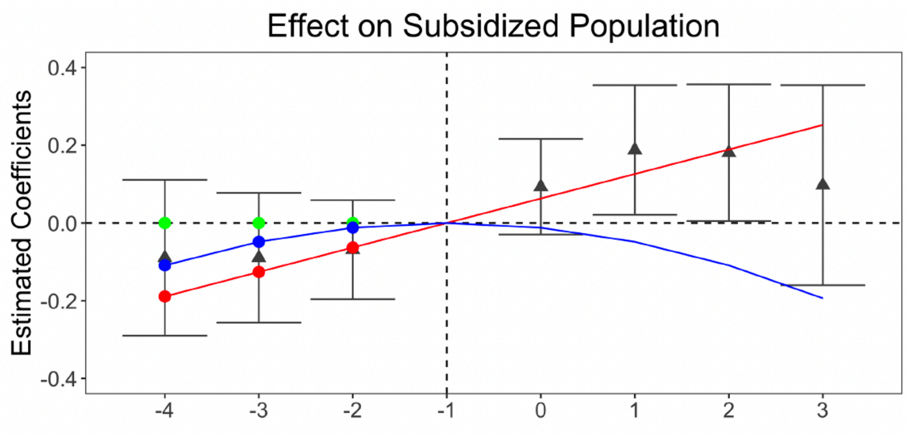
- In most DiD applications we have several periods before anyone was treated.
- We can test whether the groups were moving in parallel prior to the treatment.
  - If so, then assumption that confounding factors are stable seems more plausible.
  - If not, then it's relatively implausible that would have magically started moving in parallel after treatment date.
- Event study plot is a common way to visualize pre-trends, which van be generated based on:
  - Dynamic TWFE (not robust with staggered timing).
  - Comparision-based estimators.
  - Imputation-based, two-stage estimators.

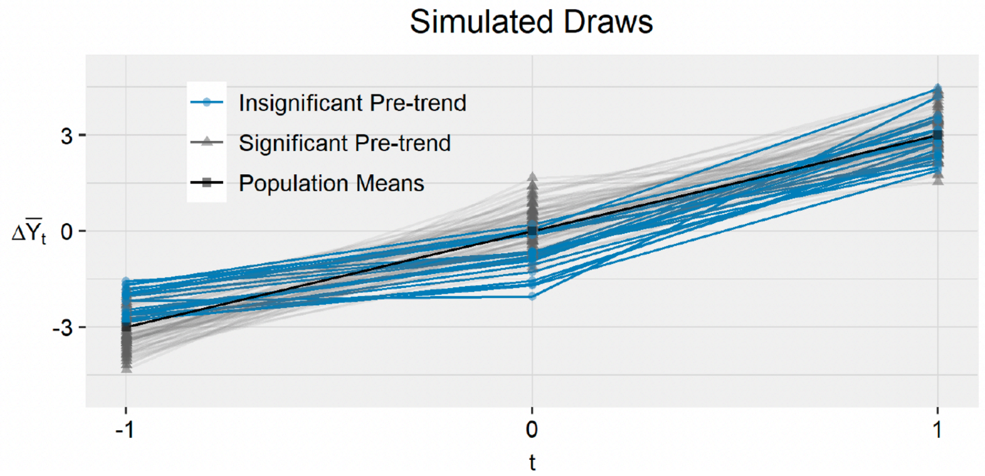
Issues with Pre-existing Trends (Roth, 2022)

- Parallel pre-trends don't necessarily imply parallel (counterfactual) post-treatment trends.
  - If other policies change at the same time as the one of interest can produce parallel pre-trends but non-parallel post-trends.
- Low power: even if pre-trends are non-zero, we may fail to detect it statistically
- Distortions from pre-testing: if we only analyze cases without statistically significant pre-trends, this introduces a form of selection bias (pre-test bias which can make things worse).
- If we fail the pre-test, what next? May still want to write a paper (especially if violation is "small").

Issues with Pre-existing Trends (Roth, 2022)

- Power issues in pre-trend testing: We can't reject zero pre-trend, but we also can't reject pre-trends that under smooth extrapolations to the post-treatment period would produce substantial bias.
- Distortions from pre-testing: If we happen to draw sample from the population where pre-trends are insignificant, the treatment effect we discover later on might be significantly biased (upwards in this example).





Solutions to Parallel Trends Testing

- Roth (2022):
  - Diagnostics of power and distortions from pre-testing.
    - Power: calculates the slope of a linear violation of parallel trends that a pre-trends test would detect a specified fraction of the time.
    - Distortions: calculates the bias that would result from only analyzing cases with statistically significant pre-trends.
  - R package 'pretrends'
- Rambachan and Roth (2022):
  - Formal sensitivity analysis that avoids pre-testing:
    - Put bounds to the unobservable post-treatment trend: how different could it be from the pre-treatment trend to invalidate the DiD estimate?
  - R package 'HonestDiD'

TUHH Institute of Entrepreneurship STARTUPENGINEER.io

Thank you for your attention!

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christoph-ihl

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