KPZ 4D

Andrea Pagnani[1], Giorgio Parisi[2]
[1] *Human Genetics Foundation (HuGeF), Via Nizza 52, I-10126, Turin, Italy*
[2] *Dipartimento di Fisica, INFN - Sezione di Roma 1, CNR-IPCF UOS Roma, Università "La Sapienza", P.le Aldo Moro 2, I-00185 Roma, Italy*

We consider the RSOS discrete surface evolution in 4D.

PACS numbers: 02.50.Ey, 05.70.Ln, 64.60.Ht, 68.35.Fx

The Kardar-Parisi-Zhang (KPZ) [1] equation is possibly the simplest, most studied, and yet not fully understood, model for out-of-equilibrium surface growth. The equation describes the time evolution of the height $h(\mathbf{r},t)$ of an interface above a $d-$dimensional substrate:

$$\partial_t h(\mathbf{r},t) = \nu\vec{\nabla}^2 h(\mathbf{r},t) + \frac{\lambda}{2}|\vec{\nabla}h(\mathbf{r},t)|^2 + \eta(\mathbf{r},t)\,, \tag{1}$$

where ν is the diffusion coefficient, λ is the strength of the non-linear growth rate term which is responsible for the $h \to -h$ symmetry breaking with respect to the growing direction, and $\eta(\mathbf{r},t)$ is a Gaussian white noise of variance (amplitude) D:

$$\langle\eta\rangle = 0 \quad , \quad \langle\eta(\mathbf{r},t)\eta(\mathbf{r}',t')\rangle = 2D\delta^d(\mathbf{r}-\mathbf{r}')\delta(t-t')\ . \tag{2}$$

The KPZ equation describe many relevant growth processes, such as the Eden model, ballistic deposition, interface growth in disordered medium. It is also related to many other physical phenomena apparently unrelated to surface growth, such as Burgers turbulence, dynamics directed polymers in random media, dissipative transport in the driven-diffusion equation [2].

The scaling properties of the height's fluctuations $w_2(L,t) = \langle h^2(\mathbf{r},t)\rangle_{\mathbf{r}} - \langle h(\mathbf{r},t)\rangle^2_{\mathbf{r}}$ (with the notation $\langle\ldots\rangle_{\mathbf{r}}$ we indicate a spatial average over a macroscopic box of linear size L over the $d-$dimensional substrate) characterize the universality class of the model. More precisely, for a system of size L, $w_2(L,t) \sim L^{2\chi} f(t/L^z)$ [2], where the scaling function is such that $f(x) \to \text{const}$ for $x \to \infty$ and $f(x) \sim x^{2\chi/z}$ for $x \to 0$. The peculiar extremal behavior of f imply that $w_2(L,t) \sim L^{2\chi}$ for $t \gg L^z$ and $w_2(L,t) \sim t^{2\chi/z}$ for $t \ll L^z$. Due to an infinitesimal tilt symmetry of Eq. (1) ($h \to h + \mathbf{r}\cdot\boldsymbol{\epsilon}$, $\mathbf{r} \to \mathbf{r} - \lambda t\boldsymbol{\epsilon}$), the two critical exponents are related by the scaling relation $\chi + z = 2$, which is believed to be valid at any dimension d.

A complete understanding of Eq. (1), and in particular the determination of the two critical exponents χ, z for any spatial dimension $d > 1$ (at $d = 1$ a fluctuation-dissipation theorem leads to the exact result $\chi = 1/2$, $z = 3/2$) , turns out to be extremely difficult for two main reasons: (i) we are dealing with an intrinsically out-of-equilibrium phenomenon where the standard equilibrium toolbox must be used with care, (ii) perturbative renormalization schemes are not adequate for describing the strong coupling regime (*i.e.* where the parameter λ is relevant). Among the unsolved theoretical issues related with Eq. (1), that of the existence of an upper critical dimension d_u, *i.e.* the substrate dimensionality d above which the fluctuation of the model become irrelevant ($\chi = 0$), is among the most controversial ones. The determination of d_u would be a most relevant achievement, since, as customary in equilibrium critical phenomena, its knowledge constitutes the first step for a controlled perturbative expansion around it. The quest for d_u has been around for more than twenty years [3–15] and the different predictions range from $d_u \approx 2.8$ to $d_u = \infty$. Analytical estimates using the mode-coupling theory which yeald exact results in $d = 1$ [16], when extended to higher dimensions, seem to hint for a $d_u = 4$ under different self-consistency schemes [5–9]. The same value for d_u is also supported by different field-theoretic approaches [3, 10–12].

At odd with what predicted by the previously mentioned field-theoretic approaches, both direct numerical integration of KPZ equation [17], and simulation of systems belonging to the KPZ universality class [13, 14, 18, 19] indicate that $d_u > 4$, while the real-space renormalization group approach [15] predicts $d_u = \infty$.

Such a long standing controversy is the consequence of the difficulties inherent to both analytical and numerical approaches. Most of the assumptions made on the functional structure of the sought solution in the different field-theoretic analysis, as well as the approximations made in the mode-coupling theories are, in general, not completely under control. On the numerical side the most severe problem is due to the fact that simulations in high spatial dimensions $d \geq 4$ are computationally very heavy, and the systems under analysis must be limited in size. As a consequence, the different fitting procedures in order to yield reliable estimates of the critical exponents, must deal with controlled finite-size scaling procedures. Under this perspective, particularly relevant is the observation that for lattice models in the KPZ universality class, a controlled asymptotic regime is achieved only when typical scale of the fluctuations is larger than the lattice spacing used in the simulations or, more precisely, for $w_2 > 1$ [9].

$$\begin{aligned} w_2 &= A_2 L^{2\chi}(1 + B_2 L^{-\omega}) \\ w_3 &= A_3 L^{3\chi}(1 + B_3 L^{-\omega}) \\ w_4 &= A_4 L^{4\chi}(1 + B_4 L^{-\omega}) \end{aligned} \tag{3}$$

We simulate $4-$dimensional lattices of volume $V = L^4$.

- Multi-surface coding
- Multi-spin coding of the lattice.

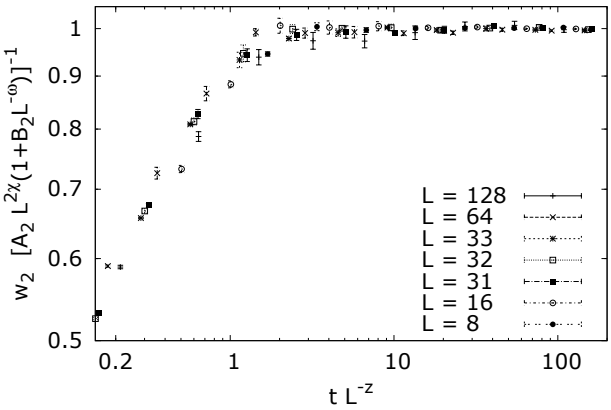

FIG. 1. Scaling plot

L	mc-sweeps	samples	type	time (h.)
8	524000	1024	MS	4
16	524000	256	MS	6
31	524000	256	MS	121
32	524000	256	MS	139
33	524000	256	MS	158
64	131000	512	MS	5376
128	512000	32	ML	7680
256	130000	3	ML	504

TABLE I. In this table we display the lattice linear size L the number of montecarlo sweeps (full lattice update), the number of samples and the simulation type (MS = multi-surface coding, ML = lattice multi-spin coding), overall computational time in hours

[1] M. Kardar, G. Parisi, and Y.-C. Zhang, Phys. Rev. Lett. **56**, 889 (Mar 1986), http://link.aps.org/doi/10.1103/PhysRevLett.56.889.

[2] F. Family and T. Vicsek, *Dynamics of fractal surfaces* (World Scientific, 1991) ISBN 9789810207212, http://books.google.it/books?id=5e8COU6flIkC; A. Barabási and H. Stanley, *Fractal concepts in surface growth* (Cambridge University Press, 1995) ISBN 9780521483186, http://books.google.it/books?id=W4SqcNr8PLYC.

[3] T. Halpin-Healy, Phys. Rev. A **42**, 711 (Jul 1990), http://link.aps.org/doi/10.1103/PhysRevA.42.711.

[4] J. Cook and B. Derrida, Journal of Physics A: Mathematical and General **23**, 1523 (1990), http://stacks.iop.org/0305-4470/23/i=9/a=017.

[5] M. Schwartz and S. F. Edwards, Phys. Rev. E **57**, 5730 (May 1998), http://link.aps.org/doi/10.1103/PhysRevE.57.5730.

[6] J. P. Bouchaud and M. E. Cates, Phys. Rev. E **47**, R1455 (Mar 1993), http://link.aps.org/doi/10.1103/PhysRevE.47.R1455; **48**, 635 (Jul 1993), http://link.aps.org/doi/10.1103/PhysRevE.48.635.2.

[7] J. P. Doherty, M. A. Moore, J. M. Kim, and A. J. Bray, Phys. Rev. Lett. **72**, 2041 (Mar 1994), http://link.aps.org/doi/10.1103/PhysRevLett.72.2041; M. A. Moore, T. Blum, J. P. Doherty, M. Marsili, J.-P. Bouchaud, and P. Claudin, **74**, 4257 (May 1995), http://link.aps.org/doi/10.1103/PhysRevLett.74.4257.

[8] Y. Tu, Phys. Rev. Lett. **73**, 3109 (Dec 1994), http://link.aps.org/doi/10.1103/PhysRevLett.73.3109.

[9] F. Colaiori and M. A. Moore, Phys. Rev. Lett. **86**, 3946 (Apr 2001), http://link.aps.org/doi/10.1103/PhysRevLett.

	χ	ω	A_2	B_2	A_3	B_3	A_4	B_4
NEW	0.2532(5)	1.14(5)	0.1720(5)	0.37(3)	0.0321(2)	-1.09(8)	0.1012(6)	0.36(4)
OLD	0.255(3)	0.98(9)	0.170(1)	0.37(3)	0.0321(2)	-0.7(1)	0.100(1)	0.46(4)

TABLE II. In this table we display the best fit values together with their statistical error of the parameters defined in Eq. (3). The first row refers to the actual data presented in this work, the second to the values in [14]

86.3946.

[10] M. Lässig, Nuclear Physics B **448**, 559 (1995), ISSN 0550-3213, http://www.sciencedirect.com/science/article/pii/055032139500268W; M. Lässig and H. Kinzelbach, Phys. Rev. Lett. **78**, 903 (Feb 1997), http://link.aps.org/doi/10.1103/PhysRevLett.78.903.

[11] H. C. Fogedby, Phys. Rev. E **73**, 031104 (Mar 2006), http://link.aps.org/doi/10.1103/PhysRevE.73.031104.

[12] L. Canet, H. Chaté, B. Delamotte, and N. Wschebor, Phys. Rev. Lett. **104**, 150601 (Apr 2010), http://link.aps.org/doi/10.1103/PhysRevLett.104.150601.

[13] J. M. Kim and J. M. Kosterlitz, Phys. Rev. Lett. **62**, 2289 (May 1989), http://link.aps.org/doi/10.1103/PhysRevLett.62.2289; D. E. Wolf and J. Kertéz, EPL (Europhysics Letters) **4**, 651 (1987), http://stacks.iop.org/0295-5075/4/i=6/a=003; L.-H. Tang, B. M. Forrest, and D. E. Wolf, Phys. Rev. A **45**, 7162 (May 1992), http://link.aps.org/doi/10.1103/PhysRevA.45.7162; T. Ala-Nissila, T. Hjelt, J. M. Kosterlitz, and O. Venäläinen, Journal of Statistical Physics **72**, 207 (1993), ISSN 0022-4715, 10.1007/BF01048047, http://dx.doi.org/10.1007/BF01048047.

[14] E. Marinari, A. Pagnani, and G. Parisi, Journal of Physics A: Mathematical and General **33**, 8181 (2000), http://stacks.iop.org/0305-4470/33/i=46/a=303; E. Marinari, A. Pagnani, G. Parisi, and Z. Rácz, Phys. Rev. E **65**, 026136 (Jan 2002), http://link.aps.org/doi/10.1103/PhysRevE.65.026136.

[15] C. Castellano, M. Marsili, and L. Pietronero, Phys. Rev. Lett. **80**, 3527 (Apr 1998), http://link.aps.org/doi/10.1103/PhysRevLett.80.3527; C. Castellano, A. Gabrielli, M. Marsili, M. A. Muñoz, and L. Pietronero, Phys. Rev. E **58**, R5209 (Nov 1998), http://link.aps.org/doi/10.1103/PhysRevE.58.R5209; C. Castellano, M. Marsili, M. A. Muñoz, and L. Pietronero, **59**, 6460 (Jun 1999), http://link.aps.org/doi/10.1103/PhysRevE.59.6460.

[16] T. Hwa and E. Frey, Phys. Rev. A **44**, R7873 (Dec 1991), http://link.aps.org/doi/10.1103/PhysRevA.44.R7873; E. Frey, U. C. Täuber, and T. Hwa, Phys. Rev. E **53**, 4424 (May 1996), http://link.aps.org/doi/10.1103/PhysRevE.53.4424.

[17] K. Moser, J. Kertsz, and D. E. Wolf, Physica A: Statistical Mechanics and its Applications **178**, 215 (1991), ISSN 0378-4371, http://www.sciencedirect.com/science/article/pii/0378437191900177.

[18] E. Perlsman and S. Havlin, EPL (Europhysics Letters) **73**, 178 (2006), http://stacks.iop.org/0295-5075/73/i=2/a=178; M. Schwartz and E. Perlsman, ArXiv e-prints(Aug. 2011), arXiv:1108.4604 [cond-mat.stat-mech].

[19] J. Kelling and G. Ódor, ArXiv e-prints(Oct. 2011), arXiv:1110.6745 [cond-mat.stat-mech].