Complex Network

Quiz 14

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Results

Consider a configuration model network that has vertices of degree 1, 2, and 3 only, in fractions p_1, p_2, and p_3, respectively
1. Find the value of the critical vertex occupation probability ϕ_c at which site percolation take place

If there are n vertices in the network model, that means:

$$n = p_1 n + p_2 n + p_3 n \tag{1}$$

Then average degree $\langle K \rangle$ is calculated as:

$$\langle K \rangle = \frac{1}{n} \sum K_i = \frac{1}{n}(p_1 n + 2p_2 n + 3p_3 n) = p_1 + 2p_2 + 3p_3 \tag{2}$$

$$\langle K^2 \rangle = \frac{1}{n} \sum_i K_i^2 = \frac{1}{n}(p_1 + 4p_2 + 9p_3)n = p_1 + 4p_2 + 9p_3 \tag{3}$$

$$\begin{aligned} \phi_c &= \frac{\langle K \rangle}{\langle K^2 \rangle - \langle K \rangle} = \frac{p_1 + 2p_2 + 3p_3}{(p_1 + 4p_2 + 9p_3) - (p_1 + 2p_2 + 3p_3)} \\ &= \frac{p_1 + 2p_2 + 3p_3}{2p_2 + 6p_3} = \frac{p_1 + 2p_2 + 6p_3 - 3p_3}{2p_2 + 6p_3} = 1 + \frac{p_1 - 3p_3}{2p_2 + 6p_3} \end{aligned} \tag{4}$$

2. Show that there is no giant cluster for any value of the occupation probability ϕ_c if $p_1 > 3p_3$. Why does this result not depend on p2 ?
The critical vertex occupation probability ϕ that a vertex is present. This must be no greater than 1. If $p_1 > 3p_3$, $\frac{p_1 - 3p_3}{2p_2 + 6p_3}$ will be greater than 0 and therefore ϕ is greater than 1, which does not make sense for probability. p_2 is in the denominator so it does not affect the result.

3. Please explain why your friends have more friends than you do.
Because when we calculate the number of friends of ourselves, every single person will be only counted once. The mean number of friends can be determined as $\frac{\sum x_i}{n}$ where x_i is individual and n is the total number of people. However, the friends have a total number of friends $\sum x_i^2$ since each individual is counted as many times as she or he has friends. That is: mean number of friends of friends $= \frac{\sum x_i^2}{\sum x_i} =$ mean(x) $+ \frac{variance(x)}{mean(x)}$. The expression indicates that the mean among friends is always at least as great as the mean among individuals because of the variance. The greater amount of mean among friends than the mean among individuals depends on the variation in the population.