Precise and Verifiable Distributed and Concurrent System Design using TLA+

Jay Parlar
@parlar
#tla-workshop

Special Thanks

- Laura Santamaria
- Ben Meyer
- Landon Jurgens
- Josh Schairbaum
- Rahman Syed
- Jordan Griege
- Freddy Knuth

Workshop, NOT a lecture

PLEASE Ask Questions

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But please save philosophical pondering until after the workshop

Useful Resources

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- TLA Toolbox 1.5.3: https://rax.io/tlaplus
  - https://github.com/tlaplus/tlaplus/releases/tag/v1.5.3

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  - https://github.com/parlarjb/tla_workshop

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  - https://github.com/parlarjb/tla_workshop
- TLA Home Page: https://rax.io/tla_home
  - http://lamport.azurewebsites.net/tla/tla.html

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- TLA Video Series: https://rax.io/tla_videos
  - http://lamport.azurewebsites.net/video/videos.html

Additional Resources

- https://learntla.com/introduction/
- http://lamport.azurewebsites.net/tla/hyperbook.html
- https://www.hillelwayne.com/post/modeling-deployments/
- http://lamport.azurewebsites.net/tla/summary.pdf
- http://lamport.azurewebsites.net/tla/book.html
- https://lorinhochstein.wordpress.com/2014/06/04/crossing-the-river-with-tla/

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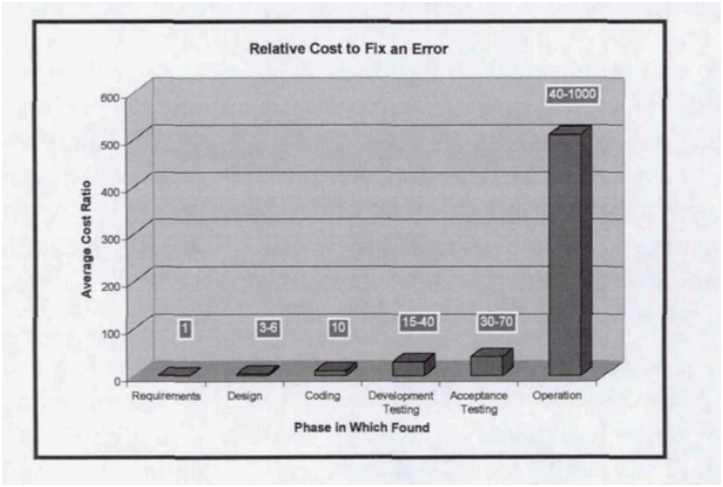
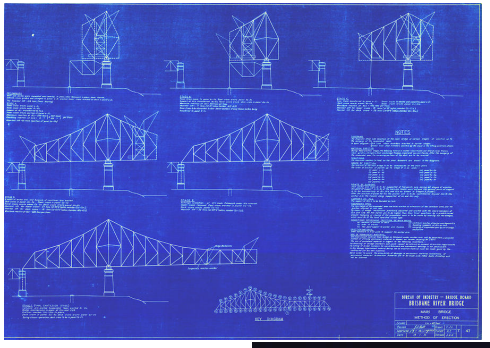
- Introduce the concepts of TLA+
- Introduce a good amount of the syntax of TLA+
- Introduce the model checker, TLC
- Provide a foundation on which you can continue learning
- Convince everyone that thinking **early** is a good thing
- World domination

6-7



"Error Cost Escalation Through the Project Life Cycle"
https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20100036670.pdf

7



8

```
algorithm ford-fulkerson is
    input: Graph G with flow capacity c,
           source node s,
           sink node t
    output: Flow f such that f is maximal from s to t

    (Note that f(u,v) is the flow from node u to node v, and c(u,v) is the flow capacity from
node u to node v)

    for each edge (u, v) in Gf do
        f(u, v) ← 0
        f(v, u) ← 0

    while there exists a path p from s to t in the residual network Gf do
        let cf be the flow capacity of the residual network Gf
        cf(p) ← min{cf(u, v) | (u, v) in p}
        for each edge (u, v) in p do
            f(u, v) ←  f(u, v) + cf(p)
            f(v, u) ← −f(u, v)

    return f
```

https://en.wikipedia.org/wiki/Pseudocode

9

“Writing is nature’s way of letting you know how sloppy your thinking is.”

- Dick Guindon

“Writing a specification helps you think clearly. Thinking clearly is hard; we can use all the help we can get. Making specification part of the design process can improve design.”

- Leslie Lamport

set the default status to "fail"
look up the message based on the error code
if the error code is valid
if doing interactive processing, display the error message
interactively and declare success
if doing command line processing, log the error message to the
command line and declare success
if the error code isn't valid, notify the user that an
internal error has been detected
return status information

https://blog.codinghorror.com/pseudocode-or-code/

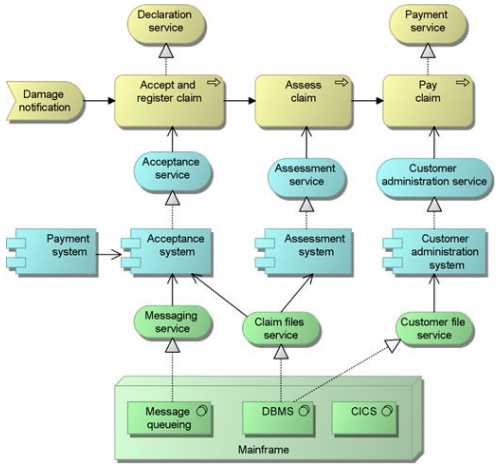
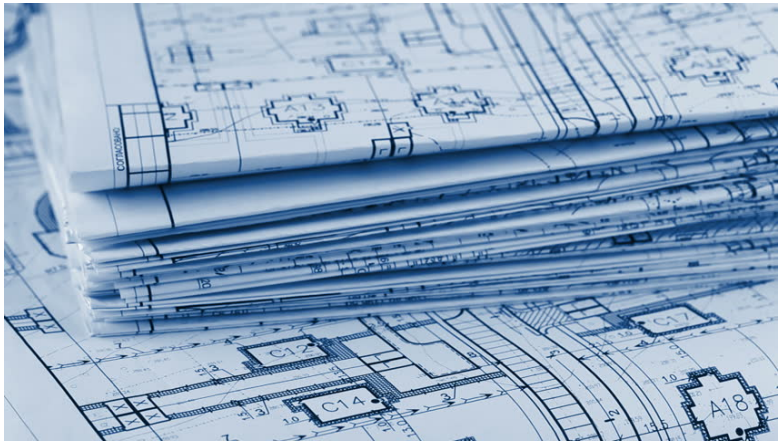


Image via https://en.wikipedia.org/wiki/ArchiMate



Dr. Leslie Lamport

- Turing Award winner
- Winner of first ever “Dijkstra Prize in Distributed Computing” award
- Byzantine Generals’ Problem
- Paxos (which powers Big Table, Google File System, etc)
- Bakery Algorithm
- LaTeX
- PlusCal / TLA+

Amazon, Microsoft, etc.

System	Components	Benefit
S3	- Fault-tolerant low-level network algorithm - Background redistribution of data	Multiple bugs found in design and optimization phases
DynamoDB	Replication and group-membership systems	Found multiple design bugs, some requiring 35-step traces
EC2	Fault tolerant replication improvement, including zero-downtime deployment	Found design bug
Xbox 360	Memory Interface	Found a bug that would hard-lock the Xbox after four hours

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17-1

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"Executive management are now proactively encouraging teams to write TLA+ specs for new features and other significant design changes. In annual planning, managers are now allocating engineering time to use TLA+."

"TLA+ is the most valuable thing that I've learned in my professional career. It has changed how I work by giving me an immensely powerful tool to find subtle flaws in system designs. It has changed how I think…"

- Chris Newcombe (former AWS Principal Engineer)

17-3

TLA+

18-1

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- “Model checker” discovers **every single possible state** of the described system, looks for “unwanted” states.
- The language is called **TLA+**; the model checker tool is called **TLC**.

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- ~500 lines of heavily-commented TLA+.
- **https://raft.github.io**

Raft

```
Next == /\ \/ \E i \in Server : Restart(i)
           \/ \E i \in Server : Timeout(i)
           \/ \E i, j \in Server : RequestVote(i, j)
           \/ \E i \in Server : BecomeLeader(i)
           \/ \E i \in Server : ClientRequest(i)
           \/ \E i \in Server : AdvanceCommitIndex(i)
           \/ \E i,j \in Server : AppendEntries(i, j)
           \/ \E m \in ValidMessage(messages) : Receive(m)
           \/ \E m \in SingleMessage(messages) : DuplicateMessage(m)
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        /\ allLogs' = allLogs \cup {log[i] : i \in Server}
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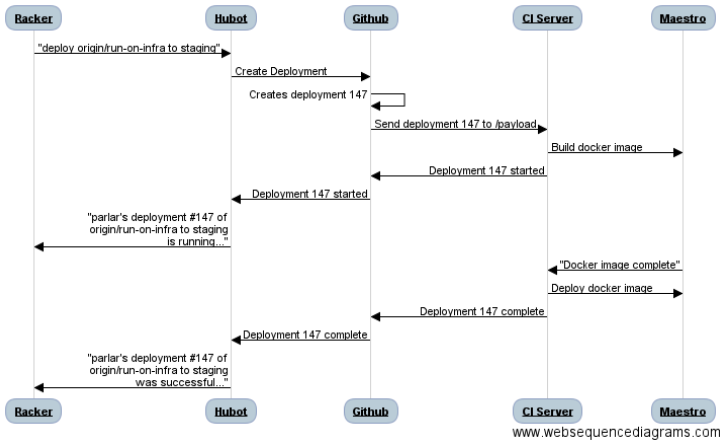
Encore Origin Deployments



```
\* This represents CI receiving a message from Maestro, when Maestro
\* has finished trying to build an image
HandleImageBuildDone(m) ==
    /\ \/ /\ m.type = PRBuildComplete
          /\ CASE m.status = "success"  -> UpdatePR(m.sha, "success", m)
               [] m.status = "error"    -> UpdatePR(m.sha, "error", m)
               [] m.status = "failure"  -> UpdatePR(m.sha, "failure", m)
       \/ /\ m.type = DeployBuildComplete
          /\ CASE m.status = "success"               -> MaestroDeploy(m.sha, m.id, m )
               [] m.status \in {"error", "failure"}  -> UpdateDeployStatus(m.sha, m.id, "failure", m)

   /\ UNCHANGED <<next_id, registry, commit_status, maestro_deploys_complete, gh_deploys_complete>>

\* Represents CI receiving a NewDeployment message from GH
HandleNewDeploy(m) ==
  /\ m.type = NewDeployment
  /\ \/ /\ m.sha \in registry
        /\ MaestroDeploy(m.sha, m.id, m) \* We already have the image, so tell Maestro to go ahead and
     \/ /\ m.sha \notin registry
        /\ BuildDeployImage(m.sha, m.id, m) \* We don't have the image, so tell Maestro to build it
  /\ UNCHANGED <<registry, next_id, commit_status, maestro_deploys_complete, gh_deploys_complete>>


\* Represents CI receiving a deployment success/failure message from Maestro. Only "success" and "fail
HandleDeployComplete(m) ==
    /\ m.type = DeployImageComplete
    /\ \/ /\ m.status = "success"
          /\ UpdateDeployStatus(m.sha, m.id, "success", m)
       \/ /\ m.status = "failure"
          /\ UpdateDeployStatus(m.sha, m.id, "failure", m)

    /\ UNCHANGED <<registry, next_id, commit_status, maestro_deploys_complete, gh_deploys_complete>>

\* Represents GH receiving a commit_status status update from CI
HandlePRStatusUpdate(m) ==
  /\ m.type = PRStatus
  /\ commit_status' = [commit_status EXCEPT ![m.sha] = m.status]
  /\ Discard(m)
```

22

Deployment Pipeline

23-1

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- “Only deploy if an image exists”

23-2

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- “All deploys must have been initiated by GitHub”

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- “All deploys must have been initiated by GitHub”
- “A single SHA can not have both a successful and failed PR build”
- “Deployment status is consistent through the different systems”
- etc. etc.


```
def has_access(self, *allowed_dept_list, **kwargs):
    ignore_core_admin = kwargs.get("ignore_core_admin", False)
    special_permissions = set(("HK_RACKER", "CORE_PERMISSION_ADMIN", "PER_EMPLOYEE_TERMINATE"))
    users_departments = set(self.departments)
    allowed_dept_list = set(allowed_dept_list)

    if special_permissions.union(allowed_dept_list):
        return True if allowed_dept_list.union(users_departments) else False

    elif ignore_core_admin is False and "CORE_ADMIN" in users_departments:
        return True

    else:
        return True if allowed_dept_list.union(users_departments) else False
```

```python
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    users_departments = set(self.departments)
    allowed_dept_list = set(allowed_dept_list)

    ignore_core_admin = True if special_permissions.union(allowed_dept_list) else ignore_core_admin
    if not ignore_core_admin and "CORE_ADMIN" in users_departments:
        return True
    else:
        return allowed_dept_list.union(users_departments)
```


PlusCal

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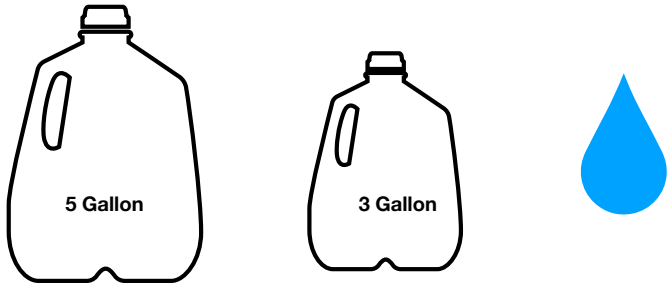
- A programming language-like layer on top of TLA+
- Some people find it easier to read and write than TLA+
- TLC compiles PlusCal code into TLA+
- Hard to learn by starting with PlusCal. Better to gain a foundation in TLA+ then take advantage of PlusCal where appropriate

How do we measure out exactly 4 gallons?

i.e. “The Die Hard Problem”



One Possible Execution

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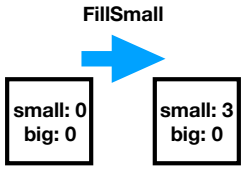
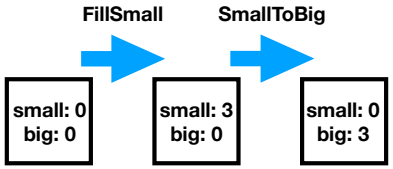


28-2

One Possible Execution



28-3

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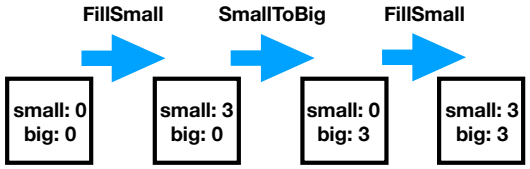

28-4

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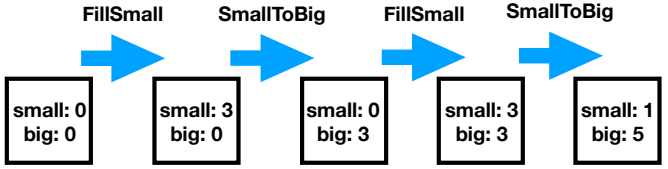

28-5

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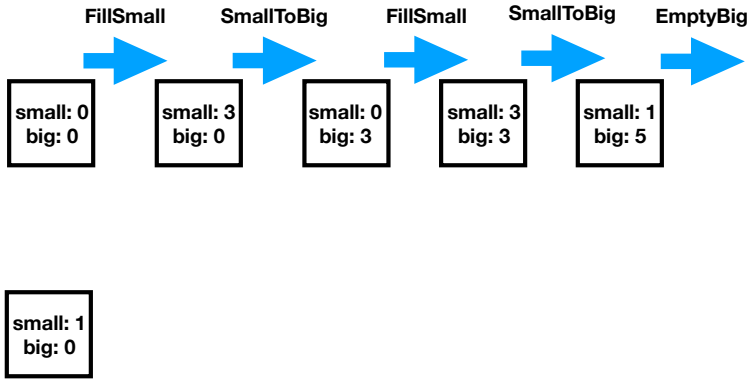

28-6

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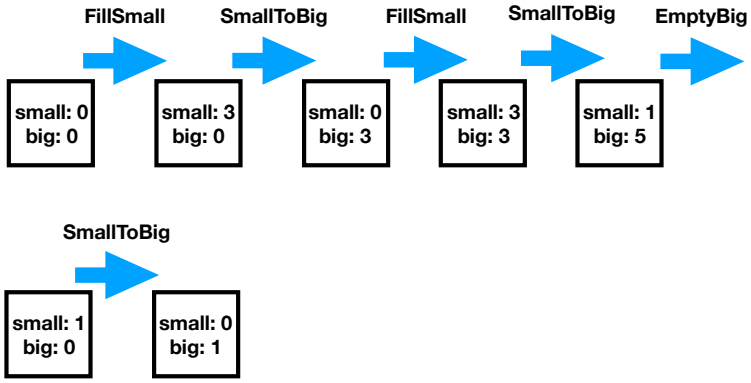

28-7

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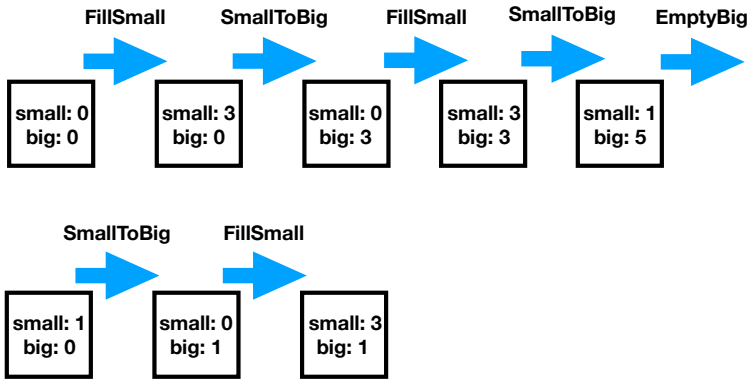

28-8

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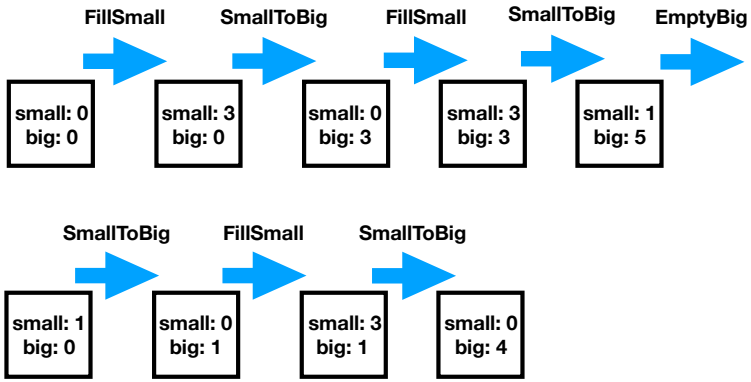

28-9

Another Possible Execution

29-1

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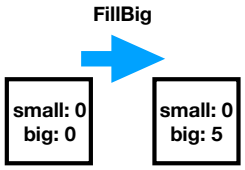

29-2

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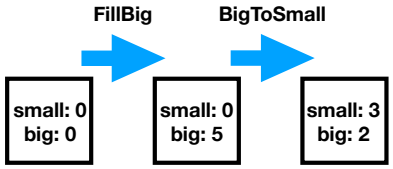

29-3

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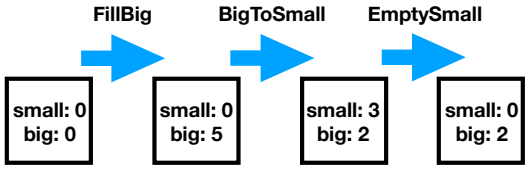

29-4

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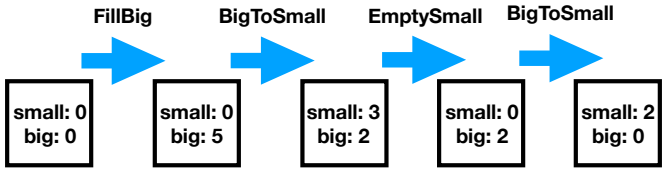
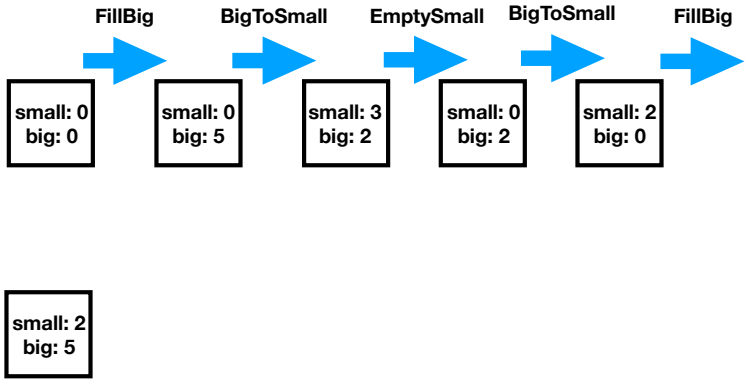


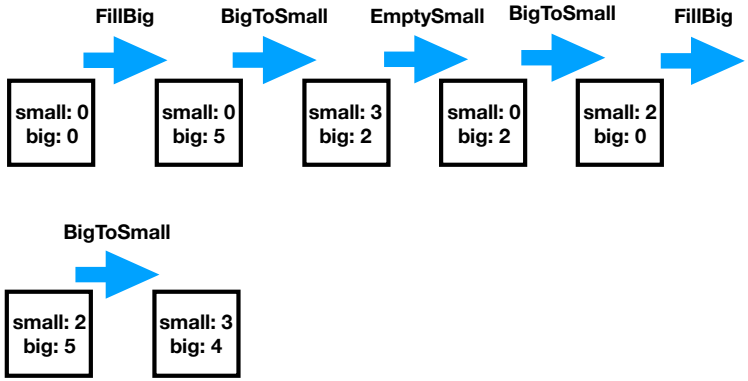
29-5

Another Possible Execution



29-6

Another Possible Execution

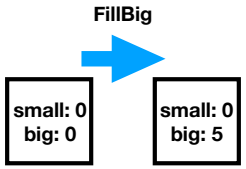


29-7

Yet Another Possible Execution

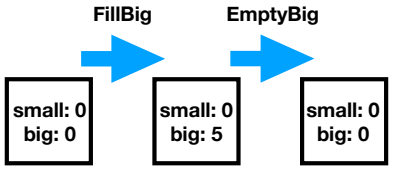
30-1

Yet Another Possible Execution



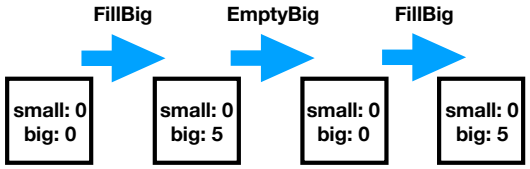
30-2

Yet Another Possible Execution



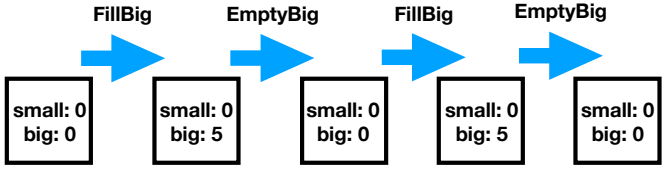
30-3

Yet Another Possible Execution



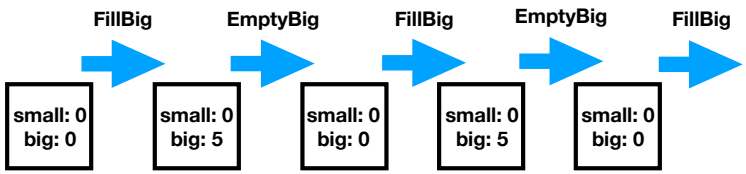
30-4

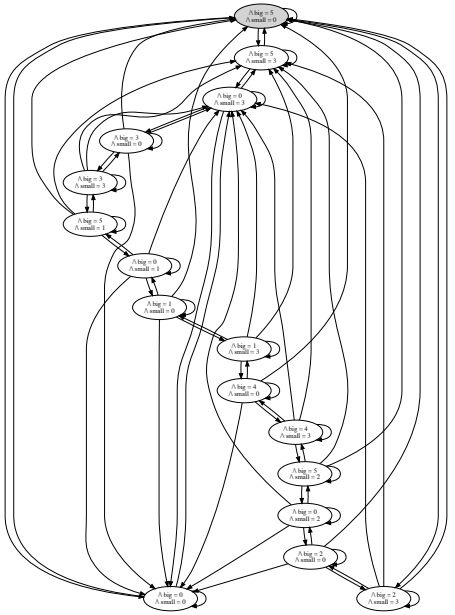
Yet Another Possible Execution



30-5

Yet Another Possible Execution





TLA+ Terms

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- A single one of those executions is called a **behaviour**.

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- A single one of those executions is called a **behaviour**.
- A behaviour is a **sequence of states**.
- A **step** is the change from one state to the next.
- A **state** is an assignment of **values** to **variables**.
- TLA+ is all about describing **all possible executions/ behaviours** of a system.

How could we describe digital systems (behaviours)?

- Programming languages
- Turing machines
- Automata
- Hardware Description Languages

We'll instead use State Machines!

A state machine is defined by three things:

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1. A listing of all of our **variables**

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A state machine is defined by three things:

1. A listing of all of our **variables**
2. All the possible **initial states**
3. The possible **next states** for any given state (i.e., state transitions), or, the **relationship** between the values of the **variables** in the current state, and their possible values in the **next state**

Die Hard State Machine

Die Hard State Machine

1. Variables: **big** and **small**

Die Hard State Machine

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2. Possible initial values: **big=0**, **small=0**

Die Hard State Machine

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3. Possible next states: …… ?

Die Hard Next States

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if (big + small) <= 5 THEN big=big+small; small=0

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if (big + small) <= 5 THEN big=big+small; small=0

ELSE big = 5; small = small - (5 - big)

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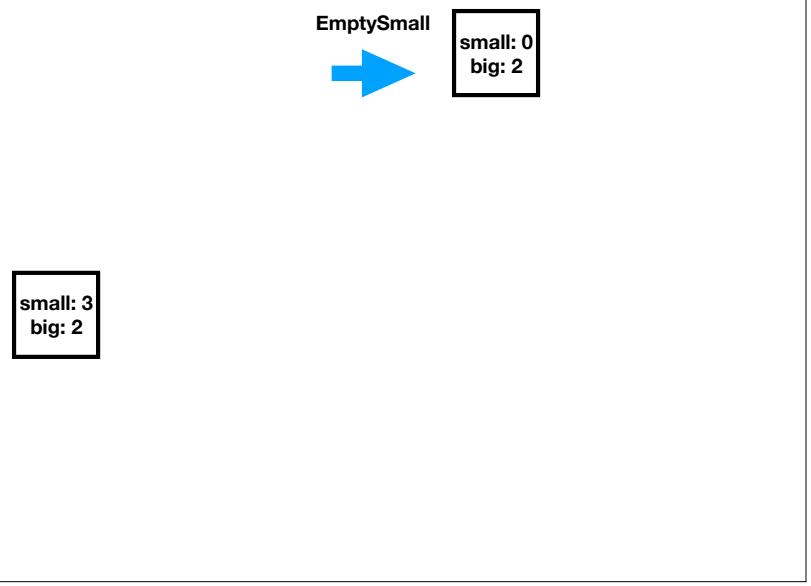
etc. etc.

Possible next states

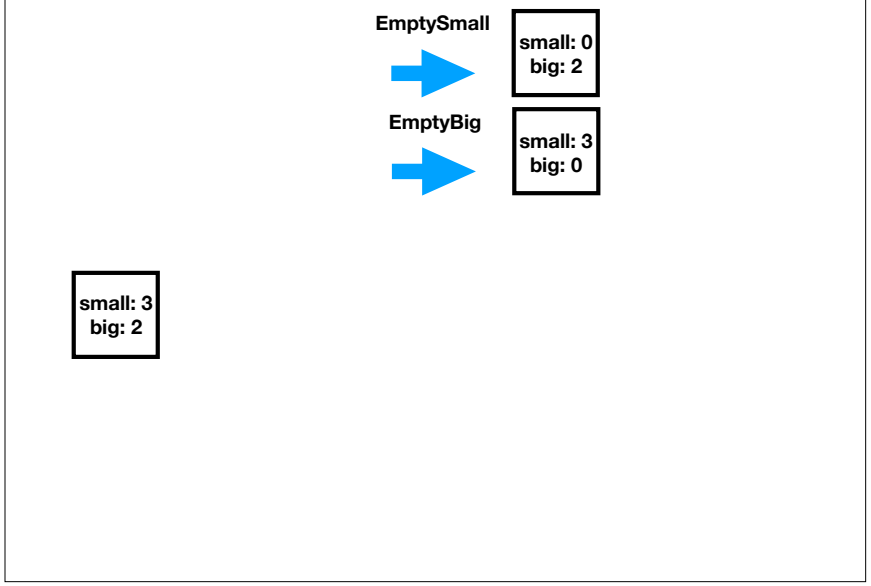
small: 3
big: 2

Possible next states



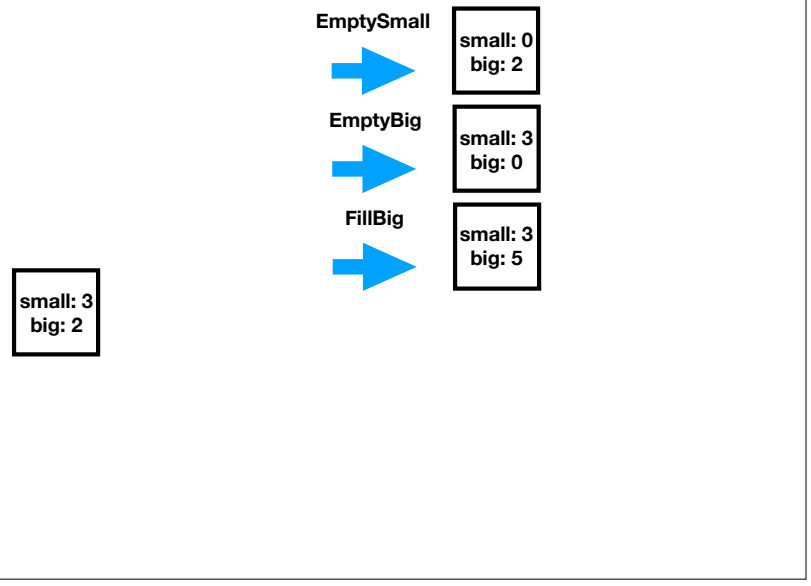
37-2

Possible next states



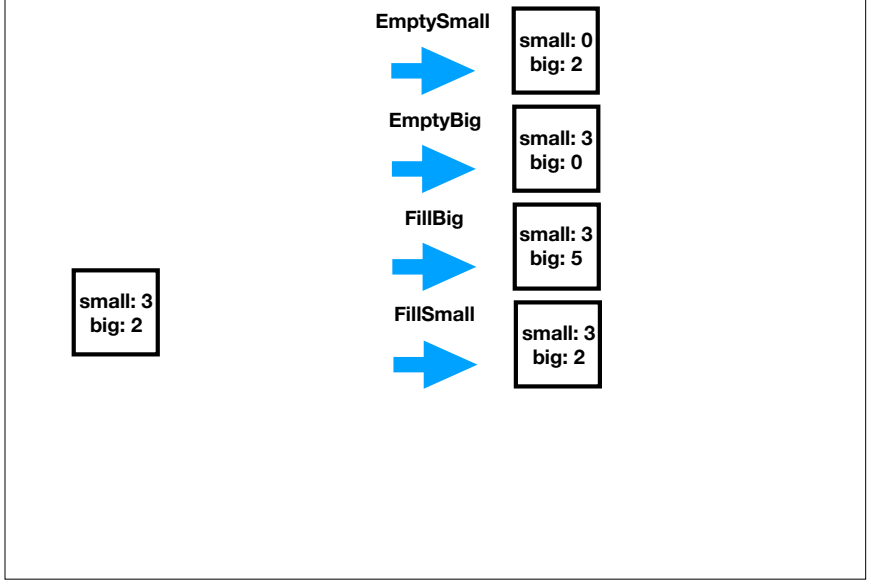
37-3

Possible next states



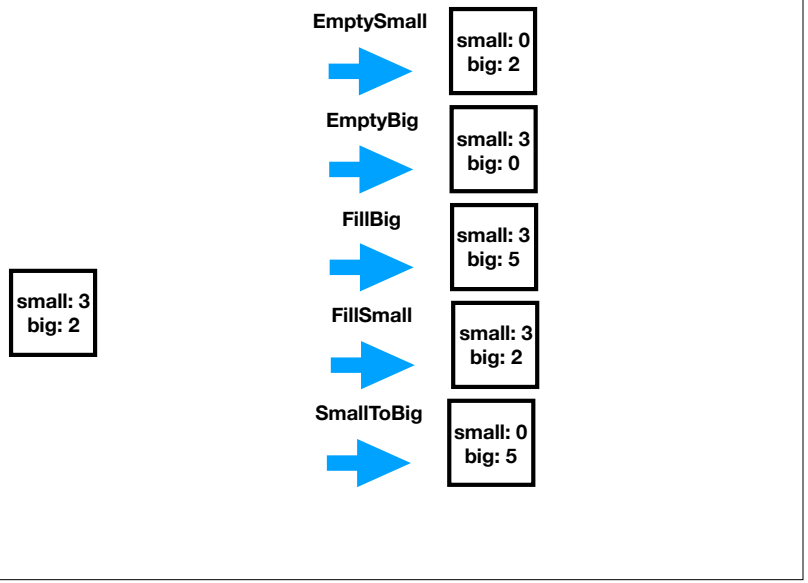
37-4

Possible next states

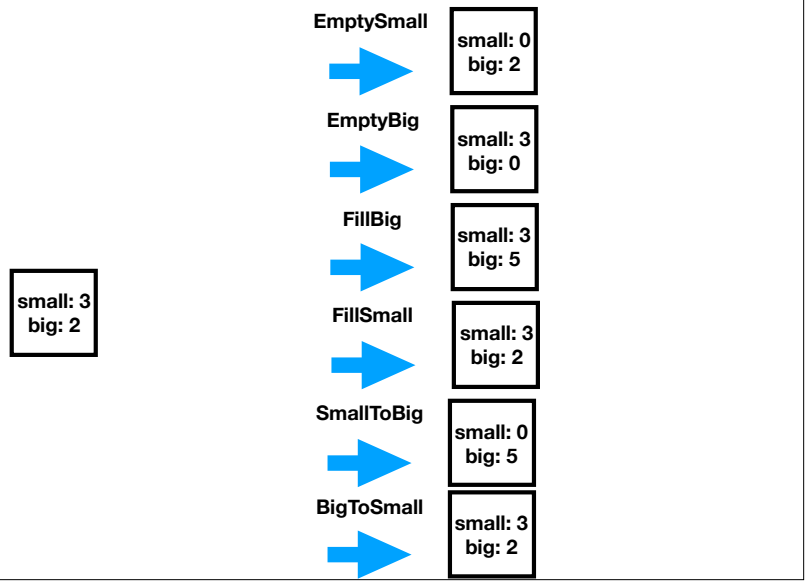


37-5

Possible next states



Possible next states




```
VARIABLES small, big

TypeOK == /\ small \in 0..3
          /\ big   \in 0..5

Init == /\ big   = 5
        /\ small = 0

FillSmall == /\ small' = 3
             /\ big'   = big

FillBig == /\ big'   = 5
           /\ small' = small

EmptySmall == /\ small' = 0
              /\ big'   = big

EmptyBig == /\ big'   = 0
            /\ small' = small

SmallToBig == IF big + small =< 5
              THEN /\ big'   = big + small
                   /\ small' = 0
              ELSE /\ big'   = 5
                   /\ small' = small - (5 - big)

BigToSmall == IF big + small =< 3
              THEN /\ big'   = 0
                   /\ small' = big + small
              ELSE /\ big'   = small - (3 - big)
                   /\ small' = 3

Next == \/ FillSmall  \/ FillBig
        \/ EmptySmall \/ EmptyBig
        \/ SmallToBig \/ BigToSmall
```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- Programming languages represent different parts of the state differently:
    - Values of variables
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    - Call stack
    - Heap
    - etc.
- Programming languages are **terrible** at representing non-determinism.

Math Time!!!

“and” / “or”

“and” / “or”

∧ Logical “AND”. Called a “conjunct”. Written in ASCII as `/\`
Written in Python as **`and`**. Written in C as **`&&`**

```
TRUE  /\ TRUE  = TRUE
TRUE  /\ FALSE = FALSE
FALSE /\ FALSE = FALSE
FALSE /\ TRUE  = FALSE
```


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TRUE  /\ TRUE  = TRUE
TRUE  /\ FALSE = FALSE
FALSE /\ FALSE = FALSE
FALSE /\ TRUE  = FALSE
```

∨ Logical “OR”. Called a “disjunct”. Written in ASCII as `\/`
Written in Python as **`or`**. Written in C as `||`

```
TRUE  \/ TRUE  = TRUE
TRUE  \/ FALSE = TRUE
FALSE \/ FALSE = FALSE
FALSE \/ TRUE  = TRUE
```

= and ≜

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= means “equality”. It is NOT an assignment operator. It is a boolean operator. It is the = you would have learned in grade 1 mathematics. It’s equivalent to Python’s **==.**

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≜ means “defined to be”. It is written as **==** in ASCII. You use it to created named definitions of things.

```
MyDef == A /\ B
```

At any time, **MyDef** will be the value of **A** and’ed with **B**

It is a little like using a macro, or an inline.

Exercise

```
TRUE \/ (TRUE /\ FALSE)

TRUE /\ (FALSE \/ (TRUE \/ FALSE))

FALSE /\ (TRUE /\ ((FALSE \/ TRUE)\/(TRUE \/ FALSE)))
```


Exercise

```
TRUE \/ (TRUE /\ FALSE)                          TRUE

TRUE /\ (FALSE \/ (TRUE \/ FALSE))

FALSE /\ (TRUE /\ ((FALSE \/ TRUE)\/(TRUE \/ FALSE)))
```


Exercise

```
TRUE \/ (TRUE /\ FALSE)                          TRUE

TRUE /\ (FALSE \/ (TRUE \/ FALSE))               TRUE

FALSE /\ (TRUE /\ ((FALSE \/ TRUE)\/(TRUE \/ FALSE)))
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Exercise

```
TRUE \/ (TRUE /\ FALSE)                          TRUE

TRUE /\ (FALSE \/ (TRUE \/ FALSE))               TRUE

FALSE /\ (TRUE /\ ((FALSE \/ TRUE)\/(TRUE \/ FALSE)))   FALSE
```

TLA + Syntax

TLA+ specs make heavy use of formulas like this

```
A /\ (B /\ ((C \/ D)\/(E /\ F))) /\ (G \/ H \/ I)
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TLA+ Syntax

In general:

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```
A /\ B /\ C /\ … /\ N      =      /\ A
                                  /\ B
                                  /\ C
                                  …
                                  /\ N
```


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```
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                                  /\ B
                                  /\ C
                                  …
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                                  \/ B
                                  \/ C
                                  …
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```


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                                  /\ B
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                                  \/ B
                                  \/ C
                                  …
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In general:

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                            /\ C
                            …
                            /\ N
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                            \/ C
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In general:

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                            /\ B
                            /\ C
                            …
                            /\ N
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                            \/ B
                            \/ C
                            …
                            \/ N
```


TLA+ Syntax

In general:

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                            /\ B
                            /\ C
                            …
                            /\ N
```

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A \/ B \/ C \/ … \/ N   =   \/ A
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                            \/ C
                            …
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```
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(We’ll come back to this.)

TLA+ Syntax

- The indentation level is meaningful.
- Expressions on the same indent level are “and”ed and “or”ed together.

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A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
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- This formula has four items at the top level, being AND’ed together. Let’s rewrite using TLA+ syntax.

TLA+ Syntax

- The indentation level is meaningful.
- Expressions on the same indent level are “and”ed and “or”ed together.

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A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
```

- This formula has four items at the top level, being AND’ed together. Let’s rewrite using TLA+ syntax.
- We’ll put /\ at the outermost indent level.

TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
```


TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)

/\ A
```

TLA+ Syntax


```
/\ A
/\ (B \/ C \/ D)
```


TLA+ Syntax


```
/\ A
/\ (B \/ C \/ D)
/\ E
```


TLA+ Syntax


```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```


TLA+ Syntax


```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

Now we can easily see all the “outermost” or “top-level” items being AND’ed together

TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
```

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

Now we can easily see all the “outermost” or “top-level” items being AND’ed together

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/\ (B \/ C \/ D)
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```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
```

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

Now we can easily see all the “outermost” or “top-level” items being AND’ed together

We have more parentheses.

TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
```

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

Now we can easily see all the “outermost” or “top-level” items being AND’ed together

We have more parentheses.

Let’s get rid of those too

TLA+ Syntax

We’ll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```


TLA+ Syntax

We’ll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

```
/\ A
```

TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
```


TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
   \/ C
```


TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
   \/ C
   \/ D
```


TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
   \/ C
   \/ D
/\ E
```

TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
   \/ C
   \/ D
/\ E
/\ /\ F
```


TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
   \/ C
   \/ D
/\ E
/\ /\ F
   /\ G
```


TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
   \/ C
   \/ D
/\ E
/\ /\ F
   /\ G
```


TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
   \/ C
   \/ D
/\ E
/\ /\ F
   /\ G
```

TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

```
/\ A
/\ \/ B
   \/ C
   \/ D
/\ E
/\ /\ F
   /\ G
```

The structure of the formula should be much more clear now.
It creates an explicit graph-like visual structure out of parentheses.
Or think of it like bullet-points, if that helps .

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```


TLA+ Syntax

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TLA+ does not allow this. Precedence between ∨ and ∧ must be made explicit with parentheses.

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A /\ B \/ C /\ D
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```
(A /\ B) \/ (C /\ D)
```

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```

TLA+ does not allow this. Precedence between ∨ and ∧ must be made explicit with parentheses.

```
(A /\ B) \/ (C /\ D)
```

Which becomes

50-4

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```

TLA+ does not allow this. Precedence between ∨ and ∧ must be made explicit with parentheses.

```
(A /\ B) \/ (C /\ D)
```

Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```

50-5

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```

TLA+ does not allow this. Precedence between ∨ and ∧ must be made explicit with parentheses.

```
(A /\ B) \/ (C /\ D)
```

Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```

50-6

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```

TLA+ does not allow this. Precedence between ∨ and ∧ must be made explicit with parentheses.

```
(A /\ B) \/ (C /\ D)
```

Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```

50-7

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```

TLA+ does not allow this. Precedence between \/ and /\ must be made explicit with parentheses.

```
(A /\ B) \/ (C /\ D)
```

Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```

50-8

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```

TLA+ does not allow this. Precedence between \/ and /\ must be made explicit with parentheses.

```
(A /\ B) \/ (C /\ D)
```

Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```

50-9

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```

TLA+ does not allow this. Precedence between \/ and /\ must be made explicit with parentheses.

```
(A /\ B) \/ (C /\ D)
```

Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```

50-10

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
```

TLA+ does not allow this. Precedence between \/ and /\ must be made explicit with parentheses.

```
(A /\ B) \/ (C /\ D)
```

Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```

50-11

Exercise

1. A \/ (B /\ (C \/ D) /\ E)

2. (A /\ B /\ (C \/ D)) /\ (E \/ F \/ G) /\ (H \/ (I /\ J))

3. (A \/ (B /\ C) \/ D) /\ E /\ F /\ (G \/ H \/ (I /\ J))

Solutions

1. A \/ (B /\ (C \/ D) /\ E)

\/ A
\/ /\ B
   /\ C \/ D
   /\ E

Solutions

1. A \/ (B /\ (C \/ D) /\ E)

\/ A
\/ /\ B
   /\ C \/ D
   /\ E

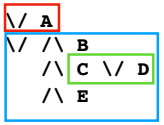
Solutions

1. A \/ (B /\ (C \/ D) /\ E)

\/ A
\/ /\ B
   /\ C \/ D
   /\ E

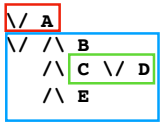
Solutions

1. A \/ (B /\ (C \/ D) /\ E)

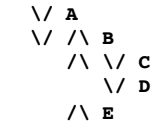


Solutions

1. A \/ (B /\ (C \/ D) /\ E)

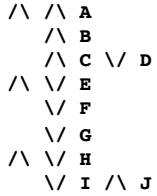


Alternate Solution



Solutions

2. (A /\ B /\ (C \/ D)) /\ (E \/ F \/ G) /\ (H \/ (I /\ J))



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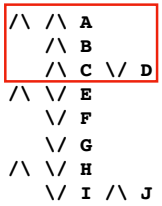

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2. (A /\ B /\ (C \/ D)) /\ (E \/ F \/ G) /\ (H \/ (I /\ J))

```
/\ /\ A
   /\ B
   /\ C \/ D
/\ \/ E
   \/ F
   \/ G
/\ \/ H
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```


Solutions

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   /\ B
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   \/ F
   \/ G
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```

Alternate Solution

```
/\ /\ A
   /\ B
   /\ \/ C
      \/ D
/\ \/ E
   \/ F
   \/ G
/\ \/ H
   \/ /\ I
      /\ J
```


Solutions

3. (A \/ (B /\ C) \/ D) /\ E /\ F /\ (G \/ H \/ (I /\ J))

```
/\ \/ A
   \/ B /\ C
   \/ D
/\ E
/\ F
/\ \/ G
   \/ H
   \/ I /\ J
```


Solutions

3. (A \/ (B /\ C) \/ D) /\ E /\ F /\ (G \/ H \/ (I /\ J))

```
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   \/ D
/\ E
/\ F
/\ \/ G
   \/ H
   \/ I /\ J
```


Solutions

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/\ F
/\ \/ G
   \/ H
   \/ I /\ J
```

Alternate Solution

```
/\ \/ A
   \/ /\ B
      /\ C
   \/ D
/\ E
/\ F
/\ \/ G
   \/ H
   \/ /\ I
      /\ J
```


Sets

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- An unordered collection of unique things

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- {“a”, “b”} = {“b”, “a”} (i.e. order does not matter)

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- {“a”, “b”} = {“b”, “a”} (i.e. order does not matter)
- The concept maps to Python’s `set()`

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      `set([“a”]).issubset(set([“a”,”b”,”c”]))`

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  - {“a”, “b”} ∪ {“c”, “d”} = {“a”, “b”, “c”, “d”}
  - {“a”, “b”} ∪ {“b”, “c”} = {“a”, “b”, “c”}

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- It “squishes together” two sets to create a new set
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  - {“a”, “b”} ∪ {“c”, “d”} = {“a”, “b”, “c”, “d”}
  - {“a”, “b”} ∪ {“b”, “c”} = {“a”, “b”, “c”}
    - Python:
      `set(["a","b"]).union(set(["b", "c"]))`

Sets

Sets

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Sets

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Sets

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Sets

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  - {“a”, “b”} ∩ {“c”, “d”} = {}
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  - Python:

Sets

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  - {“a”, “b”} ∩ {“c”, “d”} = {}
  - {“a”, “b”} ∩ {“b”, “c”} = {“b”}
  - Python:
    - `set([“a”, “b”]).intersection(set([“b”, “c”]))`

Sets

Sets

- **..** is roughly equivalent to Python's **`range()`** function

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  - 1..10 = {1,2,3,4,5,6,7,8,9,10}

Sets

- **..** is roughly equivalent to Python's **`range()`** function
  - 1..10 = {1,2,3,4,5,6,7,8,9,10}
  - In Python: **`set(range(1,11))`**

Exercises

(TRUE or FALSE for each)

{“a”, “b”} ⊆ {“b”, “a”, “c”}

{1,2,3} ⊆ ({1,2,3} ∩ {2,3})

“a” ∈ (({“b”, “c”, “d”} ∩ {“e”, “f”}) ∪ {“a”})

(“a” ∈ {“a”, “b”}) ∨ (“b” ∈ {“c”, “d”})

Exercises

(TRUE or FALSE for each)

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def exists(S):
  for x in S:
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      return True
  return False

exists(set([1,2,3,4,5]))  # False
```


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is roughly equivalent to the following Python expressions

```
def exists(S):
  for x in S:
    if x > 5:
      return True
  return False

exists(set([1,2,3,4,5]))  # False

any(map(lambda x: x > 5, [1,2,3,4,5])) #False
```

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  - ∀ x∈{1,2,3,4} : x > 0 is TRUE

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```
def for_all(S):
  for x in S:
    if not (x > 5):
      return False
  return True
```


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is roughly equivalent to the following Python expressions

```
def for_all(S):
  for x in S:
    if not (x > 5):
      return False
  return True

for_all(set([1,2,3,4,5]))  # False
```


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$\forall x \in \{1,2,3,4,5\} : x > 5$

is roughly equivalent to the following Python expressions

```
def for_all(S):
  for x in S:
    if not (x > 5):
      return False
  return True

for_all(set([1,2,3,4,5]))  # False


all(map(lambda x: x > 5, [1,2,3,4,5]))  # False
```

Exercises

$\exists\, x \in \{1,2,3,4\} : (x > 3) \wedge (x < 4)$

$\exists\, x \in \{1,2,3,4\} : \exists\, y \in \{5, 6, 7\} : x < y$

$\forall\, x \in \{1,2,3\} : \{x\} \subseteq \{1, 2\}$

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Set constructors

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{ x ∈ {1,2,3,4,5} : x > 3 } = {4,5}

This is equivalent to the following Python expressions:

```
set([x for x in [1,2,3,4,5] if x > 3])
```


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{ x ∈ {1,2,3,4,5} : x > 3 } = {4,5}

This is equivalent to the following Python expressions:

```
set([x for x in [1,2,3,4,5] if x > 3])

set(filter(lambda x: x > 3, [1,2,3,4,5]))
```

Set constructors

Set constructors

{ e : x ∈ S } is like a **`map()`** function, applying **e** to every element of **S**.

{ x * 2 : x ∈ {1,2,3,4,5}} = {2,4,6,8,10}

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This is roughly equivalent to the following Python expressions:

```
set([x*2 for x in [1,2,3,4,5]])

set(map(lambda x: x*2, [1,2,3,4,5]))
```


Exercises

1. Write a set constructor that creates a set of elements from **{1,2,3,4,5}**, consisting of the elements that are greater than **1** and less than **5** (i.e. we want **{2, 3, 4}**).

2. Write a set constructor that creates a set of elements from **{1,2,3,4,5}**, consisting of the elements that, when multiplied by **3**, are greater than **10** (i.e. we want **{4, 5}**).

3. Write a set constructor that creates a set of elements consisting of each element of **{1, 2, 3, 4, 5}** added to itself and then subtracting **1** (i.e. we want **{1, 3, 5, 7, 9}**).

Solutions

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1. { x ∈ {1,2,3,4,5} : ∧ x > 1 ∧ x < 5 } **or** { x ∈ {1,2,3,4,5} : x > 1 ∧ x < 5 }

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2. { x ∈ {1,2,3,4,5} : x*3 > 10}

70-3

Solutions

1. { x ∈ {1,2,3,4,5} : ∧ x > 1 ∧ x < 5 } **or** { x ∈ {1,2,3,4,5} : x > 1 ∧ x < 5 }

2. { x ∈ {1,2,3,4,5} : x*3 > 10}

3. { x+x-1 : x ∈ {1,2,3,4,5}}

70-4

71

Back to State Machines

72-1

Back to State Machines

Recall that a state machine is defined by three things:

Back to State Machines

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Back to State Machines

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Back to State Machines

Recall that a state machine is defined by three things:

1. The **variables**
2. The possible **initial values** of the variables
3. The possible **next states** from any given state (the relationship between the **values** of the **variables** in the current state and their possible **values** in the **next state**)

A small C program

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
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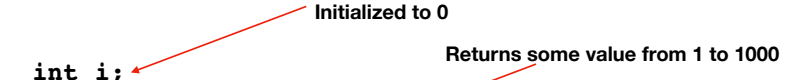


```
int i;
void main () {
    i = someNumber();
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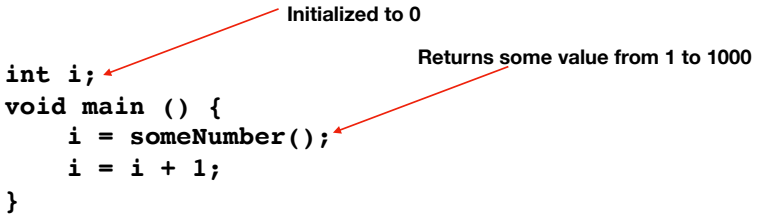
}
```

Let's exactly represent this C program as a state machine

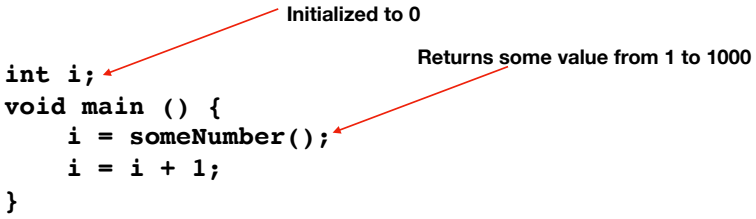
A small C program



Let's exactly represent this C program as a state machine

- Variables: **i**

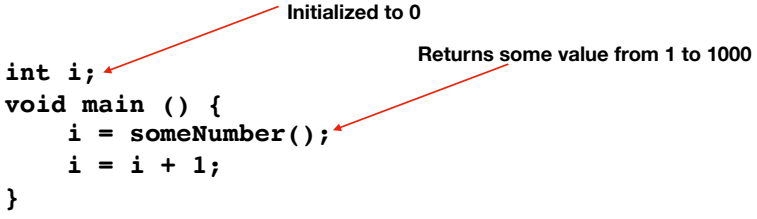
A small C program



Let's exactly represent this C program as a state machine

- Variables: **i**
- Possible initial values: **i = 0**

A small C program



Let's exactly represent this C program as a state machine

- Variables: **i**
- Possible initial values: **i = 0**
- Next states for some given state… let's see!

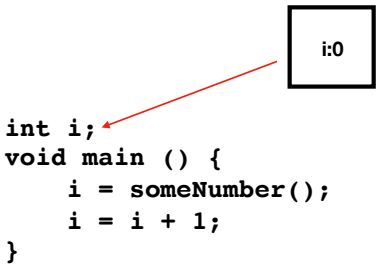
A small C program - example behaviour

Assume that for this particular *behaviour,* `someNumber()` returns **42**.

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```

A small C program - example behaviour

Assume that for this particular *behaviour,* **`someNumber()`** returns **42**.



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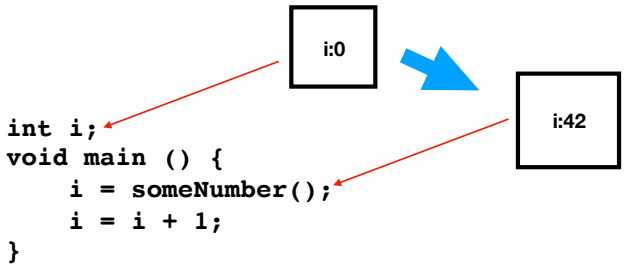

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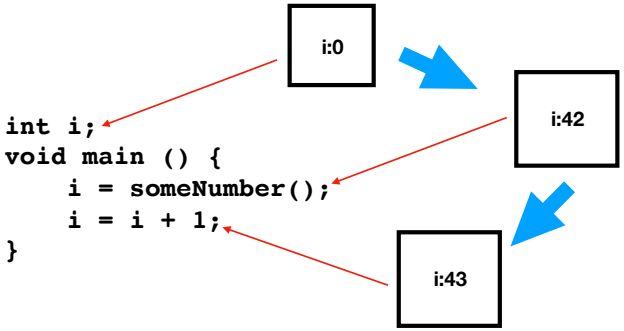

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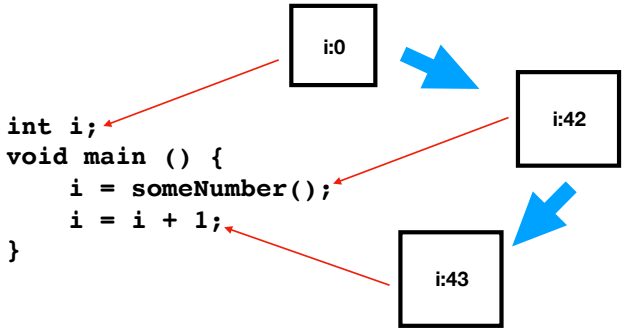
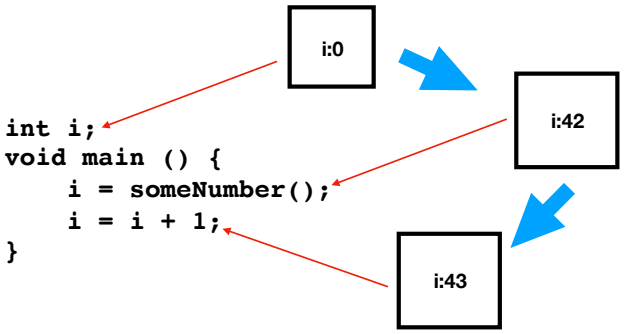


So we know that for the state **[i:42]**, the only next possible state is **[i:43]**.

A small C program - example behaviour

Assume that for this particular *behaviour,* **`someNumber()`** returns **42**.



So we know that for the state **[i:42]**, the only next possible state is **[i:43]**.

And for the state **[i:43]**, the only next state is “program termination”.

A small C program - example behaviour

Now let’s look at a behaviour where **`someNumber()`** returned **43**.

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```


A small C program - example behaviour

Now let’s look at a behaviour where **`someNumber()`** returned **43**.

A small C program - example behaviour

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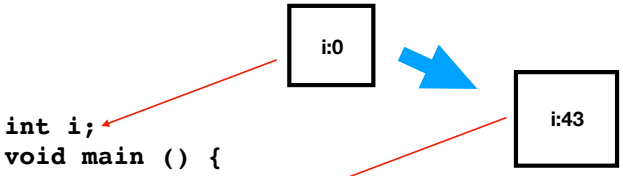

A small C program - example behaviour

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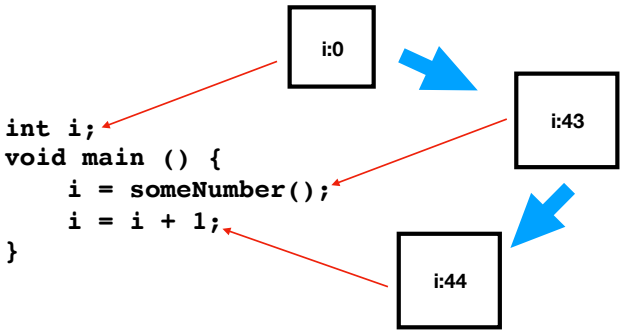
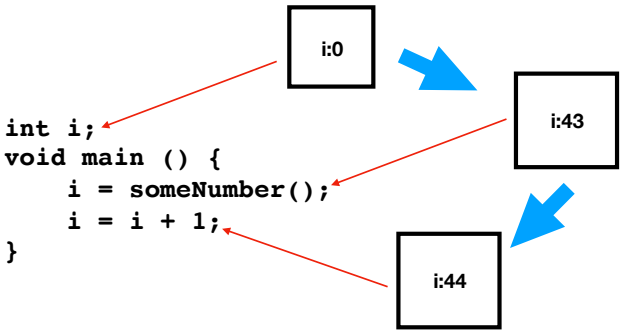


A small C program - example behaviour

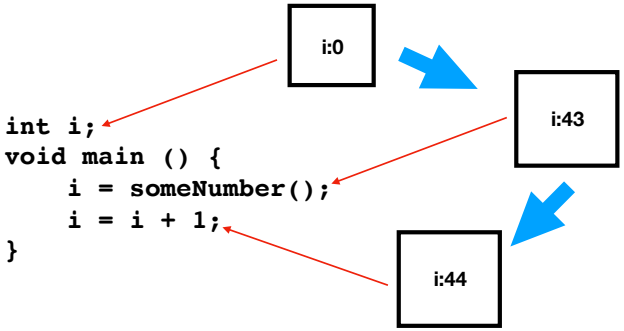
Now let's look at a behaviour where **`someNumber()`** returned **43**.



In this case, a state of **[i:43]** has a next state of **[i:44]**.

A small C program - example behaviour

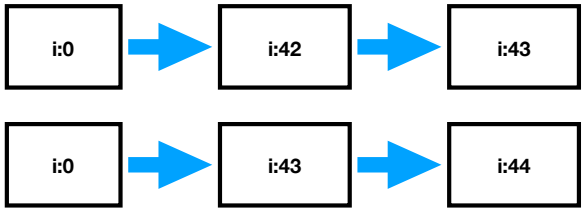
Now let's look at a behaviour where **`someNumber()`** returned **43**.



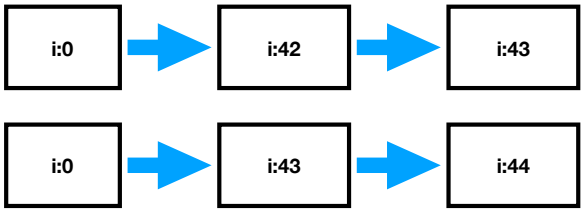
In this case, a state of **[i:43]** has a next state of **[i:44]**.

But that's different than the next state for **[i:43]** in the last behaviour!

43



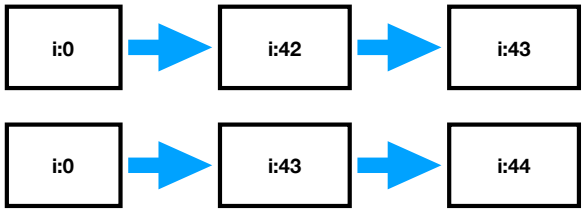
43



Thus representing the state of the program with just the variable **i** is not sufficient.

76-2

43

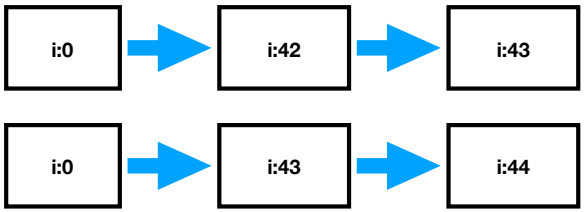


Thus representing the state of the program with just the variable **i** is not sufficient.

The two behaviours require different **next values** for **i** for the same **current value** of 43

76-3

43



Thus representing the state of the program with just the variable **i** is not sufficient.

The two behaviours require different **next values** for **i** for the same **current value** of 43

We must be able to exactly represent the program using the variables of our state machine.

76-4

A small C program - example behaviour

We can't represent this simple program using just the variable **i**

We need a second variable, **pc**, to represent "control state", i.e. where in the program we are

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```

77-1

A small C program - example behaviour

We can't represent this simple program using just the variable **i**

We need a second variable, **pc**, to represent "control state", i.e. where in the program we are


```
int i;
void main () {
    i = someNumber();
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```


A small C program - example behaviour

We can't represent this simple program using just the variable **i**

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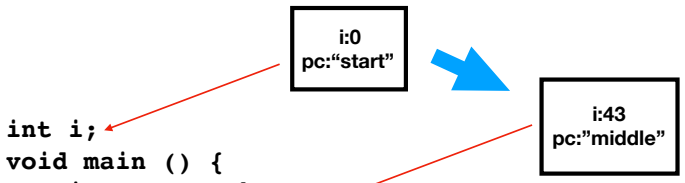


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```


A small C program - example behaviour

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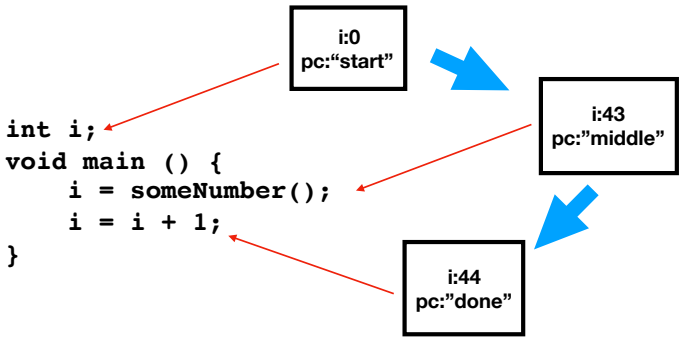


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int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```


A small C program

Let's make this a bit more formal

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```

A small C program

Let's make this a bit more formal

1. Variables: **i, pc**

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```


A small C program

Let's make this a bit more formal

1. Variables: **i, pc**
2. Initial values: **i=0, pc="start"**

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```


A small C program

Let's make this a bit more formal

1. Variables: **i, pc**
2. Initial values: **i=0, pc="start"**
3. **Next state** for a given state:

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    i = someNumber();
    i = i + 1;
}
```

```
if current value of pc = "start"
  then
    next value of i ∈ 1..1000
    next value of pc = "middle"
  else if current value of pc = "middle"
    then
      next value of i = current value of i + 1
      next value of pc = "done"
    else
      no next values
```

A small C program

Let's make this a bit more formal

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A small C program

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A small C program

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```


A small C program

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A small C program

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      next value of pc = "done"
    else
      no next values
```

i:0
pc:"start"

A small C program

```
if current value of pc = "start"
  then
    next value of i ∈ 1..1000
    next value of pc = "middle"
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      no next values
```

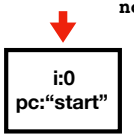

A small C program

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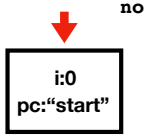

A small C program

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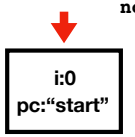

A small C program

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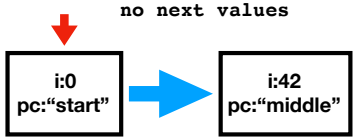

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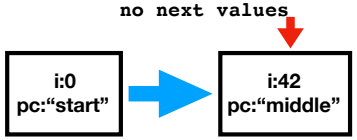

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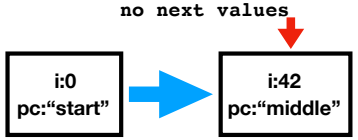

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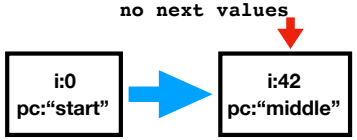

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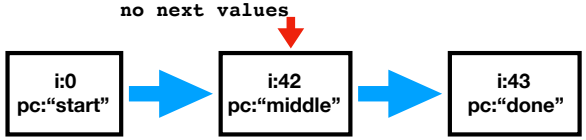

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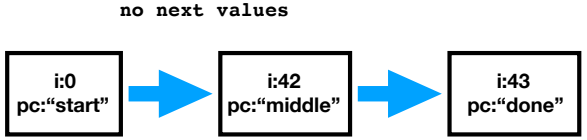

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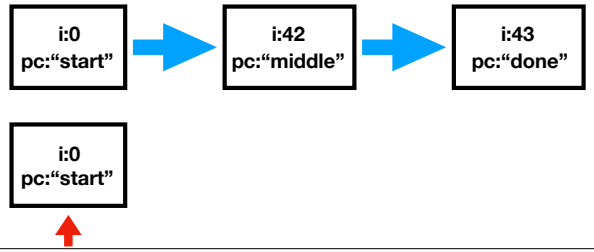

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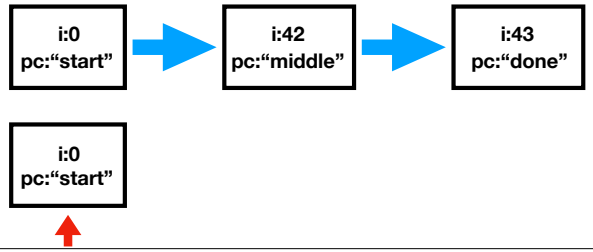

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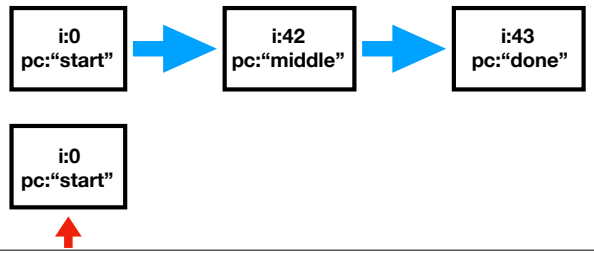

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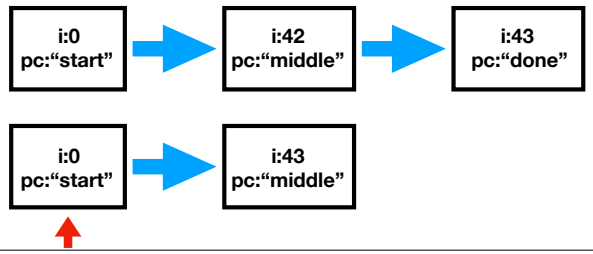

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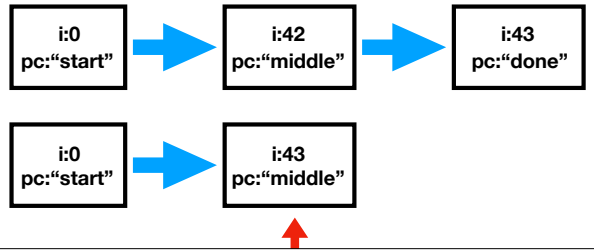

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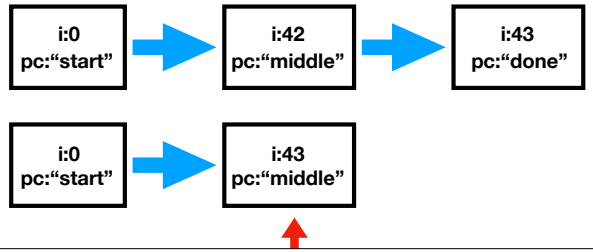

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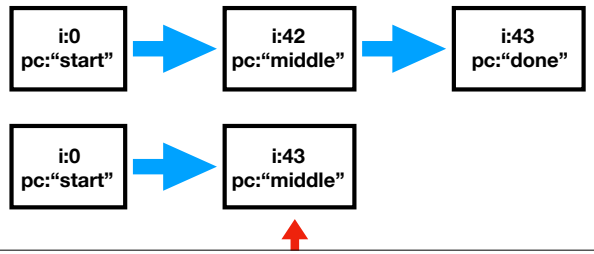

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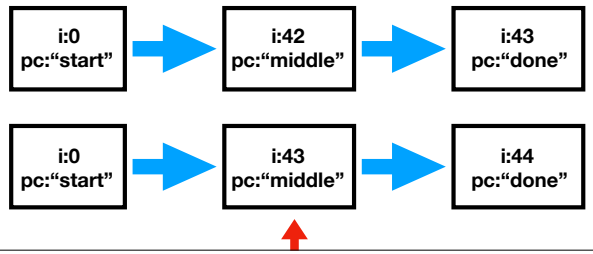

Improving the next state formula

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pc replaces current value of pc

Improving the next state formula

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```

pc replaces current value of pc

i replaces current value of i

Improving the next state formula

```
if pc = "start"
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Improving the next state formula

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Improving the next state formula

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```

i′ replaces next value of i

Improving the next state formula

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    then
      next value of i = i + 1
      next value of pc = "done"
    else
      no next values
```

i′ replaces next value of i

pc′ replaces next value of pc

Improving the next state formula

```
if pc = "start"
  then
    next value of i ∈ 1..1000
    next value of pc = "middle"
  else if pc = "middle"
    then
      next value of i = i + 1
      next value of pc = "done"
    else
      no next values
```

i′ replaces next value of i

pc′ replaces next value of pc

These are called "**primed**" variables, and represent the value in the next state

Improving the next state formula

```
if pc = "start"
  then
    i′ ∈ 1..1000
    pc′ = "middle"
  else if pc = "middle"
    then
      i′ = i + 1
      pc′ = "done"
    else
      no next values
```


= doesn't mean assignment!!!

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This means:

"If in the current state pc is equal to "start", then a valid next state is any state in which i is any value between 1 and 1000, and pc is the string "middle"

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    i' ∈ 1..1000
    pc' = "middle"
  else if pc = "middle"
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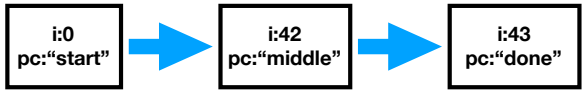
      i' = i + 1
      pc' = "done"
    else
      no next values
```

Again, NOT assignment, we are specifying the next possible states in a state machine.

State pairs

```
if pc = "start"
  then
    i' ∈ 1..1000
    pc' = "middle"
  else if pc = "middle"
    then
      i' = i + 1
      pc' = "done"
    else
      no next values
```



State pairs

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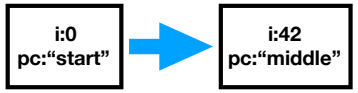

Current state **Next state**

State pairs

```
if pc = "start"
  then
    i' ∈ 1..1000
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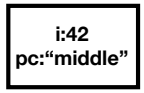

Current state **Next state**

State pairs

```
if pc = "start"
  then
    i' ∈ 1..1000
    pc' = "middle"
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    then
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```

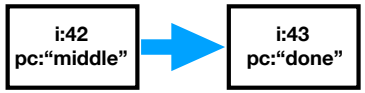

Current state Next state

State pairs

State pairs

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    2. **Initial values** of the variables.

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    3. Possible **next state** for a given state, i.e., relationship between **values** of **variables** in the current state and their **values** in the **next state**.

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  1. **Variables**.
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- That third point is really a **formula** that defines all possible current<->next state pairs (See your cheat sheet for a reminder of what a **formula** is).
- This is SUPER important. Please stop me now with questions if you don't understand!

A little bit more formal

```
if pc = "start"
  then
    i' ∈ 1..1000
    pc' = "middle"
  else if pc = "middle"
    then
      i' = i + 1
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```

This is pretty formal, but to turn it into real TLA+, we need to go a bit further. This is a formula, it must always return a TRUE or FALSE

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The "**then**" and "**else**" clauses must return TRUE or FALSE

A little bit more formal

```
if pc = "start"
  then
    i' ∈ 1..1000
    pc' = "middle"
  else if pc = "middle"
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```

`(i' ∈ 1..1000) /\ (pc' = "middle")` replaces `i' ∈ 1..1000` `pc' = "middle"`

A little bit more formal

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`(i' = i + 1) /\ (pc' = "done")` replaces `i' = i + 1` `pc' = "done"`

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```
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    else
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```

FALSE replaces no next values

A little bit more formal

```
if pc = "start"
  then
    (i' ∈ 1..1000) /\ (pc' = "middle")
  else if pc = "middle"
    then
      (i' = i + 1) /\ (pc' = "done")
    else
      FALSE
```


Looking pretty good…

But we're not there just yet. Our formula is good, but we can make it better.

A little bit more formal

```
if pc = "start"
  then
    (i' ∈ 1..1000) /\ (pc' = "middle")
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if pc = "start"
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    else
      FALSE
```

When does this formula return TRUE?

A little bit more formal

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if pc = "start"
  then
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  else if pc = "middle"
    then
      (i' = i + 1) /\ (pc' = "done")
    else
      FALSE
```

When does this formula return TRUE?

There are two cases

Case 1

```
if pc = "start"
  then
    (i' ∈ 1..1000) /\ (pc' = "middle")
  else if pc = "middle"
    then
      (i' = i + 1) /\ (pc' = "done")
    else
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```


Case 1

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if pc = "start"
  then
    (i' ∈ 1..1000) /\ (pc' = "middle")
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When **`pc="start"`** in the current state

Case 1

```
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When **pc="start"** in the current state

And **(i' ∈ 1..1000) /\ (pc' = "middle")** in the next state

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When **pc="start"** in the current state

And **(i' ∈ 1..1000) /\ (pc' = "middle")** in the next state

Or in other words:

Case 1

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Case 2

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if pc = "start"
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    else
      FALSE
```


Case 2

```
if pc = "start"
  then
    (i' ∈ 1..1000) /\ (pc' = "middle")
  else if pc = "middle"
    then
      (i' = i + 1) /\ (pc' = "done")
    else
      FALSE
```

When **pc="middle"** in the current state

Case 2

```
if pc = "start"
  then
    (i' ∈ 1..1000) /\ (pc' = "middle")
  else if pc = "middle"
    then
      (i' = i + 1) /\ (pc' = "done")
    else
      FALSE
```

When **pc="middle"** in the current state

Case 2

```
if pc = "start"
  then
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    else
      FALSE
```

When **pc="middle"** in the current state

And **(i' = i + 1) /\ (pc' = "done")** in the next state

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Rewrite!

```
if pc = "start"
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```

Rewrite!

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if pc = "start"
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```

becomes

Rewrite!

```
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```

becomes

```
((pc="start") /\ ((i' ∈ 1..1000) /\ (pc' = "middle")))
```


Rewrite!

```
if pc = "start"
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becomes

```
((pc="start") /\ ((i' ∈ 1..1000) /\ (pc' = "middle")))
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103-6

Rewrite!

```
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becomes

```
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```

103-7

Rewrite!

```
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    else
      FALSE
```

becomes

```
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```

103-8

Rewrite!

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becomes

```
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103-9

Rewrite!

```
if pc = "start"
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becomes

```
((pc="start") /\ ((i' ∈ 1..1000) /\ (pc' = "middle")))
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```

(We'll turn this into TLA syntax on the next slide.)

TLA+ Syntax Replacement

```
((pc="start") /\ ((i' ∈ 1..1000) /\ (pc' = "middle")))
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TLA+ Syntax Replacement

```
((pc="start") /\ ((i' ∈ 1..1000) /\ (pc' = "middle")))
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TLA+ Syntax Replacement

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becomes

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\/ /\ pc = "start"
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\/ /\ pc = "middle"
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```

Comparison

```
int i;
void main () {
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    i = i + 1;
}
```


Comparison

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```

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\/ /\ pc = "start"
   /\ i' ∈ 1..1000
   /\ pc' = "middle"
\/ /\ pc = "middle"
   /\ i' = i + 1
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- Think about how `someNumber()` would actually have to be implemented.

Notes

- In C, = means “assignment”. In math, it means equality, like you learned in grade 1. What does “assignment” actually mean? It’s a very complicated concept.
- `someNumber()` returning a value from 1 to 1000 is very hard to specify in C. It represents non-determinism. Almost no programming languages were designed to express non-determinism.
- Think about how `someNumber()` would actually have to be implemented.
- Non-determinism in math is easy. i’ $\in$ 1..1000 is non-determinism. “i can be any value from 1 to 1000 in the next state”.

An aside

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- It **can** be specified, but it's not where the power lies.
- The point of this exercise has been to go step by step from something you already understand to full TLA+ syntax.
- This spec is 6 lines long. The TLA+ spec is longer. Usually, TLA+ specs are orders of magnitude **shorter** than the systems they specify.

Next State

Next State

```
\/ /\ pc = "start"
   /\ i' ∈ 1..1000
   /\ pc' = "middle"
\/ /\ pc = "middle"
   /\ i' = i + 1
   /\ pc' = "done
```


Next State

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\/ /\ pc = "start"
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- This formula defines all the valid state pairs.

Next State

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- This formula defines all the valid state pairs.
- It does NOT say "**if pc='middle' then set i to i + 1 and set pc to 'done'** ".

Next State

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\/ /\ pc = "start"
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   /\ pc' = "done
```

- This formula defines all the valid state pairs.
- It does NOT say "**if pc='middle' then set i to i + 1 and set pc to 'done'** ".
- So… What's actually going on? What is this really saying?

Behaviours

Behaviours

- Think back to our Die Hard example

Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours

Behaviours

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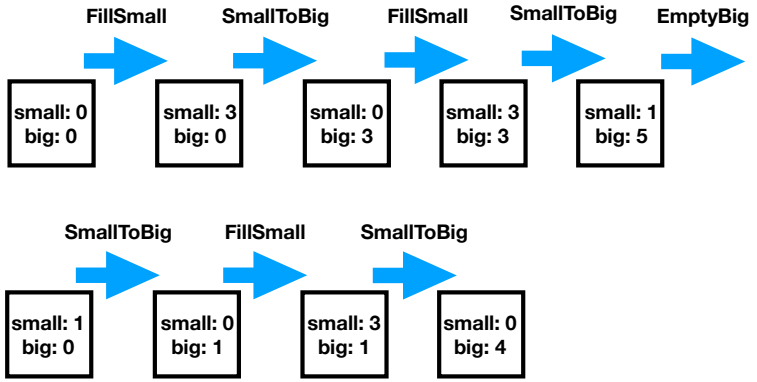
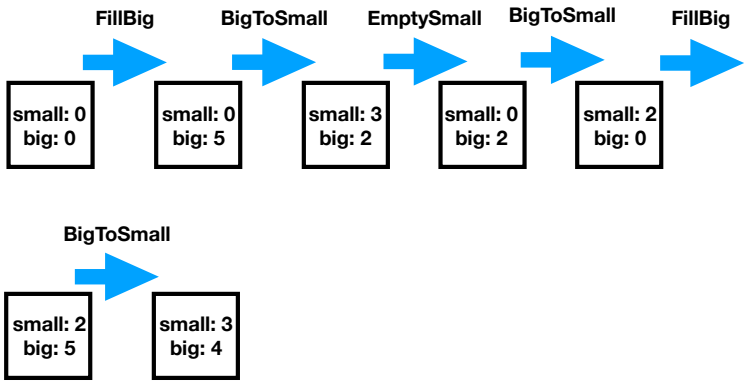
- We showed three possible behaviours



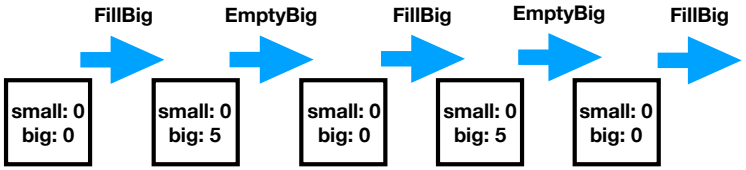
Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours



Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours



Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours

- An INFINITE number of behaviours are possible

Behaviours

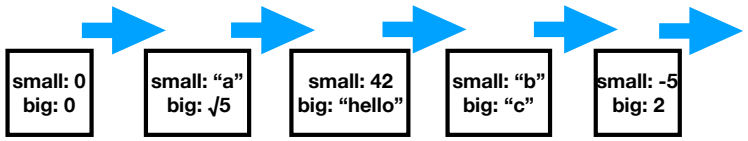
- Think back to our Die Hard example
- We showed three possible behaviours

- An INFINITE number of behaviours are possible
- Not only behaviours that Fill/Empty/Pour, but an infinite number of values can be assigned to **big** and **small**

Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours

- An INFINITE number of behaviours are possible
- Not only behaviours that Fill/Empty/Pour, but an infinite number of values can be assigned to **big** and **small**



Behaviours

Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.

110-2

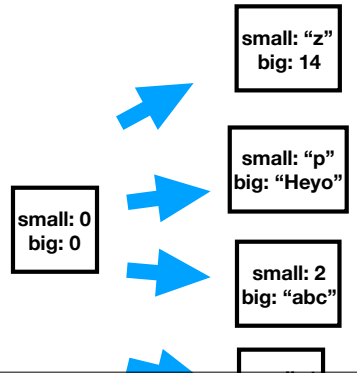
Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.
- From any **state**, an infinite number of **next states** are possible.

110-3

Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.
- From any **state**, an infinite number of **next states** are possible.



110-4

Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.
- From any **state**, an infinite number of **next states** are possible.
- The point of our **formula** is to restrict which of those **state pairs** are permitted. All the combinations of allowed state pairs make up our possible behaviours.

110-5

Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.
- From any **state**, an infinite number of **next states** are possible.
- The point of our **formula** is to restrict which of those **state pairs** are permitted. All the combinations of allowed state pairs make up our possible behaviours.
- This is the foundational concept of TLA+.

Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.
- From any **state**, an infinite number of **next states** are possible.
- The point of our **formula** is to restrict which of those **state pairs** are permitted. All the combinations of allowed state pairs make up our possible behaviours.
- This is the foundational concept of TLA+.
- The formula restricts which subset of infinite behaviours is allowed, by specifying the allowed transitions from one state to another.

Behaviours

```
\/ /\ pc = "start"
   /\ i' ∈ 1..1000
   /\ pc' = "middle"
\/ /\ pc = "middle"
   /\ i' = i + 1
   /\ pc' = "done
```

Behaviours

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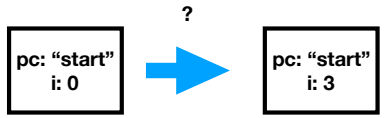

Behaviours

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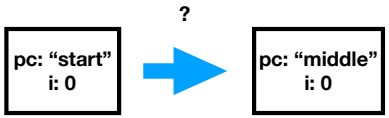

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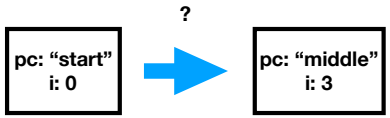

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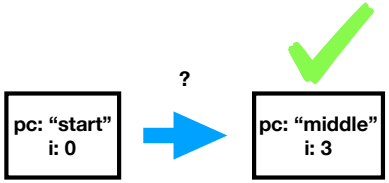

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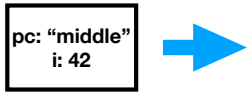

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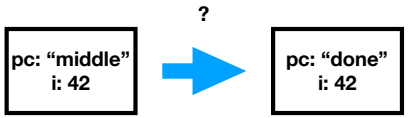

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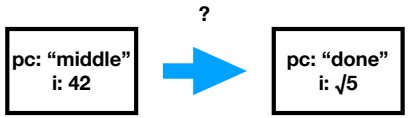

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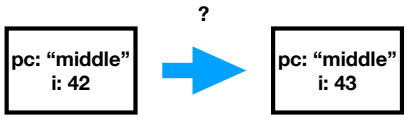

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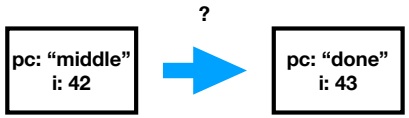

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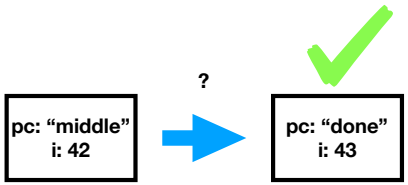


Final Spec for our C Program

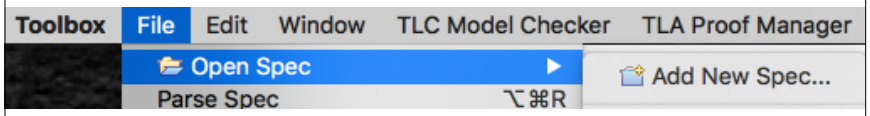
Final Spec for our C Program

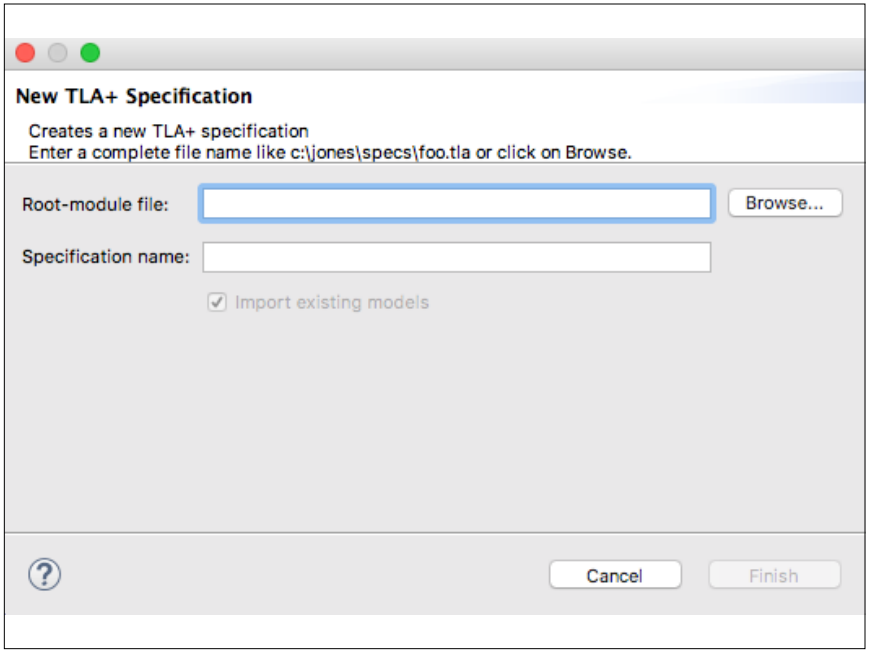
```
---- MODULE SimpleProgram ----

EXTENDS Integers
VARIABLES i, pc

Init == (pc="start") /\ (i=0)

Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done
```



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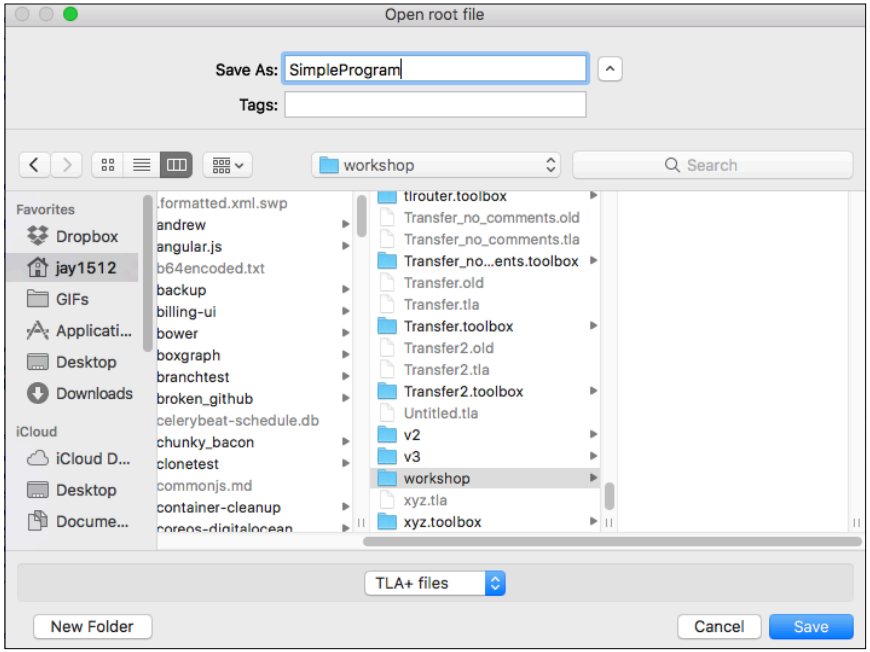

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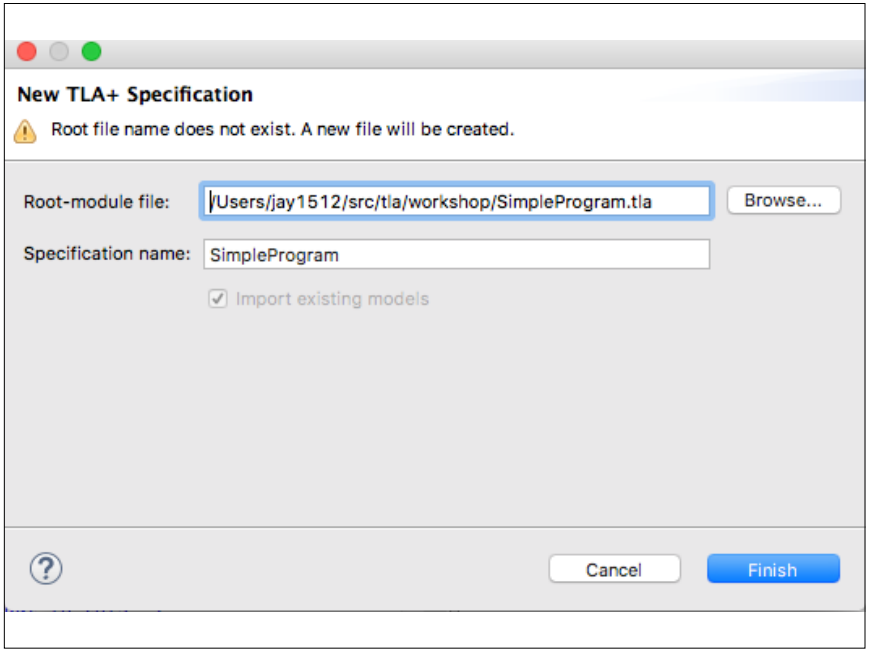

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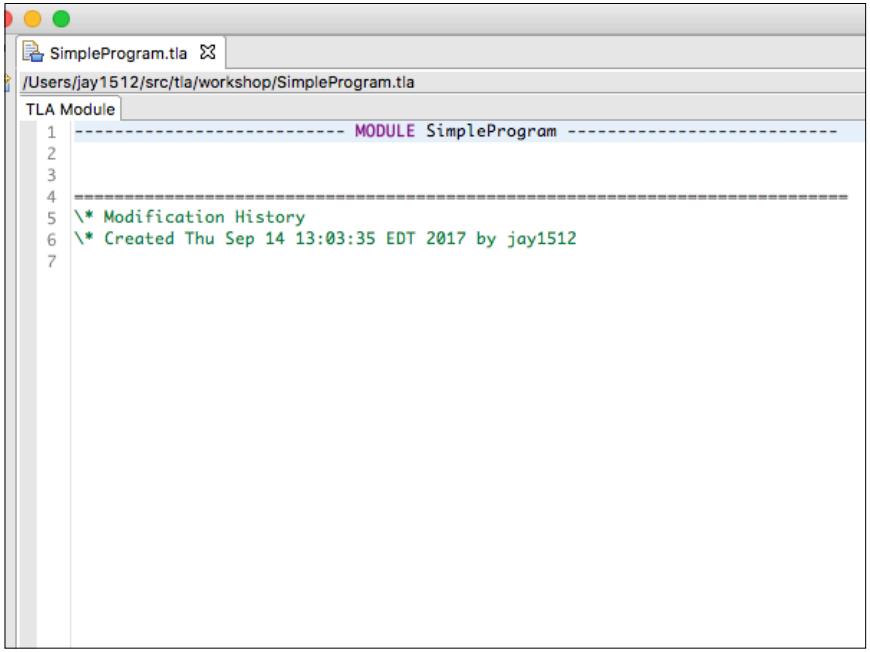


118

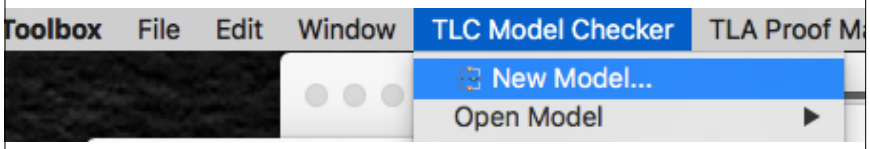
/Users/jay1512/src/tla/workshop/SimpleProgram.tla

TLA Module

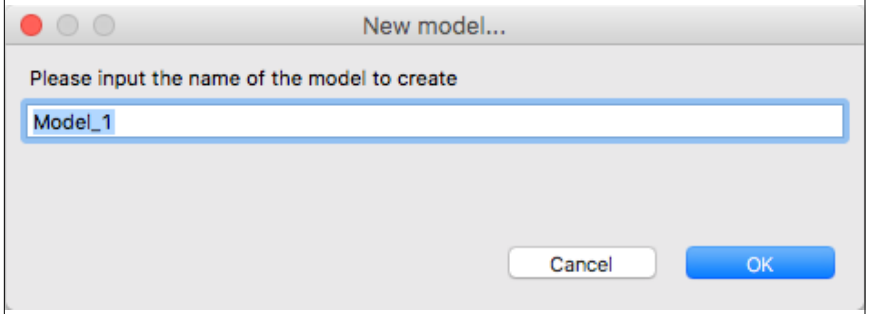
```
1  ---------------------------- MODULE SimpleProgram -------------
2  EXTENDS Integers
3  VARIABLES i, pc
4
5  Init == (pc="start") /\ (i=0)
6
7  Next ==
8        \/ /\ pc = "start"
9           /\ i' \in 1..1000
10          /\ pc' = "middle"
11       \/ /\ pc = "middle"
12          /\ i' = i + 1
13          /\ pc' = "done"
14
15 ============================================================
16 \* Modification History
17 \* Last modified Thu Sep 14 13:04:45 EDT 2017 by jay1512
```

In GitHub at `specs/SimpleProgram.tla`

119



120



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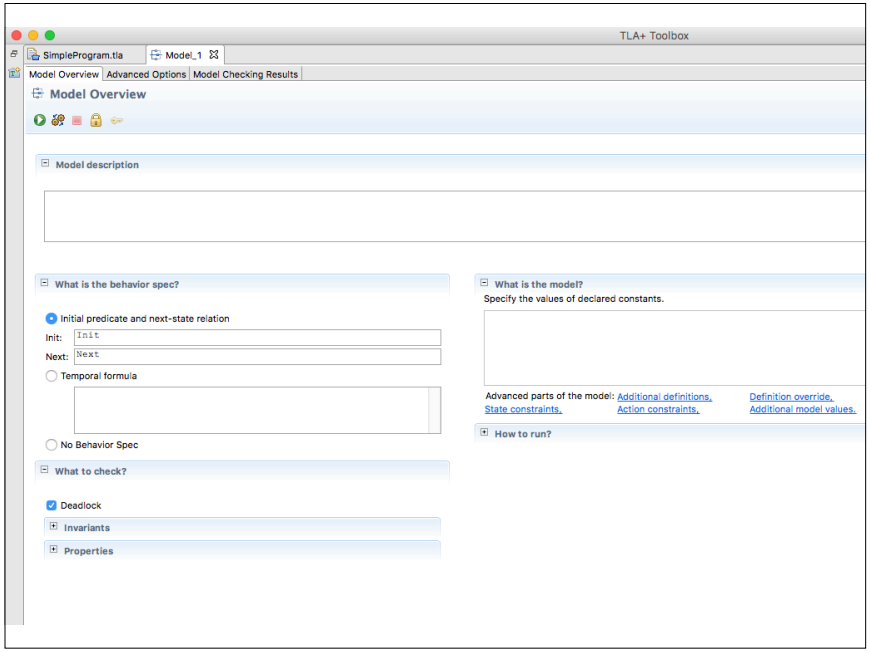
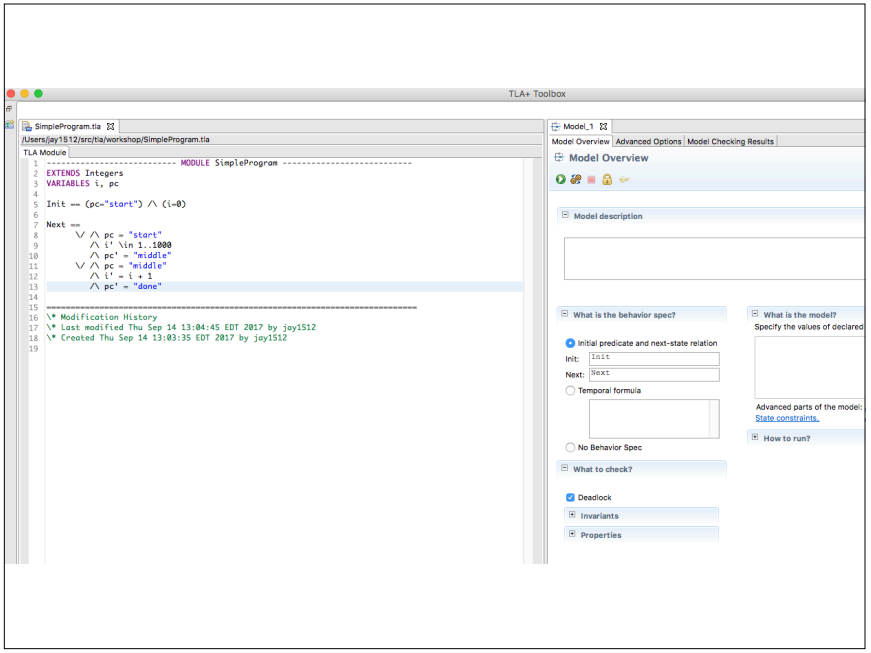


122



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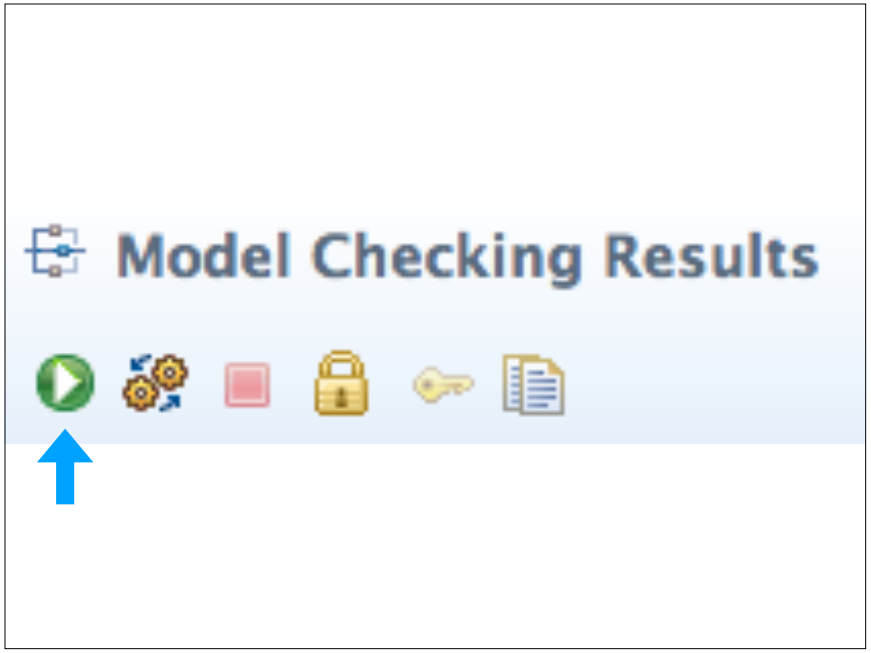
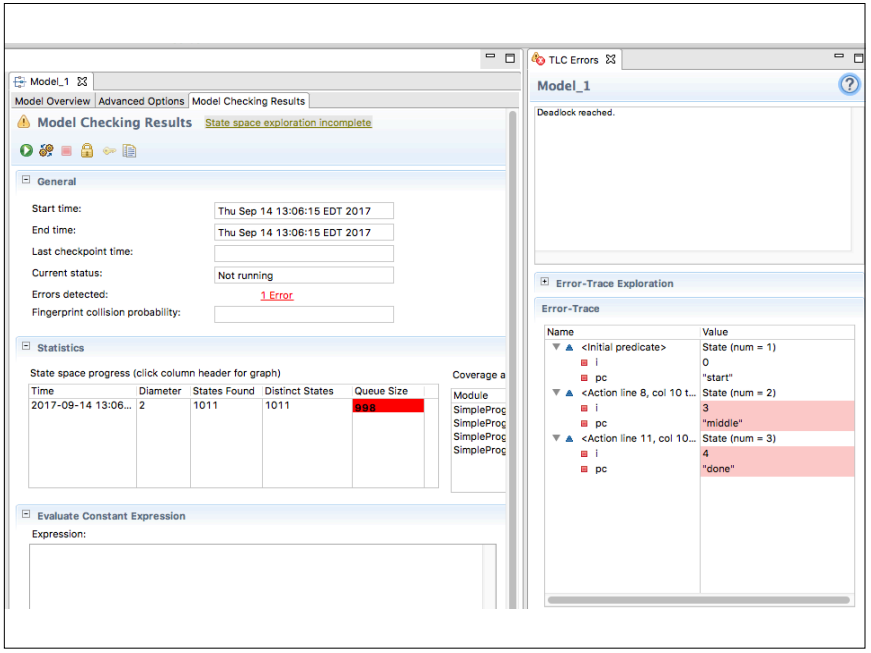
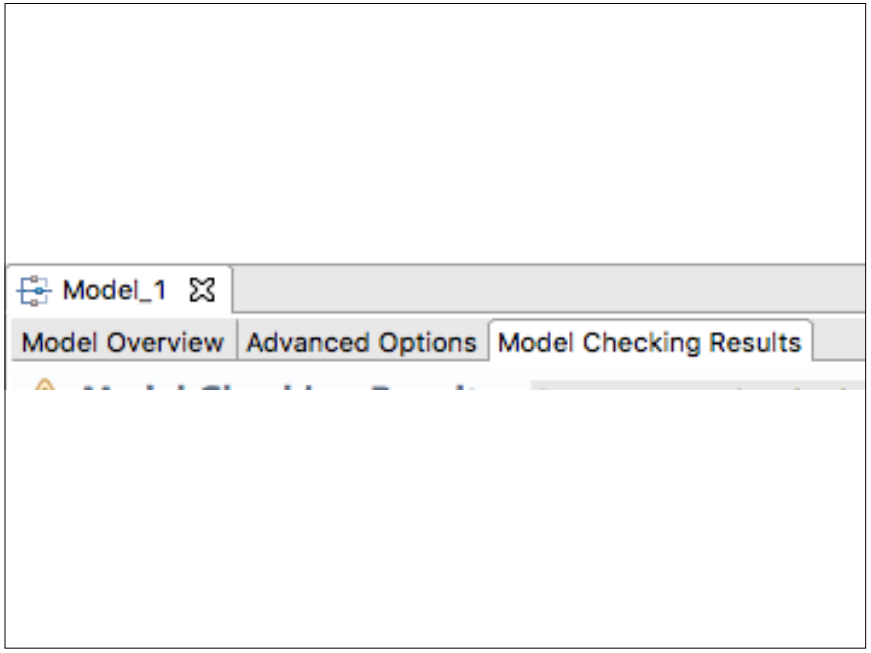


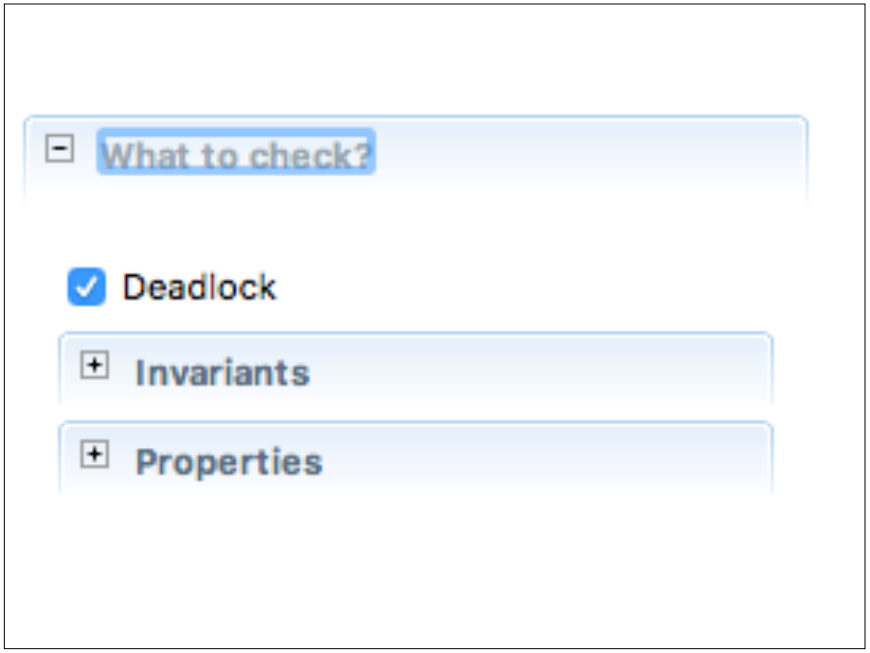
124



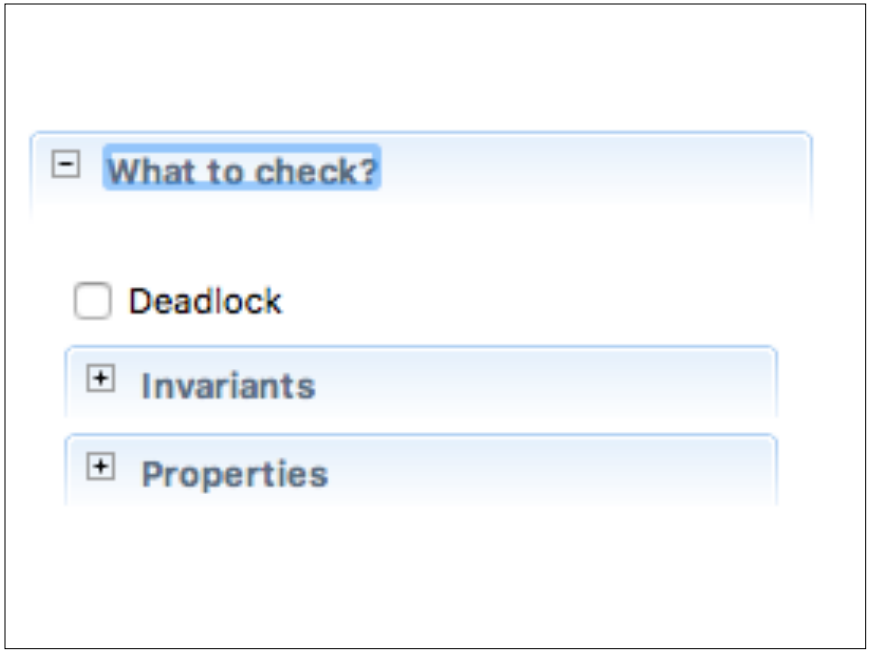
125



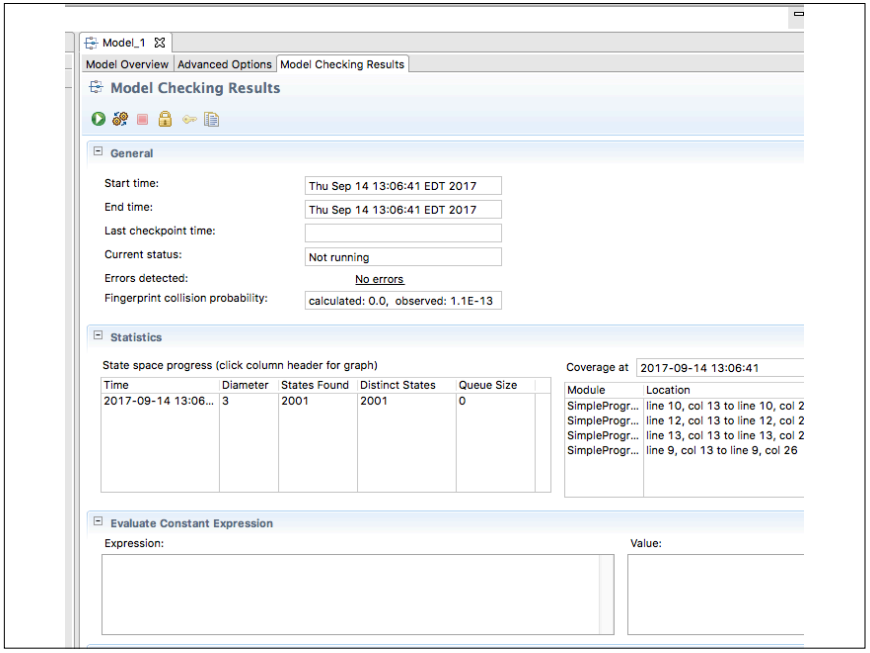
126



127



128



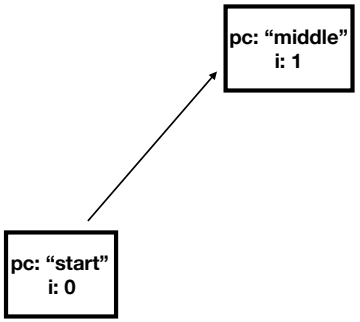
129

All Possible State Transitions

pc: “start”
i: 0

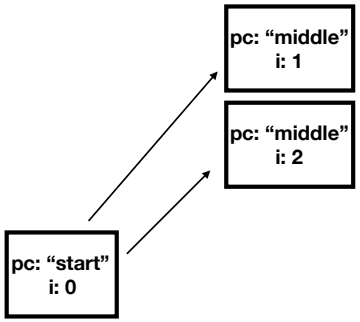
130-1

All Possible State Transitions



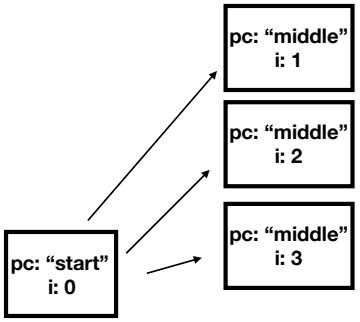
130-2

All Possible State Transitions



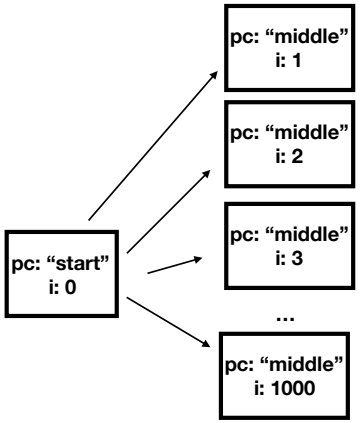
130-3

All Possible State Transitions



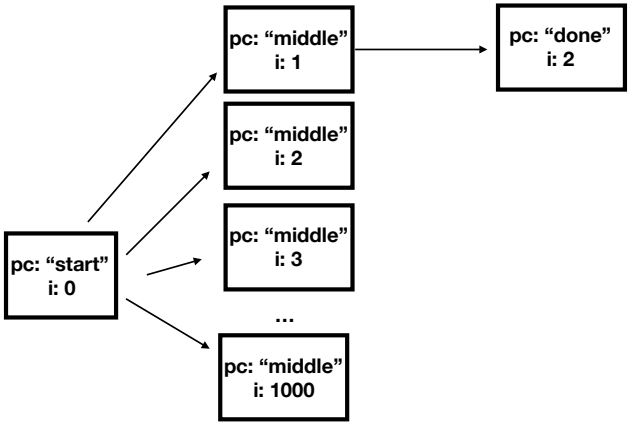
130-4

All Possible State Transitions



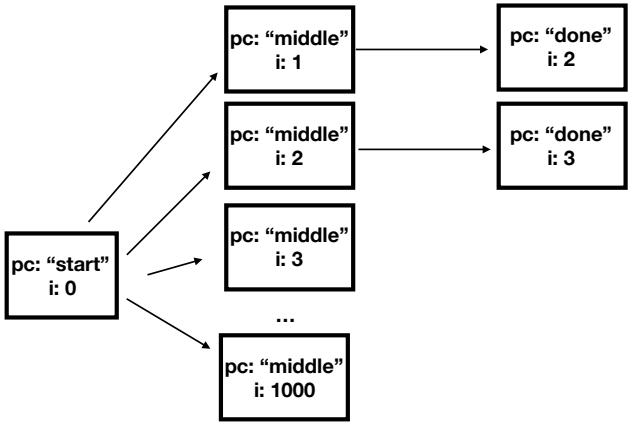
130-5

All Possible State Transitions



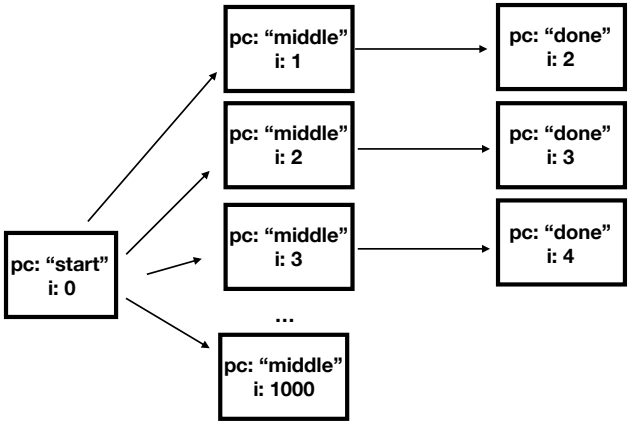
130-6

All Possible State Transitions



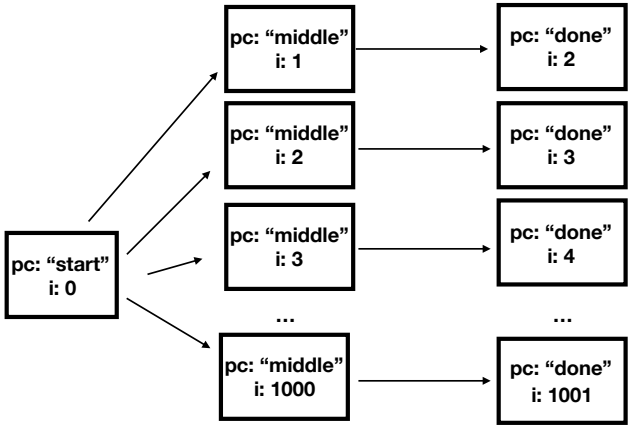
130-7

All Possible State Transitions



130-8

All Possible State Transitions



130-9

Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up!
TLA+ allows for the same thing

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done
```


Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up!
TLA+ allows for the same thing

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done
```


Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up!
TLA+ allows for the same thing

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done
```


Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up!
TLA+ allows for the same thing

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done


Pick ==
      /\ pc = "start"
      /\ i' \in 1..1000
      /\ pc' = "middle"
```

Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up!
TLA+ allows for the same thing

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done"

Pick ==
      /\ pc = "start"
      /\ i' \in 1..1000
      /\ pc' = "middle"

Add ==
      /\ pc = "middle"
      /\ i' = i + 1
      /\ pc' = "done"
```


Sub-definitions

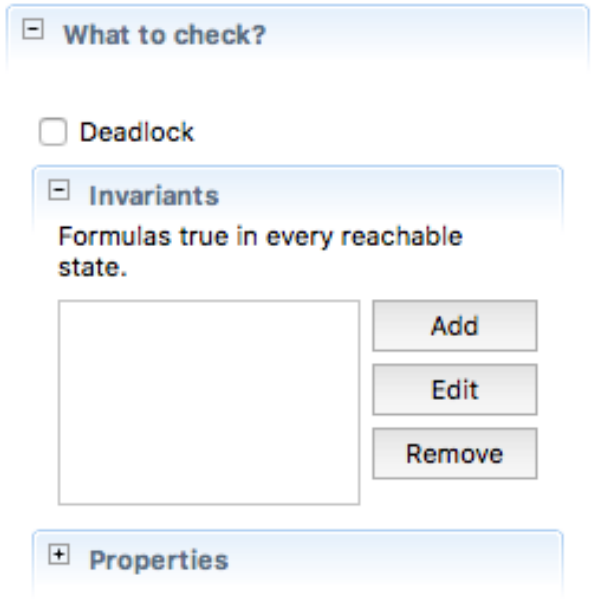
When programming, we don't stuff all our code into a single subroutine. We split it up!
TLA+ allows for the same thing

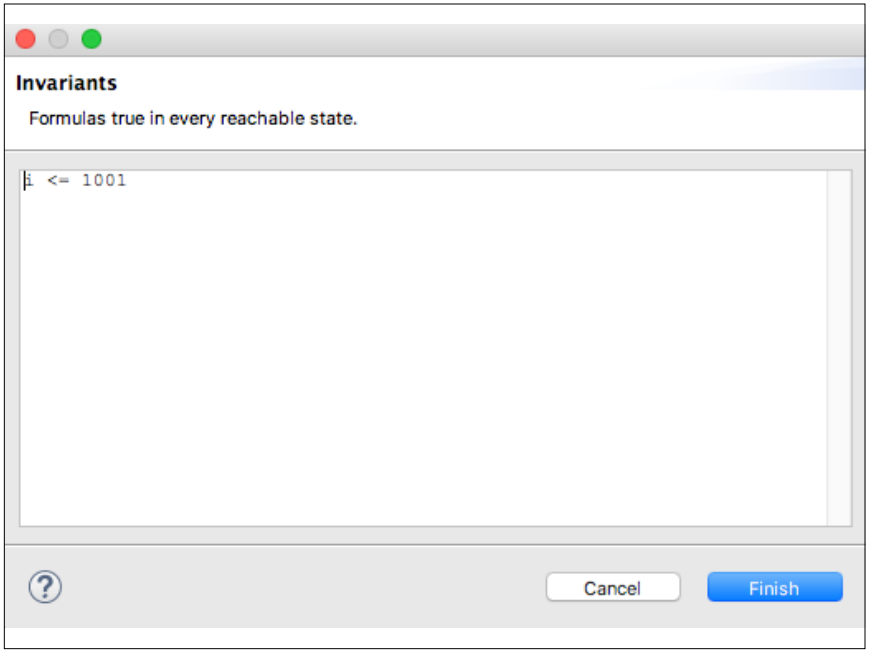
```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done"

Pick ==
      /\ pc = "start"
      /\ i' \in 1..1000
      /\ pc' = "middle"

Add ==
      /\ pc = "middle"
      /\ i' = i + 1
      /\ pc' = "done"

Next == Pick \/ Add
```



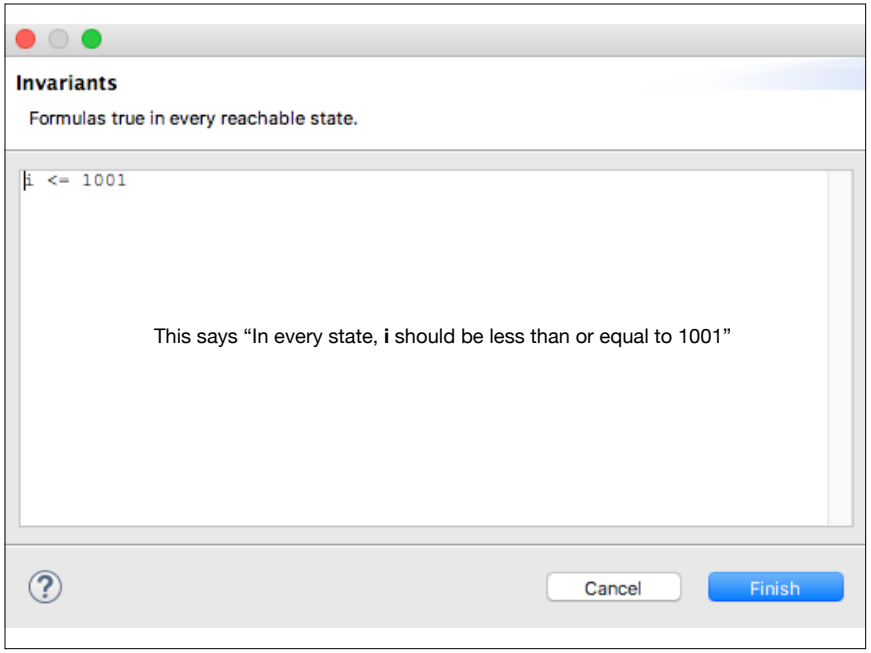
```
EXTENDS Integers
VARIABLES i, pc

Init == (pc="start") /\ (i=0)

Pick ==
   /\ pc = "start"
   /\ i' \in 1..1000
   /\ pc' = "middle"

Add ==
   /\ pc = "middle"
   /\ i' = i + 1
   /\ pc' = "done"

Next == Pick \/ Add
```

specs/SimpleProgram2.tla





134-1



134-2

Run the model again, everything should still be fine

135

Change the Add action

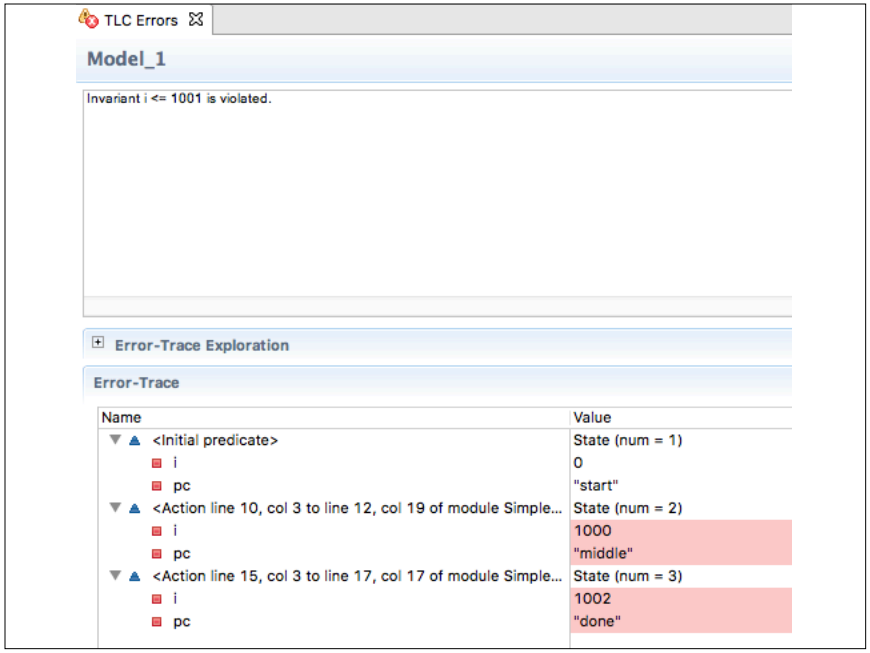
```
Add ==
  /\ pc = "middle"
  /\ i' = i + 2
  /\ pc' = "done"
```

136-1

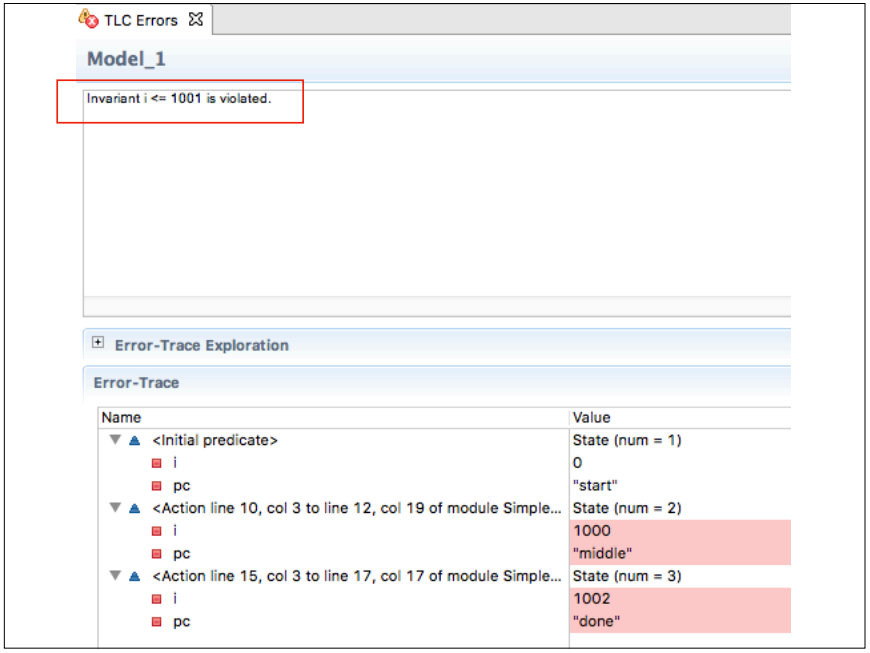
Change the Add action

```
Add ==
   /\ pc = "middle"
   /\ i' = i + 2
   /\ pc' = "done"
```

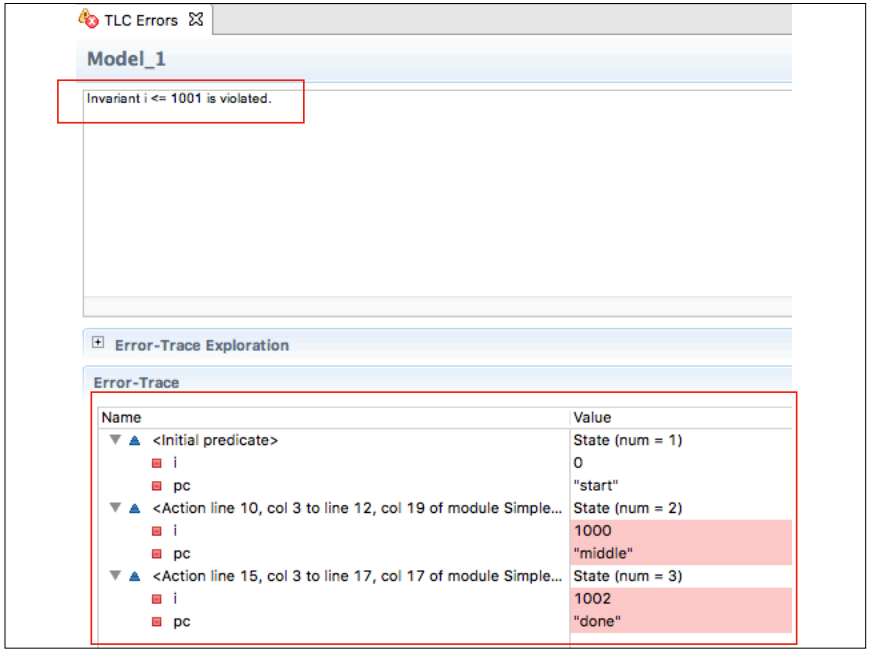
136-2



137-1

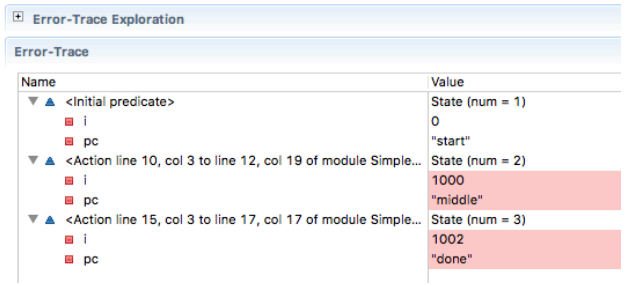


137-2



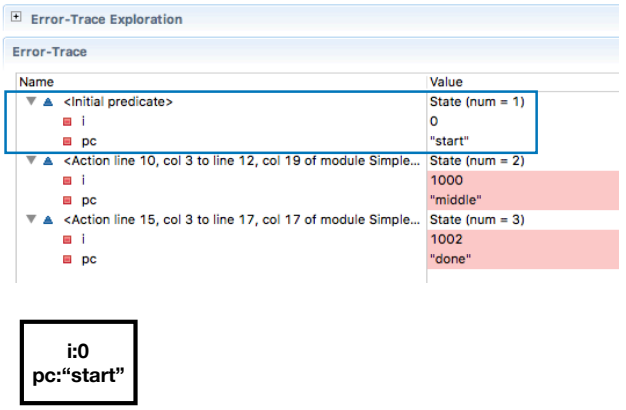
137-3

Error Trace



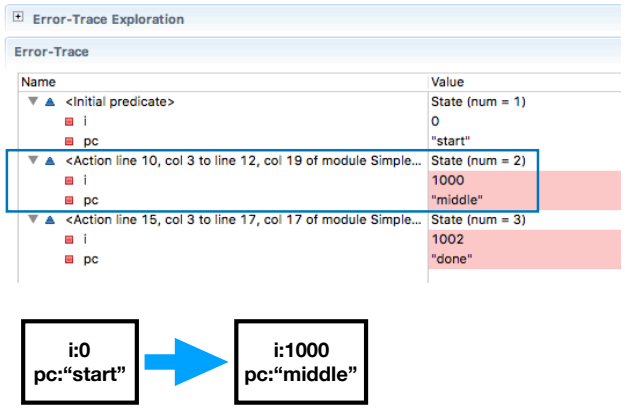
138-1

Error Trace



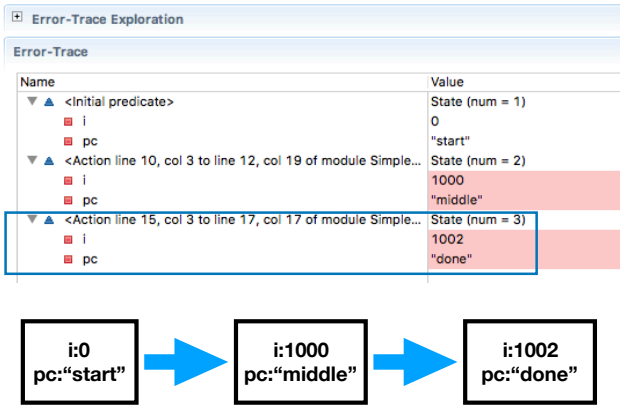
138-2

Error Trace



138-3

Error Trace



138-4

Die Hard Variables

```
VARIABLES big, small
```


Die Hard Initial States

```
Init == (big = 0) /\ (small = 0)
```


Die Hard Next States

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

3.We can always empty **small**, i.e. in the next state **small=0**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

3.We can always empty **small**, i.e. in the next state **small=0**

4.We can always empty **big**, i.e. in the next state **big=0**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

3.We can always empty **small**, i.e. in the next state **small=0**

4.We can always empty **big**, i.e. in the next state **big=0**

5.We can always pour **small** into **big**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

3.We can always empty **small**, i.e. in the next state **small=0**

4.We can always empty **big**, i.e. in the next state **big=0**

5.We can always pour **small** into **big**

6. We can always pour **big** into **small**

Die Hard Next States

```
FillSmall
FillBig
EmptySmall
EmptyBig
SmallToBig
BigToSmall
```


FillSmall

FillSmall

```
FillSmall == /\ small' = 3
```


FillSmall

```
FillSmall == /\ small' = 3
             /\ big' = big
```


EmptySmall

EmptySmall

```
EmptySmall == /\ small' = 0
```

EmptySmall

```
EmptySmall == /\ small' = 0
              /\ big' = big
```


SmallToBig

- Two possible cases:
  1. There **is** room in **big** for all the water in **small**
  2. There **is not** room in **big** for all the water in **small**

SmallToBig

Case 1: Enough room in **big**

We put all the water from the **small** jug into the **big** jug, which has the effect of emptying the **small** jug

SmallToBig

Case 1: Enough room in **big**

We put all the water from the **small** jug into the **big** jug, which has the effect of emptying the **small** jug

```
/\ big' = big + small
```

SmallToBig

Case 1: Enough room in **big**

We put all the water from the **small** jug into the **big** jug, which has the effect of emptying the **small** jug

```
/\ big' = big + small
/\ small' = 0
```

147-3

SmallToBig

Case 2: NOT enough room in **big**

We pour what we can from **small** into **big**. **big** gets filled by this action. **small** is emptied by however much was poured out

148-1

SmallToBig

Case 2: NOT enough room in **big**

We pour what we can from **small** into **big**. **big** gets filled by this action. **small** is emptied by however much was poured out

```
/\ big' = 5
```

148-2

SmallToBig

Case 2: NOT enough room in **big**

We pour what we can from **small** into **big**. **big** gets filled by this action. **small** is emptied by however much was poured out

```
/\ big' = 5
/\ small' = small - (5 - big)
```

148-3

SmallToBig

Putting it all together

SmallToBig

Putting it all together

```
SmallToBig == IF big + small < 5
```


SmallToBig

Putting it all together

```
SmallToBig == IF big + small < 5
              THEN /\ big' = big + small
```


SmallToBig

Putting it all together

```
SmallToBig == IF big + small < 5
              THEN /\ big' = big + small
                   /\ small' = 0
```

SmallToBig

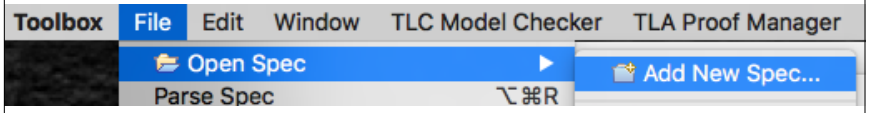
Putting it all together

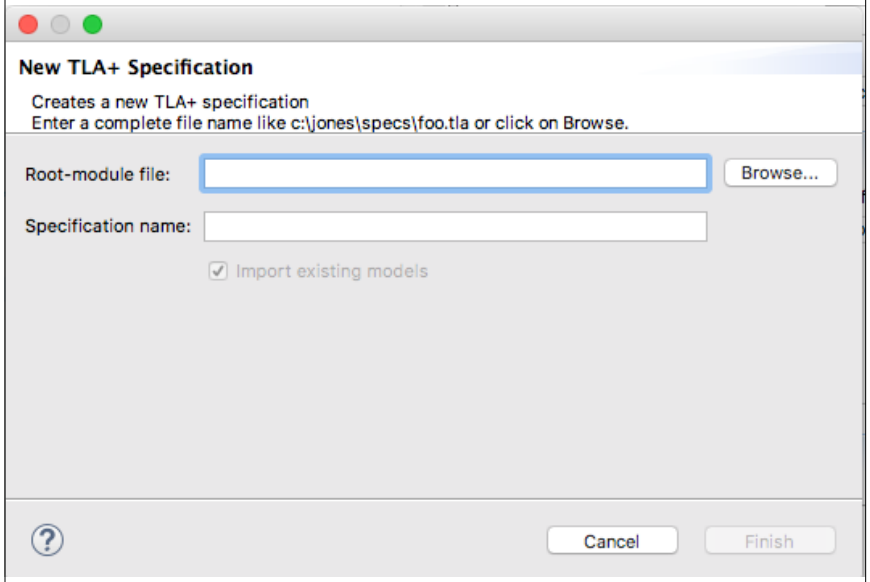
```
SmallToBig == IF big + small < 5
              THEN /\ big' = big + small
                   /\ small' = 0
              ELSE /\ small' = small - (5 - big)
```

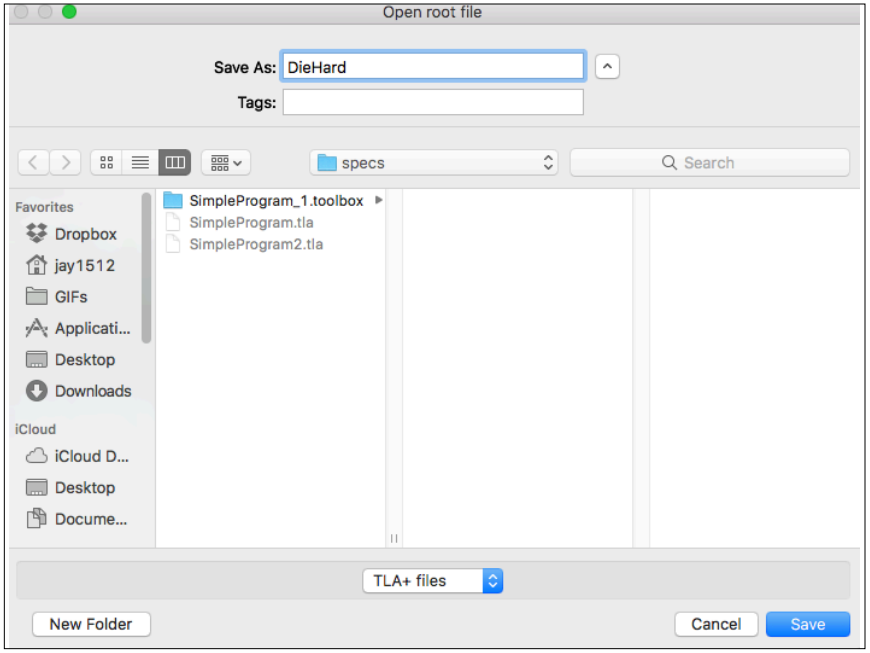

SmallToBig

Putting it all together

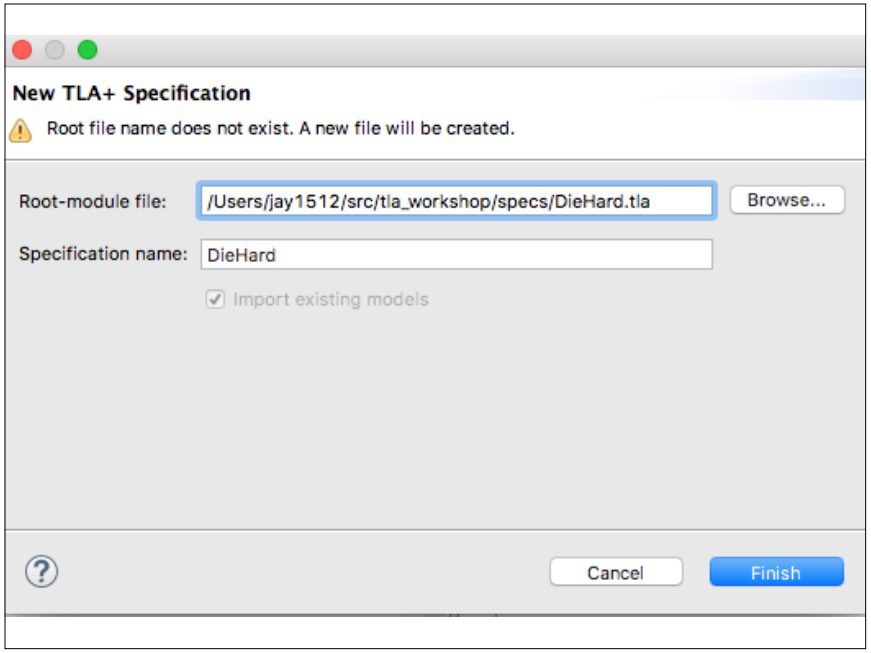
```
SmallToBig == IF big + small < 5
              THEN /\ big' = big + small
                   /\ small' = 0
              ELSE /\ small' = small - (5 - big)
                   /\ big' = 5
```

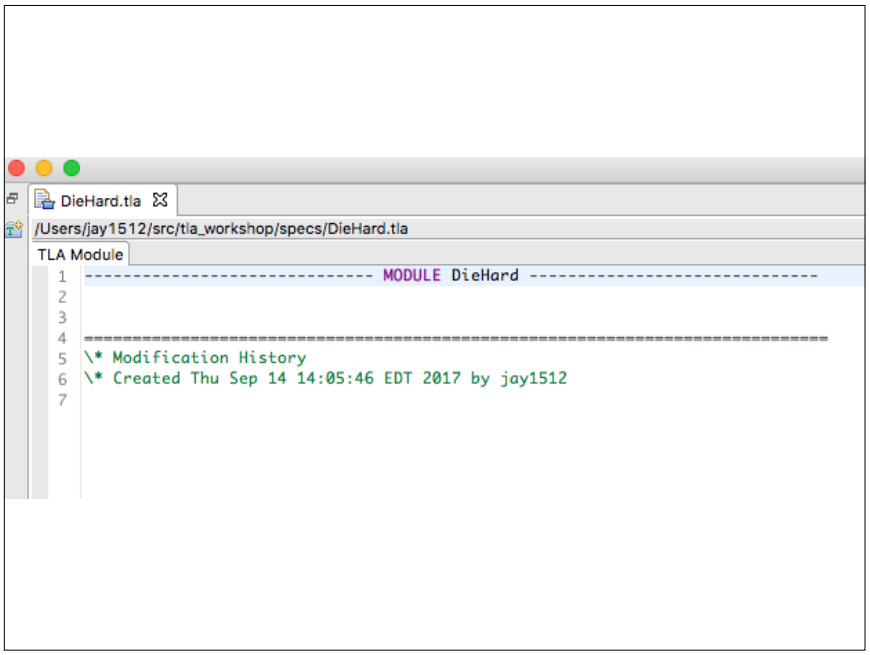




152



153

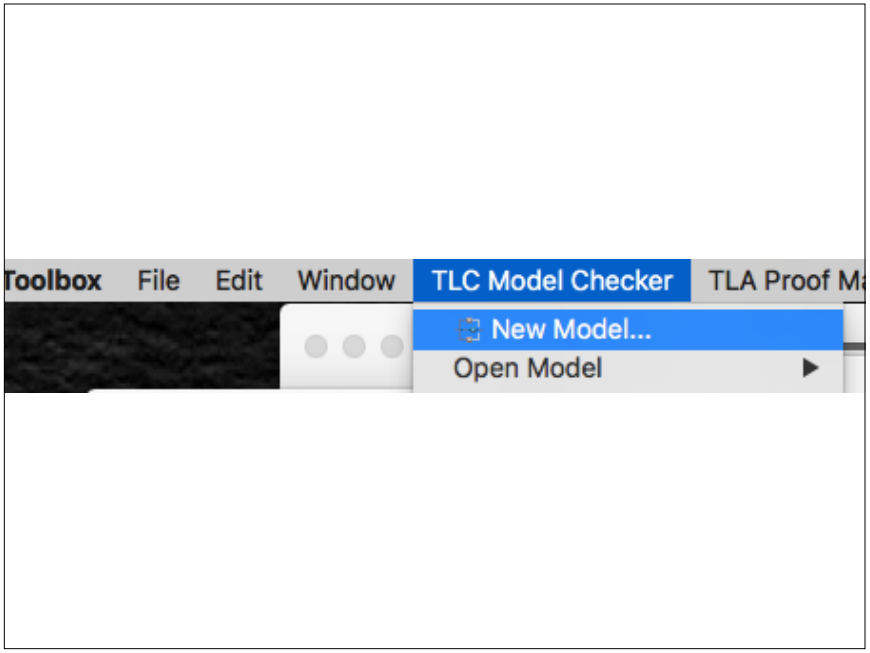


154

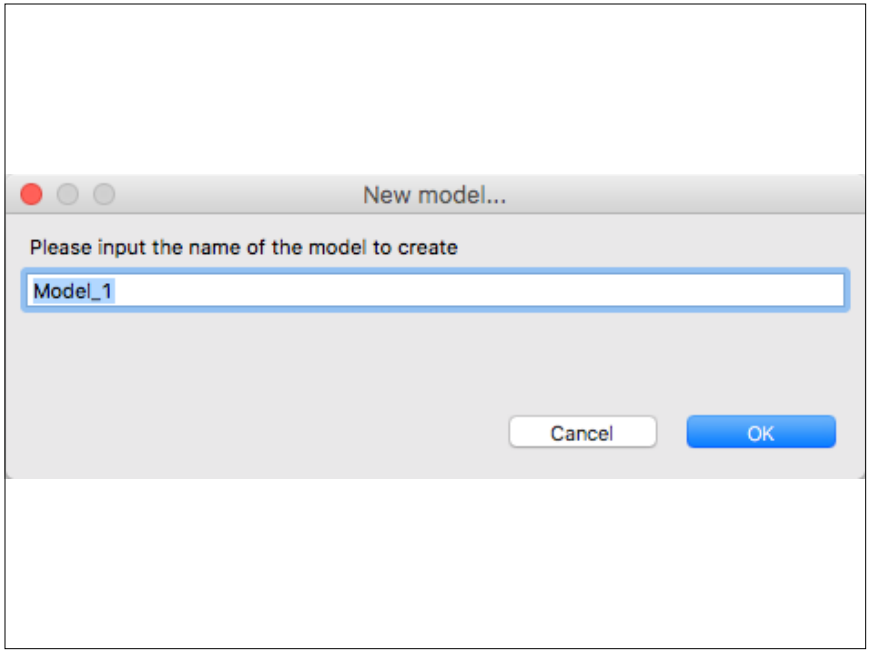
```
---------- MODULE DieHard ----------
EXTENDS Integers

VARIABLES small, big

TypeOK == /\ small \in 0..3
          /\ big   \in 0..5

Init == /\ big   = 0
        /\ small = 0

FillSmall == /\ small' = 3
             /\ big'   = big

FillBig ==

EmptySmall == /\ small' = 0
              /\ big'   = big

EmptyBig ==

SmallToBig == IF big + small =< 5
              THEN /\ big'   = big + small
                   /\ small' = 0
              ELSE /\ big'   = 5
                   /\ small' = small - (5 - big)

BigToSmall ==

Next == \/ FillSmall
        \/ FillBig
        \/ EmptySmall
        \/ EmptyBig
        \/ SmallToBig
        \/ BigToSmall
```

specs/DieHard_incomplete.tla

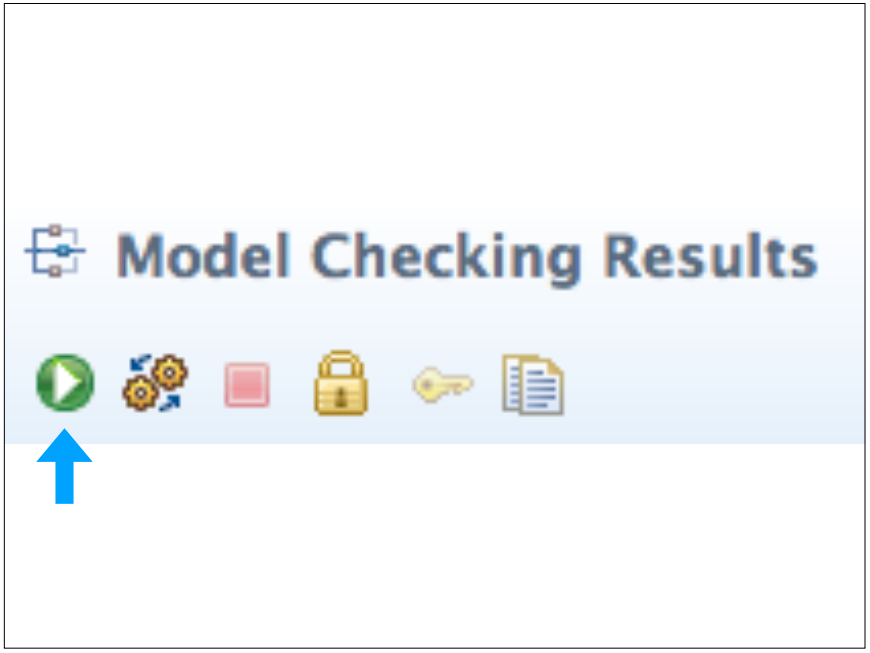
155



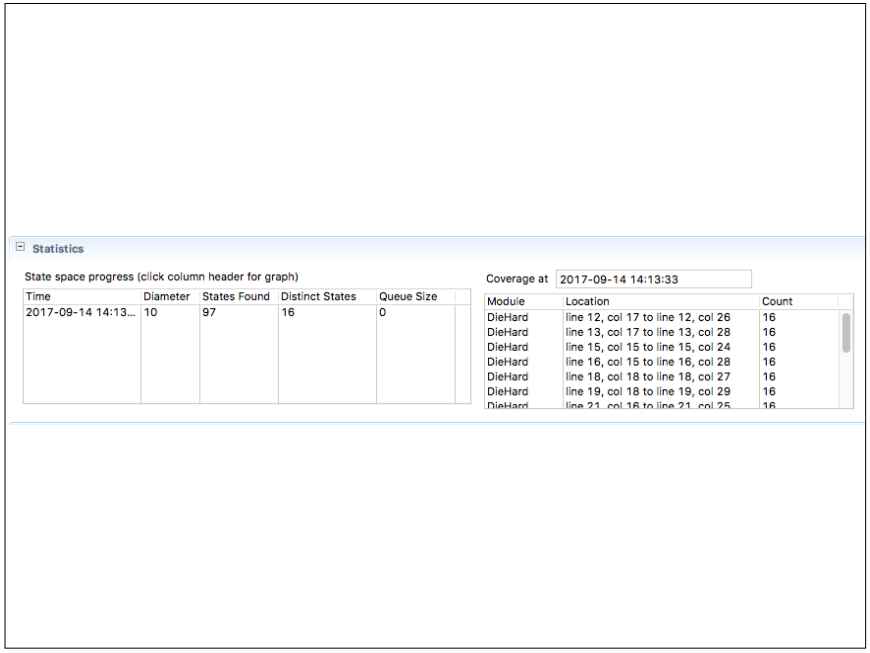
156



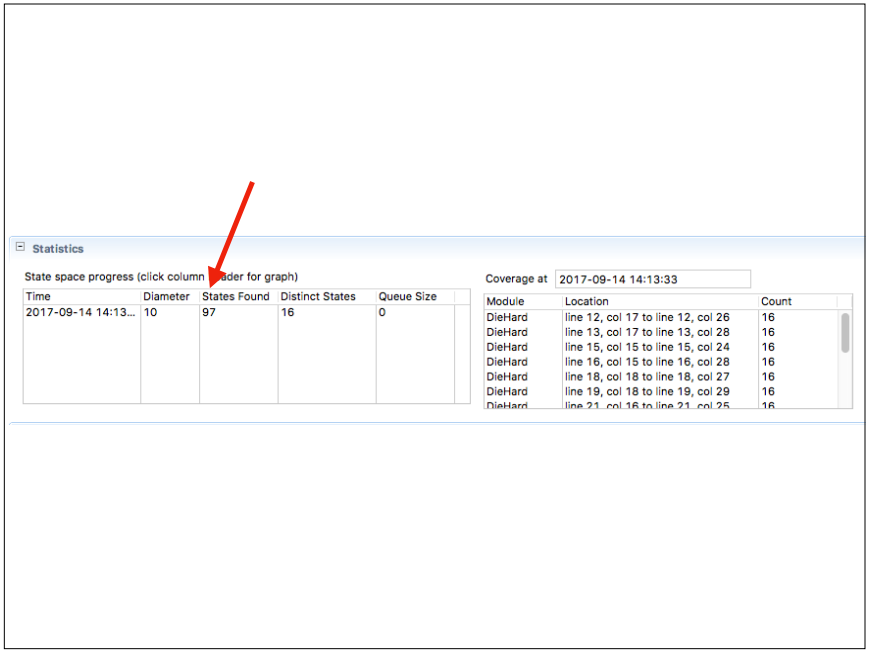
157



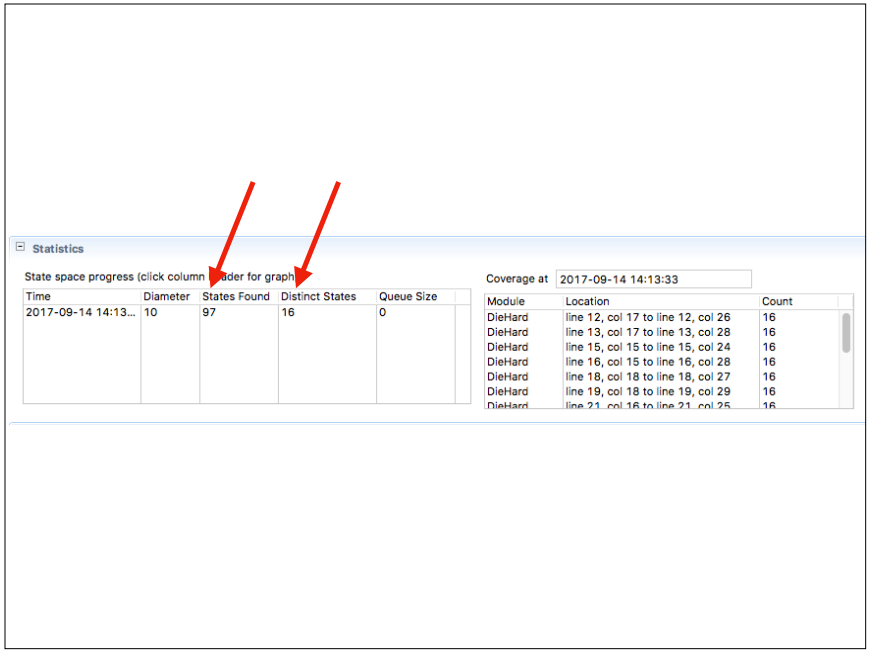
158



159-1



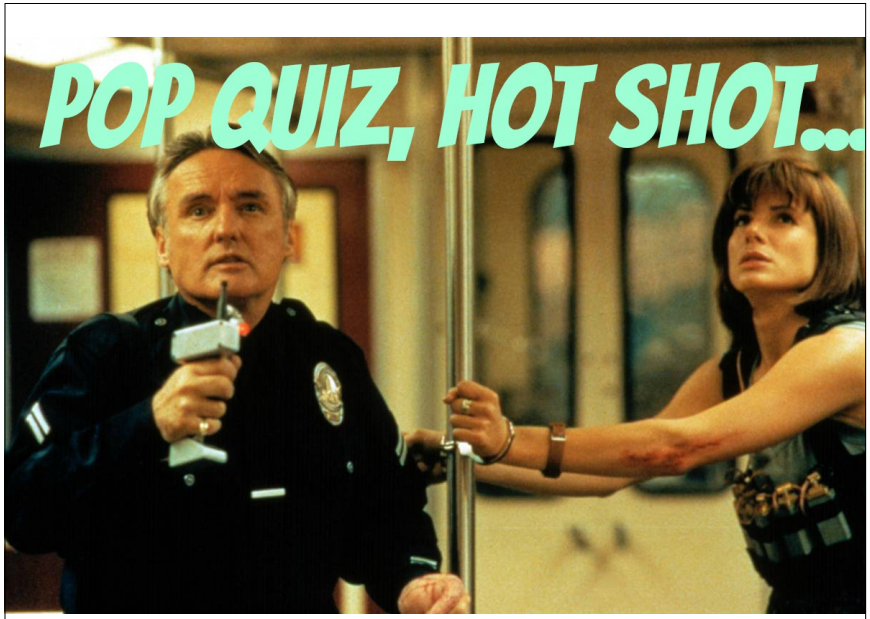
159-2



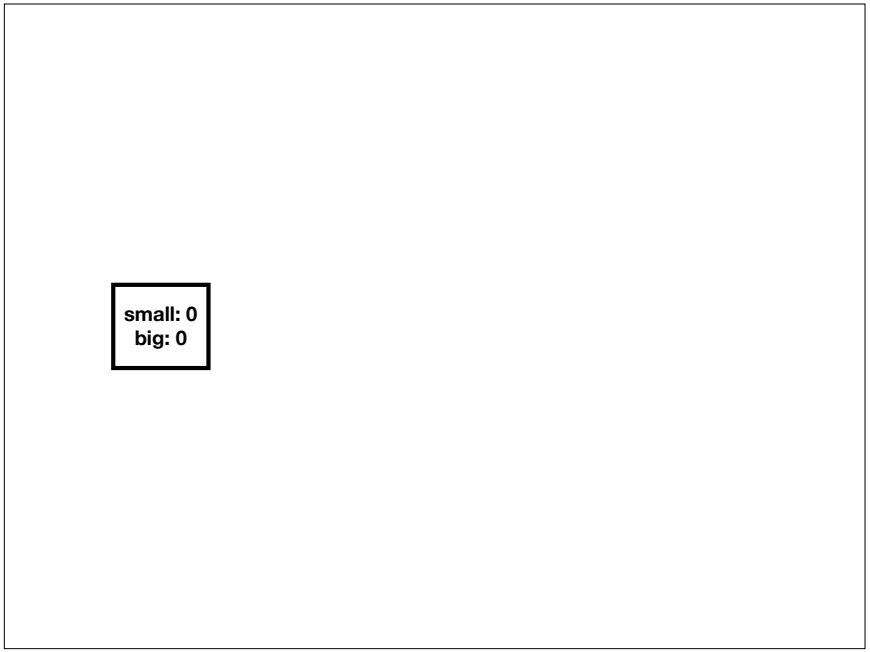
159-3

```
EXTENDS Integers
VARIABLES small, big

TypeOK == /\ small \in 0..3
          /\ big   \in 0..5

Init == /\ big   = 0
        /\ small = 0

FillSmall == /\ small' = 3
             /\ big'   = big

FillBig == /\ big'   = 5
           /\ small' = small

EmptySmall == /\ small' = 0
              /\ big'   = big

EmptyBig == /\ big'   = 0
            /\ small' = small

SmallToBig == IF big + small =< 5
              THEN /\ big'   = big + small
                   /\ small' = 0
              ELSE /\ big'   = 5
                   /\ small' = small - (5 - big)

BigToSmall == IF big + small =< 3
              THEN /\ big'   = 0
                   /\ small' = big + small
              ELSE /\ big'   = small - (3 - big)
                   /\ small' = 3

Next == \/ FillSmall
        \/ FillBig
        \/ EmptySmall
        \/ EmptyBig
        \/ SmallToBig
        \/ BigToSmall
```

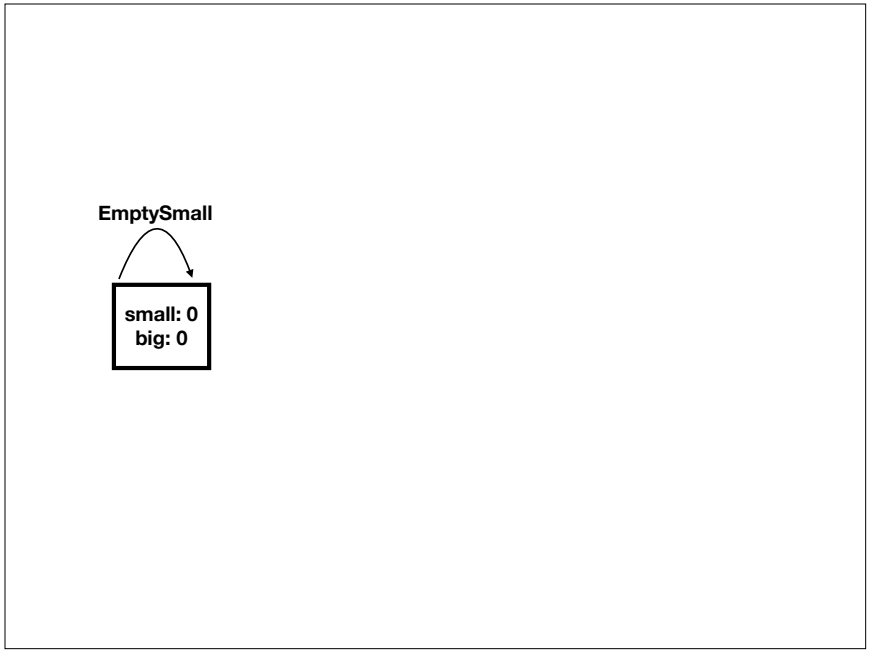
160



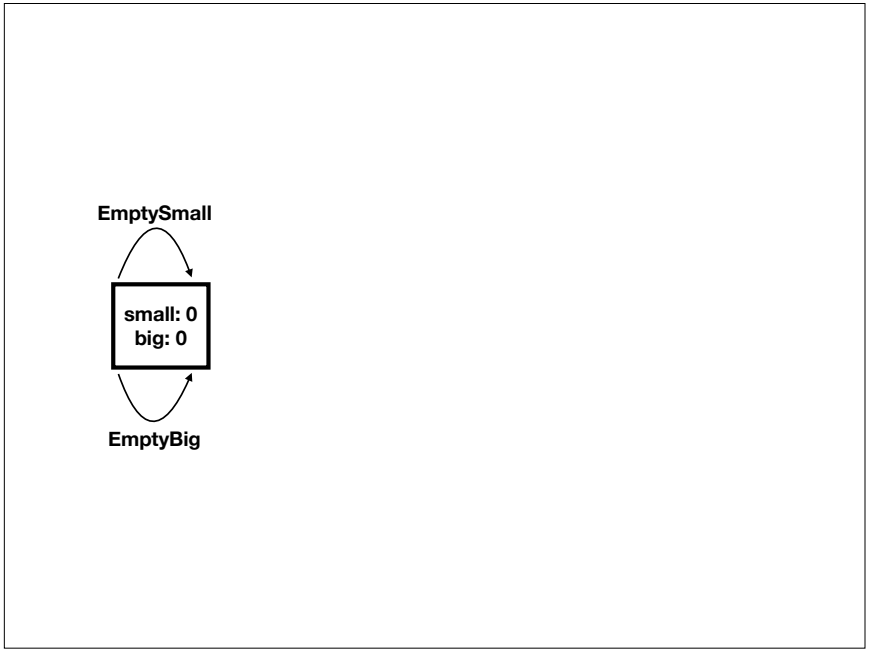
161



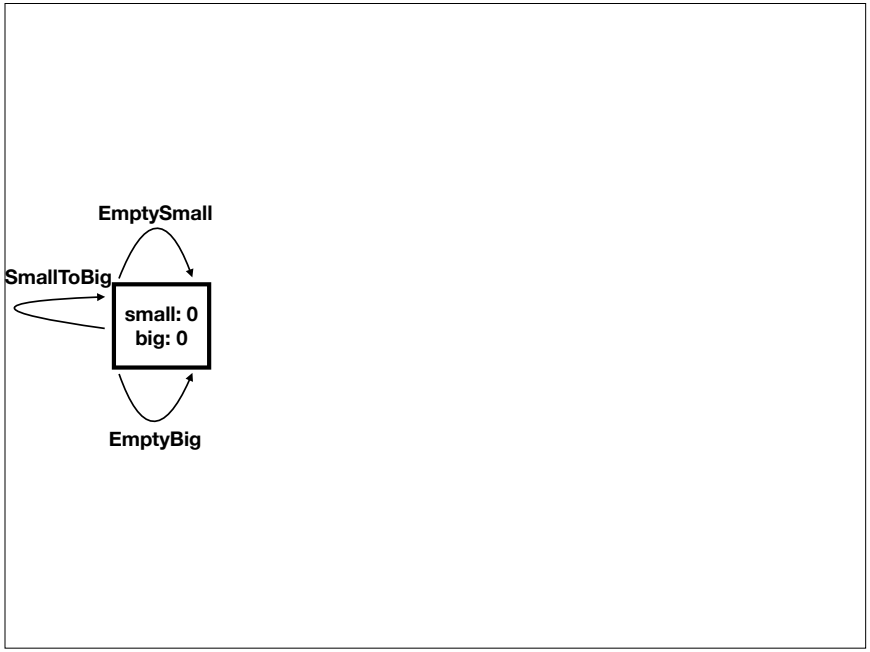
162-1



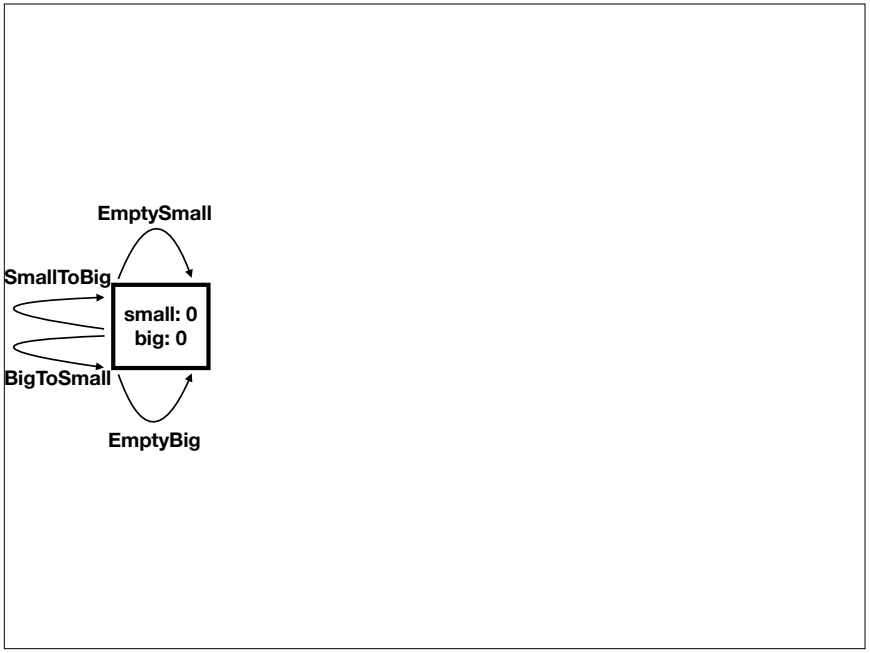
162-2



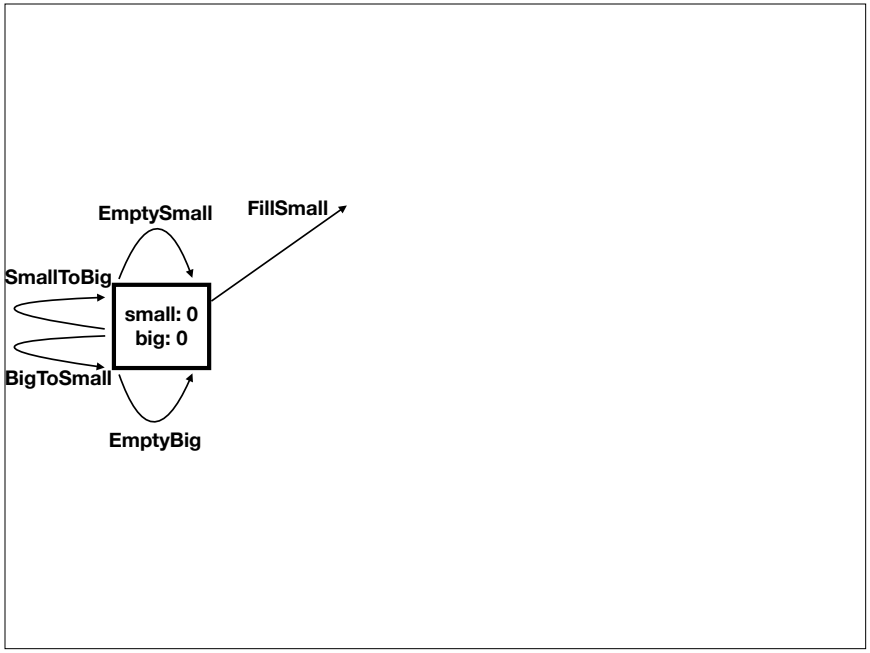
162-3



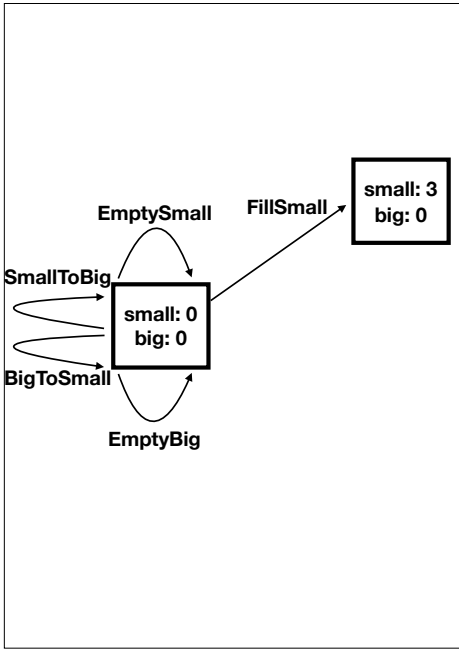
162-4



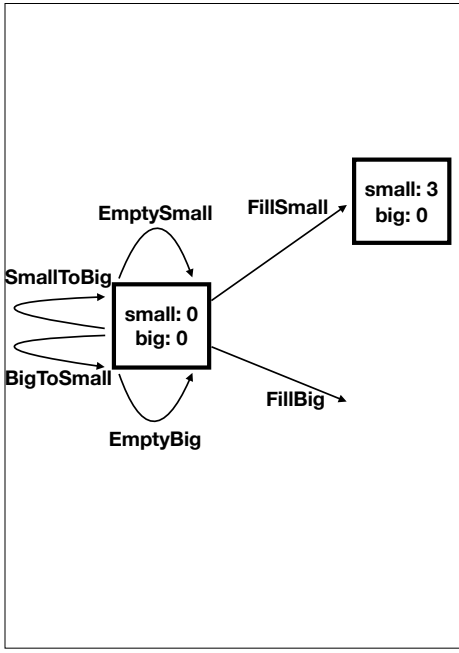
162-5



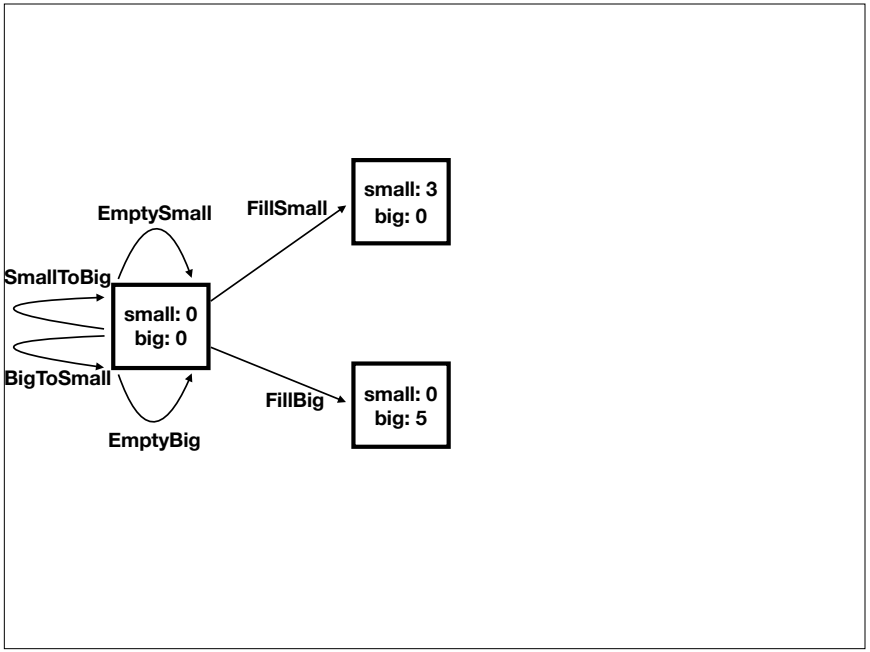
162-6



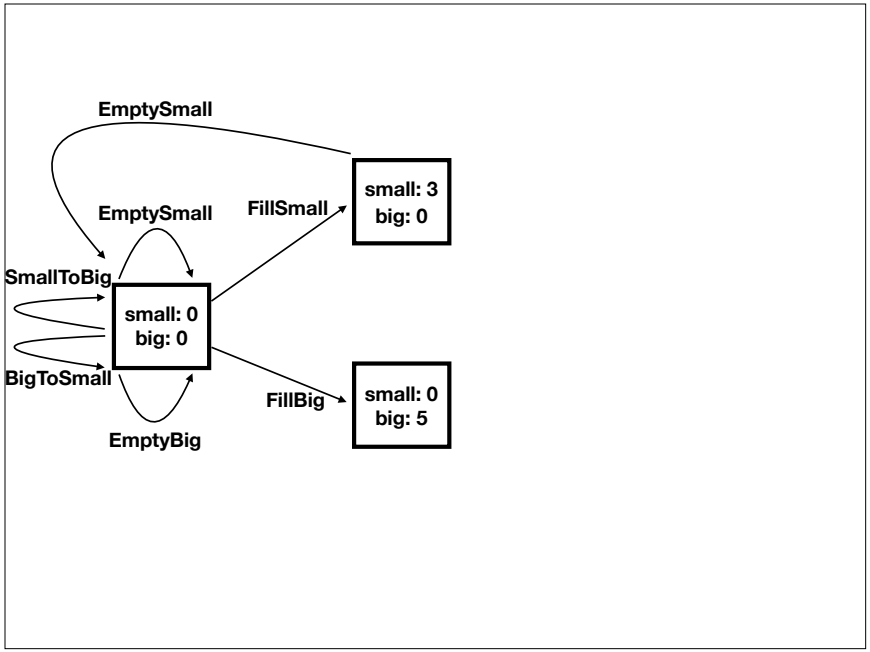
162-7



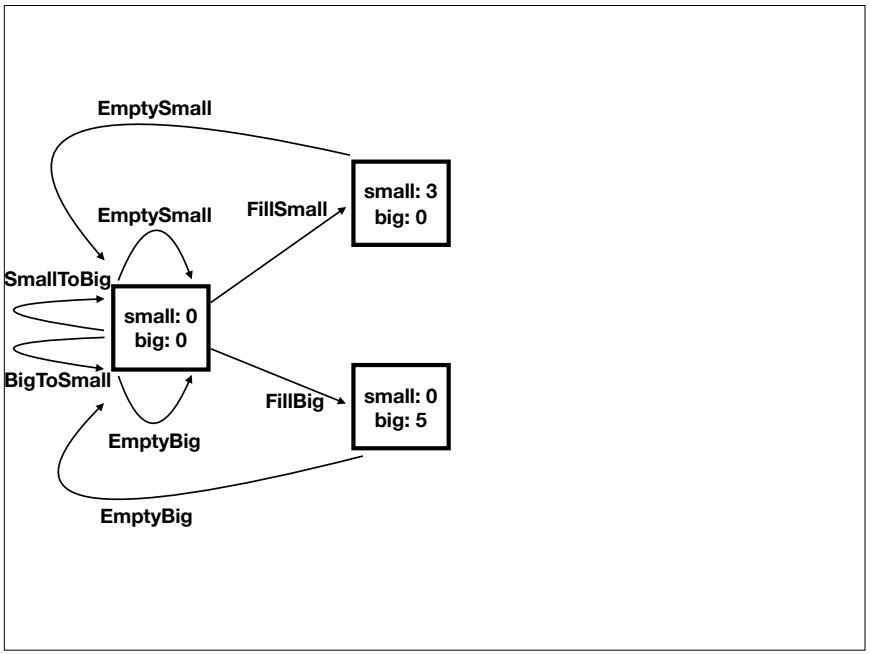
162-8



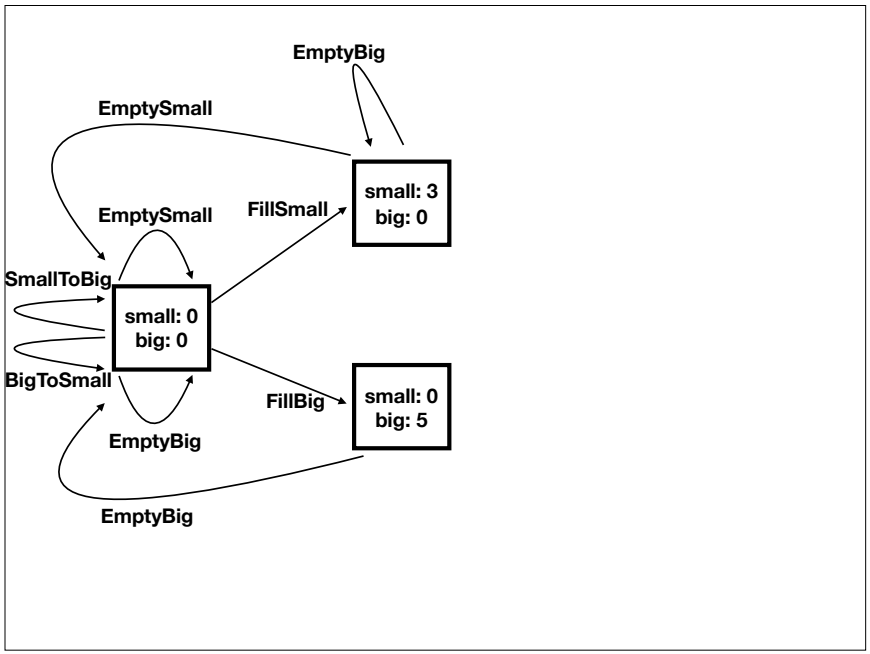
162-9



162-10



162-11



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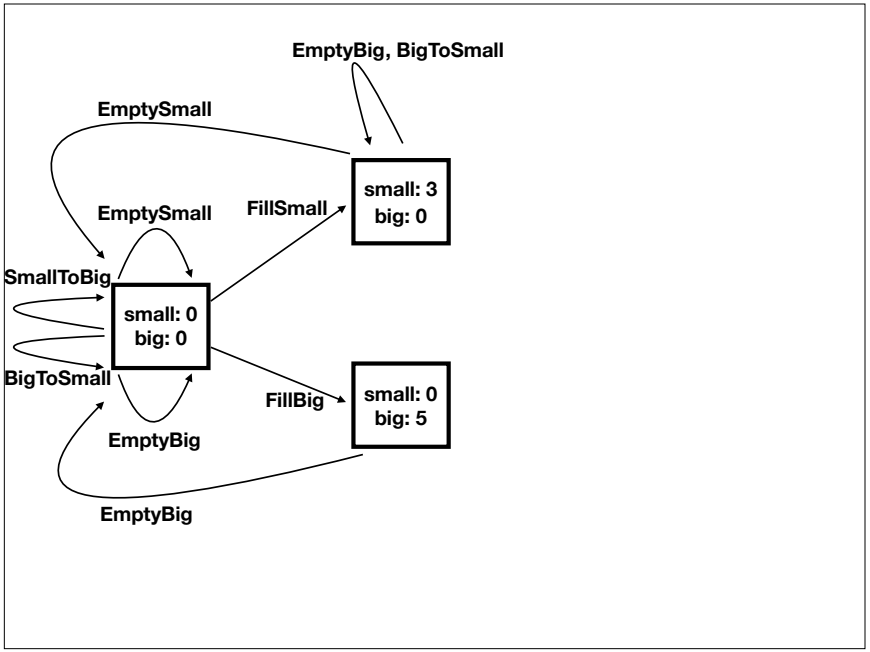

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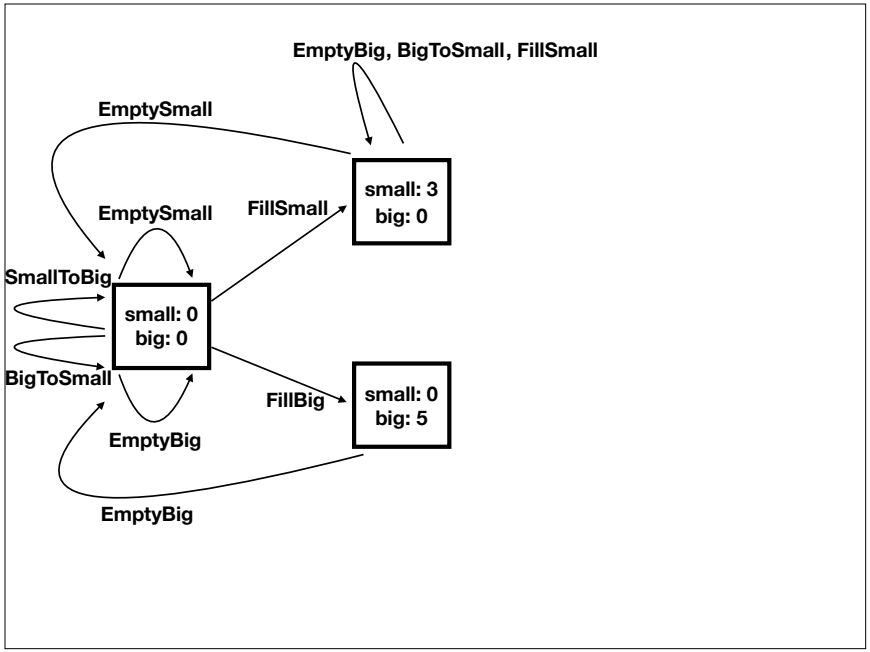

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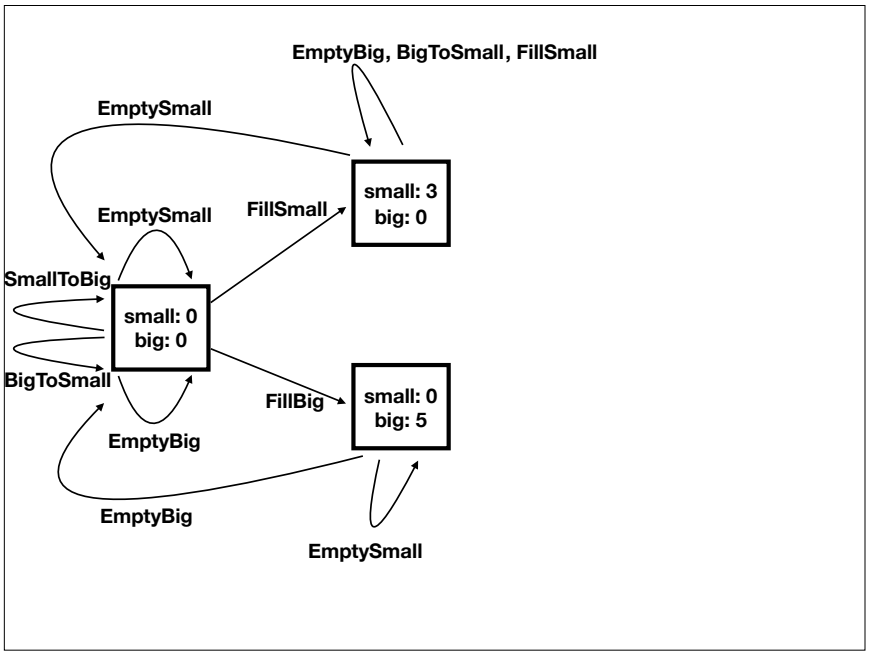

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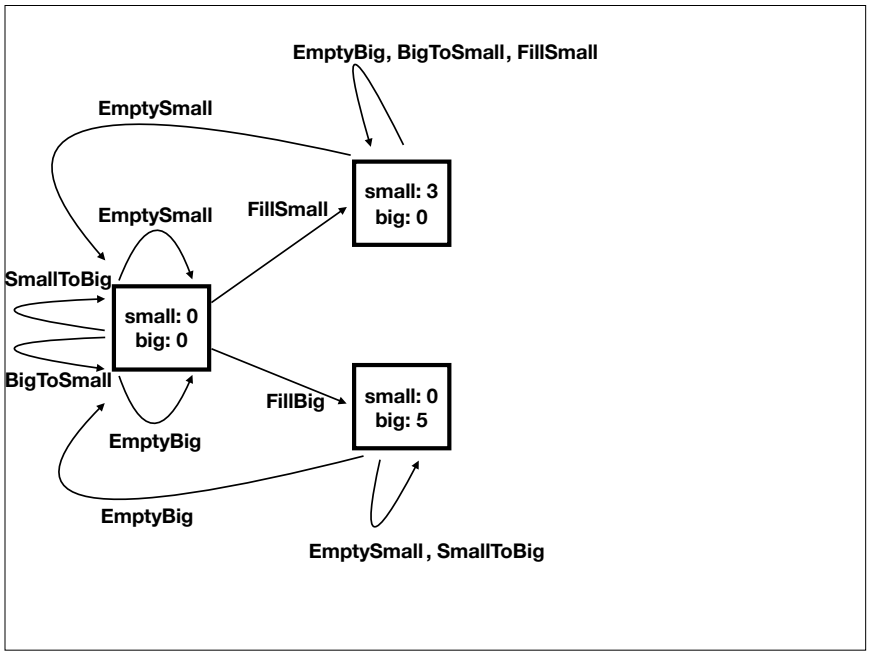

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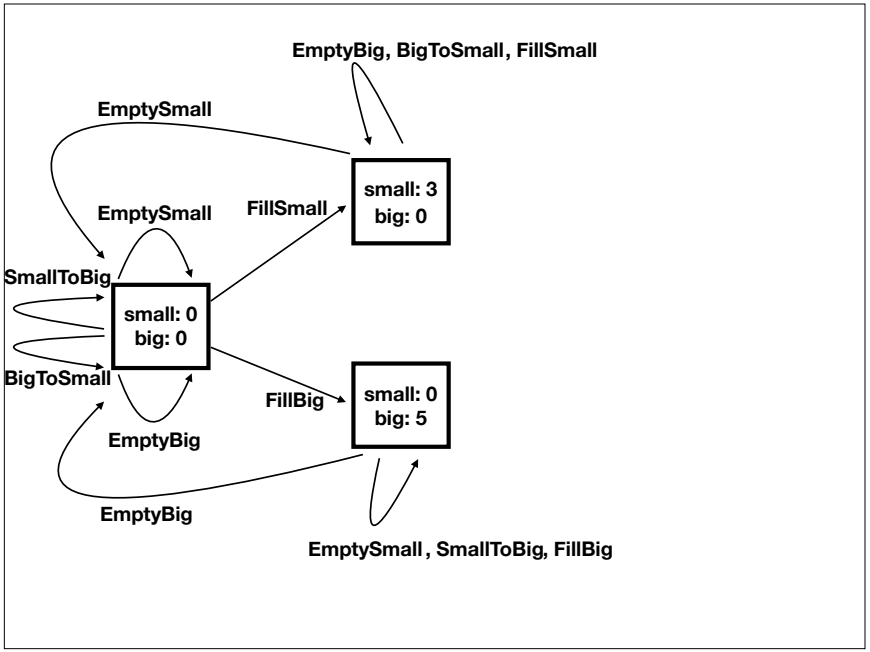

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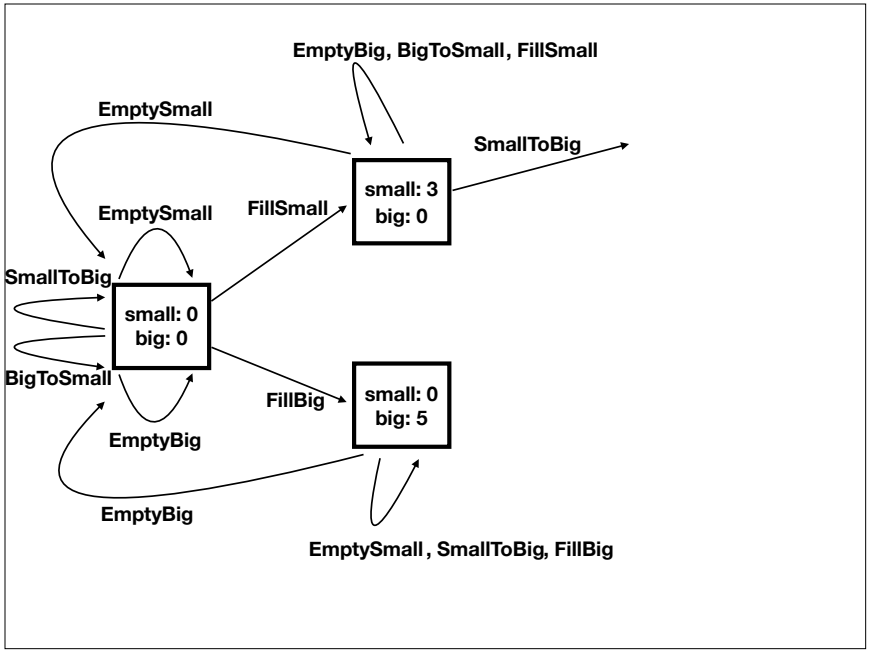

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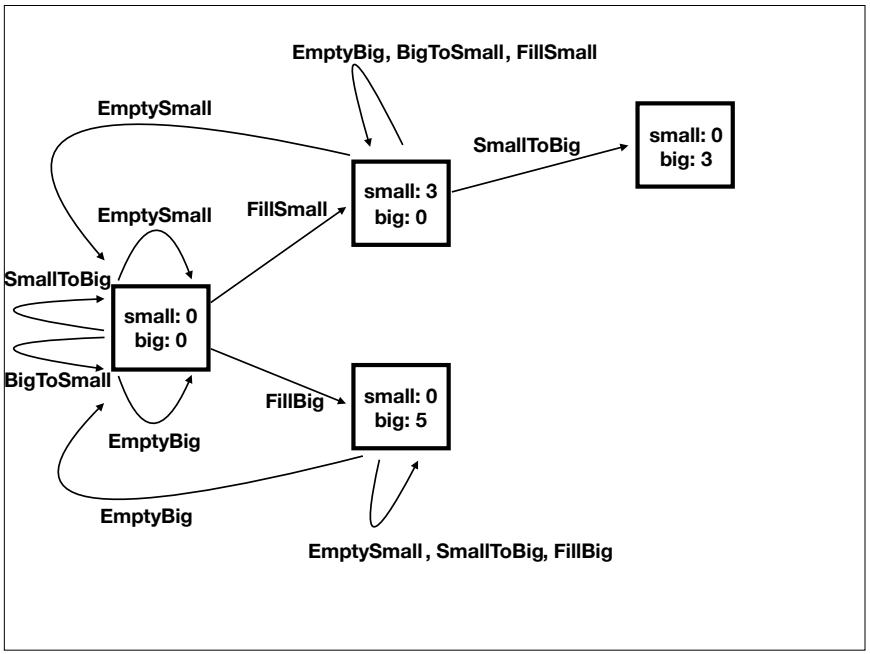

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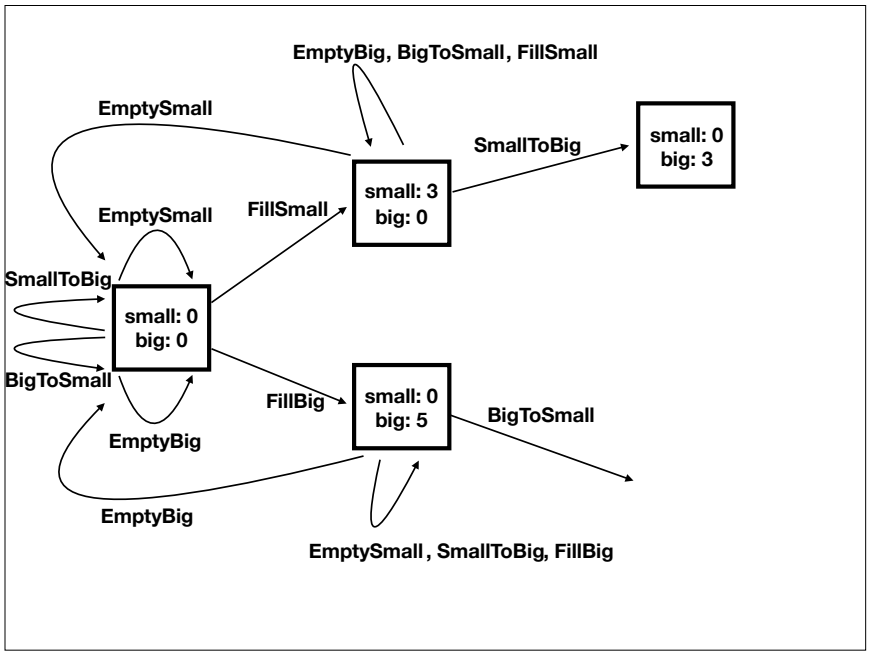

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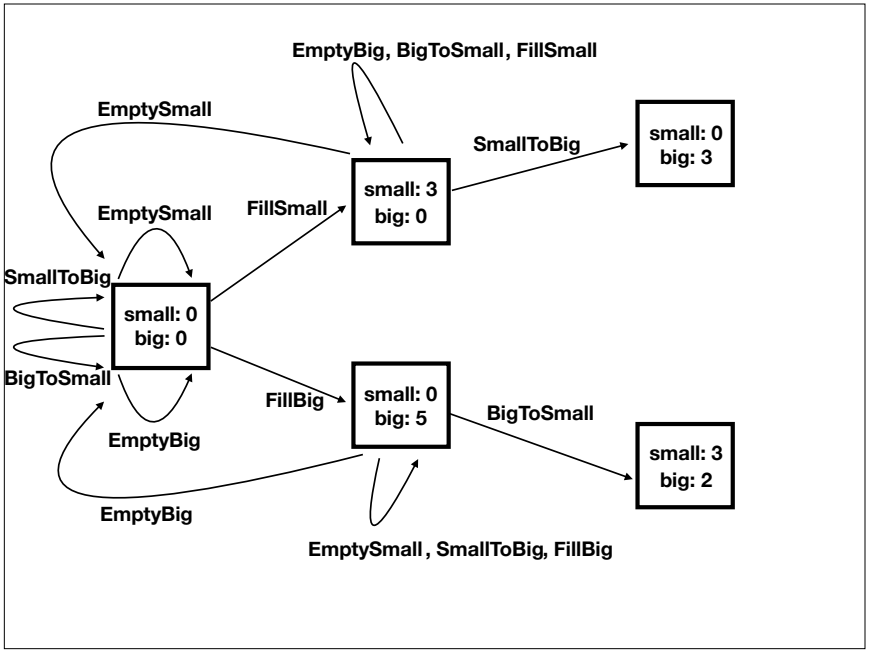

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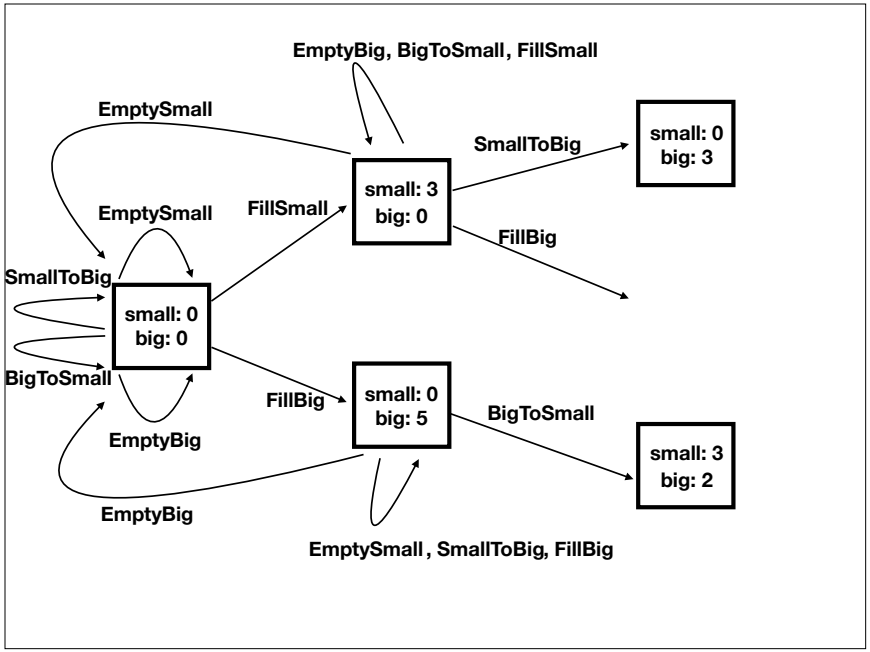

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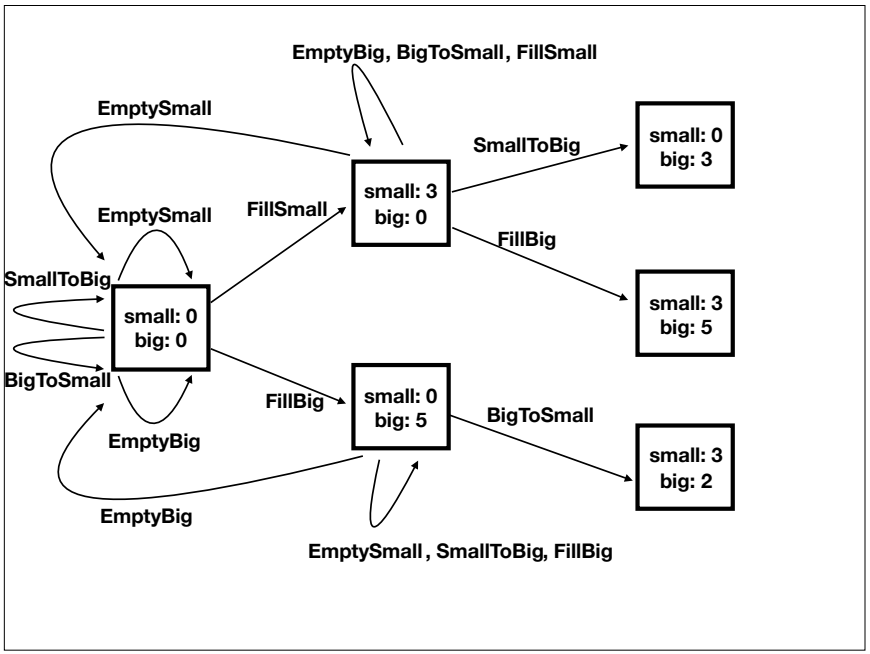

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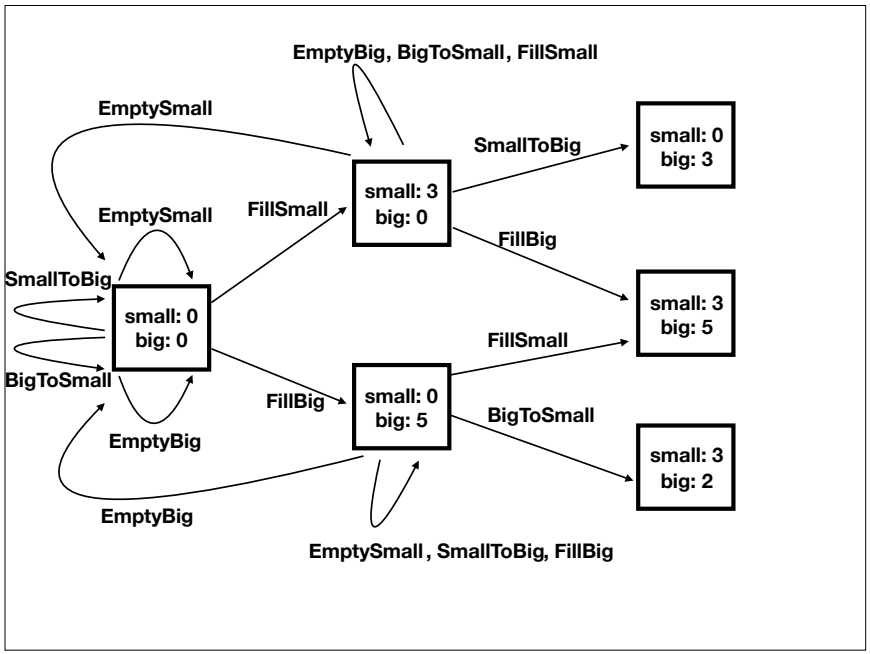

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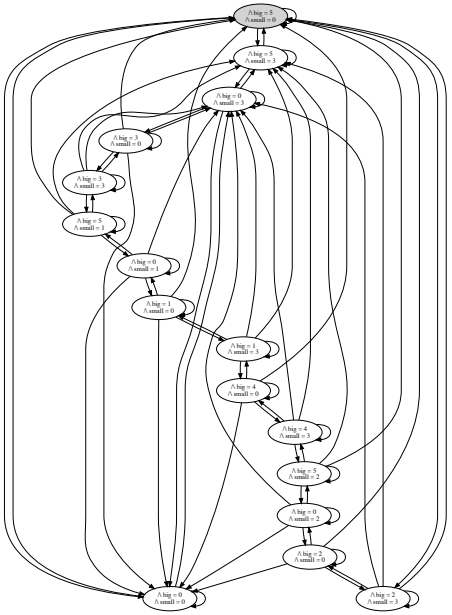

163

Die Hard

164-1

Die Hard

- Given the restrictions determined by our formula, TLC has found a total of 16 **unique** states.

164-2

Die Hard

- Given the restrictions determined by our formula, TLC has found a total of 16 **unique** states.
- Remember that a state is a particular assignment of **values** to our **variables**. Thus, 16 unique and valid combinations of values assigned to **big** and **small**.

164-3

Die Hard

- Given the restrictions determined by our formula, TLC has found a total of 16 **unique** states.
- Remember that a state is a particular assignment of **values** to our **variables**. Thus, 16 unique and valid combinations of values assigned to **big** and **small**.
- …. and so what? What does this get us? Who cares?

Invariants

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- It's time to explore the true power of TLA+ and TLC

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- An invariant is "**something that should always be true**"

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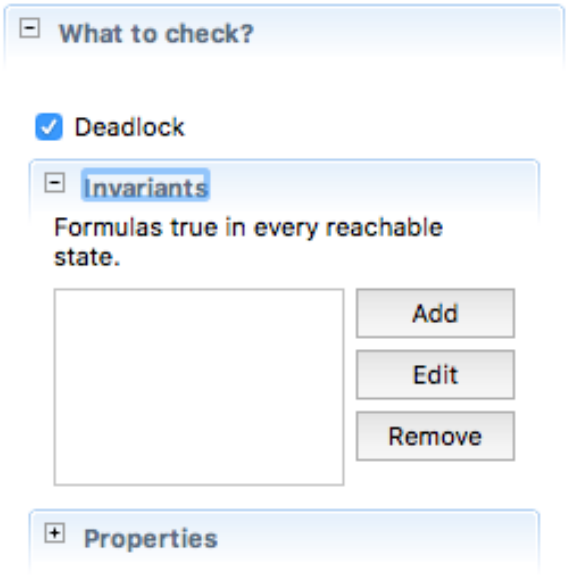
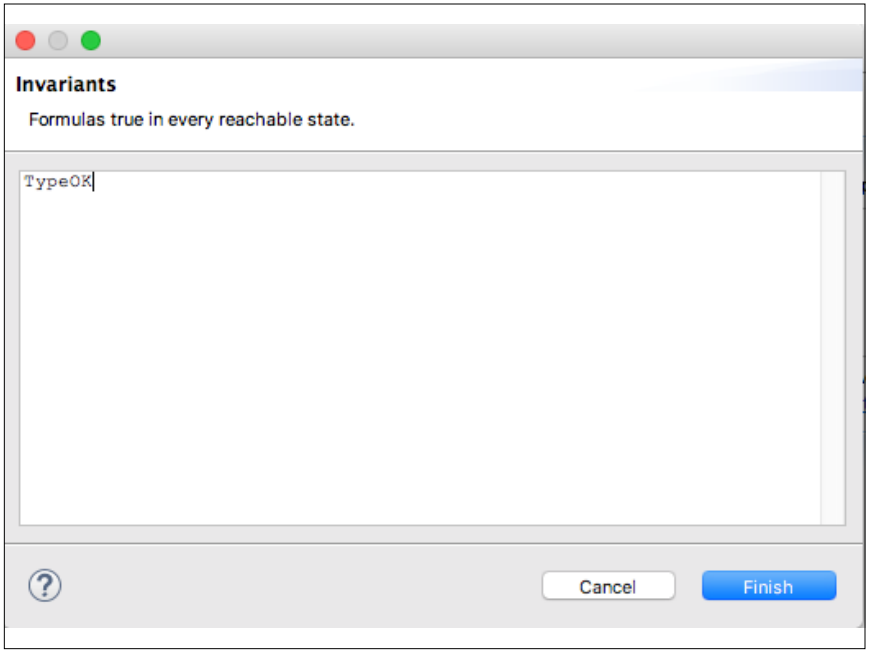
Invariants

- It's time to explore the true power of TLA+ and TLC
- An invariant is "**something that should always be true**"
  - Or "**something that should never occur**"
- TLC excels at generating every possible state that can occur given your formula
- You can also tell it to check that a certain thing is **always** true (or **never** true) in any of those states, i.e. check invariants





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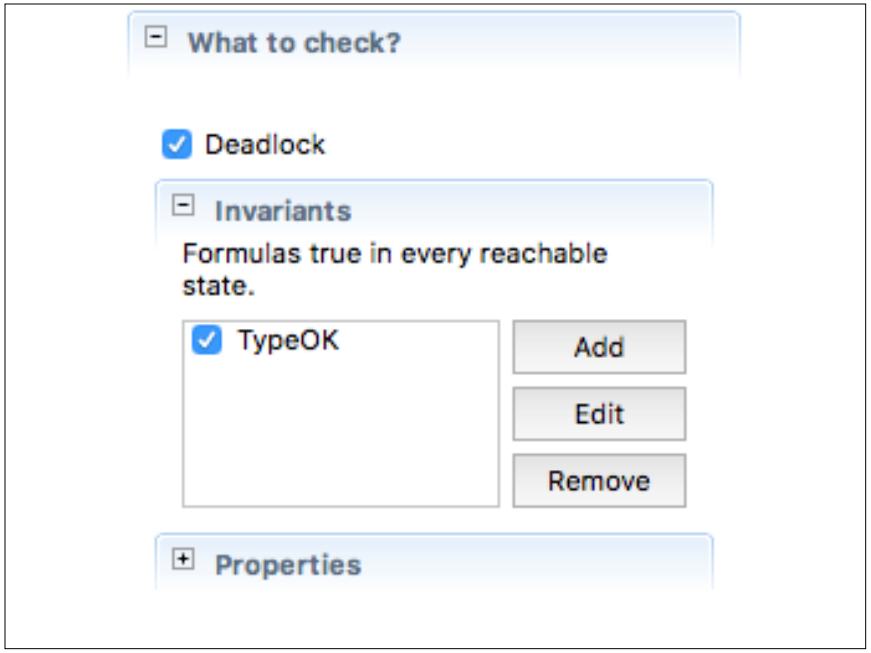

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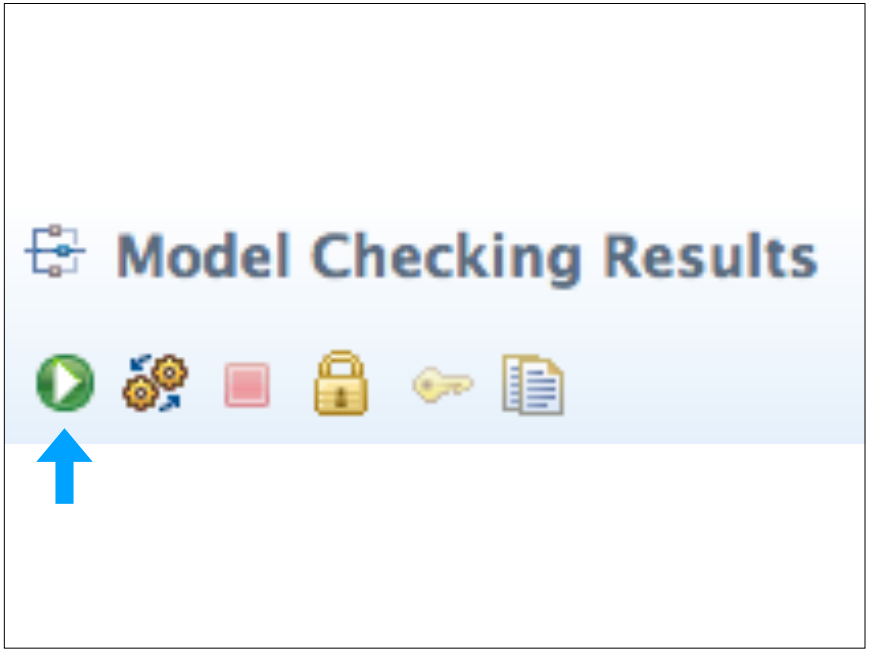

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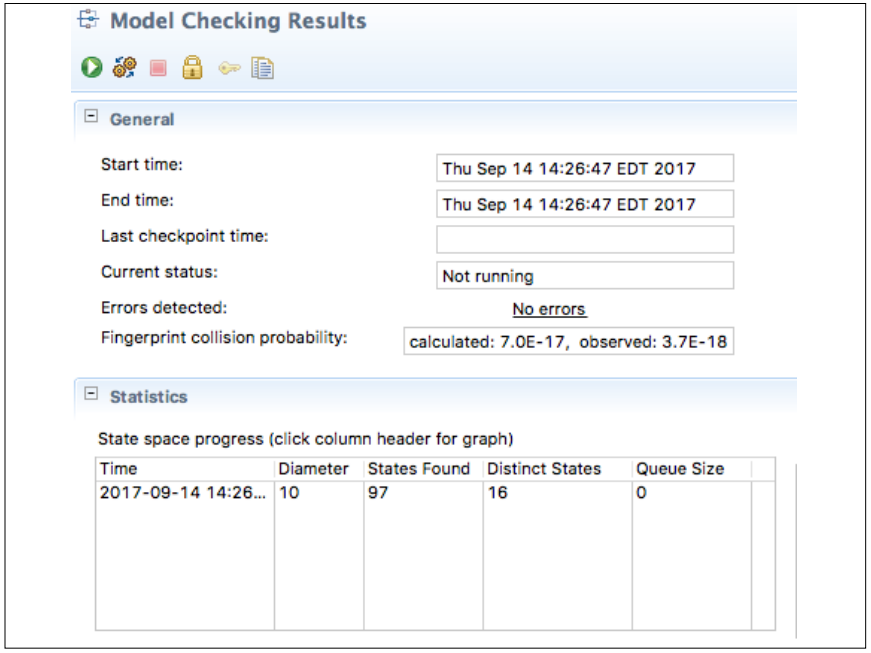
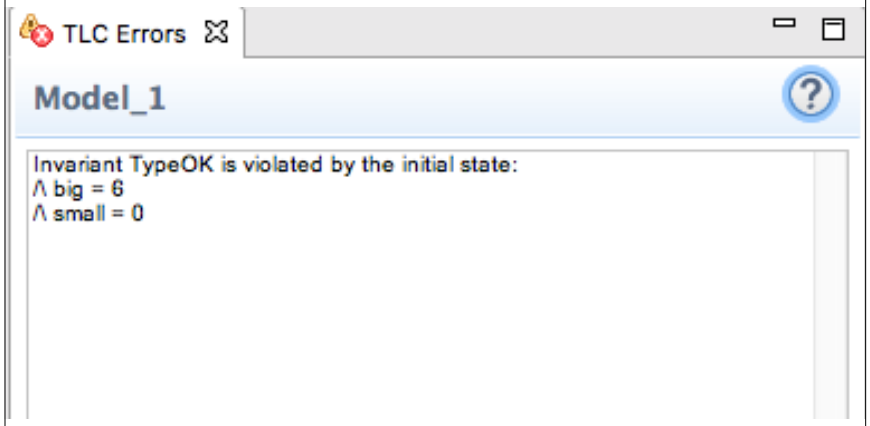


170

```
Init == /\ big   = 6
        /\ small = 0
```

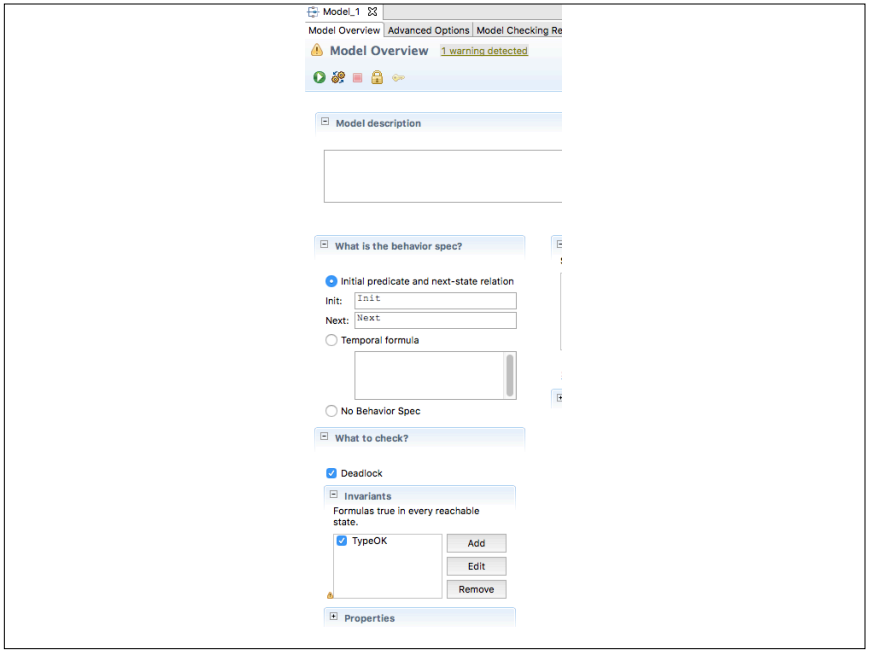



TypeOK

```
TypeOK == /\ small \in 0..3
          /\ big   \in 0..5
```



```
Init == /\ big   = 5
        /\ small = 0
```



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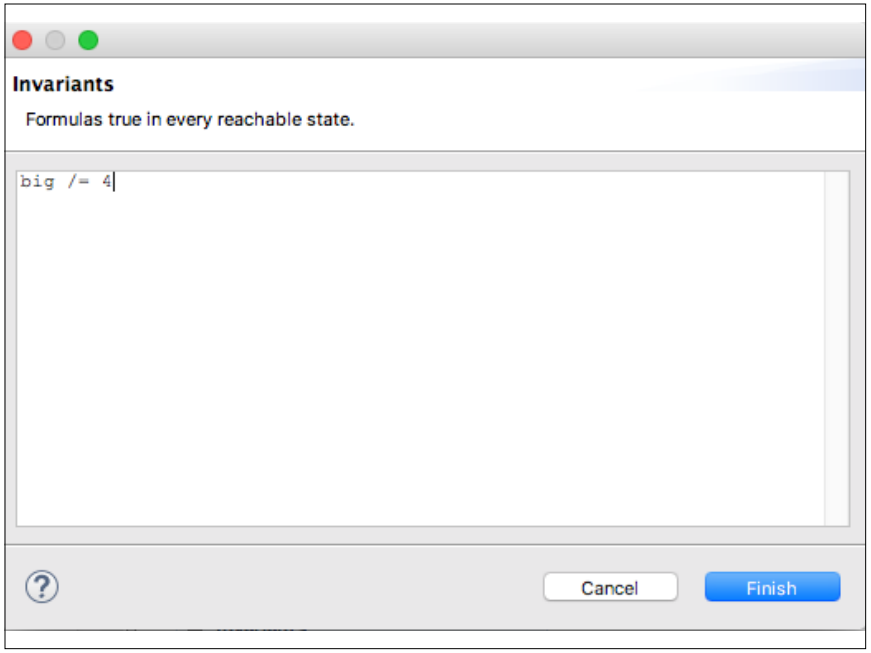

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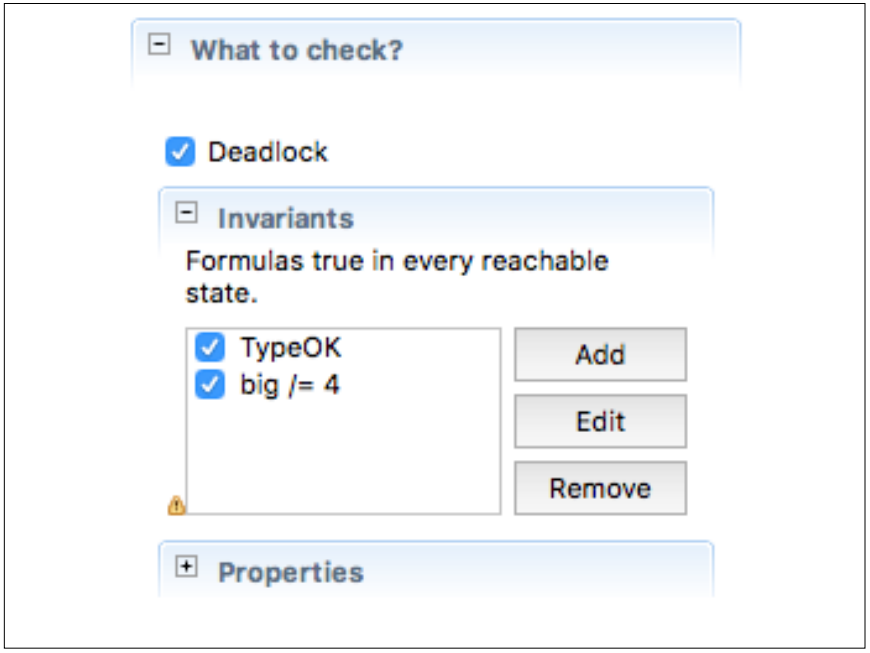

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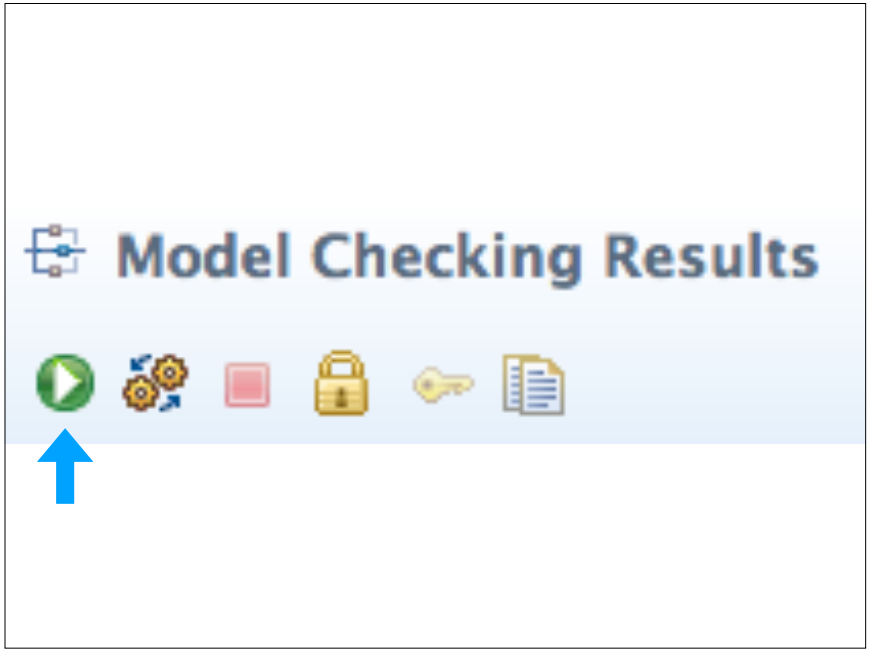

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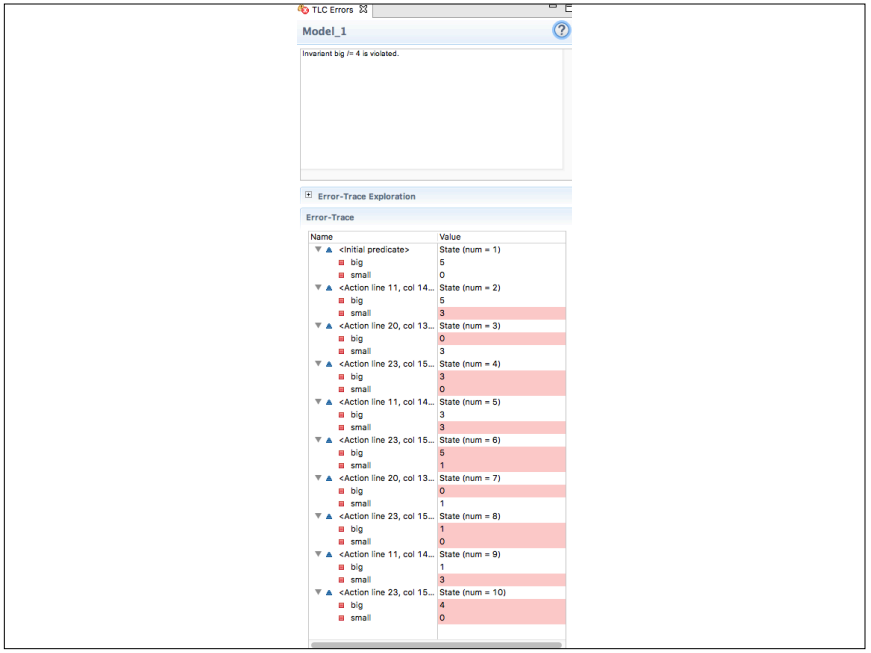
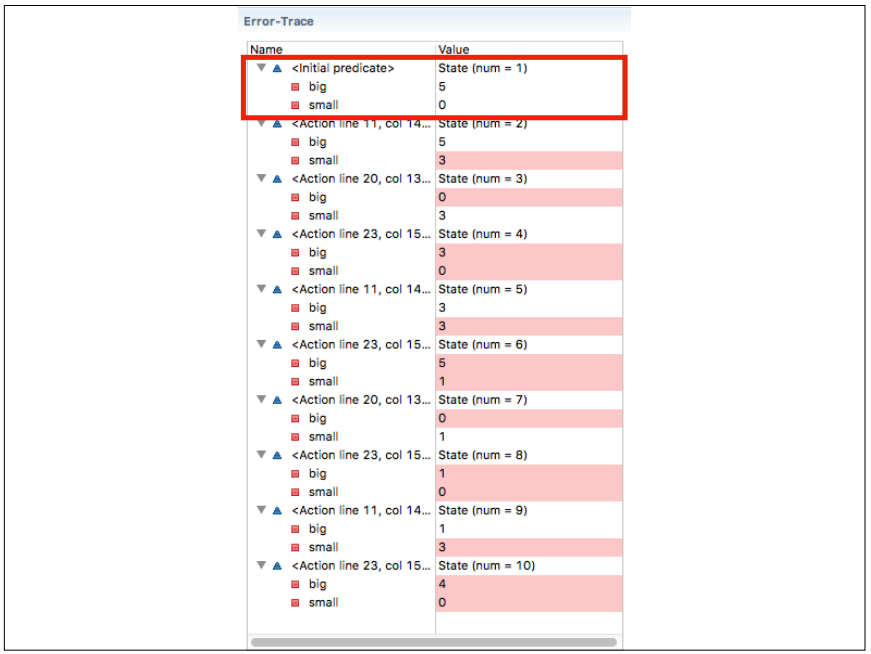


179

180-1



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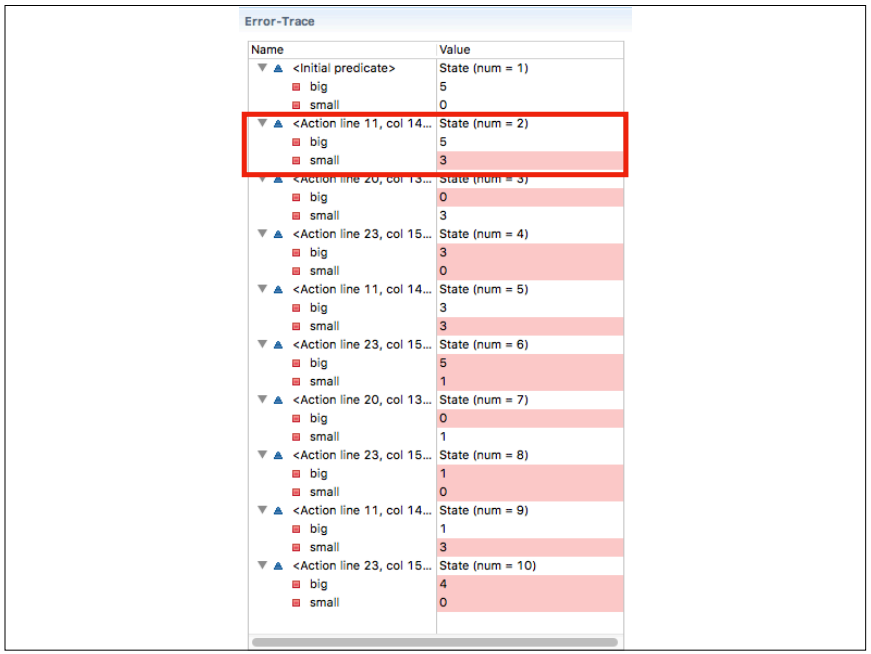

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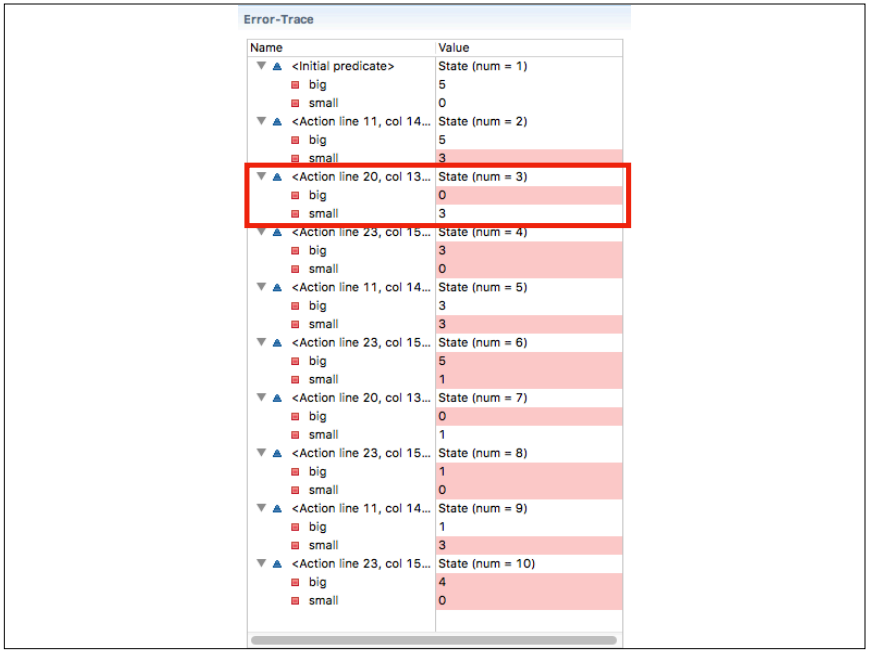

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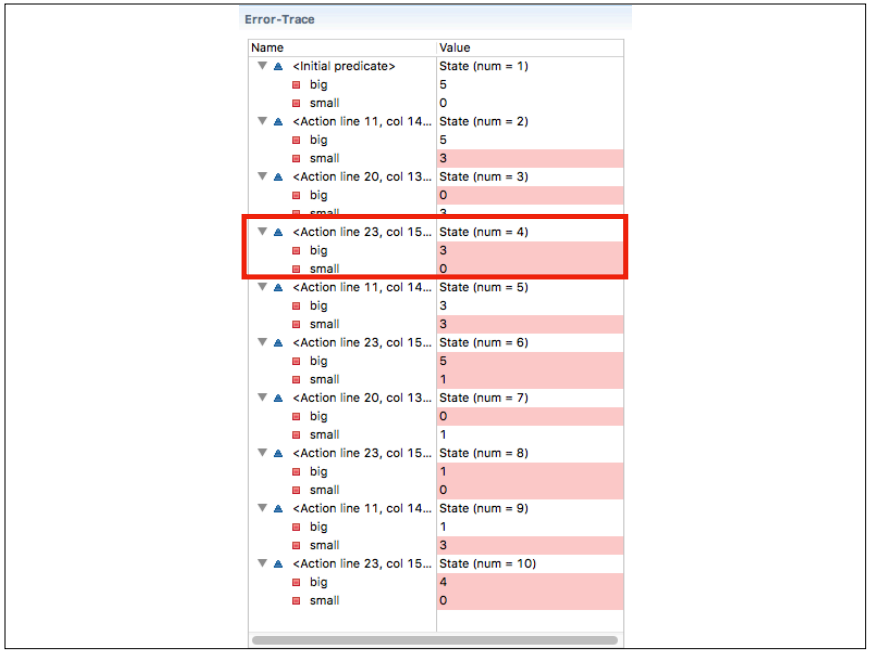

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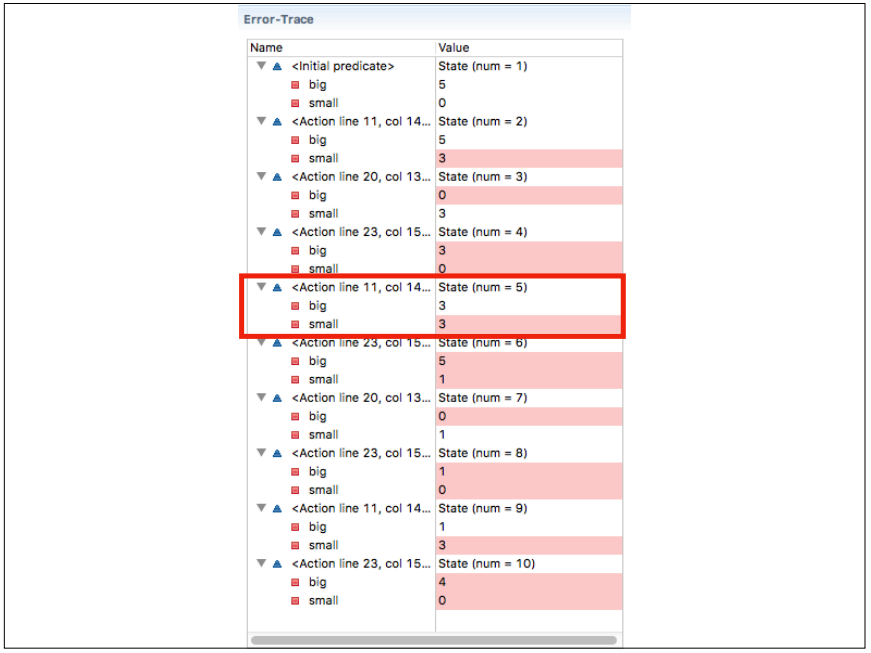

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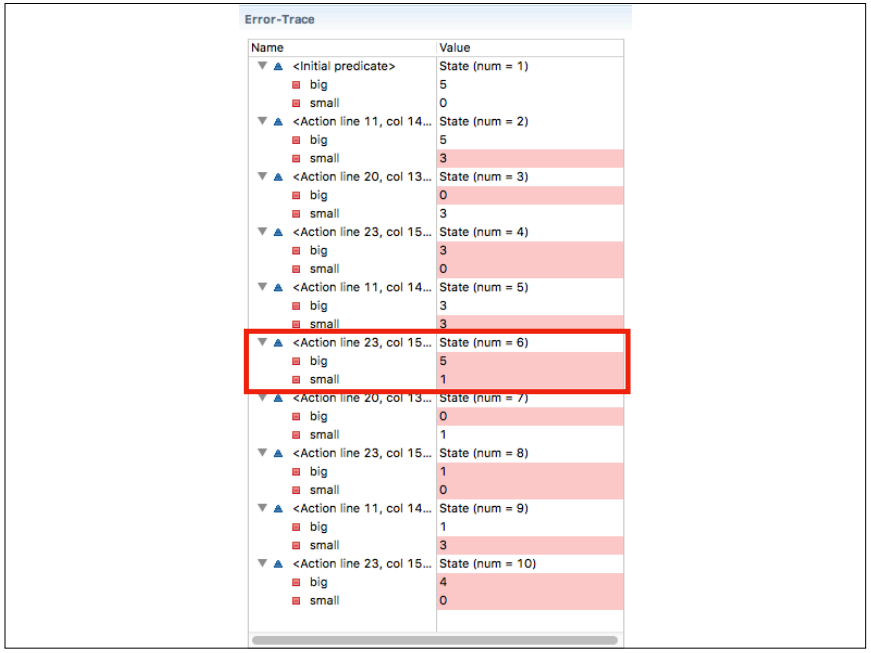

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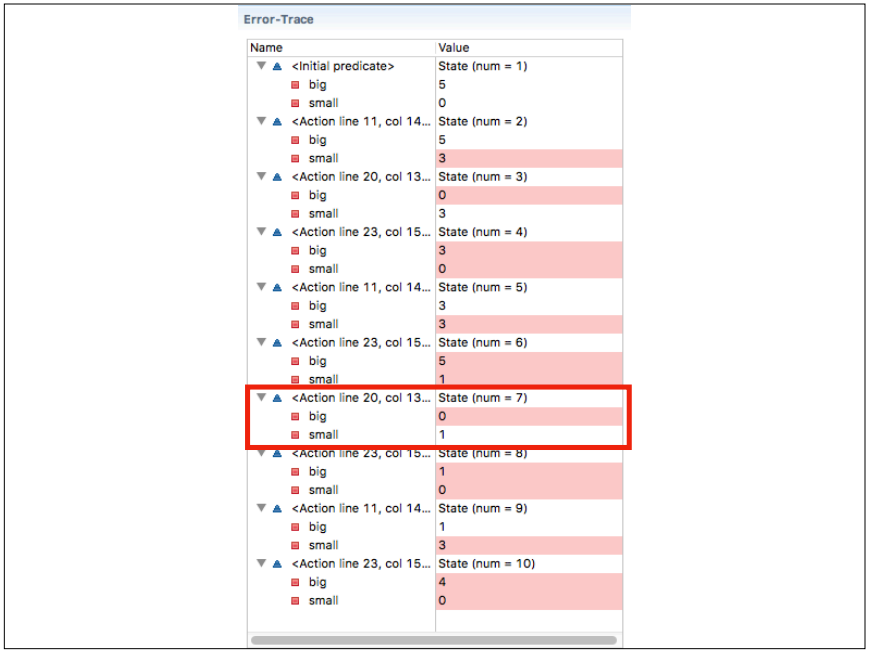

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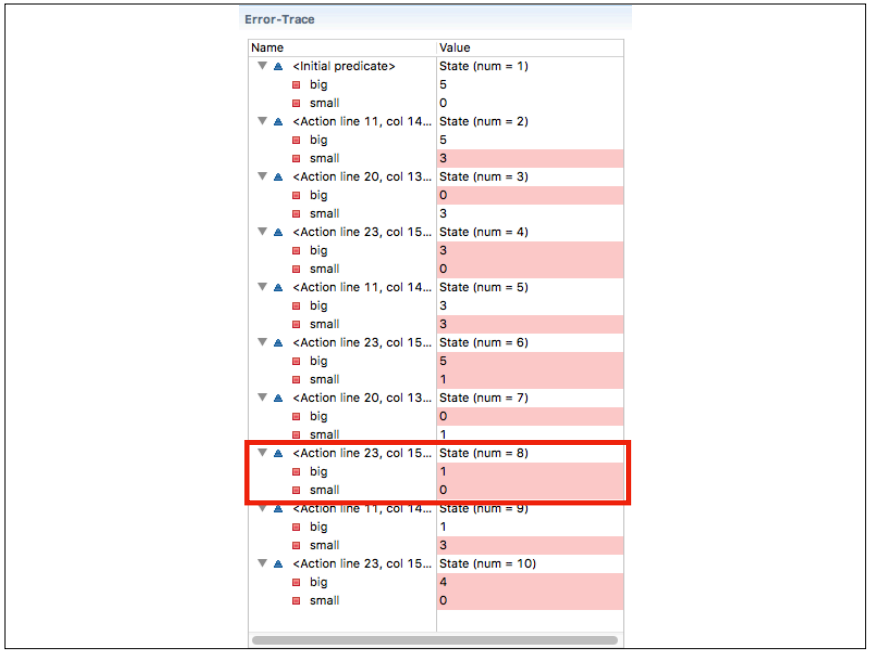

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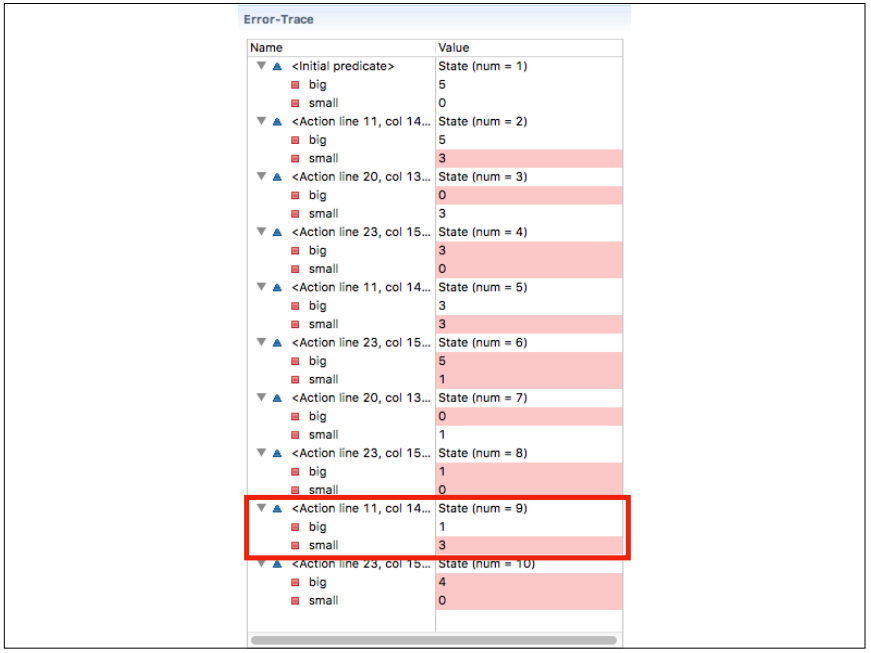

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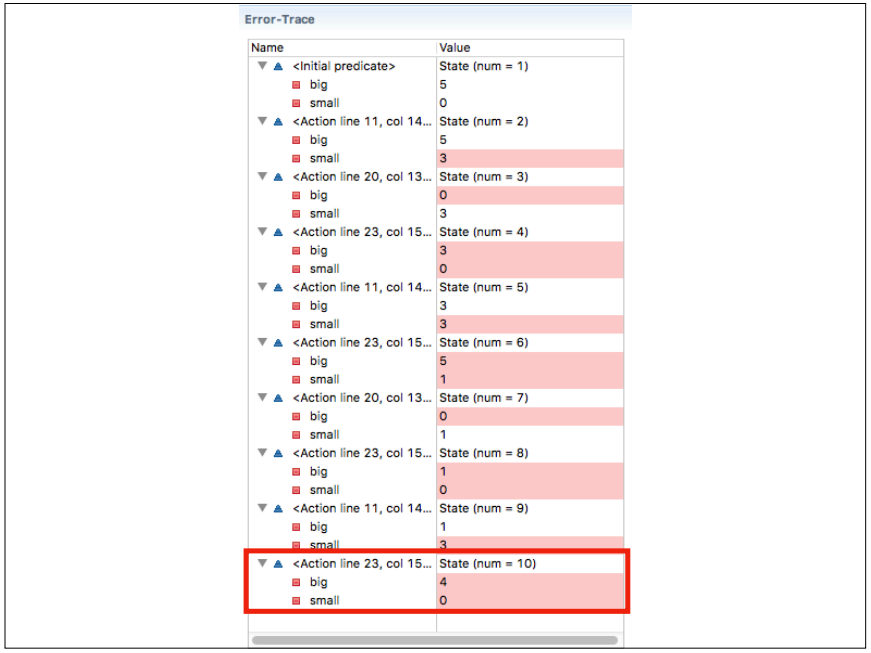

180-11

Woah

Woah

- This is amazingly powerful.

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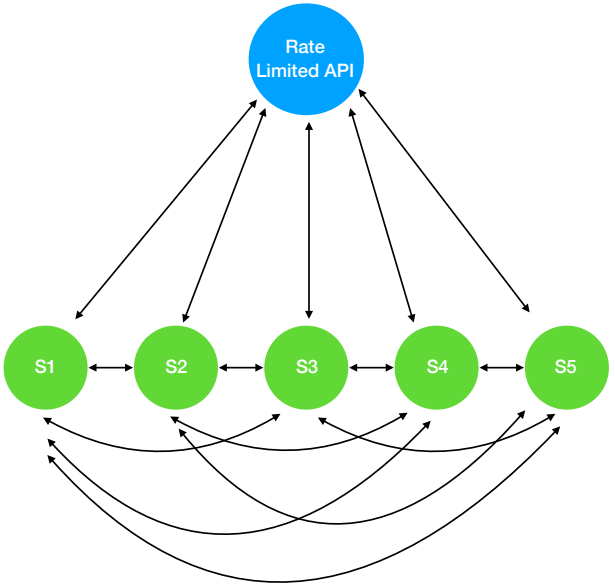
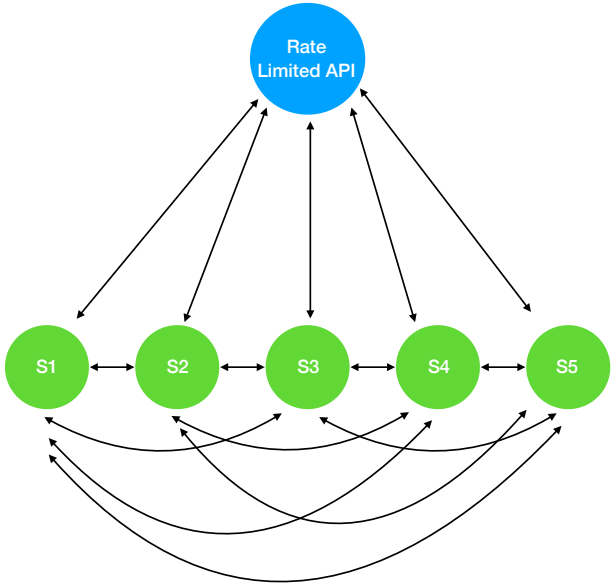
Woah

- This is amazingly powerful.
- TLC tries every single possible combination of states (i.e. behaviour/execution), and checks invariants for every state in every behaviour.
- The entire state history is kept, providing the best “stack trace” you’ve ever seen.
- For concurrent and distributed systems, this allows for detection of incredibly subtle interactions, race conditions, etc.





https://learntla.com/concurrency/example/

AWS Fault-Tolerant Replication Algorithm

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- Engineer took an existing designed and implemented algorithm at AWS, which had been analyzed for months with dozens of pages of informal proofs.

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- Wrote a TLA+ specification for the algorithm.

AWS Fault-Tolerant Replication Algorithm

- Engineer took an existing designed and implemented algorithm at AWS, which had been analyzed for months with dozens of pages of informal proofs.
- Wrote a TLA+ specification for the algorithm.
- TLC found three serious bugs, one of which had a 35+-step trace.

Transaction Commit

Transaction Commit

- Consider a replicated database.

Transaction Commit

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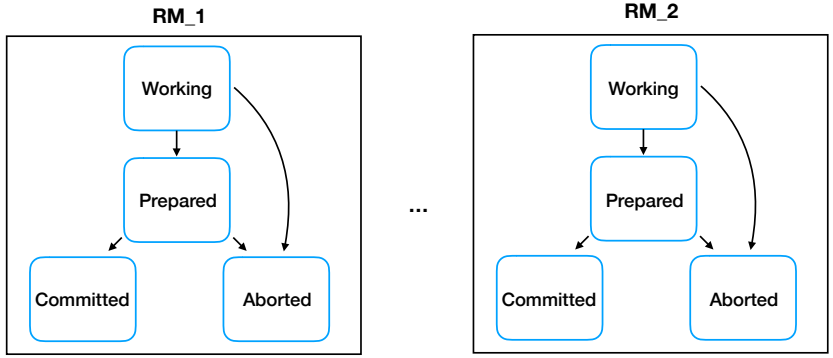
- Multiple “resource managers,” each carrying a full replica.

Transaction Commit

- Consider a replicated database.
- Multiple “resource managers,” each carrying a full replica.
- For any transaction sent to the database, all resource managers **must come to a consensus** on whether the transaction should be committed or aborted.

Resource Managers

Resource Managers



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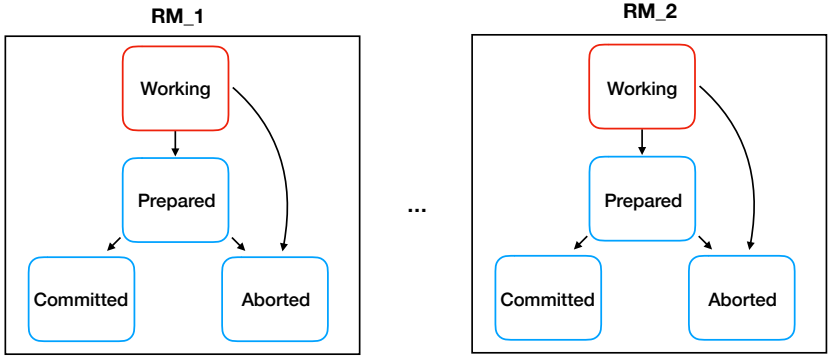
Resource Managers



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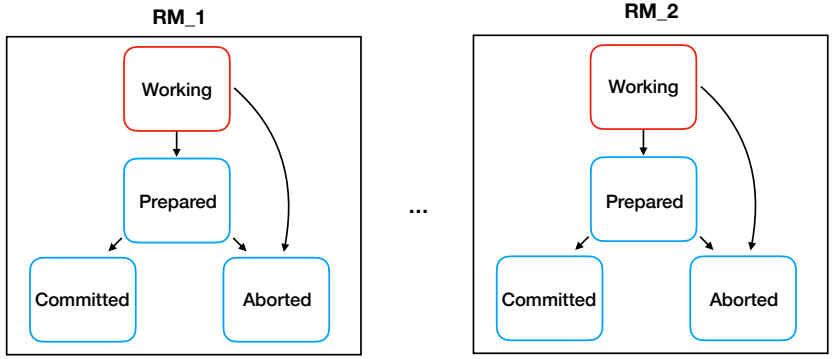
Resource Managers



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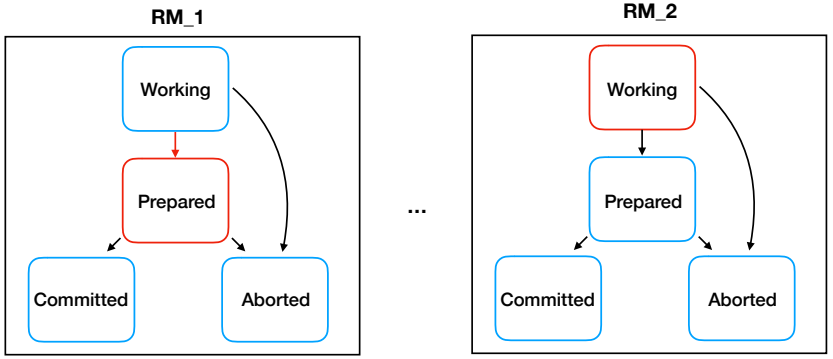
Resource Managers



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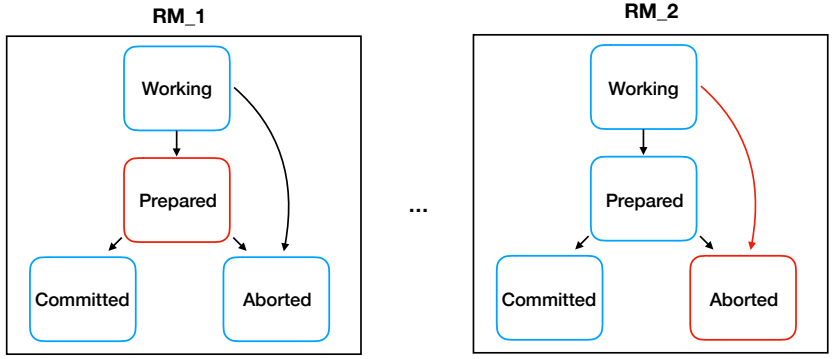
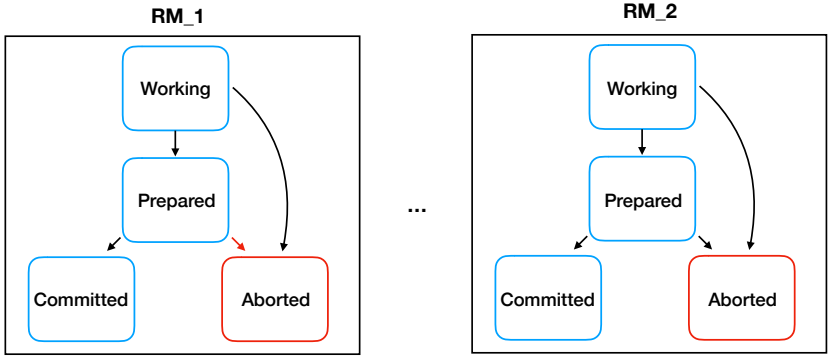
Resource Managers



Resource Managers



Resource Managers

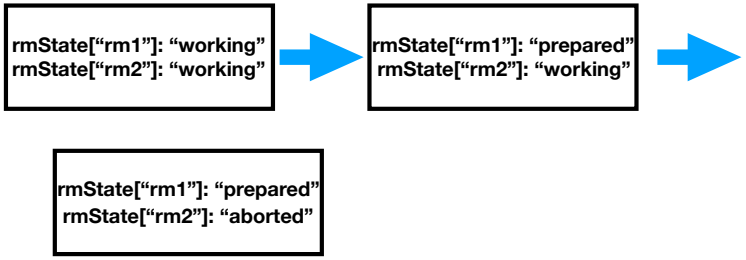
rmState["rm1"]: "working"
rmState["rm2"]: "working"

Resource Managers

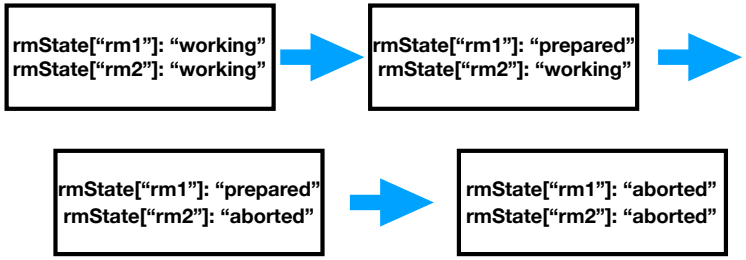
rmState["rm1"]: "working"
rmState["rm2"]: "working"

rmState["rm1"]: "prepared"
rmState["rm2"]: "working"

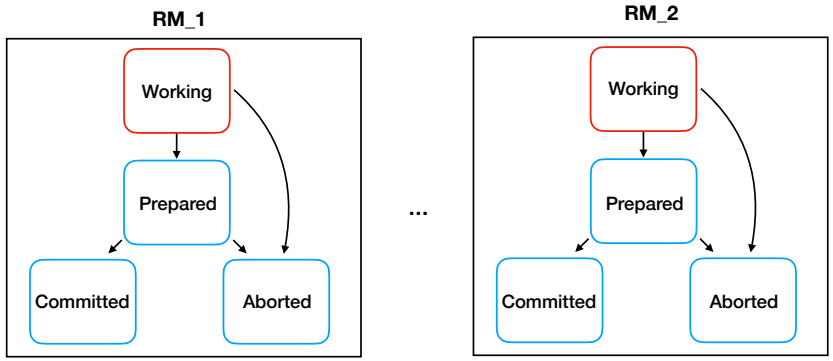
Resource Managers



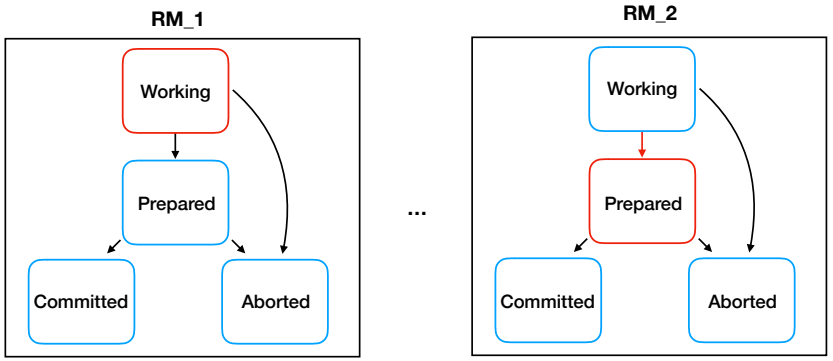
Resource Managers



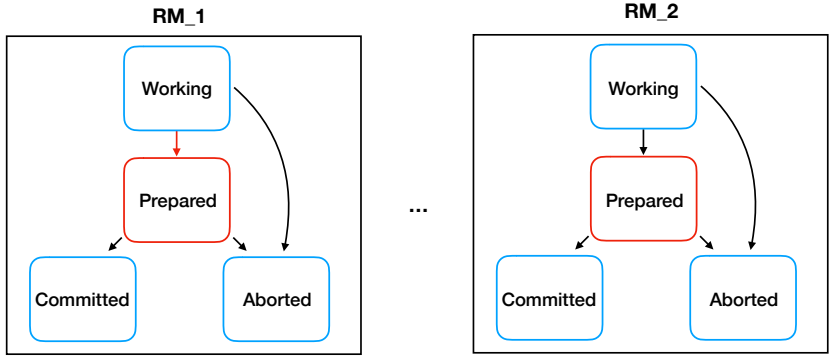
Resource Managers



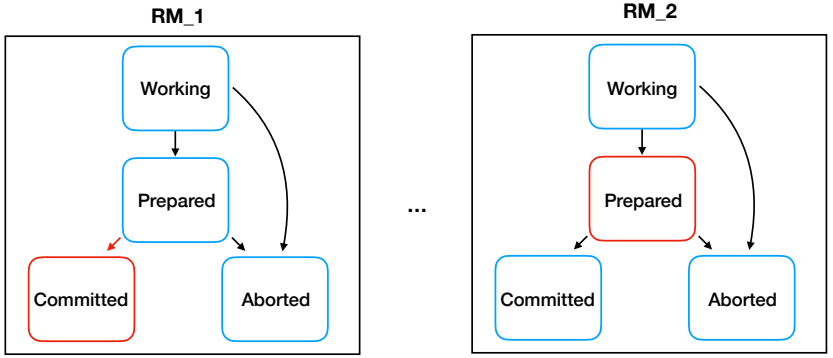
Resource Managers



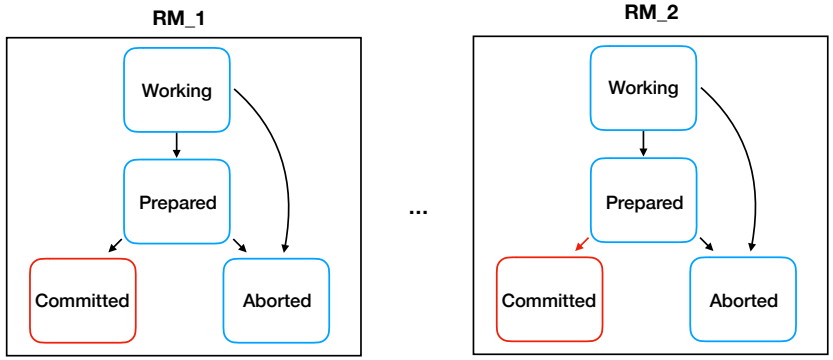
Resource Managers



Resource Managers



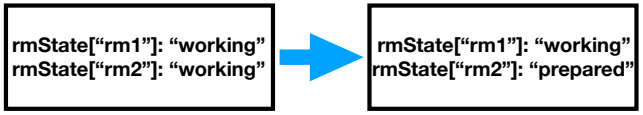
Resource Managers



Resource Managers

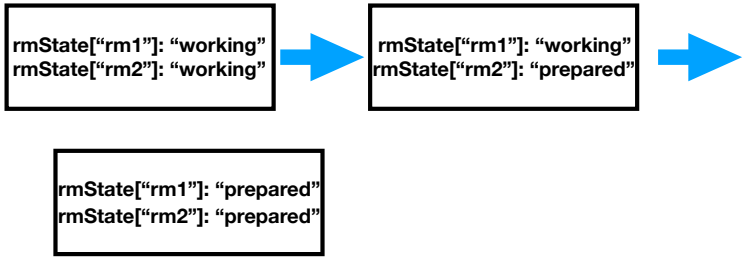
rmState[“rm1”]: “working”
rmState[“rm2”]: “working”

Resource Managers



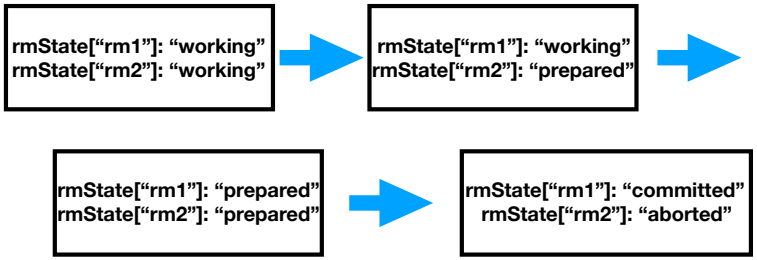
198-2

Resource Managers



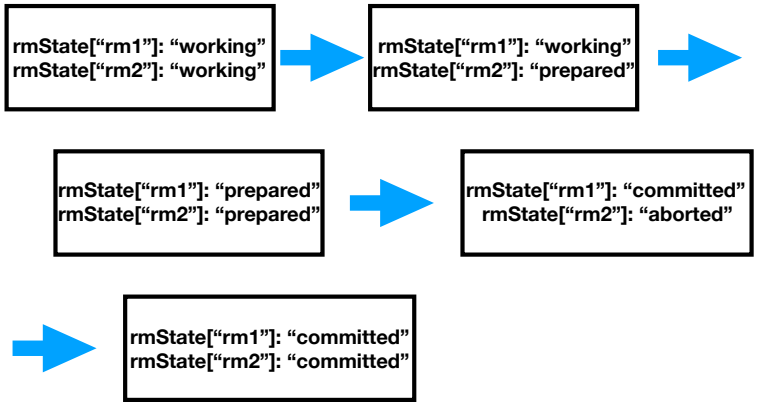
198-3

Resource Managers



198-4

Resource Managers



198-5

Resource Managers

Resource Managers

- We just looked at two possible behaviours.

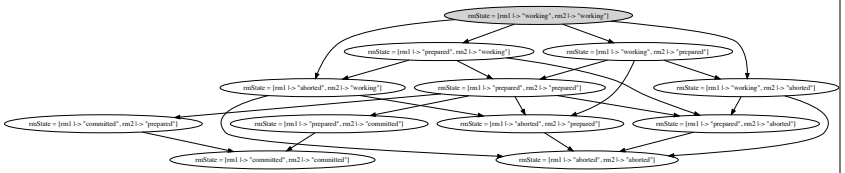
Resource Managers

- We just looked at two possible behaviours.
- How many in total are possible?

Resource Managers

- We just looked at two possible behaviours.
- How many in total are possible?
- Lots and lots! And with ever additional Resource Manager added, it grows and grows.

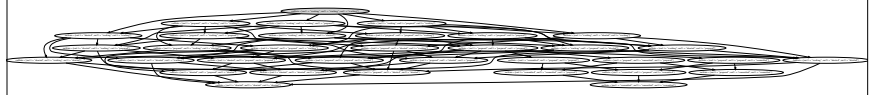
2 Resource Managers



12 Distinct States

200

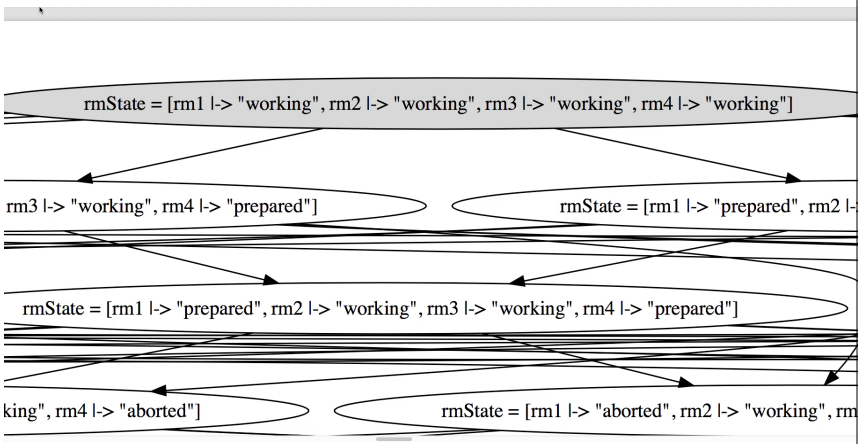
3 Resource Managers



34 Distinct States

201

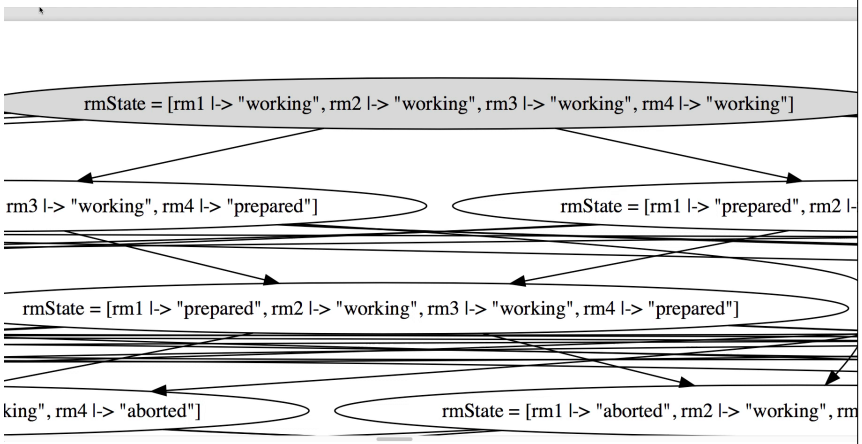
4 Resource Managers



96 Distinct States

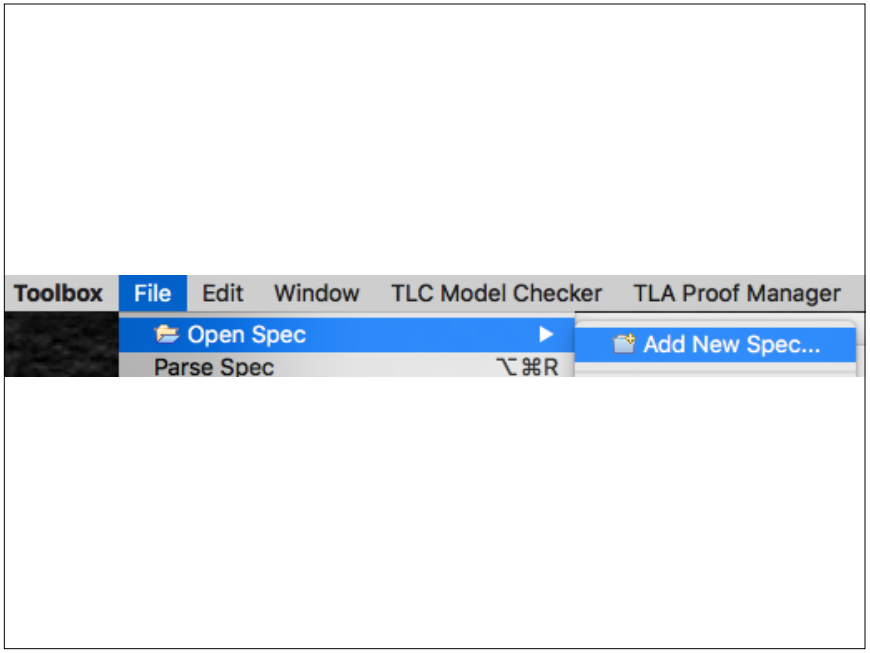
202-1

4 Resource Managers

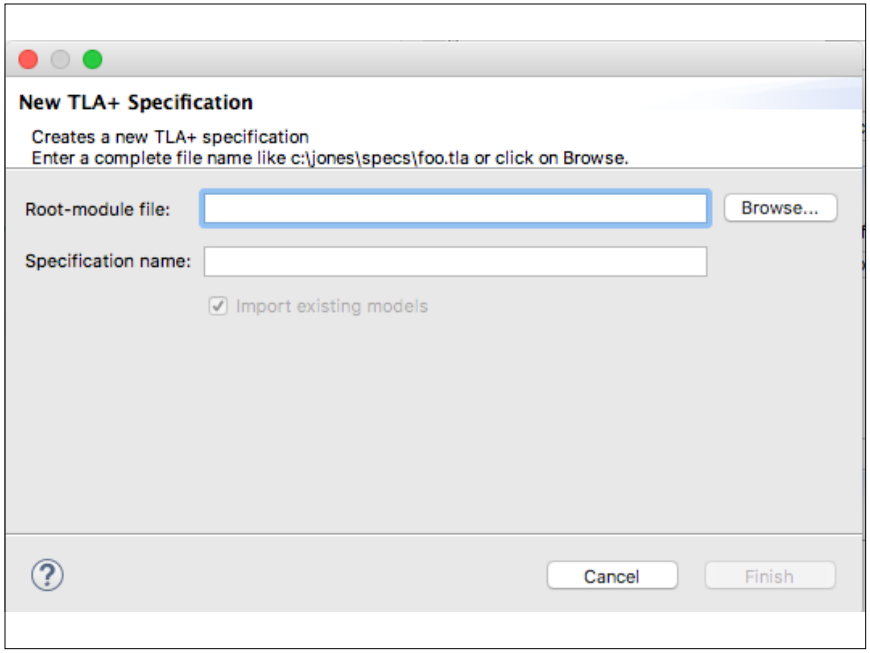


96 Distinct States

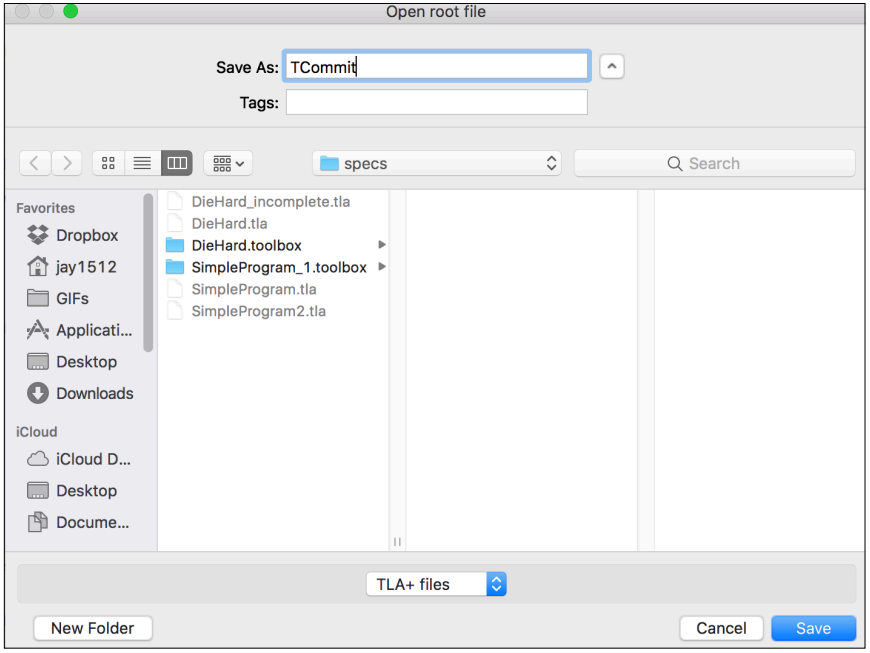
202-2



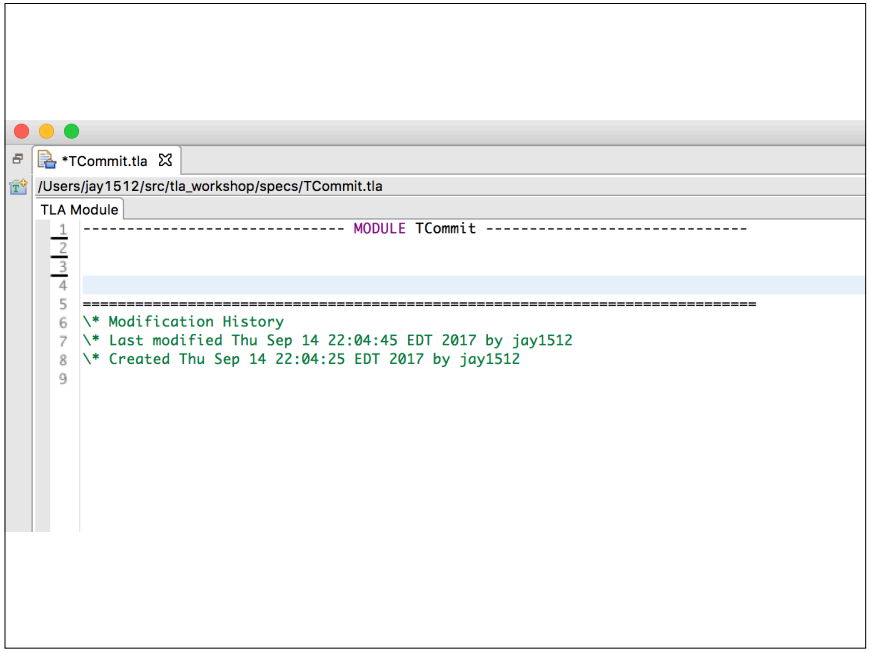
203



204



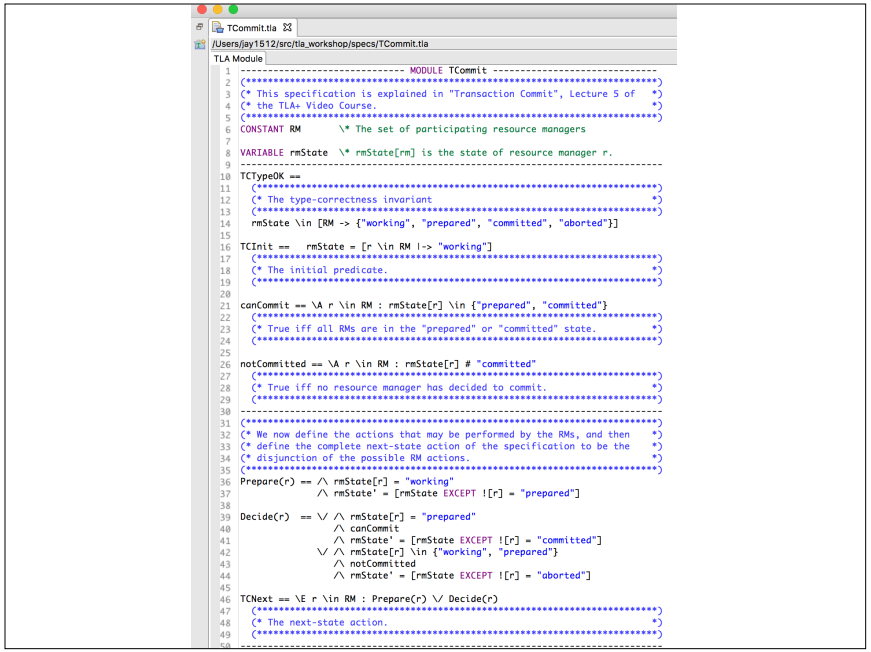
205



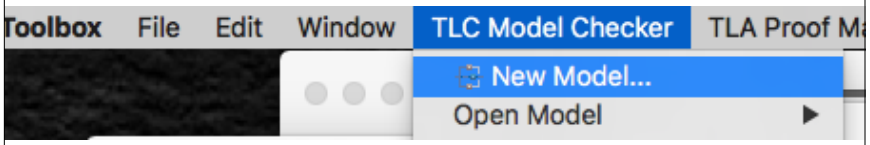
206

specs/TCommit.tla

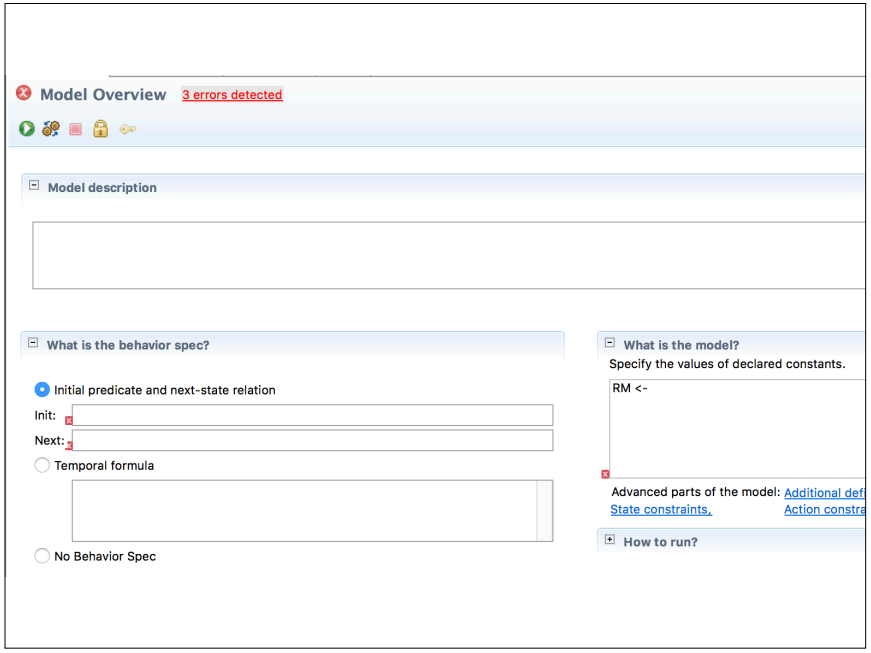
207



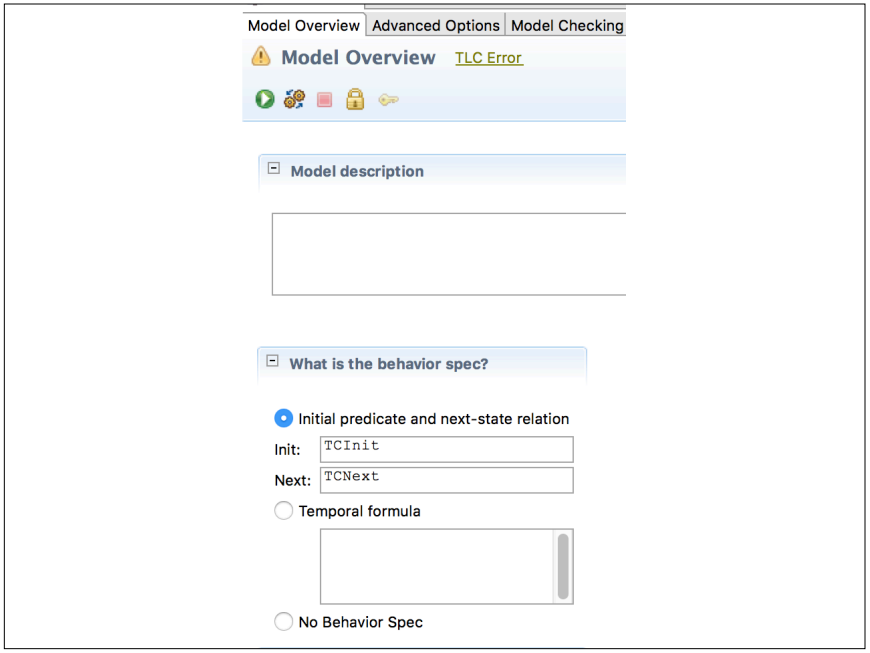
208



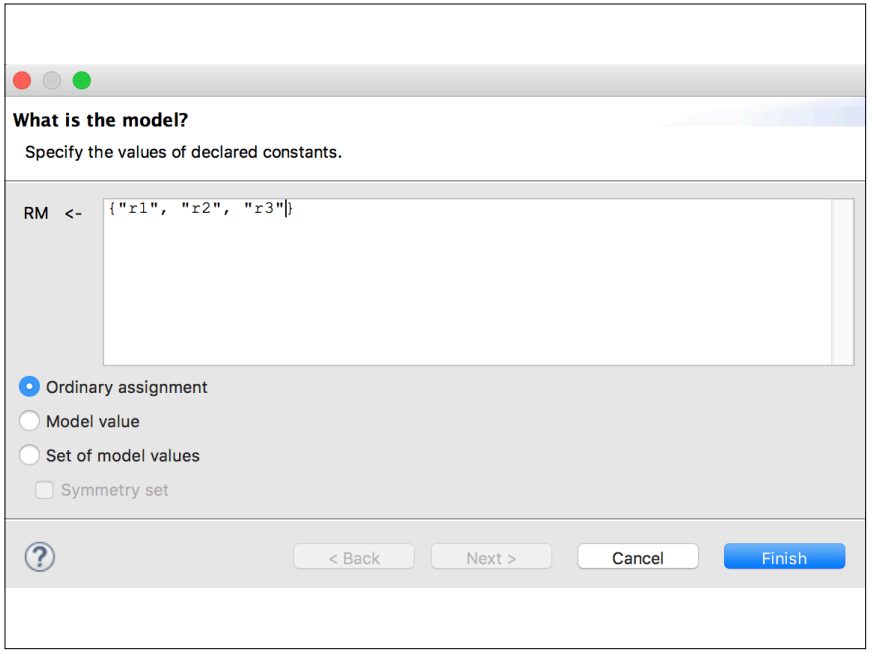
209



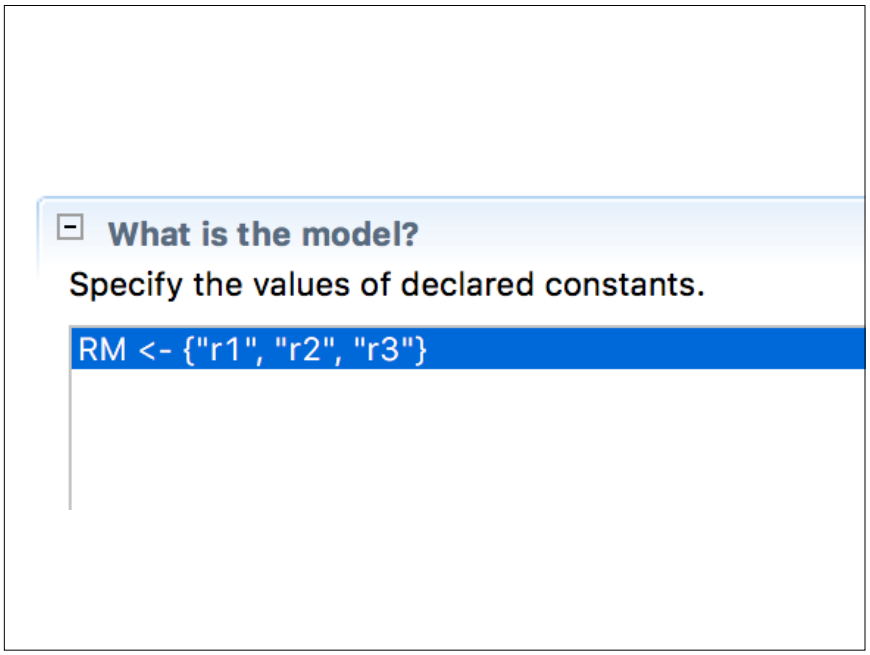
210



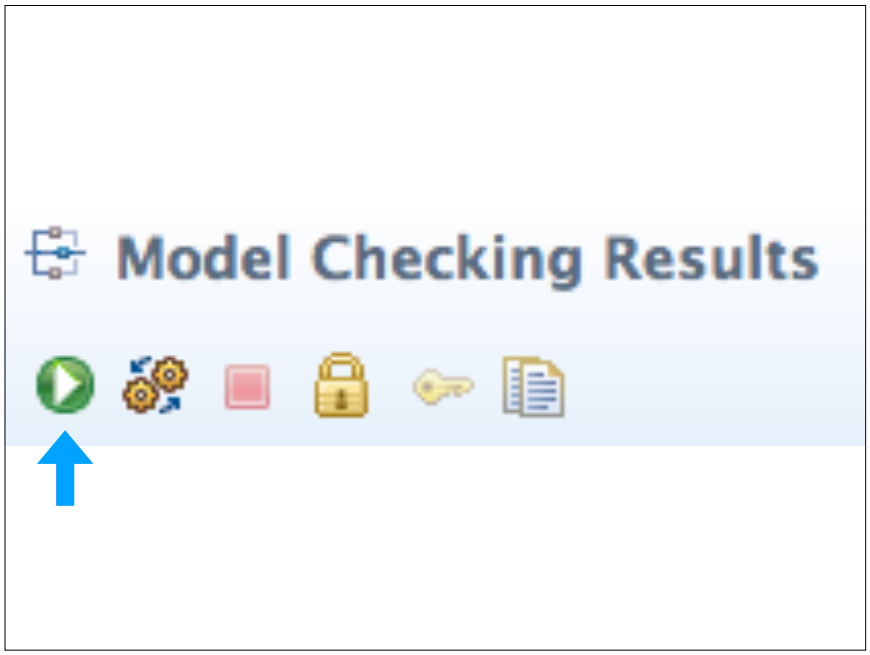
211



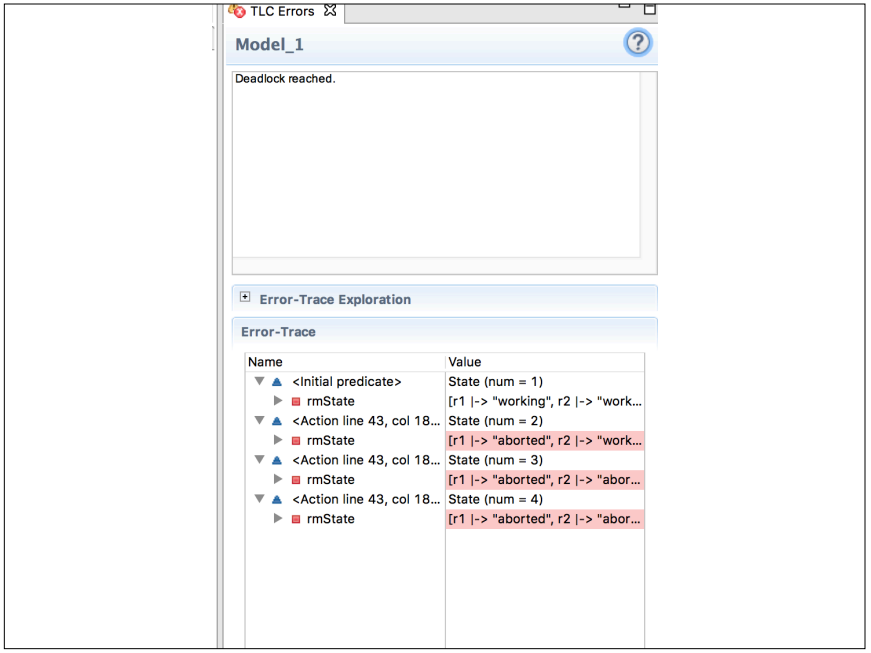
212



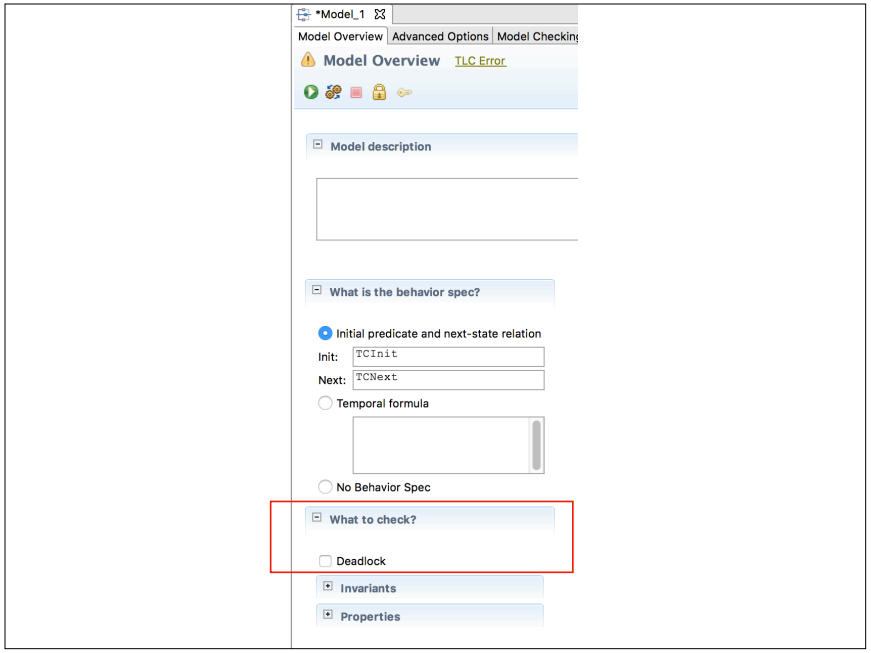
213



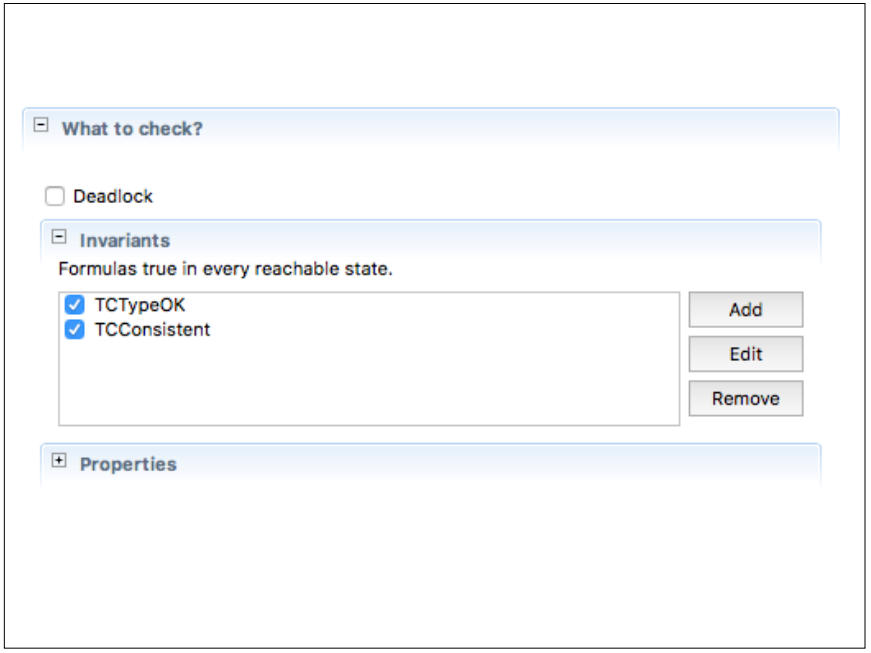
214



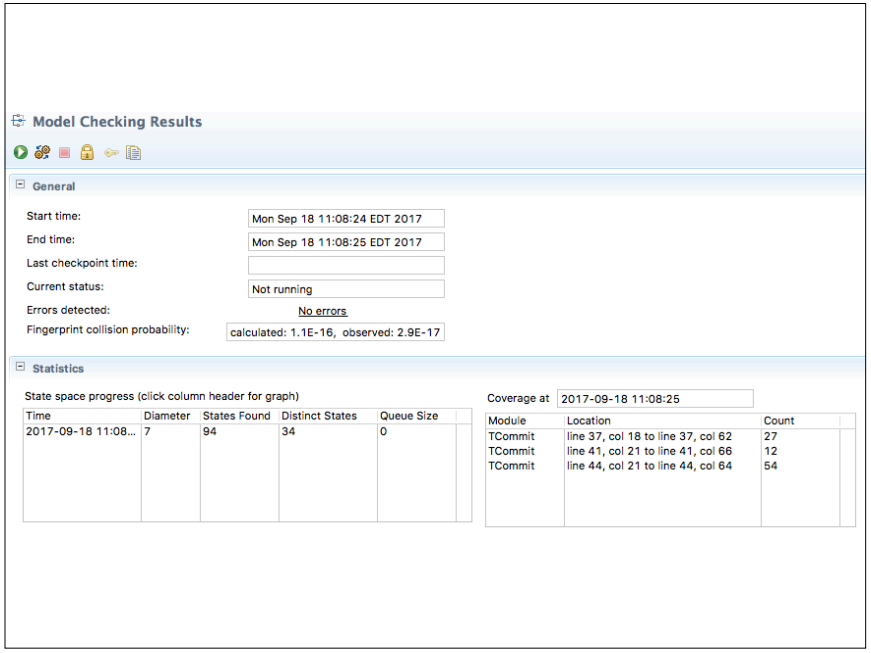
215



216



217



218

Let's try breaking something!


```
Decide(r)  == \/ /\ rmState[r] = "prepared"
                 \* /\ canCommit
                 /\ rmState' = [rmState EXCEPT ![r] = "committed"]
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



```
Decide(r)  == \/ /\ rmState[r] = "prepared"
                 \* /\ canCommit
                 /\ rmState' = [rmState EXCEPT ![r] = "committed"]
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```

- The ***** is a TLA+ single-line comment, we're temporarily removing the **canCommit** requirement.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Decide(r)  == \/ /\ rmState[r] = "prepared"
                 \* /\ canCommit
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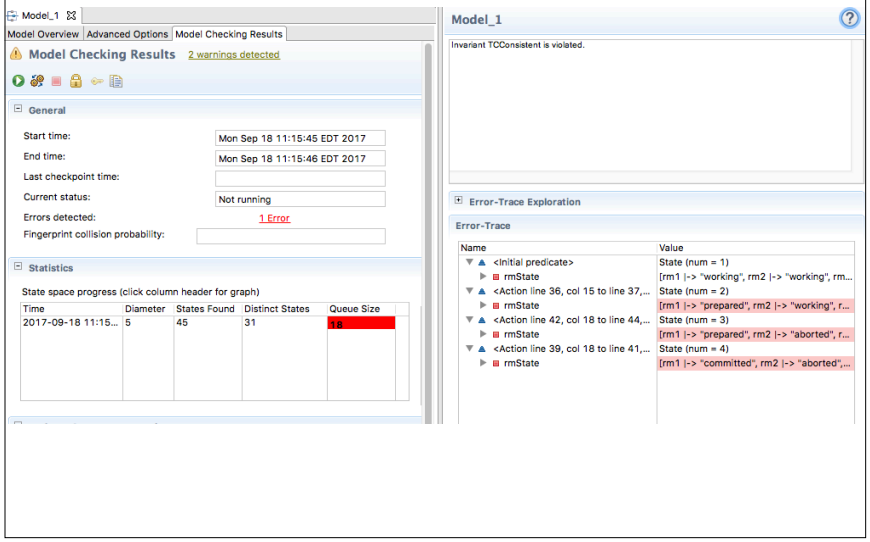
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```

- The ***** is a TLA+ single-line comment, we're temporarily removing the **canCommit** requirement.
- This means that **canCommit** no longer needs to be true to allow a state transition where some RM **r** goes from "prepared" to "committed."

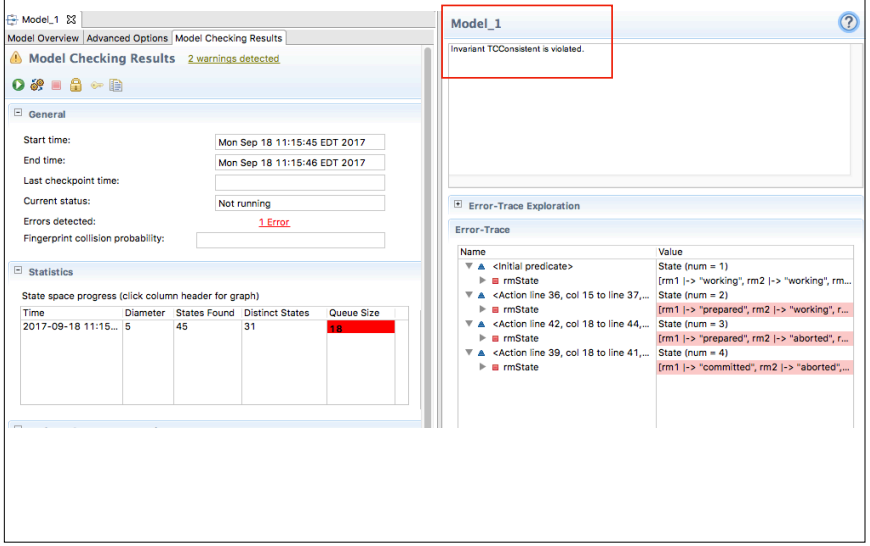
```
Decide(r)  == \/ /\ rmState[r] = "prepared"
                 \* /\ canCommit
                 /\ rmState' = [rmState EXCEPT ![r] = "committed"]
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```

- The ***** is a TLA+ single-line comment, we're temporarily removing the **canCommit** requirement.
- This means that **canCommit** no longer needs to be true to allow a state transition where some RM **r** goes from "prepared" to "committed."
- There could now be some other RM in an "aborted" state, and the specification would permit this transition.

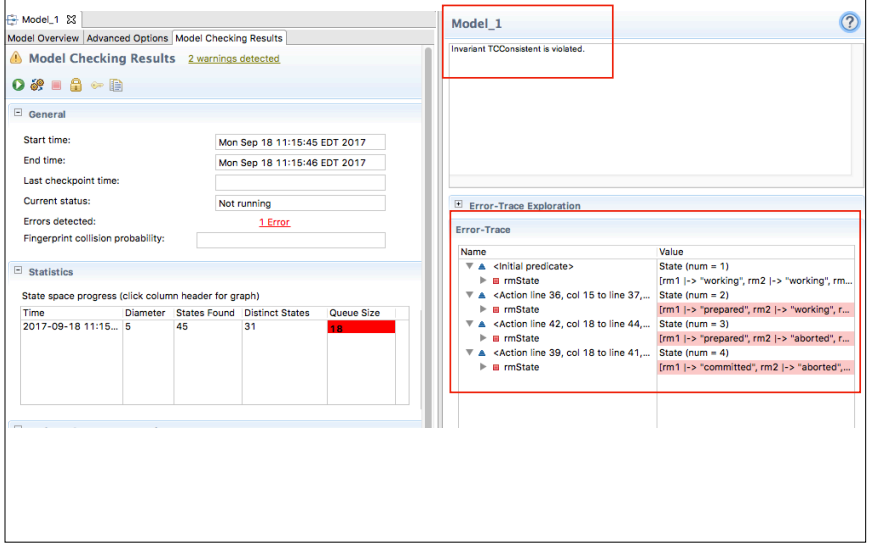
220-4



221-1



221-2



221-3

Error-Trace

Name	Value
<Initial predicate>	State (num = 1)
rmState	[rm1 |-> "working", rm2 |-> "working", rm3 |-> "working"]
<Action line 36, col 15 to line 37, col 62 of module...	State (num = 2)
rmState	[rm1 |-> "prepared", rm2 |-> "working", rm3 |-> "working"]
<Action line 42, col 18 to line 44, col 64 of module...	State (num = 3)
rmState	[rm1 |-> "prepared", rm2 |-> "aborted", rm3 |-> "working"]
<Action line 39, col 18 to line 41, col 66 of module...	State (num = 4)
rmState	[rm1 |-> "committed", rm2 |-> "aborted", rm3 |-> "working"]

222

Error-Trace

Name	Value
<Initial predicate>	State (num = 1)
rmState	[rm1 |-> "working", rm2 |-> "working", rm3 |-> "working"]
<Action line 36, col 15 to line 37, col 62 of module...	State (num = 2)
rmState	[rm1 |-> "prepared", rm2 |-> "working", rm3 |-> "working"]
<Action line 42, col 18 to line 44, col 64 of module...	State (num = 3)
rmState	[rm1 |-> "prepared", rm2 |-> "aborted", rm3 |-> "working"]
<Action line 39, col 18 to line 41, col 66 of module...	State (num = 4)
rmState	[rm1 |-> "committed", rm2 |-> "aborted", rm3 |-> "working"]

Double-click this row

223

```
30 ----------------------------------------------------------------------------
31 (***************************************************************************)
32 (* We now define the actions that may be performed by the RMs, and then    *)
33 (* define the complete next-state action of the specification to be the    *)
34 (* disjunction of the possible RM actions.                                 *)
35 (***************************************************************************)
36 Prepare(r) == /\ rmState[r] = "working"
37              /\ rmState' = [rmState EXCEPT ![r] = "prepared"]
38
39 Decide(r)  == \/ /\ rmState[r] = "prepared"
40                  \* /\ canCommit
41                  /\ rmState' = [rmState EXCEPT ![r] = "committed"]
42               \/ /\ rmState[r] \in {"working", "prepared"}
43                  /\ notCommitted
44                  /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
45
```

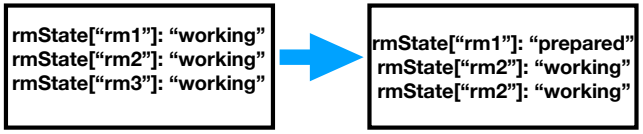
224

Resource Managers

rmState[“rm1”]: “working”
rmState[“rm2”]: “working”
rmState[“rm3”]: “working”

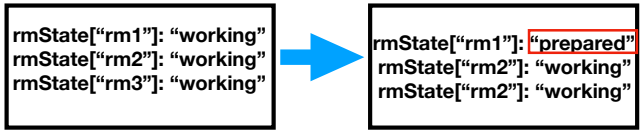
225-1

Resource Managers



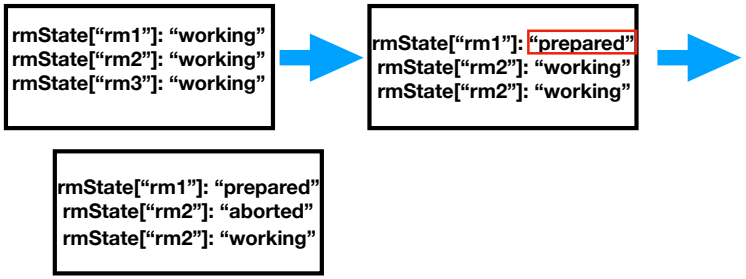
225-2

Resource Managers



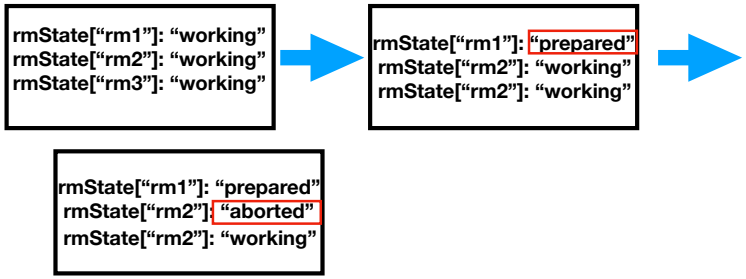
225-3

Resource Managers



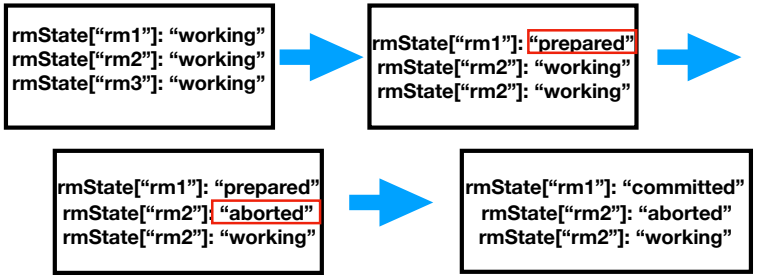
225-4

Resource Managers

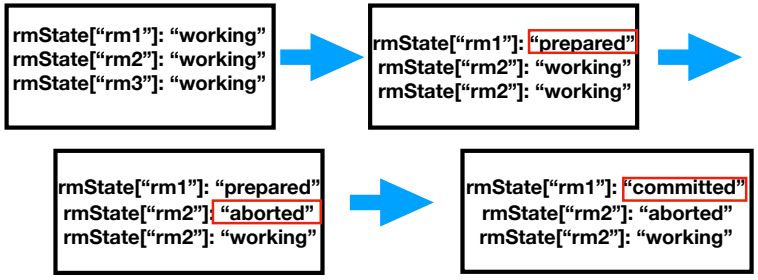


225-5

Resource Managers



Resource Managers



Syntax time

Functions

Functions

- A TLA+ function is a *true* mathematical function. The term "function" we use in most programming languages is completely wrong. Those things should generally be called "sub-routines"

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- The set of **keys** are called the DOMAIN, the set of **values** are the RANGE or IMAGE

Functions

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```
{i: i*2 for i in [2,3,4]}
```


Functions

- The value for some key can be accessed using the same syntax as array/dictionary access in Python

```
f == [ x \in {1,2,3} |-> x * 5 ]
```

```
f[1] = 5
f[2] = 10
f[3] = 15
```


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Exercise:

If we have the following function **f**

```
f == [x \in {1,2,3} |-> x*5]
```

How could you find the range (i.e. the set of values) of this function?

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Answer:

```
{ f[i] : i \in DOMAIN f }
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- Huh?

Sets of functions

```
[{2,3} -> {"a", “b”, “c”}] =   { (2 :> "a" @@ 3 :> "a"),
                                 (2 :> "a" @@ 3 :> "b"),
                                 (2 :> "a" @@ 3 :> "c"),
                                 (2 :> "b" @@ 3 :> "a"),
                                 (2 :> "b" @@ 3 :> "b"),
                                 (2 :> "b" @@ 3 :> "c"),
                                 (2 :> "c" @@ 3 :> "a"),
                                 (2 :> "c" @@ 3 :> "b"),
                                 (2 :> "c" @@ 3 :> "c") }
```

That output is **roughly** equivalent to the following Python set of dictionaries:

```
{ {2:”a”, 3:“a"},
  {2:”a", 3:“b"},
  {2:”a", 3:“c"},
  {2:”b", 3:“a”},
  {2:”b", 3:“b"},
  {2:”b", 3:“c"},
  {2:”c", 3:“a"},
  {2:”c", 3:“b"},
  {2:”c", 3:“c”} }
```


:> and @@

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[{2,3} -> {"a", “b”}] =  { (2 :> "a" @@ 3 :> "a"),
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```

:> is used as the as the separator between **keys** and **values**
@@ is used to concatenate together two functions (remember, a function is a set of key/value pairs)

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- Tuples are just syntactic sugar for functions! The keys are the integers from `1..N`, and the values are whatever you write inside the `<< >>`

- Equivalent to the Python tuple `("a","b","c")`, except TLA+ is 1-index

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t = [f EXCEPT ![4] = 30]
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- You can read that as:
  - **t** is the same as the function as **f**, except with the **value** for the **key 4** being **30**

EXCEPT

```
f == [x \in {2,4,6} |-> x*4]
t == [f EXCEPT ![4] = 30]
```

A Python equivalent

```
import copy
f = {2: 8, 4: 16, 6: 24}
t = copy.deepcopy(f)
t[4] = 30
```


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- **Tuples** are syntactic sugar over functions.


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CONSTANT RM       \* The set of participating resource managers

VARIABLE rmState  \* rmState[rm] is the state of resource manager r.
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Let's assume that RM will be the set {"r1", "r2"}


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  [r1 |-> "aborted", r2 |-> "committed"],
  [r1 |-> "committed", r2 |-> "waiting"],
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  - Where `Integers` is the set of all integer values


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- Thus only one possible initial state is permitted

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  - At every state, an infinite number of next states are theoretically possible. Our formula identifies which next states are **actually** possible, by applying the infinite set of next states to the formula and seeing which ones come out as TRUE

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TCNext == \E r \in RM : Prepare(r) \/ Decide(r)
```

- Let's look at `Prepare(r)` to get some clarity

```
Prepare(r) == /\ rmState[r] = "working"
              /\ rmState' = [rmState EXCEPT ![r] = "prepared"]
```

- This says:
  - This formula is true for a pair of states if in the **current** state, resource manager **r** is "working"

Next state formula

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TCNext == \E r \in RM : Prepare(r) \/ Decide(r)
```

- Let's look at `Prepare(r)` to get some clarity

```
Prepare(r) == /\ rmState[r] = "working"
              /\ rmState' = [rmState EXCEPT ![r] = "prepared"]
```

- This says:
  - This formula is true for a pair of states if in the **current** state, resource manager **r** is "working"
  - And if in the **next** state, the state of resource manager **r** is "prepared", and all the other resource managers have the same state as they do right now

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Prepare(r) == /\ rmState[r] = "working"
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```

rmState: [r1 |-> "prepared",
r2 |-> "working"]

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Prepare(r) == /\ rmState[r] = "working"
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rmState: [r1 |-> "prepared",
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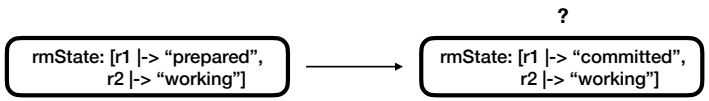

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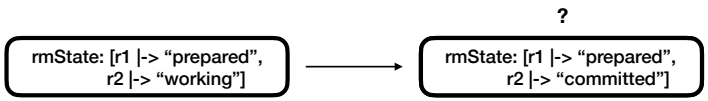

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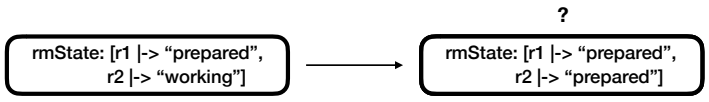

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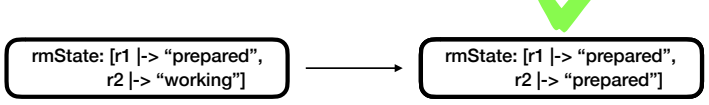

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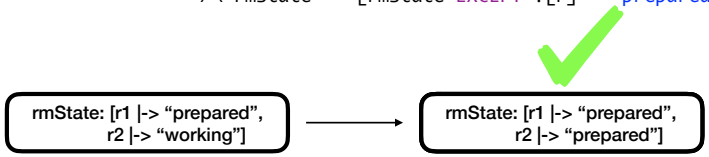

- The parts of the formula with no primed variables are called **enabling** conditions

Next state formula

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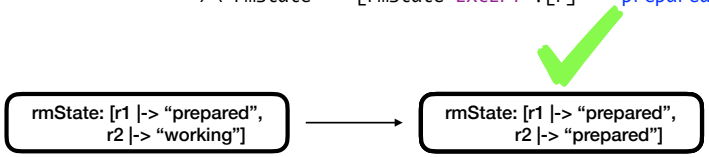

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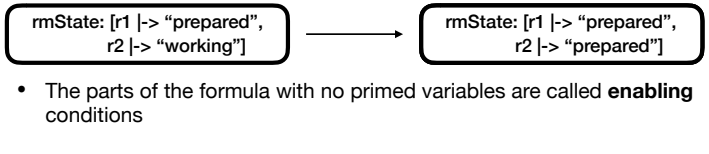

- The parts of the formula with no primed variables are called **enabling** conditions
- They describe what the **current state** must look like to be able to use this action

Next state formula

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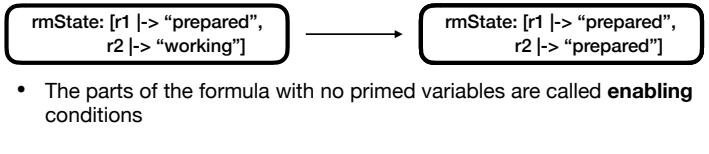

- The parts of the formula with no primed variables are called **enabling** conditions
- They describe what the **current state** must look like to be able to use this action
- Any formula with **primed variables** is called an **action**

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
                 /\ canCommit
                 /\ rmState' = [rmState EXCEPT ![r] = "committed"]
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
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Next state formula

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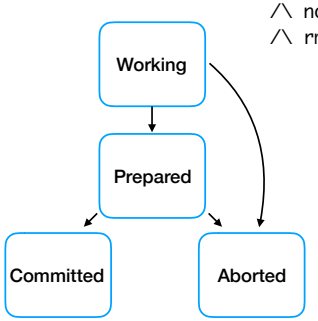

Next state formula

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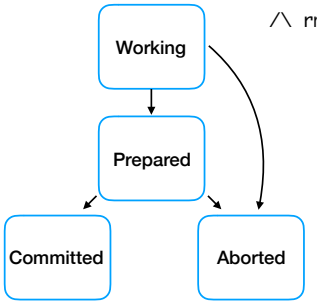

- This formula is true when either

Next state formula

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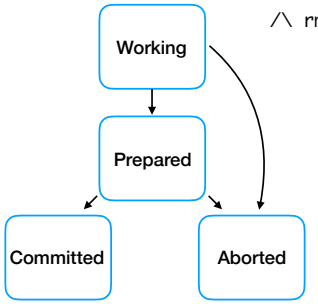

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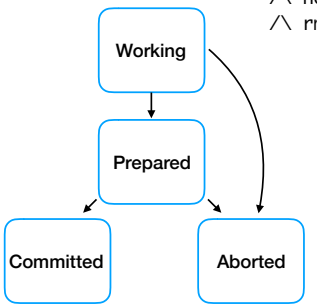

- This formula is true when either
  - RM **r** is in state “prepared”

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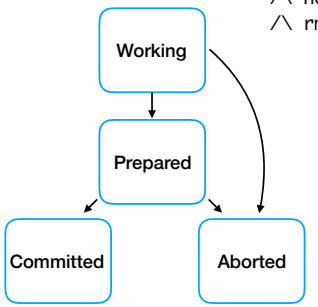

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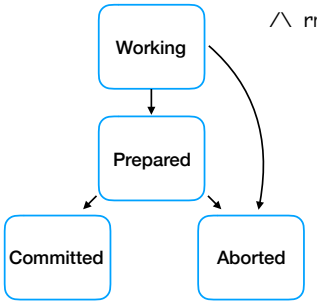

- This formula is true when either
  - RM **r** is in state “prepared”
  - And **canCommit** is true

Next state formula

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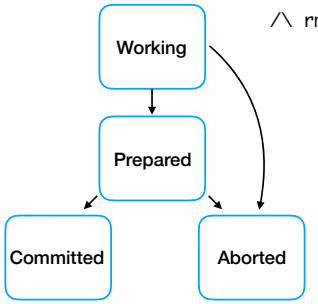

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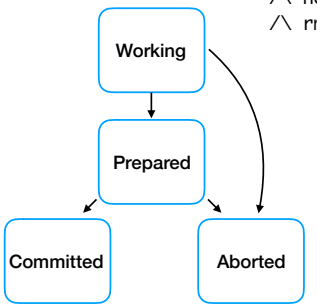

- This formula is true when either
  - RM **r** is in state “prepared”
  - And **canCommit** is true
  - And that RM **r** is “committed” in the next state

248-9

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
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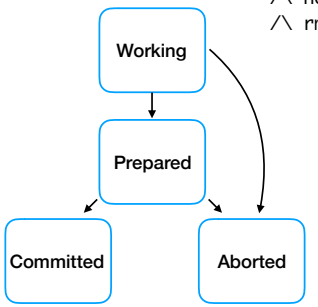
```



- This formula is true when either
  - RM **r** is in state “prepared”
  - And **canCommit** is true
  - And that RM **r** is “committed” in the next state

248-10

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
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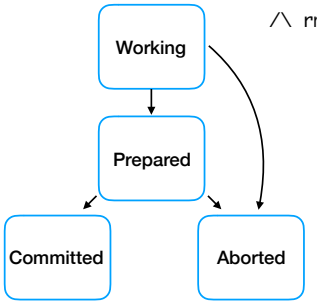

249-1

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
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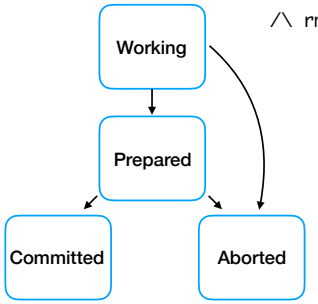

- or when

249-2

Next state formula

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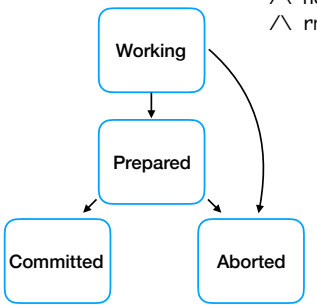

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Next state formula

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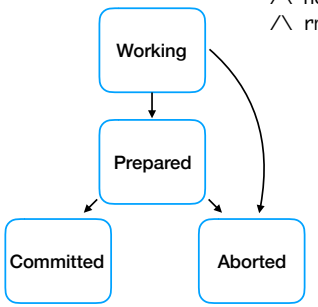

- or when
  - RM **r** is currently “working” or “prepared”

Next state formula

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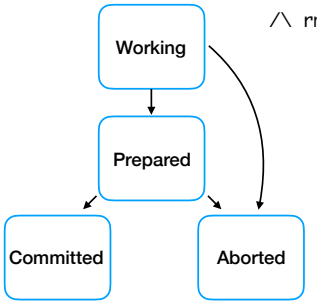

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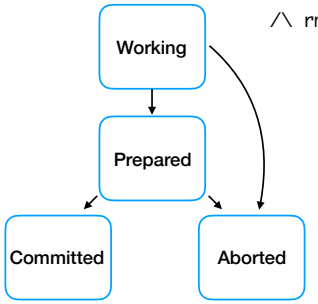

- or when
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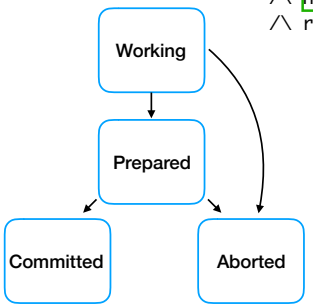

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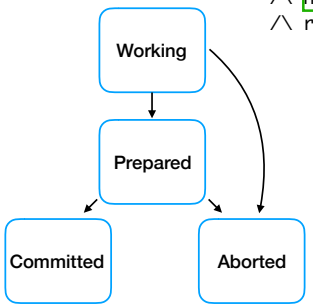

- or when
  - RM **r** is currently “working” or “prepared”
  - And **notCommitted** is true
  - And in the next state RM **r** is “aborted”

Next state formula

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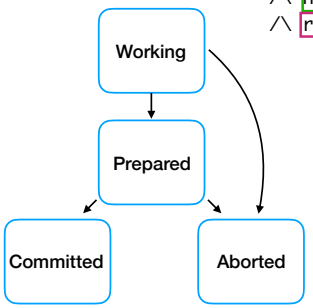

- or when
  - RM **r** is currently “working” or “prepared”
  - And **notCommitted** is true
  - And in the next state RM **r** is “aborted”

canCommit

```
canCommit == \A r \in RM : rmState[r] \in {"prepared", "committed"}
```

canCommit

```
canCommit == \A r \in RM : rmState[r] \in {"prepared", "committed"}
```

Python Equivalents

```
RM = [“rm1”, “rm2”] # a constant
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canCommitStates = [“prepared”, “committed”] # a constant

...

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notCommitted

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```

"Not equal." You can also use **/=**

notCommitted

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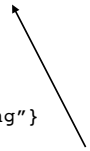

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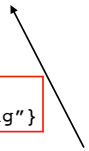

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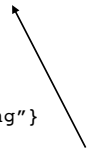

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- If **multiple r** in RM satisfy that, TLC will create behaviours for all of them
- TLC always explores every possible state

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  \A r1, r2 \in RM : ~ /\ rmState[r1] = "aborted"
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```

Python equivalent

```
def TCConsistent():
    for r1 in RM:
        for r2 in RM:
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                return False
    return True
```

TCommit Recap

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  - It is never allowed that one RM is committed and another is aborted
- Our spec says **nothing** about how to actually implement such a thing, it only specifies the desired behaviours
- Now we can write a new spec which presents a solution to this problem
  - (There are **LOTS** of algorithms that solve this problem, we’ll just look at one)

Two Phase Commit

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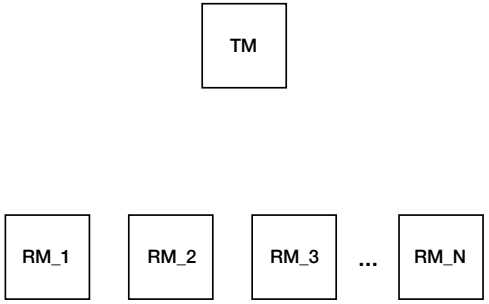
- Introduces a “Transaction Manager” (TM) to coordinate with the RMs
- The TM becomes the “source of truth” on whether a transaction should commit or abort

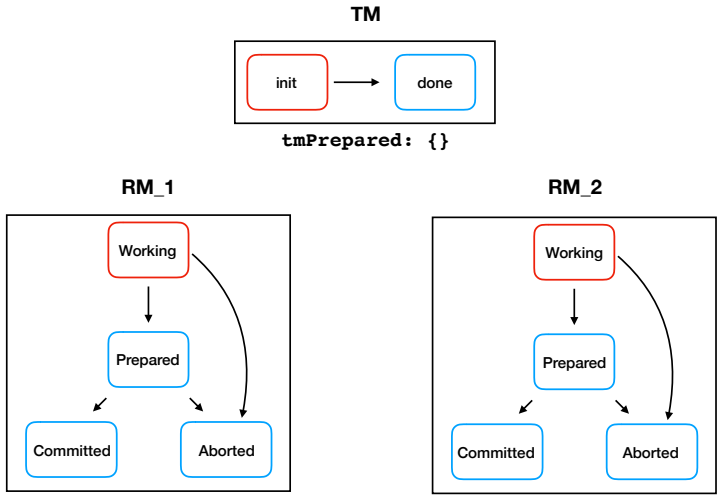
Transaction Manager

Transaction Manager



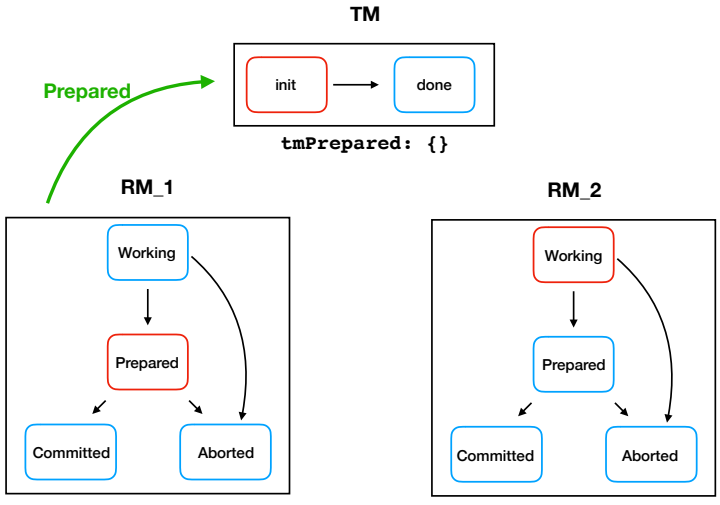
257-2

Transaction Manager



258

Transaction Manager



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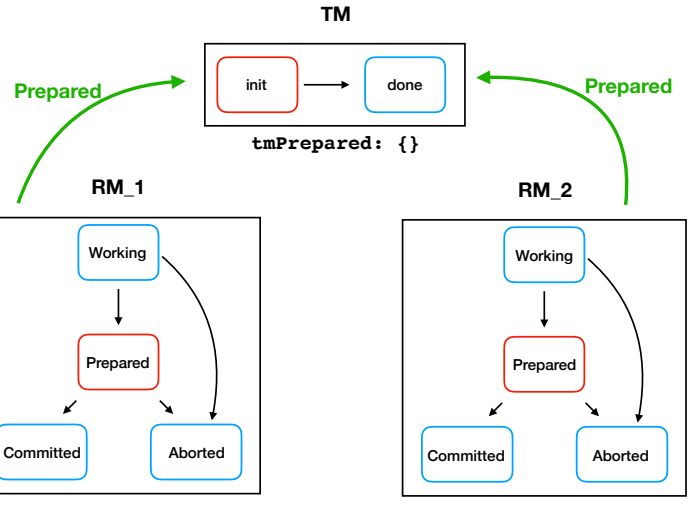
Transaction Manager



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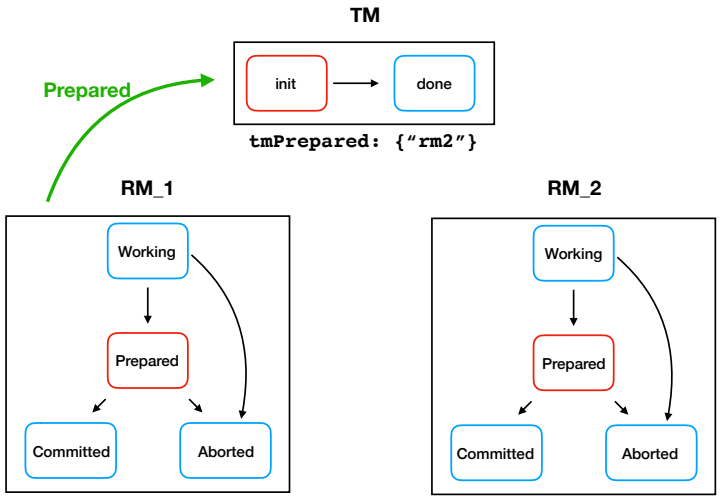
Transaction Manager



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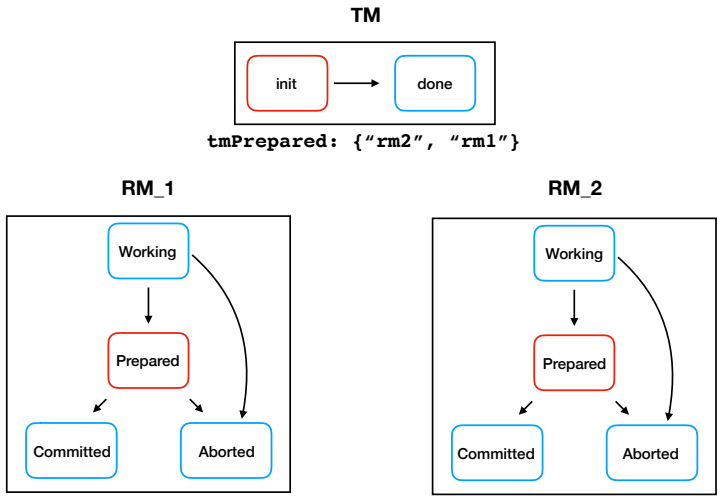

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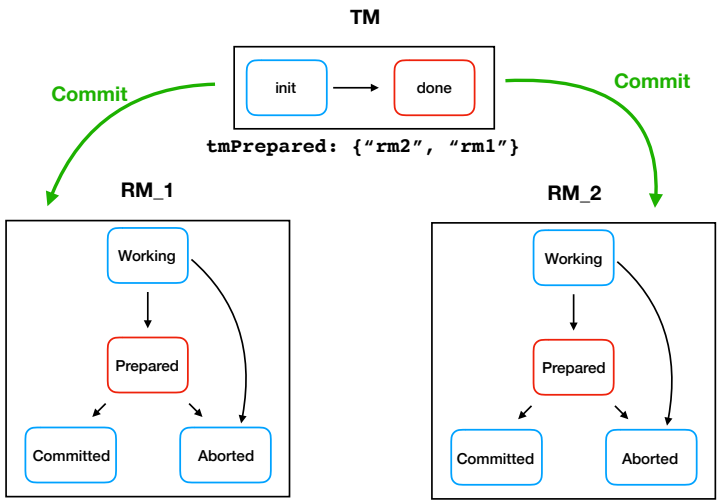

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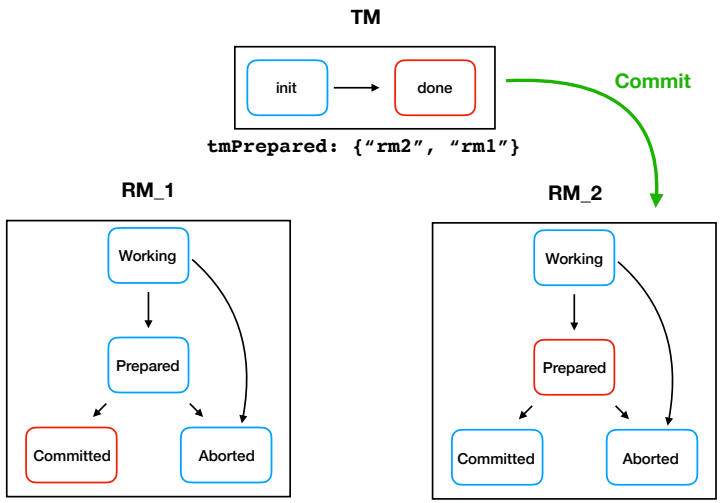

264

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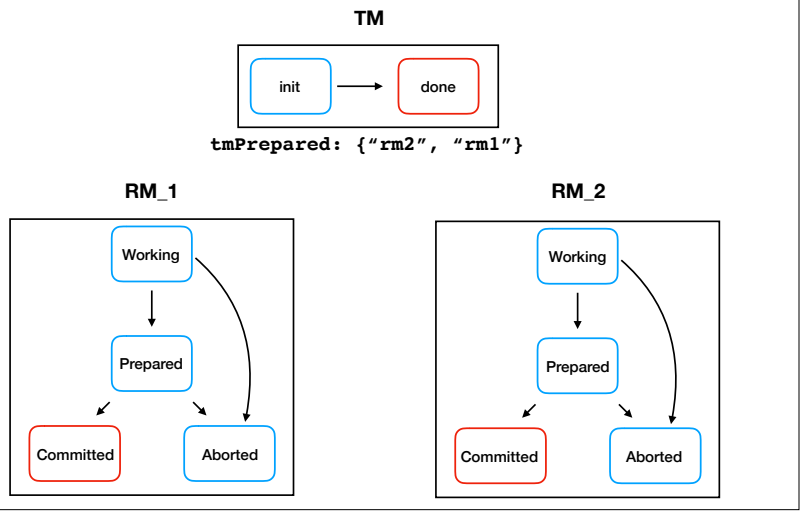

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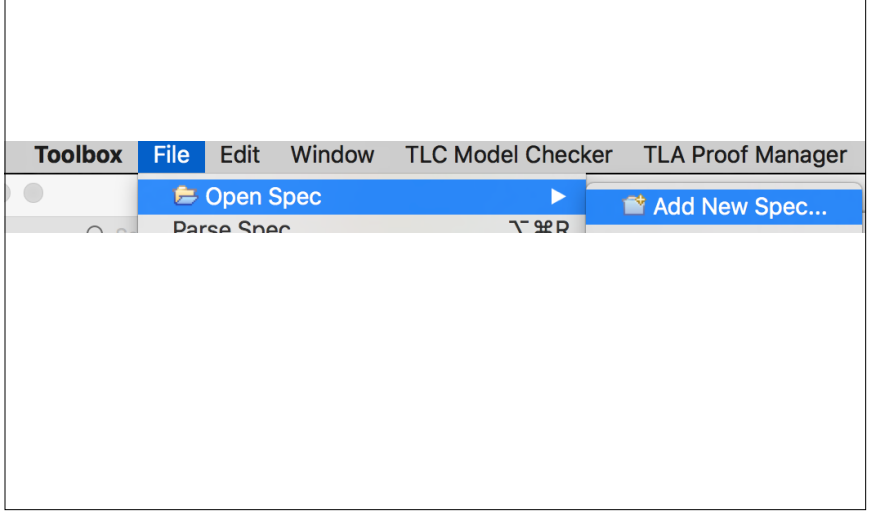
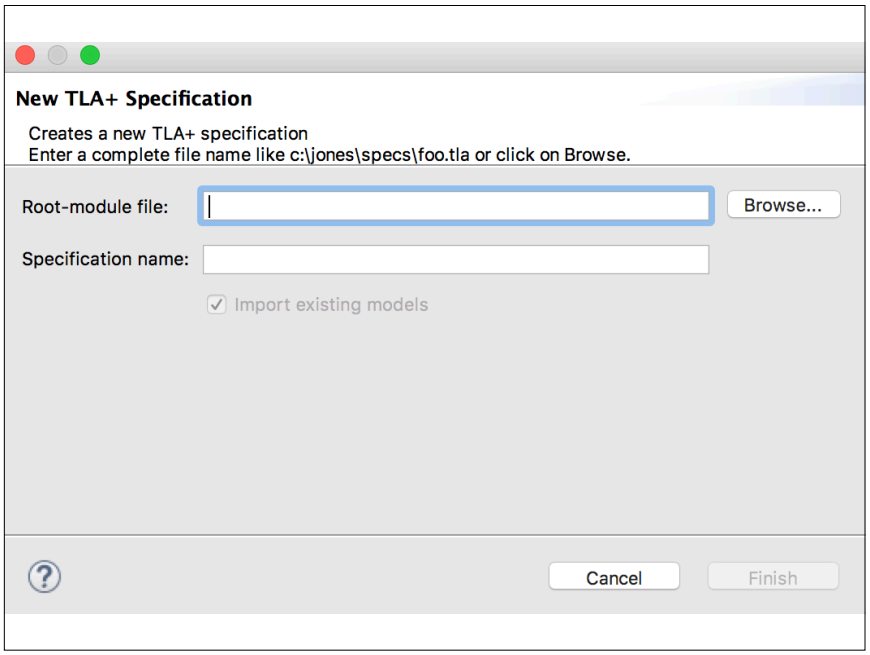
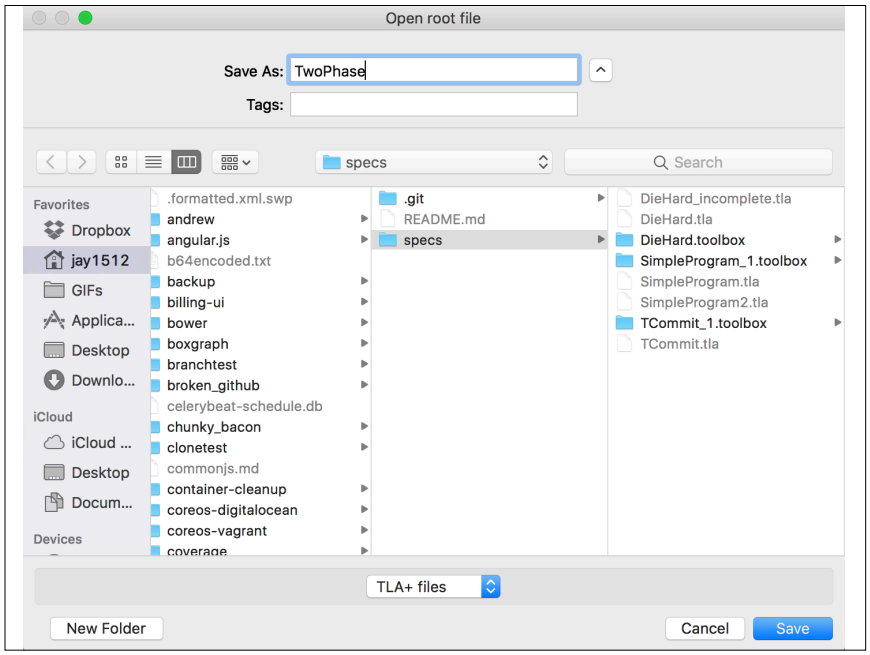


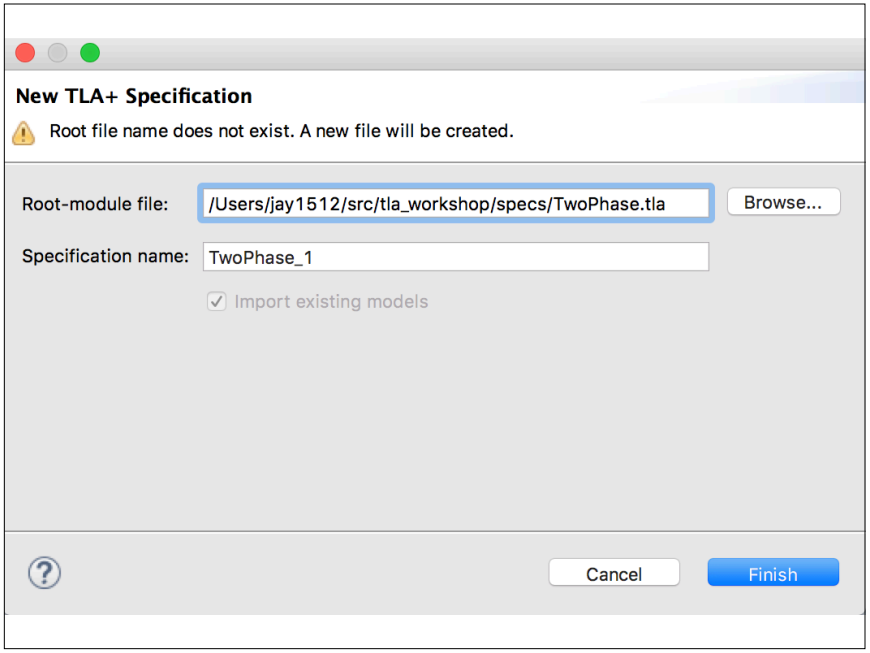
266



267



268



269

```
---------------------------- MODULE TwoPhase ----------------------------
(***************************************************************************)
(* This specification is discussed in "Two-Phase Commit", Lecture 6 of the *)
(* TLA+ Video Course.  It describes the Two-Phase Commit protocol, in      *)
(* which a transaction manager (TM) coordinates the resource managers      *)
(* (RMs) to implement the Transaction Commit specification of module       *)
(* TCommit.  In this specification, RMs spontaneously issue Prepared       *)
(* messages.  We ignore the Prepare messages that the TM can send to the   *)
(* RMs.                                                                    *)
(*                                                                         *)
(* For simplicity, we also eliminate Abort messages sent by an RM when it  *)
(* decides to abort.  Such a message would cause the TM to abort the       *)
(* transaction, an event represented here by the TM spontaneously deciding *)
(* to abort.                                                               *)
(***************************************************************************)
CONSTANT RM  \* The set of resource managers

VARIABLES
  rmState,       \* rmState[r] is the state of resource manager r.
  tmState,       \* The state of the transaction manager.
  tmPrepared,    \* The set of RMs from which the TM has received "Prepared"
                 \* messages.
  msgs
    (*************************************************************************)
    (* In the protocol, processes communicate with one another by sending    *)
    (* messages.  For simplicity, we represent message passing with the      *)
    (* variable msgs whose value is the set of all messages that have been   *)
    (* sent.  A message is sent by adding it to the set msgs.  An action     *)
    (* that, in an implementation, would be enabled by the receipt of a      *)
    (* certain message is here enabled by the presence of that message in    *)
    (* msgs.  For simplicity, messages are never removed from msgs.  This    *)
    (* allows a single message to be received by multiple receivers.         *)
    (* Receipt of the same message twice is therefore allowed; but in this   *)
    (* particular protocol, that's not a problem.                            *)
    (*************************************************************************)

Messages ==
  (***************************************************************************)
  (* The set of all possible messages.  Messages of type "Prepared" are      *)
  (* sent from the RM indicated by the message's rm field to the TM.         *)
  (* Messages of type "Commit" and "Abort" are broadcast by the TM, to be    *)
  (* received by all RMs.  The set msgs contains just a single copy of      *)
  (* such a message.                                                         *)
  (***************************************************************************)
  [type : {"Prepared"}, rm : RM]  \cup  [type : {"Commit", "Abort"}]
```

specs/TwoPhase.tla

270

Records

- Records are another syntactic sugar on top of functions, for cases where you want all the keys to be strings

271-1

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```
r == [nodes |-> {1,2}, edges |-> {“a”, “b”}, cost |-> 5]
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271-2

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r == [nodes |-> {1,2}, edges |-> {“a”, “b”}, cost |-> 5]
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r = {“nodes”: set([1,2]), “edges”: set([“a”, “b”]), “cost”: 5}
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```
[nodes: {1,2}, edges: {"a","b","c"}]
```

```
{ [nodes |-> 1, edges |-> "a"],
  [nodes |-> 1, edges |-> "b"],
  [nodes |-> 1, edges |-> "c"],
  [nodes |-> 2, edges |-> "a"],
  [nodes |-> 2, edges |-> "b"],
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```

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set_of_records = set()
for node in [1,2]:
    for edge in [“a”,”b”,”c”]:
        set_of_records.add({“nodes”: node, “edges”, edge})
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```
{  {“nodes”: 1, “edges”: ”a”},
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   {“nodes”: 2, ”edges”: “a"},
   {“nodes”: 2, ”edges”: “b"},
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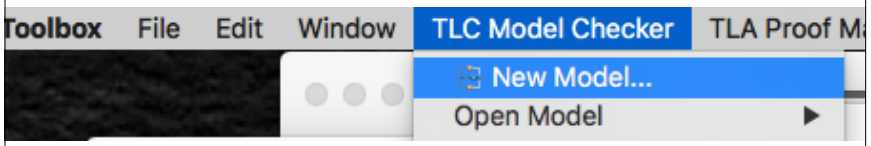
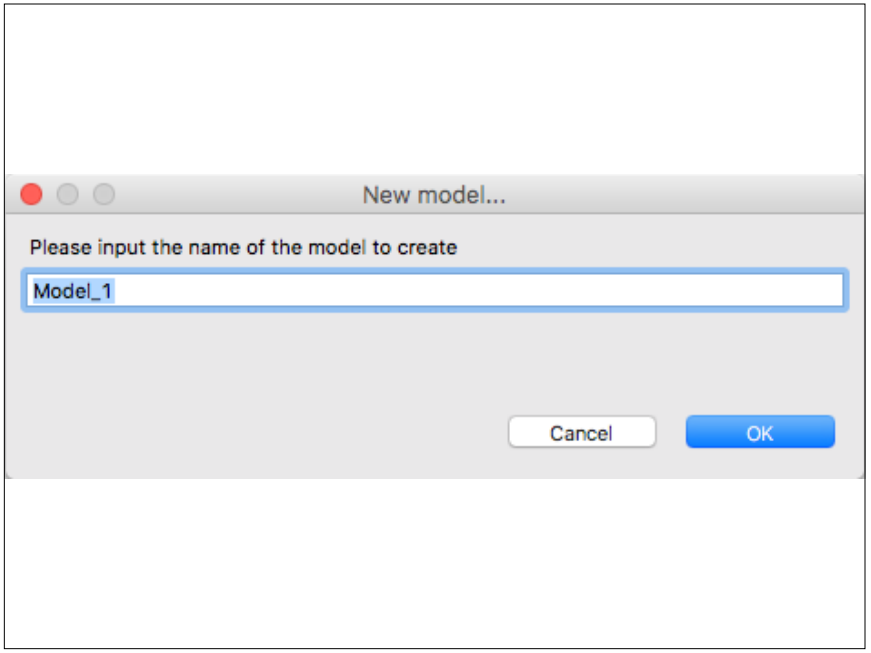
```



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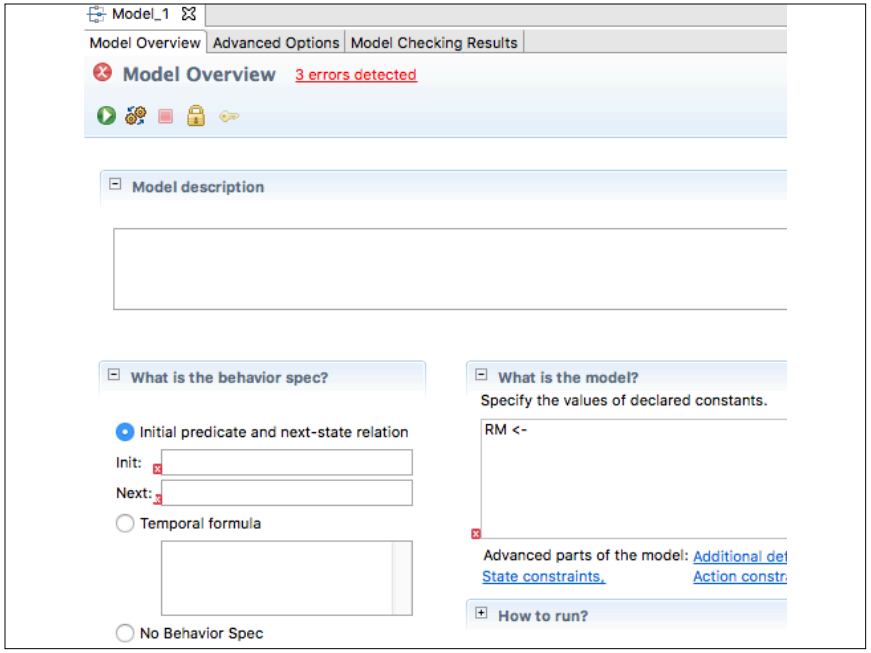
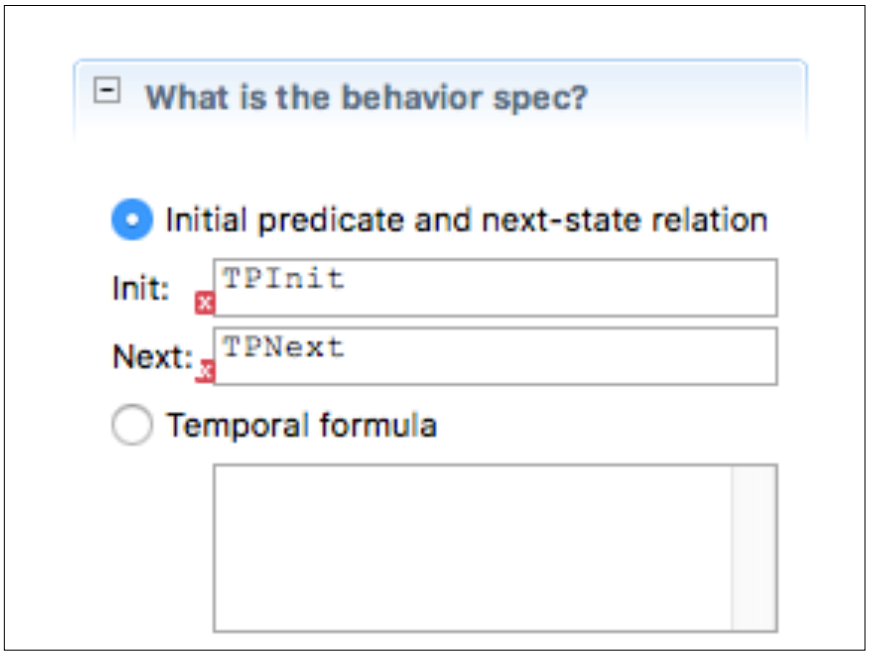
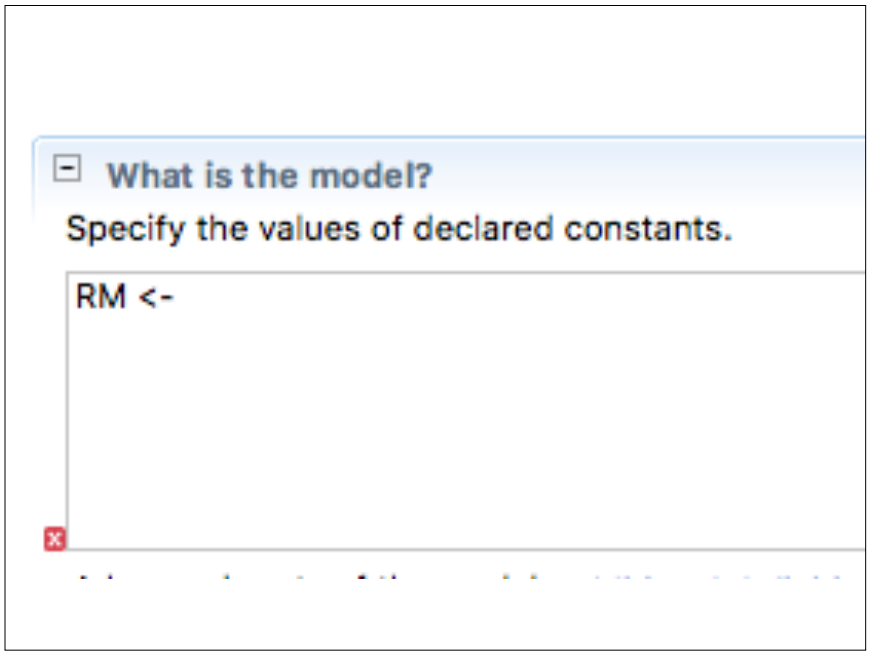


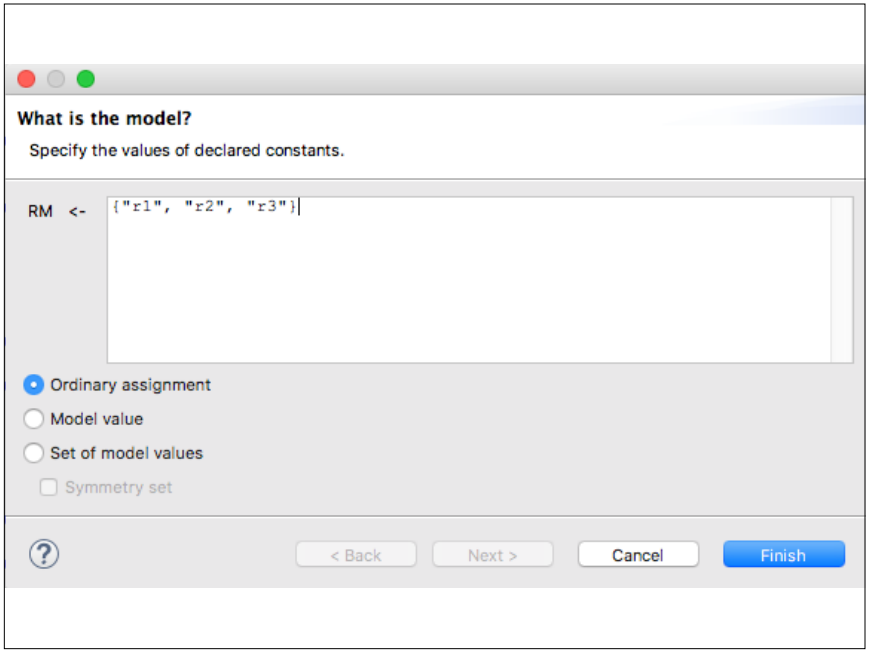
276



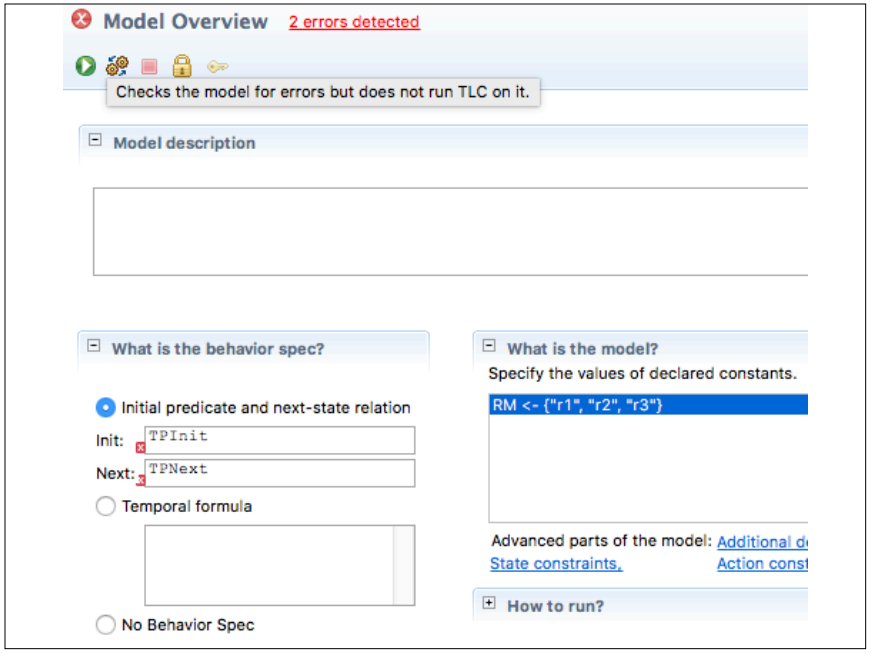
277



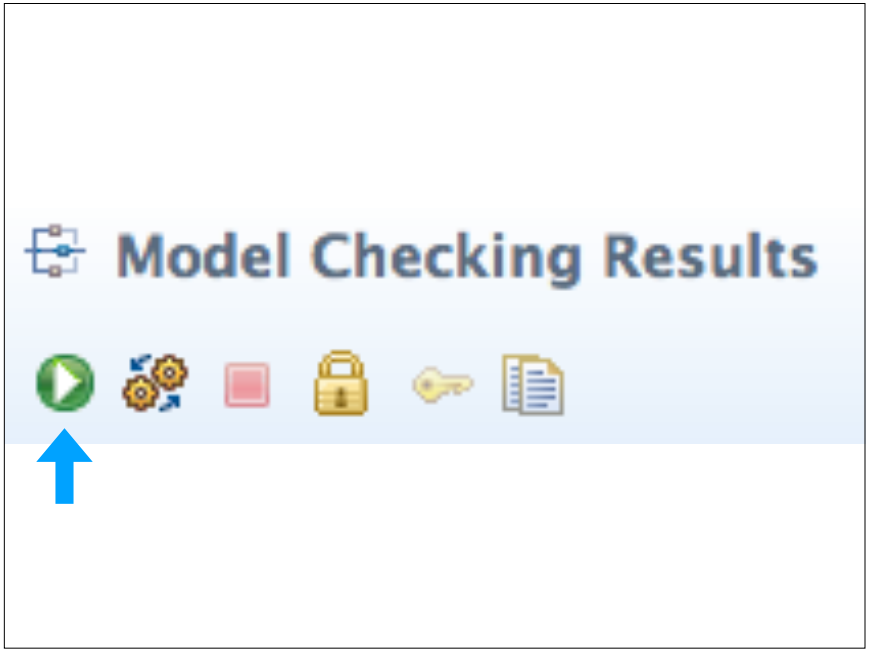
278



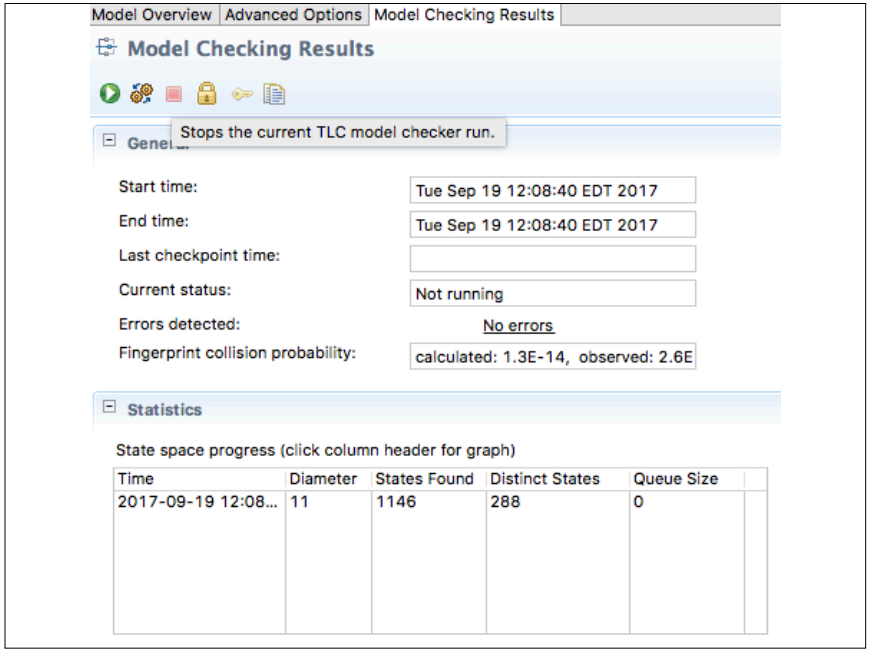
279



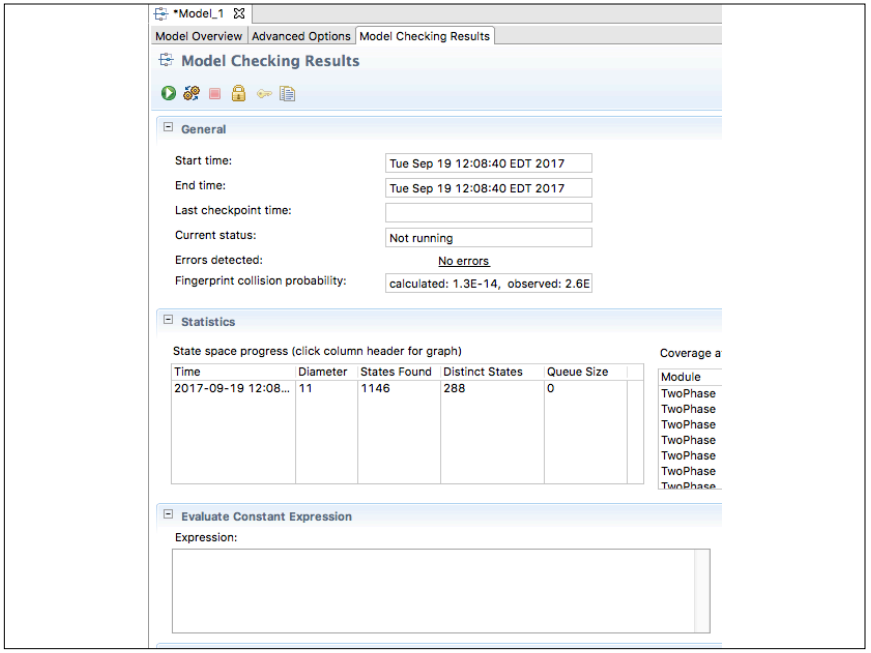
280



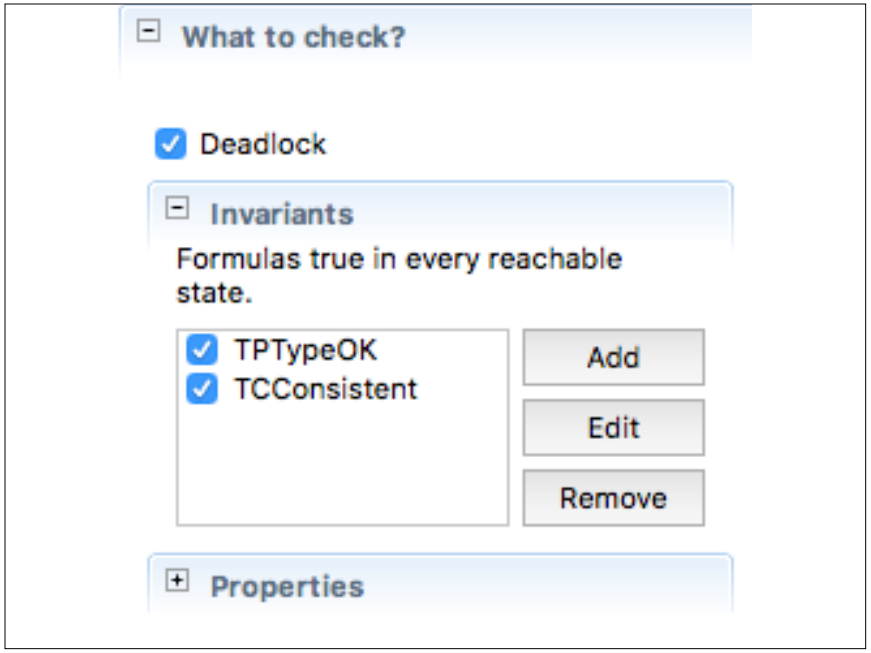
281



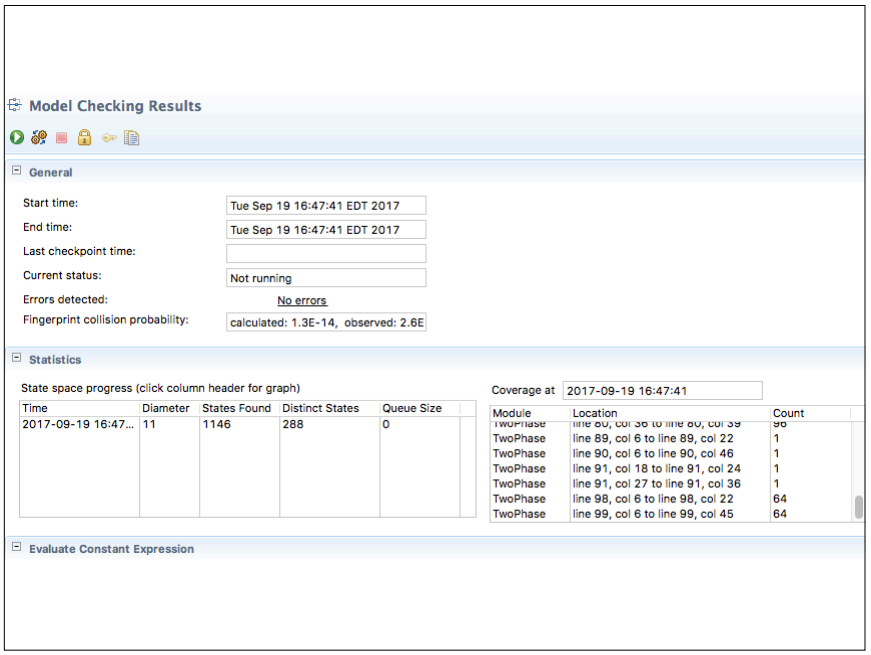
282



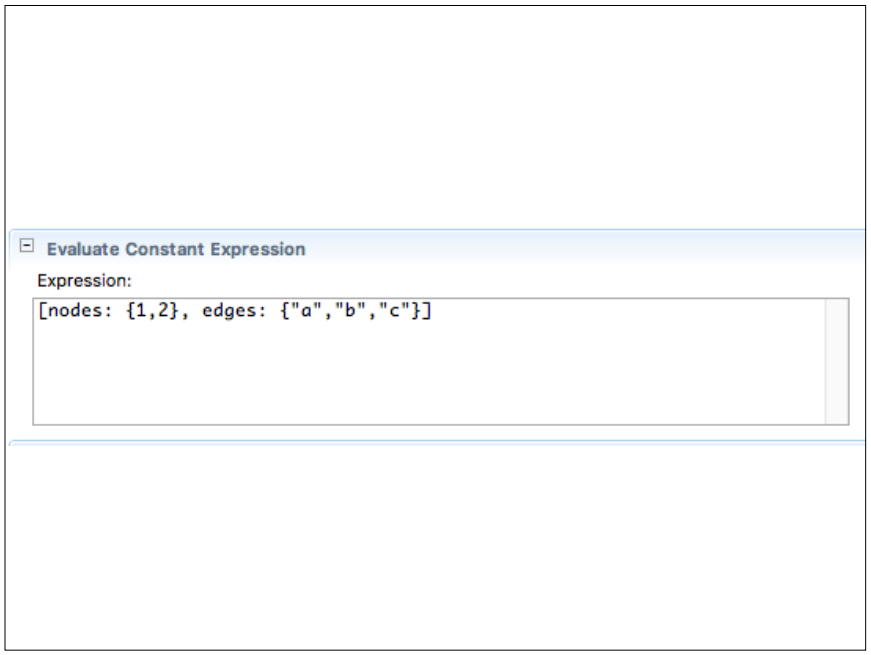
283



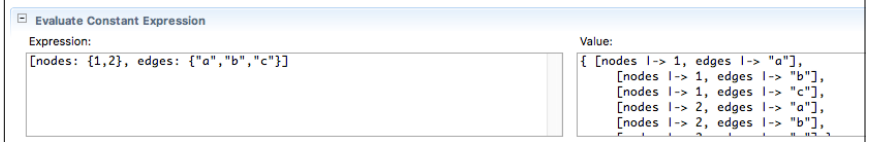
284



285



286



TwoPhase

```
CONSTANT RM  \* The set of resource managers

VARIABLES
  rmState,       \* rmState[r] is the state of resource manager r.
  tmState,       \* The state of the transaction manager.
  tmPrepared,    \* The set of RMs from which the TM has received "Prepared"
                 \* messages.
  msgs
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- **msgs** starts as an empty set

TwoPhase

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Messages ==
  (***************************************************************************)
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  (* sent from the RM indicated by the message's rm field to the TM.       *)
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  (* received by all RMs.  The set msgs contains just a single copy of     *)
  (* such a message.                                                       *)
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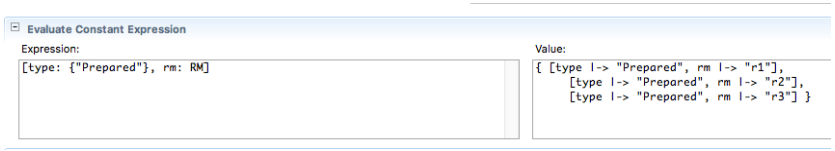

TwoPhase

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TwoPhase

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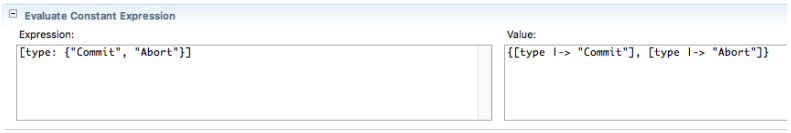

290-3

TwoPhase

```
Messages ==
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  (* The set of all possible messages.  Messages of type "Prepared" are    *)
  (* sent from the RM indicated by the message's rm field to the TM.       *)
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  (* such a message.                                                       *)
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```

291-1

TwoPhase

```
Messages ==
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  (* received by all RMs.  The set msgs contains just a single copy of     *)
  (* such a message.                                                       *)
  (***************************************************************************)
  [type : {"Prepared"}, rm : RM]  \union  [type : {"Commit", "Abort"}]
```

291-2

TwoPhase

```
Messages ==
  (***************************************************************************)
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  (* sent from the RM indicated by the message's rm field to the TM.       *)
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  (* received by all RMs.  The set msgs contains just a single copy of     *)
  (* such a message.                                                       *)
  (***************************************************************************)
  [type : {"Prepared"}, rm : RM]  \union  [type : {"Commit", "Abort"}]
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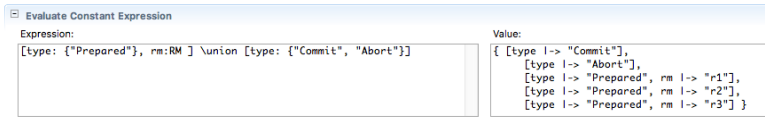

291-3

TPTypeOK

```
TPTypeOK ==
  (***************************************************************************)
  (* The type-correctness invariant                                          *)
  (***************************************************************************)
  /\ rmState \in [RM -> {"working", "prepared", "committed", "aborted"}]
  /\ tmState \in {"init", "done"}
  /\ tmPrepared \subseteq RM
  /\ msgs \subseteq Messages
```


TPTypeOK

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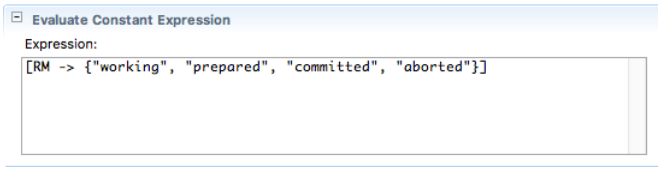

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  /\ tmState \in {"init", "done"}
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  /\ msgs \subseteq Messages
```

```
{ [r1 |-> "working", r2 |-> "working", r3 |-> "working"],
    [r1 |-> "working", r2 |-> "working", r3 |-> "prepared"],
    [r1 |-> "working", r2 |-> "working", r3 |-> "committed"],
    [r1 |-> "working", r2 |-> "working", r3 |-> "aborted"],
    [r1 |-> "working", r2 |-> "prepared", r3 |-> "working"],
    [r1 |-> "working", r2 |-> "prepared", r3 |-> "prepared"],
    [r1 |-> "working", r2 |-> "prepared", r3 |-> "committed"],
    [r1 |-> "working", r2 |-> "prepared", r3 |-> "aborted"],
    [r1 |-> "working", r2 |-> "committed", r3 |-> "working"],
    [r1 |-> "working", r2 |-> "committed", r3 |-> "prepared"],
    [r1 |-> "working", r2 |-> "committed", r3 |-> "committed"],
    [r1 |-> "working", r2 |-> "committed", r3 |-> "aborted"],
    [r1 |-> "aborted", r2 |-> "prepared", r3 |-> "aborted"],
    [r1 |-> "aborted", r2 |-> "committed", r3 |-> "working"],
```

TPTypeOK

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\subseteq means “subset or equal”

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msgs

msgs

- The way we will model message transfer is by keeping a global set of all messages ever sent, **msgs**

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- Messages will be added to this set, but never removed
- The protocol behaves correctly in the face of messages never being removed
- This will make more sense after reading through the spec!

TPInit

```
TPInit ==
  (***************************************************************************)
  (* The initial predicate.                                                  *)
  (***************************************************************************)
  /\ rmState = [r \in RM |-> "working"]
  /\ tmState = "init"
  /\ tmPrepared   = {}
  /\ msgs = {}
```


TPInit

```
TPInit ==
  (***************************************************************************)
  (* The initial predicate.                                                  *)
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  /\ rmState = [r \in RM |-> "working"]
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TPInit

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```

Evaluate this expression. What do you get?

TPInit

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  /\ msgs = {}
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Evaluate this expression. What do you get?

Value:

```
[r1 |-> "working", r2 |-> "working", r3 |-> "working"]
```


TPInit

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  (***************************************************************************)
  (* The initial predicate.                                                  *)
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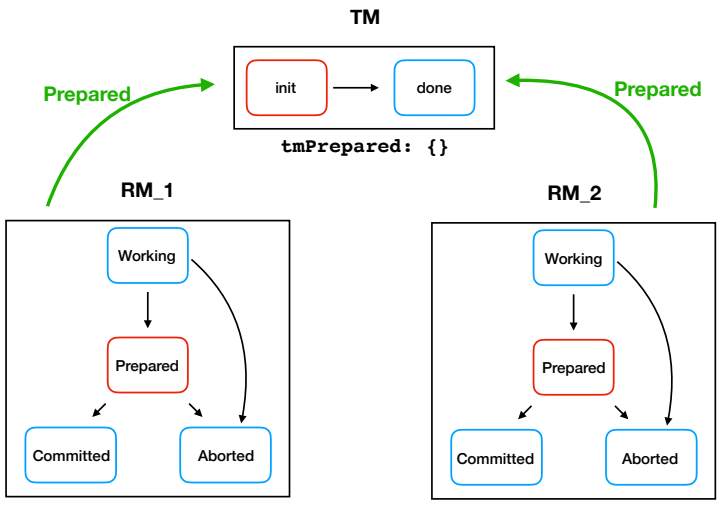
Evaluate this expression. What do you get?

Value:

```
[r1 |-> "working", r2 |-> "working", r3 |-> "working"]
```

This is a single function, where the DOMAIN is made up of resource managers, and the value associated to each is “working”

TMRcvPrepared(r)



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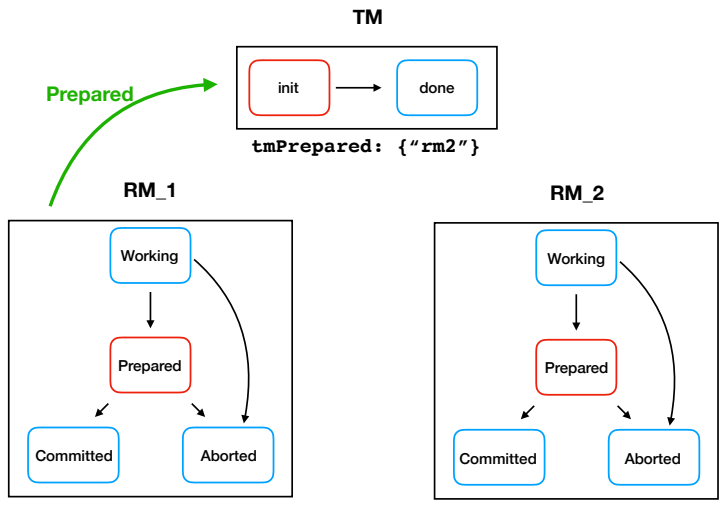

TMRcvPrepared(r)

```
TMRcvPrepared(r) ==
  /\ tmState = "init"
  /\ [type |-> "Prepared", rm |-> r] \in msgs
  /\ tmPrepared' = tmPrepared \union {r}
  /\ UNCHANGED <<rmState, tmState, msgs>>
```

This formula describes the TM receiving a “Prepared” message from a RM

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`tmState = “init"` means that the TM must be in the “init” state

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This formula has primed and unprimed variables, and is therefore an “action”

What are the enabling conditions?

`tmState = “init"` means that the TM must be in the “init” state

`[type |-> "Prepared", rm |-> r] \in msgs` means that a “Prepared” message from RM **r** must be present in the **msgs** set

TMRcvPrepared(r)

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This says that "in the next state, **tmPrepared** will be the result of taking the UNION of the current **tmPrepared** and **r**"

ex. If **r** is **"rm3"**, and **tmPrepared = {"rm1"}**, then in the next state, **tmPrepared** will be **{"rm1", "rm3"}**

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This is new

It says: “In the next state, **rmState** and **tmState** and **msgs** will be unchanged”

It is syntactic sugar for:

```
/\ rmState’ = rmState
/\ tmState’ = tmState
/\ msgs’ = msgs
```


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- Why would that be a valid next state if we didn’t have the **UNCHANGED**?
- Because the possible set of next states is infinite, and the purpose of our formula is to identify the “valid” ones

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  /\ rmState[r] = "working"
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- It is valid for an RM **r** whenever that **r** is in a “working” state
- In the new state:
  - **rmState** has that **RM** set to “prepared”
  - And a new message has been added to **msgs**, specifically, the message from RM **r** to the TM, telling it that it is prepared

TPNext

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  \/ TMCommit \/ TMAbort
  \/ \E r \in RM :
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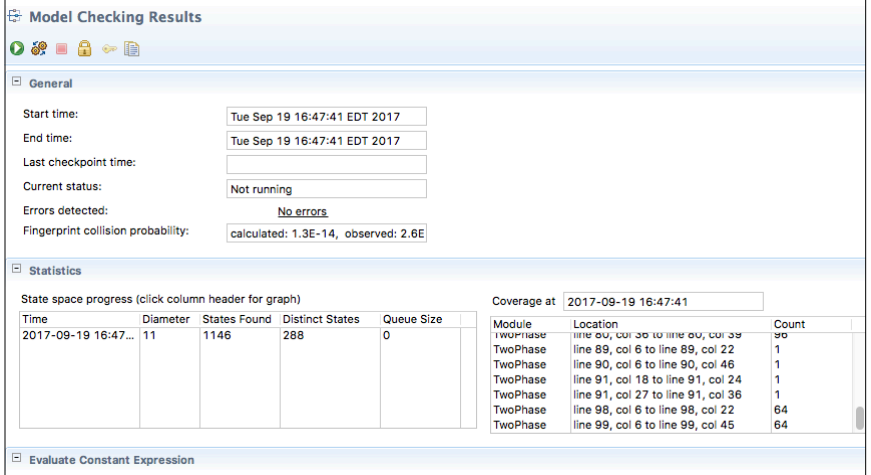
- This "imports" all variable declarations, constants, and definitions from the **TCommit** module
- This is how we got access to the **TCConsistent** invariant
- Because the model checker still passes with the **TCConsistent** invariant enabled, it means that the **TwoPhase** specification "implements" the **TCommit** specification

Exercise

- Replace the contents of your TwoPhase.tla with the contents of TwoPhase_incomplete.tla
- Try to write the correct definitions for
  - TMCommit
  - TMAbort
  - RMChooseToAbort
  - RMRcvCommitMsg
  - RMRcvAbortMsg



The End?

- This brings us to the end of Video 6 in Lamport's series
  - TLA Video Series: https://rax.io/tla_videos
  - http://lamport.azurewebsites.net/video/videos.html
- You now know **most** of the important TLA+ syntax, but not all
  - `LET/IN` is an important one, so is `CHOOSE`
- I recommend watching Lamport's videos, as they'll offer a slightly different perspective and will solidify your understanding
- Thanks!