CN Quiz 4

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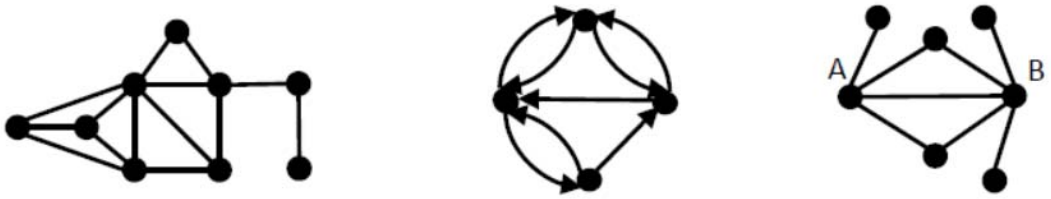
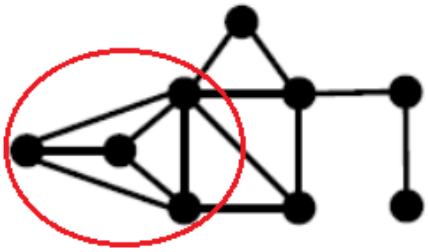
Consider these three networks:



a. Find a 3-core in the first network.

There is a 3-core showing below, each node of the subset is connected to at least 3 others:



b. What is the reciprocity of the second network?

There are 8 directed edges in the graph. So, m = 8 while $\mathrm{r} = \frac{1}{\mathrm{m}}\sum_{ij} \boldsymbol{A_{ij}A_{ji}} = \frac{\mathbf{1}}{\boldsymbol{m}} \boldsymbol{TrA^2}$

There are 3 pairs of loop of length two in the network, which means $\boldsymbol{TrA^2 = 6}$

So, we have $\mathrm{r} = \frac{6}{8} = \frac{3}{4}$

c. What is the cosine similarity of vertices A and B in the third network?

$$\boldsymbol{\sigma_{ij} = cos\theta = \frac{\sum_k A_{ik}A_{kj}}{\sqrt{\sum_k A_{ik}^2}\sqrt{\sum_k A_{jk}^2}}}$$

Since this is an unweighted simple graph, we can change the expression to:

$$= \frac{\boldsymbol{n_{ij}}}{\sqrt{\boldsymbol{k_i k_j}}} = \frac{\mathbf{2}}{\sqrt{\mathbf{4 * 5}}} = \frac{\sqrt{\mathbf{5}}}{\mathbf{5}}$$

of which n_{ij} is the number of common neighbors of A, B. And k_i , k_j stand respectively for the degree of A and B. In conclusion, the cosine similarity is $\frac{\sqrt{5}}{5}$