CN Quiz 3

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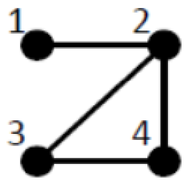

$$A=\begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

Random walks on the above graph：
pi(t): probability that the walk is at vertex i at time t

$$\mathbf{p}_i(t) = \sum_j \frac{A_{ij}}{k_j} p_j(t-1)$$
$$\mathbf{p(t) = AD^{-1}p(t-1)}$$

1.find $\mathbf{AD^{-1}}$

$$\mathbf{D^{-1}} = \begin{pmatrix} 1/k_1 & 0 & 0 & \cdots \\ 0 & 1/k_2 & 0 & \cdots \\ 0 & 0 & 1/k_3 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$
$$\mathbf{AD^{-1}} = \begin{pmatrix} 0 & 1/k_2 & 0 & 0 \\ 1/k_1 & 0 & 1/k_3 & 1/k_4 \\ 0 & 1/k_2 & 0 & 1/k_4 \\ 0 & 1/k_2 & 1/k_3 & 0 \end{pmatrix}$$

Since we have $\mathbf{k_1 = 1, k_2 = 3, k_3 = 2, k_4 = 2}$. So we can find $\mathbf{AD^{-1}}$:

$$\mathbf{AD^{-1}} = \begin{pmatrix} 0 & 1/3 & 0 & 0 \\ 1 & 0 & 1/2 & 1/2 \\ 0 & 1/3 & 0 & 1/2 \\ 0 & 1/3 & 1/2 & 0 \end{pmatrix}$$

2.find p1(∞), p2(∞), p3(∞), and p4(∞)

According to the result of $\mathbf{AD^{-1}}$ **,** we can have:

$$\begin{cases} \mathbf{p_1}(\infty) = 0 + \frac{1}{3}p_2(\infty) + 0 + 0 \\ \mathbf{p_2}(\infty) = p_1(\infty) + 0 + \frac{1}{2}p_3(\infty) + \frac{1}{2}p_4(\infty) \\ \mathbf{p_3}(\infty) = 0 + \frac{1}{3}p_2(\infty) + 0 + \frac{1}{2}p_4(\infty) \\ \mathbf{p_4}(\infty) = 0 + \frac{1}{3}p_2(\infty) + \frac{1}{2}p_3(\infty) + 0 \\ \mathbf{p_1}(\infty) + \mathbf{p_2}(\infty) + \mathbf{p_3}(\infty) + \mathbf{p_4}(\infty) = 1 \end{cases}$$

As a result:

$$\mathbf{p_1}(\infty) = \frac{1}{8}, \mathbf{p_2}(\infty) = \frac{3}{8}, \mathbf{p_3}(\infty) = \frac{1}{4}, \mathbf{p_4}(\infty) = \frac{1}{4}$$