Precise and Verifiable Distributed and Concurrent System Design using TLA+

Jay Parlar
@parlar
#tla-workshop

Special Thanks

- Laura Santamaria
- Ben Meyer
- Landon Jurgens
- Josh Schairbaum
- Rahman Syed
- Jordan Griege
- Freddy Knuth

Workshop, NOT a lecture

PLEASE Ask Questions

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But please save philosophical pondering until after the workshop

Useful Resources

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- TLA Toolbox 1.5.3: https://rax.io/tlaplus
  - https://github.com/tlaplus/tlaplus/releases/tag/v1.5.3

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  - https://github.com/parlarjb/tla_workshop

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- TLA Home Page: https://rax.io/tla_home
  - http://lamport.azurewebsites.net/tla/tla.html

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- TLA Video Series: https://rax.io/tla_videos
  - http://lamport.azurewebsites.net/video/videos.html

Additional Resources

- https://learntla.com/introduction/
- http://lamport.azurewebsites.net/tla/hyperbook.html
- https://www.hillelwayne.com/post/modeling-deployments/
- http://lamport.azurewebsites.net/tla/summary.pdf
- http://lamport.azurewebsites.net/tla/book.html
- https://lorinhochstein.wordpress.com/2014/06/04/crossing-the-river-with-tla/

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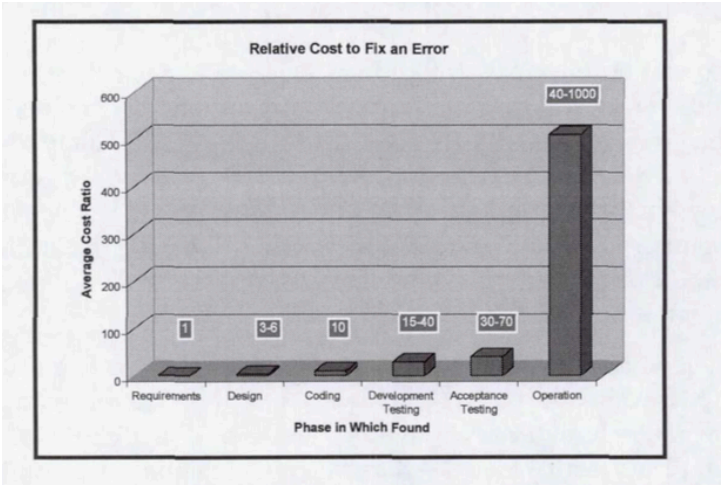
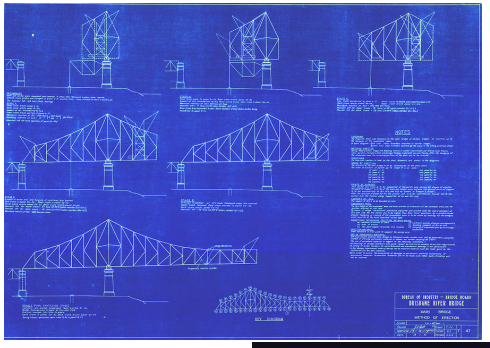
- Introduce the model checker, TLC
- Provide a foundation on which you can continue learning
- Convince everyone that thinking **early** is a good thing
- World domination

6-7



"Error Cost Escalation Through the Project Life Cycle"
https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/20100036670.pdf

7



8

```
algorithm ford-fulkerson is
    input: Graph G with flow capacity c,
           source node s,
           sink node t
    output: Flow f such that f is maximal from s to t

    (Note that f(u,v) is the flow from node u to node v, and c(u,v) is the flow capacity from
node u to node v)

    for each edge (u, v) in Gf do
        f(u, v) ← 0
        f(v, u) ← 0

    while there exists a path p from s to t in the residual network Gf do
        let cf be the flow capacity of the residual network Gf
        cf(p) ← min{cf(u, v) | (u, v) in p}
        for each edge (u, v) in p do
            f(u, v) ←  f(u, v) + cf(p)
            f(v, u) ← −f(u, v)

    return f
```

https://en.wikipedia.org/wiki/Pseudocode

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“Writing a specification helps you think clearly. Thinking clearly is hard; we can use all the help we can get. Making specification part of the design process can improve design.”

- Leslie Lamport

set the default status to "fail"
look up the message based on the error code
if the error code is valid
if doing interactive processing, display the error message
interactively and declare success
if doing command line processing, log the error message to the
command line and declare success
if the error code isn't valid, notify the user that an
internal error has been detected
return status information

https://blog.codinghorror.com/pseudocode-or-code/

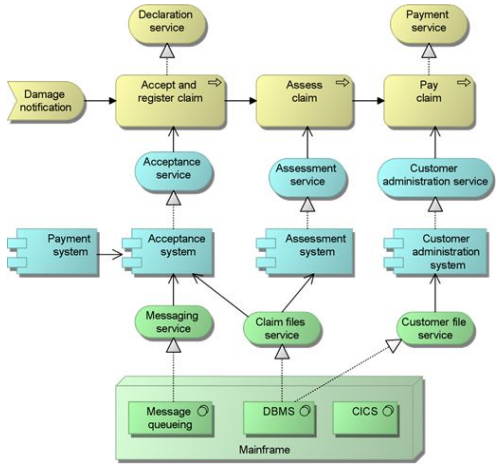
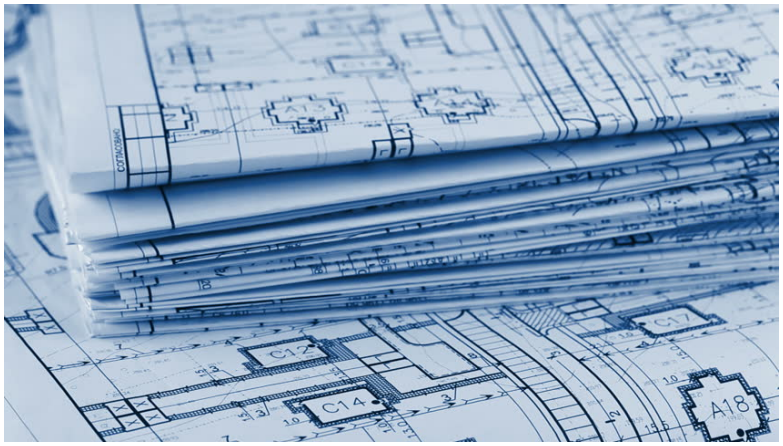


Image via https://en.wikipedia.org/wiki/ArchiMate



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Dr. Leslie Lamport

- Turing Award winner
- Winner of first ever “Dijkstra Prize in Distributed Computing” award
- Byzantine Generals’ Problem
- Paxos (which powers Big Table, Google File System, etc)
- Bakery Algorithm
- LaTeX
- PlusCal / TLA+

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15-1

15-2

Amazon, Microsoft, etc.

System	Components	Benefit
S3	- Fault-tolerant low-level network algorithm - Background redistribution of data	Multiple bugs found in design and optimization phases
DynamoDB	Replication and group-membership systems	Found multiple design bugs, some requiring 35-step traces
EC2	Fault tolerant replication improvement, including zero-downtime deployment	Found design bug
Xbox 360	Memory Interface	Found a bug that would hard-lock the Xbox after four hours

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"TLA+ is the most valuable thing that I've learned in my professional career. It has changed how I work by giving me an immensely powerful tool to find subtle flaws in system designs. It has changed how I think…"

- Chris Newcombe (former AWS Principal Engineer)

17-3

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- Every digital system can be modelled as a set of variables and actions.
- “Model checker” discovers **every single possible state** of the described system, looks for “unwanted” states.
- The language is called **TLA+**; the model checker tool is called **TLC**.

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- **https://raft.github.io**

Raft

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           \/ \E i, j \in Server : RequestVote(i, j)
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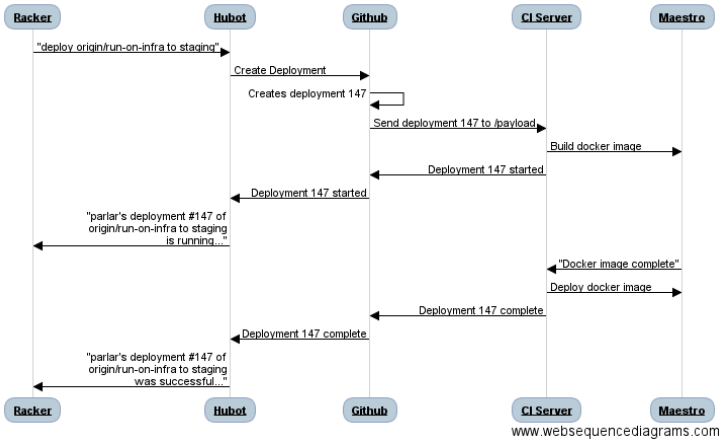
Encore Origin Deployments




```
\* This represents CI receiving a message from Maestro, when Maestro
\* has finished trying to build an image
HandleImageBuildDone(m) ==
    /\ \/ /\ m.type = PRBuildComplete
          /\ CASE m.status = "success"  -> UpdatePR(m.sha, "success", m)
                  [] m.status = "error"    -> UpdatePR(m.sha, "error", m)
                  [] m.status = "failure"  -> UpdatePR(m.sha, "failure", m)
       \/ /\ m.type = DeployBuildComplete
          /\ CASE m.status = "success"              -> MaestroDeploy(m.sha, m.id, m )
                  [] m.status \in {"error", "failure"}  -> UpdateDeployStatus(m.sha, m.id, "failure", m)

    /\ UNCHANGED <<next_id, registry, commit_status, maestro_deploys_complete, gh_deploys_complete>>

\* Represents CI receiving a NewDeployment message from GH
HandleNewDeploy(m) ==
  /\ m.type = NewDeployment
  /\ \/ /\ m.sha \in registry
        /\ MaestroDeploy(m.sha, m.id, m) \* We already have the image, so tell Maestro to go ahead and
     \/ /\ m.sha \notin registry
        /\ BuildDeployImage(m.sha, m.id, m) \* We don't have the image, so tell Maestro to build it
  /\ UNCHANGED <<registry, next_id, commit_status, maestro_deploys_complete, gh_deploys_complete>>


\* Represents CI receiving a deployment success/failure message from Maestro. Only "success" and "fail
HandleDeployComplete(m) ==
    /\ m.type = DeployImageComplete
    /\ \/ /\ m.status = "success"
          /\ UpdateDeployStatus(m.sha, m.id, "success", m)
       \/ /\ m.status = "failure"
          /\ UpdateDeployStatus(m.sha, m.id, "failure", m)

    /\ UNCHANGED <<registry, next_id, commit_status, maestro_deploys_complete, gh_deploys_complete>>

\* Represents GH receiving a commit_status status update from CI
HandlePRStatusUpdate(m) ==
  /\ m.type = PRStatus
  /\ commit_status' = [commit_status EXCEPT ![m.sha] = m.status]
  /\ Discard(m)
```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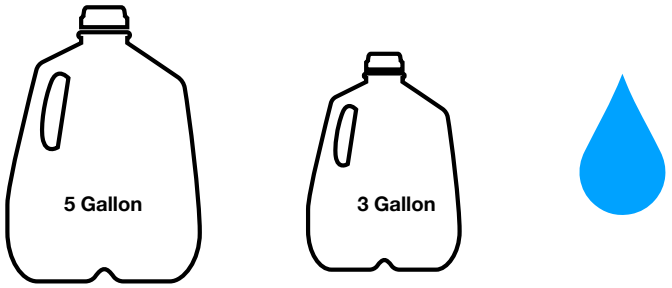
- Some people find it easier to read and write than TLA+.
- TLC compiles PlusCal code into TLA+.
- Hard to learn by starting with PlusCal. Better to gain a foundation in TLA+ then take advantage of PlusCal where appropriate.

How do we measure out exactly 4 gallons?

i.e. “The Die Hard Problem”



One Possible Execution

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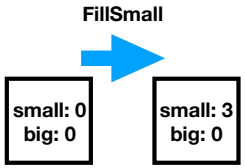

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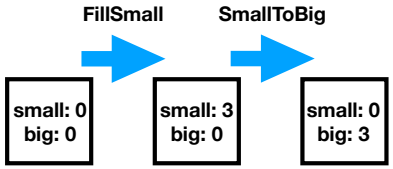

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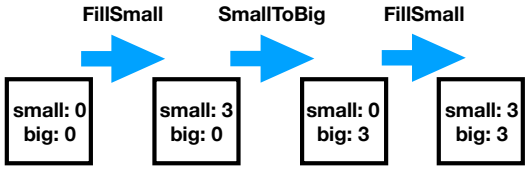

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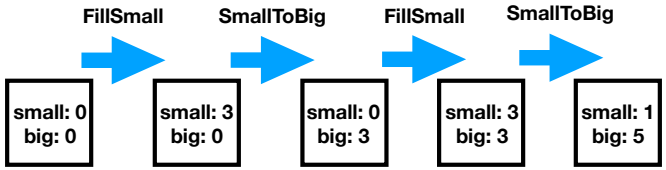

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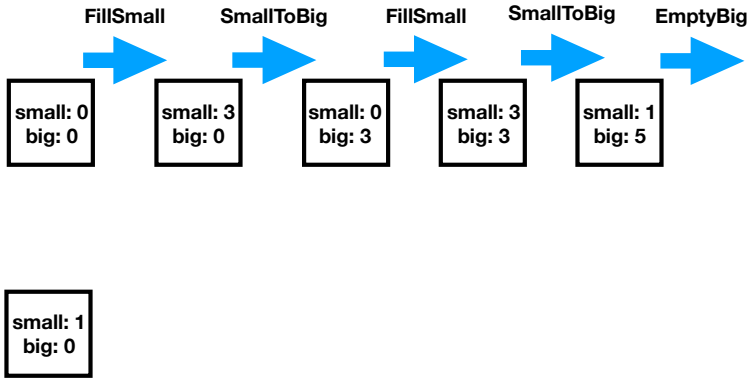

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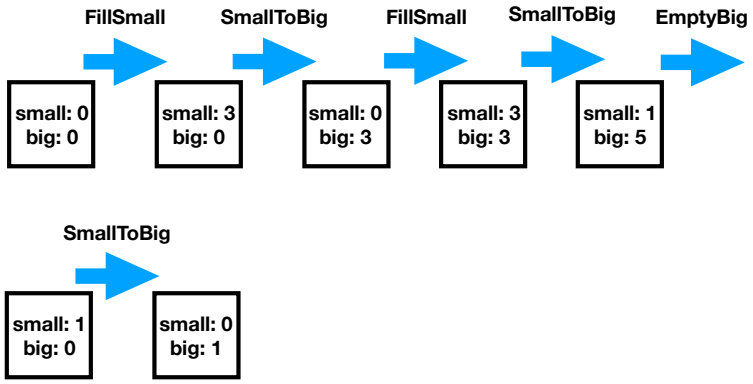

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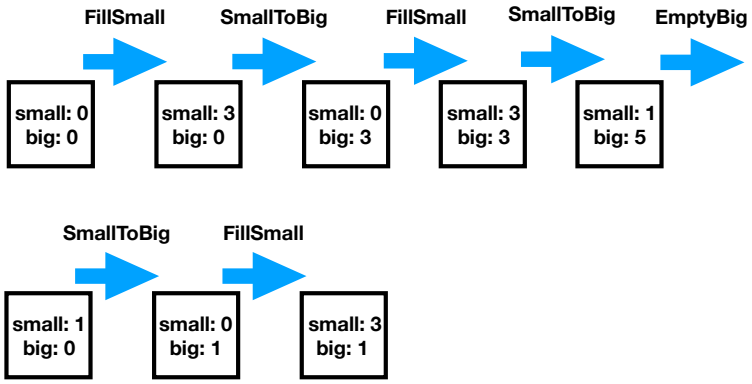

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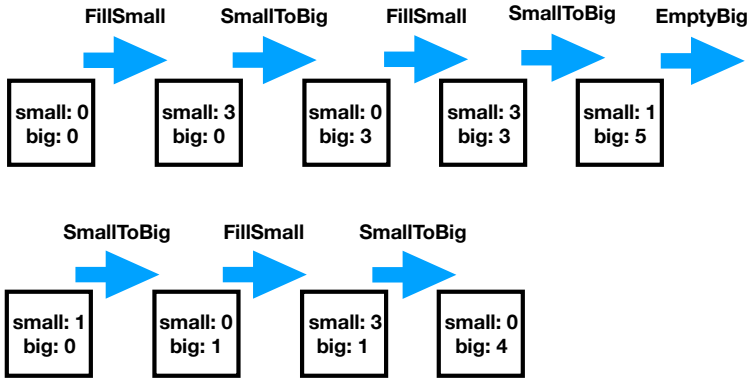

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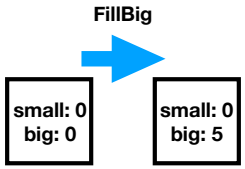

27-2

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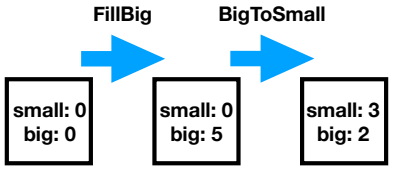

27-3

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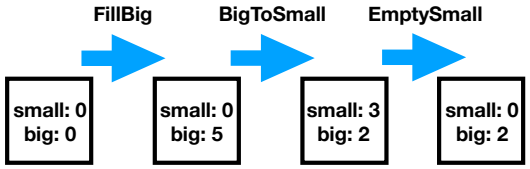

27-4

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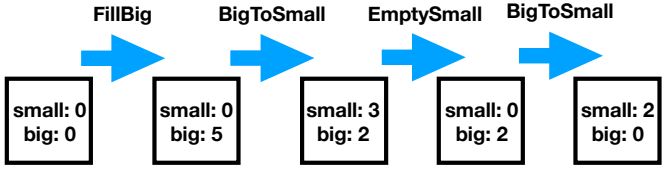

27-5

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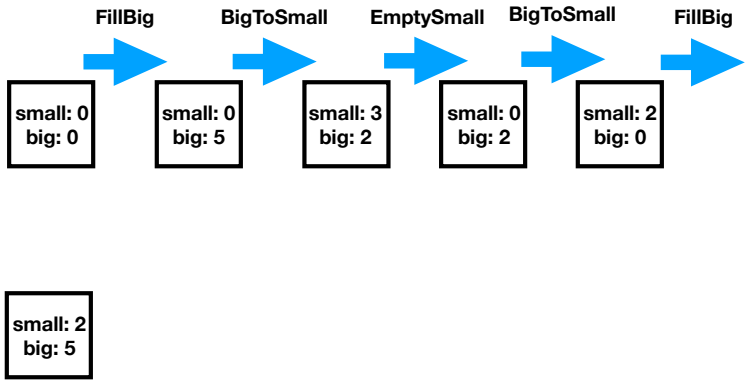

27-6

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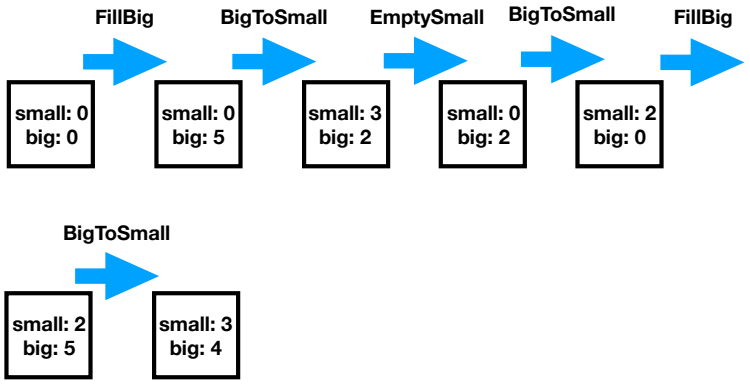

27-7

Yet Another Possible Execution

28-1

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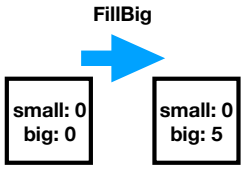
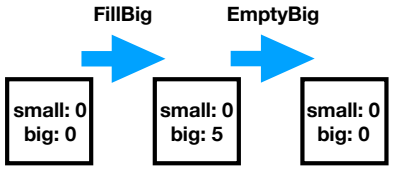


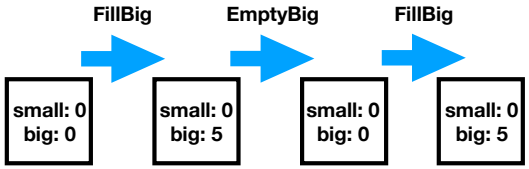
28-2

Yet Another Possible Execution



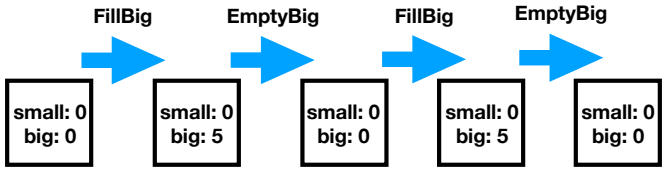
28-3

Yet Another Possible Execution



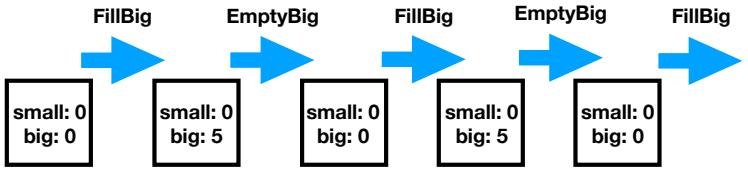
28-4

Yet Another Possible Execution

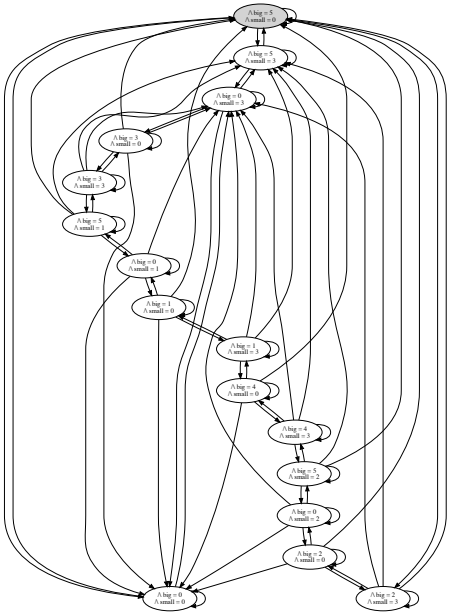


28-5

Yet Another Possible Execution



28-6



29

TLA+ Terms

30-1

TLA+ Terms

- A single one of those executions is called a **behaviour**.

30-2

TLA+ Terms

- A single one of those executions is called a **behaviour**.
- A behaviour is a **sequence of states**.

30-3

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TLA+ Terms

- A single one of those executions is called a **behaviour**.
- A behaviour is a **sequence of states**.
- A **step** is the change from one state to the next.
- A **state** is an assignment of **values** to **variables**.
- TLA+ is all about describing **all possible executions/ behaviours** of a system.

How could we describe digital systems (behaviours)?

- Programming languages
- Turing machines
- Automata
- Hardware Description Languages

We'll instead use State Machines!

A state machine is defined by three things:

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A state machine is defined by three things:

1. A listing of all of our **variables**.

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2. All the possible **initial states**.

We'll instead use State Machines!

A state machine is defined by three things:

1. A listing of all of our **variables**.
2. All the possible **initial states**.
3. The possible **next states** for any given state (i.e., state transitions), or, the **relationship** between the values of the **variables** in the current state, and their possible values in the **next state**.

Die Hard State Machine

Die Hard State Machine

1. Variables: **big** and **small**

Die Hard State Machine

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Die Hard State Machine

1. Variables: **big** and **small**
2. Possible initial values: **big=0**, **small=0**
3. Possible next states: …… ?

Die Hard Next States

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2.We can always fill **big**, i.e., in the next state **big = 5**

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5.We can always pour small into big, i.e., in the next state:

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if (big + small) <= 5 THEN big=big+small; small=0

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if (big + small) <= 5 THEN big=big+small; small=0

ELSE big = 5; small = small - (5 - big)

Die Hard Next States

1.We can always fill **small**, i.e., in the next state **small=3**

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if (big + small) <= 5 THEN big=big+small; small=0

ELSE big = 5; small = small - (5 - big)

etc. etc.

Possible next states

small: 3
big: 2

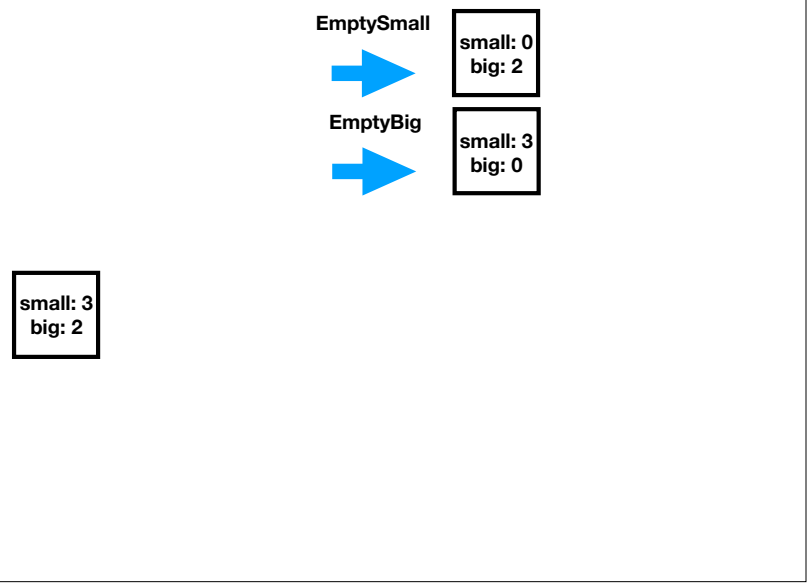
Possible next states

EmptySmall

small: 0
big: 2

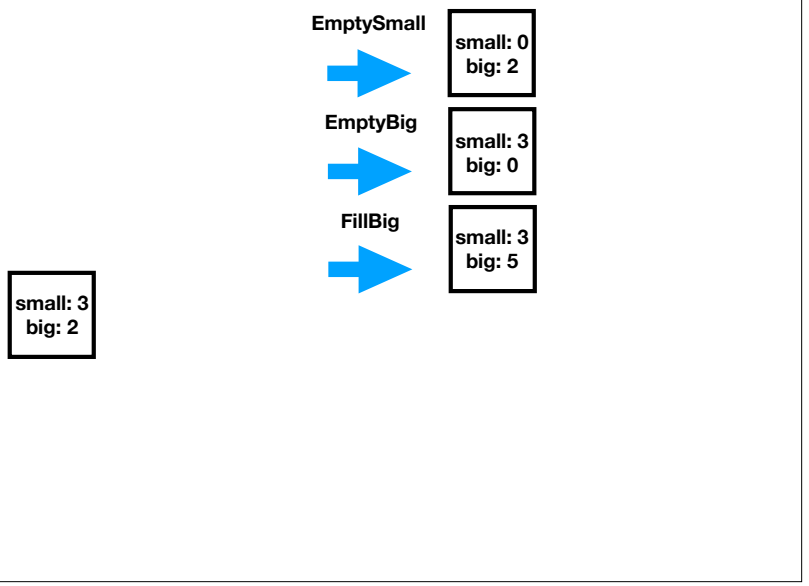
small: 3
big: 2

Possible next states



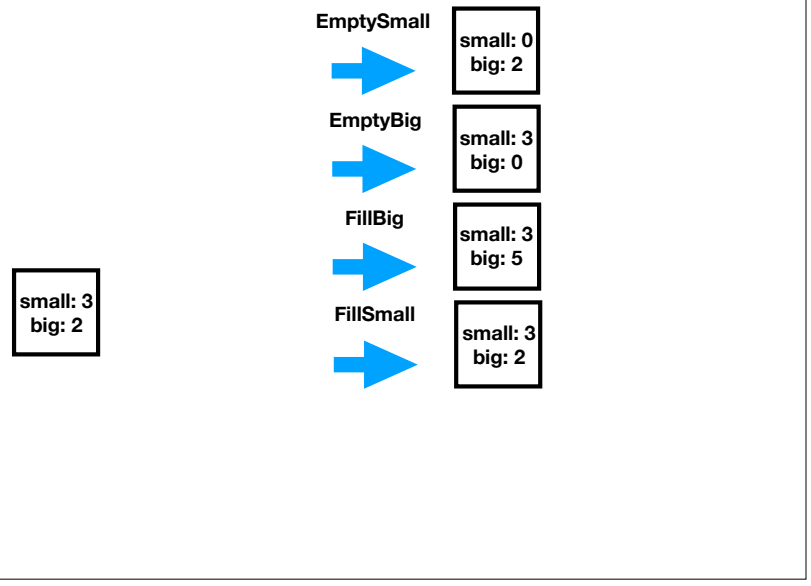
35-3

Possible next states



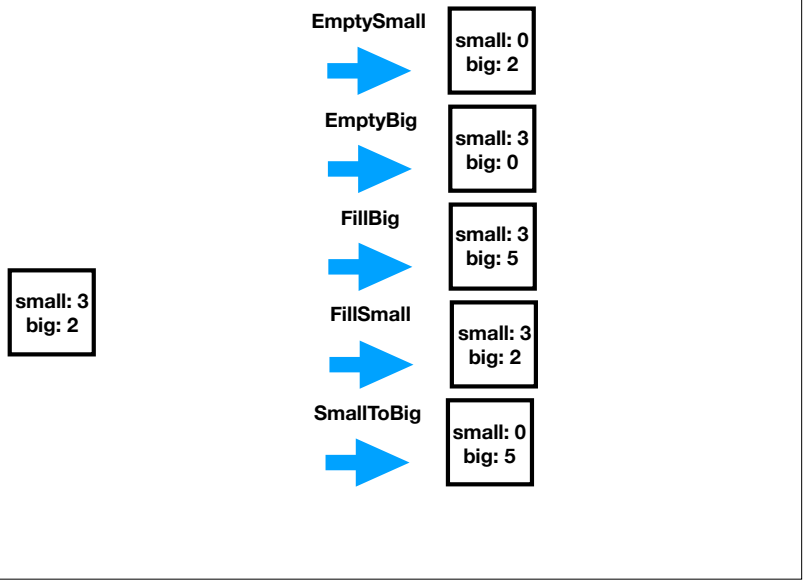
35-4

Possible next states



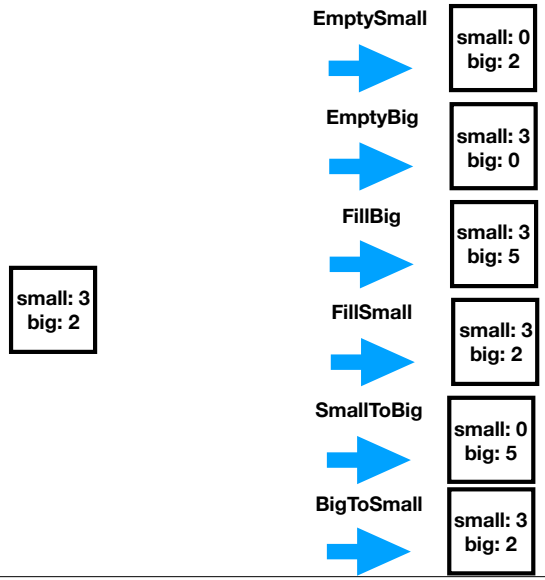
35-5

Possible next states



35-6

Possible next states




```
VARIABLES small, big

TypeOK == /\ small \in 0..3
          /\ big   \in 0..5

Init == /\ big   = 5
        /\ small = 0

FillSmall == /\ small' = 3
             /\ big'   = big

FillBig == /\ big'   = 5
           /\ small' = small

EmptySmall == /\ small' = 0
              /\ big'   = big

EmptyBig == /\ big'   = 0
            /\ small' = small

SmallToBig == IF big + small =< 5
              THEN /\ big'   = big + small
                   /\ small' = 0
              ELSE /\ big'   = 5
                   /\ small' = small - (5 - big)

BigToSmall == IF big + small =< 3
              THEN /\ big'   = 0
                   /\ small' = big + small
              ELSE /\ big'   = small - (3 - big)
                   /\ small' = 3

Next == \/ FillSmall  \/ FillBig
        \/ EmptySmall \/ EmptyBig
        \/ SmallToBig \/ BigToSmall
```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- Programming languages represent different parts of the state differently:
    - Values of variables
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    - Call stack
    - Heap
    - etc.
- Programming languages are **terrible** at representing non-determinism.

Math Time!!!

“and” / “or”

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$\wedge$ Logical “AND”. Called a “conjunct”. Written in ASCII as `/\`
Written in Python as **`and`**. Written in C as **`&&`**

```
TRUE  /\ TRUE  = TRUE
TRUE  /\ FALSE = FALSE
FALSE /\ FALSE = FALSE
FALSE /\ TRUE  = FALSE
```


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$\vee$ Logical “OR”. Called a “disjunct”. Written in ASCII as `\/`
Written in Python as **`or`**. Written in C as **`||`**

```
TRUE  \/ TRUE  = TRUE
TRUE  \/ FALSE = TRUE
FALSE \/ FALSE = FALSE
FALSE \/ TRUE  = TRUE
```


= and $\triangleq$

= and ≜

= means "equality". It is NOT an assignment operator. It is a boolean operator. It is the = you would have learned in grade 1 mathematics. It's equivalent to Python's **==.**

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```
MyDef == A /\ B
```

At any time, **MyDef** will be the value of **A** and'ed with **B**

It is a little like using a macro, or an inline.

Exercise

```
TRUE \/ (TRUE /\ FALSE)
```

```
TRUE /\ (FALSE \/ (TRUE \/ FALSE))
```

```
FALSE /\ (TRUE /\ ((FALSE \/ TRUE)\/(TRUE \/ FALSE)))
```

Exercise

```
TRUE \/ (TRUE /\ FALSE)                                    TRUE

TRUE /\ (FALSE \/ (TRUE \/ FALSE))

FALSE /\ (TRUE /\ ((FALSE \/ TRUE)\/(TRUE \/ FALSE)))
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Exercise

```
TRUE \/ (TRUE /\ FALSE)                                    TRUE

TRUE /\ (FALSE \/ (TRUE \/ FALSE))                         TRUE

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Exercise

```
TRUE \/ (TRUE /\ FALSE)                                    TRUE

TRUE /\ (FALSE \/ (TRUE \/ FALSE))                         TRUE

FALSE /\ (TRUE /\ ((FALSE \/ TRUE)\/(TRUE \/ FALSE)))      FALSE
```


TLA + Syntax

TLA+ specs make heavy use of formulas like this

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A /\ (B /\ ((C \/ D)\/(E /\ F))) /\ (G \/ H \/ I)
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It's like using a bulleted-list of /\ or \/ symbols

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TLA+ Syntax

In general:

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```
A /\ B /\ C /\ … /\ N      =      /\ A
                                  /\ B
                                  /\ C
                                  …
                                  /\ N
```


TLA+ Syntax

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```
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                                  /\ B
                                  /\ C
                                  …
                                  /\ N

A \/ B \/ C \/ … \/ N      =      \/ A
                                  \/ B
                                  \/ C
                                  …
                                  \/ N
```


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In general:

```
A /\ B /\ C /\ … /\ N      =      /\ A
                                  /\ B
                                  /\ C
                                  …
                                  /\ N

A \/ B \/ C \/ … \/ N      =      \/ A
                                  \/ B
                                  \/ C
                                  …
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```


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```
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                                  /\ B
                                  /\ C
                                  …
                                  /\ N

A \/ B \/ C \/ … \/ N      =      \/ A
                                  \/ B
                                  \/ C
                                  …
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```

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A /\ B /\ C /\ … /\ N  =
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/\ B
/\ C
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/\ N
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```
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In general:

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```
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```
(A /\ B) \/ (C /\ D)
```

(We’ll come back to this.)

TLA+ Syntax

- The indentation level is meaningful.
- Expressions on the same indent level are “and”ed and “or”ed together.

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
```


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- This formula has four items at the top level, being AND’ed together. Let’s rewrite using TLA+ syntax.

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```

- This formula has four items at the top level, being AND’ed together. Let’s rewrite using TLA+ syntax.
- We’ll put /\ at the outermost indent level.

TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
```


TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)

/\ A
```


TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)

/\ A
/\ (B \/ C \/ D)
```

TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
```

```
/\ A
/\ (B \/ C \/ D)
/\ E
```


TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
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/\ A
/\ (B \/ C \/ D)
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/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

Now we can easily see all the “outermost” or “top-level” items being AND’ed together

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/\ (F /\ G)
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Now we can easily see all the “outermost” or “top-level” items being AND’ed together

We have more parentheses.

TLA+ Syntax

```
A /\ (B \/ C \/ D) /\ E /\ (F /\ G)
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/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

Now we can easily see all the “outermost” or “top-level” items being AND’ed together

We have more parentheses.

Let’s get rid of those too

TLA+ Syntax

We’ll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```


TLA+ Syntax

We’ll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

```
/\ A
```


TLA+ Syntax

We’ll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)
```

```
/\ A
/\ \/ B
```

TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
   \/ C
```


TLA+ Syntax

We'll create indent levels for each of these

```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
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   \/ C
   \/ D
```


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/\ A
/\ \/ B
   \/ C
   \/ D
/\ E
```


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```
/\ A
/\ (B \/ C \/ D)
/\ E
/\ (F /\ G)

/\ A
/\ \/ B
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   \/ D
/\ E
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```

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/\ A
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   \/ C
   \/ D
/\ E
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   /\ G
```


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```

```
/\ A
/\ \/ B
   \/ C
   \/ D
/\ E
/\ /\ F
   /\ G
```

The structure of the formula should be much more clear now.
It creates an explicit graph-like visual structure out of parentheses.
Or think of it like bullet-points, if that helps .

TLA+ Syntax

What about this? How would this look?

```
A /\ B \/ C /\ D
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\/ /\ C
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```
\/ /\ A
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   /\ D
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\/ /\ C
   /\ D
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Which becomes

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\/ /\ A
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\/ /\ C
   /\ D
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Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```


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```
(A /\ B) \/ (C /\ D)
```

Which becomes

```
\/ /\ A
   /\ B
\/ /\ C
   /\ D
```


Exercise

1. `A \/ (B /\ (C \/ D) /\ E)`

2. `(A /\ B /\ (C \/ D)) /\ (E \/ F \/ G) /\ (H \/ (I /\ J))`

3. `(A \/ (B /\ C) \/ D) /\ E /\ F /\ (G \/ H \/ (I /\ J))`

Solutions

1. A \/ (B /\ (C \/ D) /\ E)

\/ A
\/ /\ B
   /\ C \/ D
   /\ E

Solutions

1. A \/ (B /\ (C \/ D) /\ E)

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Solutions

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Solutions

1. A \/ (B /\ (C \/ D) /\ E)

```
\/ A
\/ /\ B
   /\ C \/ D
   /\ E
```

Alternate Solution

```
\/ A
\/ /\ B
   /\ \/ C
      \/ D
   /\ E
```


Solutions

2. (A /\ B /\ (C \/ D)) /\ (E \/ F \/ G) /\ (H \/ (I /\ J))

```
/\ /\ A
   /\ B
   /\ C \/ D
/\ \/ E
   \/ F
   \/ G
/\ \/ H
   \/ I /\ J
```


Solutions

2. (A /\ B /\ (C \/ D)) /\ (E \/ F \/ G) /\ (H \/ (I /\ J))

```
/\ /\ A
   /\ B
   /\ C \/ D
/\ \/ E
   \/ F
   \/ G
/\ \/ H
   \/ I /\ J
```

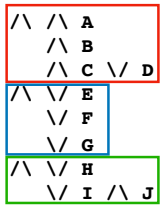

Solutions

2. (A /\ B /\ (C \/ D)) /\ (E \/ F \/ G) /\ (H \/ (I /\ J))

```
/\ /\ A
   /\ B
   /\ C \/ D
/\ \/ E
   \/ F
   \/ G
/\ \/ H
   \/ I /\ J
```

Solutions

2. (A /\ B /\ (C \/ D)) /\ (E \/ F \/ G) /\ (H \/ (I /\ J))



Solutions

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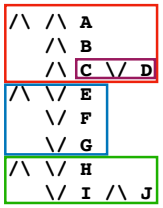

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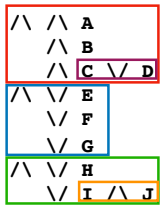

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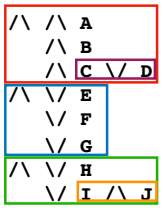

Alternate Solution

```
/\ /\ A
   /\ B
   /\ \/ C
      \/ D
/\ \/ E
   \/ F
   \/ G
/\ \/ H
   \/ /\ I
      /\ J
```

Solutions

3. (A \/ (B /\ C) \/ D) /\ E /\ F /\ (G \/ H \/ (I /\ J))

```
/\ \/ A
   \/ B /\ C
   \/ D
/\ E
/\ F
/\ \/ G
   \/ H
   \/ I /\ J
```


Solutions

3. (A \/ (B /\ C) \/ D) /\ E /\ F /\ (G \/ H \/ (I /\ J))

```
/\ \/ A
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/\ F
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```


Solutions

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Sets

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- {“a”, “b”} = {“b”, “a”} (i.e. order does not matter).
- The concept maps to Python’s `set()`.

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  - {“a”, “b”} ∪ {“b”, “c”} = {“a”, “b”, “c”}
    - Python:
      `set([“a”,”b”]).union(set([“b”, “c”]))`

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  - {“a”, “b”} ∩ {“b”, “c”} = {“b”}
  - Python:
    - `set([“a”, “b”]).intersection(set([“b”, “c”]))`

Sets

Sets

- **`..`** is roughly equivalent to Python’s **`range()`** function

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  - 1..10 = {1,2,3,4,5,6,7,8,9,10}

Sets

- **..** is roughly equivalent to Python's **`range()`** function
  - 1..10 = {1,2,3,4,5,6,7,8,9,10}
  - In Python: **`set(range(1,11))`**

Exercises

(TRUE or FALSE for each)

{“a”, “b”} ⊆ {“b”, “a”, “c”}

{1,2,3} ⊆ ({1,2,3} ∩ {2,3})

“a” ∈ (({“b”, “c”, “d”} ∩ {“e”, “f”}) ∪ {“a”})

(“a” ∈ {“a”, “b”}) ∨ (“b” ∈ {“c”, “d”})

Exercises

(TRUE or FALSE for each)

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“a” ∈ (({“b”, “c”, “d”} ∩ {“e”, “f”}) ∪ {“a”})

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“a” ∈ (({“b”, “c”, “d”} ∩ {“e”, “f”}) ∪ {“a”}) TRUE

(“a” ∈ {“a”, “b”}) ∨ (“b” ∈ {“c”, “d”}) TRUE

“There exists”

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- The usual form is ∃ x ∈ S : P(x)

“There exists”

- ∃ means “there exists”, written as **\E** in ASCII
- The usual form is ∃ x ∈ S : P(x)
  - This means “there exists some x in the set S such that P(x) is TRUE”

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  - ∃ x∈{1,2,3,4} : x > 3 is TRUE
  - ∃ x∈{1,2,3,4} : x > 5 is FALSE

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is roughly equivalent to the following Python expressions

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$\exists\, x \in \{1,2,3,4,5\} : x > 5$

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```
def exists(S):
  for x in S:
    if x > 5:
      return True
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“There exists”

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is roughly equivalent to the following Python expressions

```
def exists(S):
  for x in S:
    if x > 5:
      return True
  return False

exists(set([1,2,3,4,5]))  # False
```


“There exists”

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is roughly equivalent to the following Python expressions

```
def exists(S):
  for x in S:
    if x > 5:
      return True
  return False

exists(set([1,2,3,4,5]))  # False

any(map(lambda x: x > 5, [1,2,3,4,5])) #False
```


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  - The entire expression evaluates to TRUE or FALSE
  - ∀ x∈{1,2,3,4} : x > 3 is FALSE
  - ∀ x∈{1,2,3,4} : x > 0 is TRUE

“For all”

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$\forall x \in \{1,2,3,4,5\} : x > 5$

is roughly equivalent to the following Python expressions

```
def for_all(S):
  for x in S:
    if not (x > 5):
      return False
  return True
```


“For all”

$\forall x \in \{1,2,3,4,5\} : x > 5$

is roughly equivalent to the following Python expressions

```
def for_all(S):
  for x in S:
    if not (x > 5):
      return False
  return True

for_all(set([1,2,3,4,5]))  # False
```


“For all”

$\forall x \in \{1,2,3,4,5\} : x > 5$

is roughly equivalent to the following Python expressions

```
def for_all(S):
  for x in S:
    if not (x > 5):
      return False
  return True

for_all(set([1,2,3,4,5]))  # False


all(map(lambda x: x > 5, [1,2,3,4,5]))  # False
```


Exercises

$\exists x \in \{1,2,3,4\} : (x > 3) \wedge (x < 4)$

$\exists x \in \{1,2,3,4\} : \exists y \in \{5, 6, 7\} : x < y$

$\forall x \in \{1,2,3\} : \{x\} \subseteq \{1, 2\}$

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$\forall x \in \{3, 4, 5\} : \wedge x > 1$
$\wedge x < 6$

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Set constructors

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{ x ∈ S : P } is like a **`filter()`** function, creating a new set consisting of the elements of **S** that satisfy **P**.

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{ x ∈ {1,2,3,4,5} : x > 3 } = {4,5}

This is equivalent to the following Python expressions:

```
set([x for x in [1,2,3,4,5] if x > 3])
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This is equivalent to the following Python expressions:

```
set([x for x in [1,2,3,4,5] if x > 3])


set(filter(lambda x: x > 3, [1,2,3,4,5]))
```


Set constructors

Set constructors

{ e : x ∈ S } is like a **`map()`** function, applying **e** to every element of **S**.

{ x * 2 : x ∈ {1,2,3,4,5}} = {2,4,6,8,10}

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This is roughly equivalent to the following Python expressions:

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{ x * 2 : x ∈ {1,2,3,4,5}} = {2,4,6,8,10}

This is roughly equivalent to the following Python expressions:

```
set([x*2 for x in [1,2,3,4,5]])

set(map(lambda x: x*2, [1,2,3,4,5]))
```

Exercises

1. Write a set constructor that creates a set of elements from **{1,2,3,4,5}**, consisting of the elements that are greater than **1** and less than **5** (i.e. we want **{2, 3, 4}**).

2. Write a set constructor that creates a set of elements from **{1,2,3,4,5}**, consisting of the elements that, when multiplied by **3**, are greater than **10** (i.e. we want **{4, 5}**).

3. Write a set constructor that creates a set of elements consisting of each element of **{1, 2, 3, 4, 5}** added to itself and then subtracting **1** (i.e. we want **{1, 3, 5, 7, 9}**).

Solutions

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1. $\{ x \in \{1,2,3,4,5\} : \wedge\ x > 1 \wedge x < 5 \}$ **or** $\{ x \in \{1,2,3,4,5\} : x > 1 \wedge x < 5 \}$

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2. $\{ x \in \{1,2,3,4,5\} : x*3 > 10\}$

Solutions

1. { x ∈ {1,2,3,4,5} : ∧ x > 1 ∧ x < 5 } **or** { x ∈ {1,2,3,4,5} : x > 1 ∧ x < 5 }

2. { x ∈ {1,2,3,4,5} : x*3 > 10}

3. { x+x-1 : x ∈ {1,2,3,4,5}}

Back to State Machines

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Recall that a state machine is defined by three things:

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1. The **variables**.

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Recall that a state machine is defined by three things:

1. The **variables**.
2. The possible **initial values** of the variables.
3. The possible **next states** from any given state (the relationship between the **values** of the **variables** in the current state and their possible **values** in the **next state**).

A small C program

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
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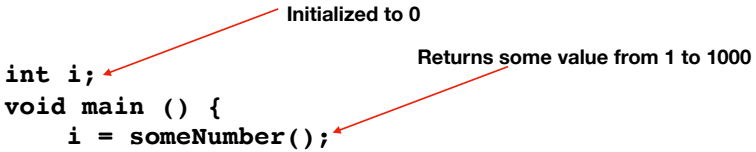


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int i;
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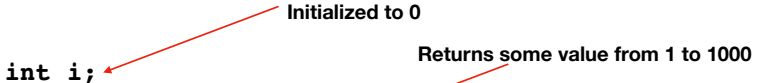


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}
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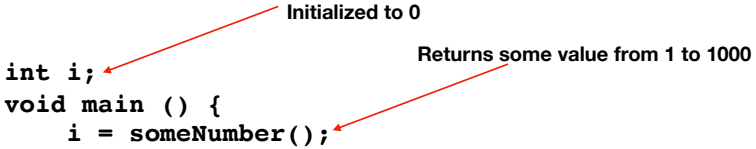
Let's exactly represent this C program as a state machine

A small C program

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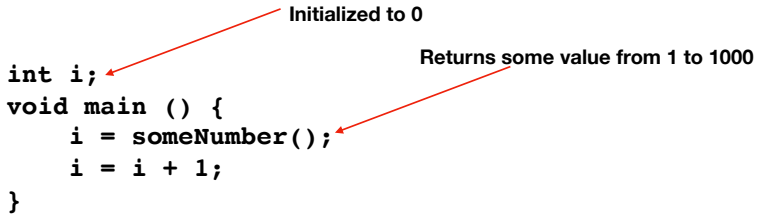
}
```

Let's exactly represent this C program as a state machine

- Variables: **i**

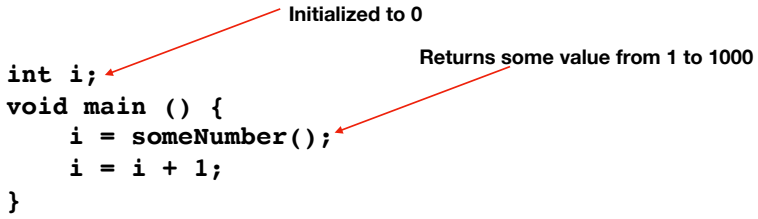
A small C program



Let's exactly represent this C program as a state machine

- Variables: **i**
- Possible initial values: **i = 0**

A small C program



Let's exactly represent this C program as a state machine

- Variables: **i**
- Possible initial values: **i = 0**
- Next states for some given state… let's see!

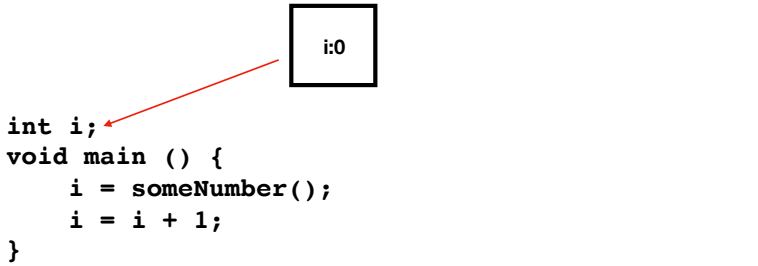
A small C program - example behaviour

Assume that for this particular *behaviour,* **`someNumber()`** returns **42**.

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```


A small C program - example behaviour

Assume that for this particular *behaviour,* **`someNumber()`** returns **42**.



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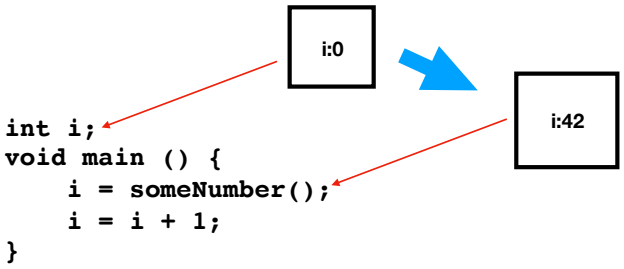

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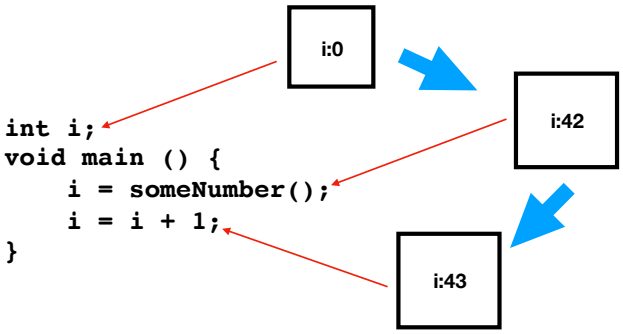

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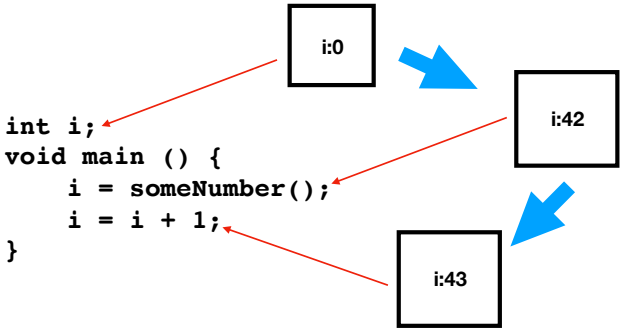
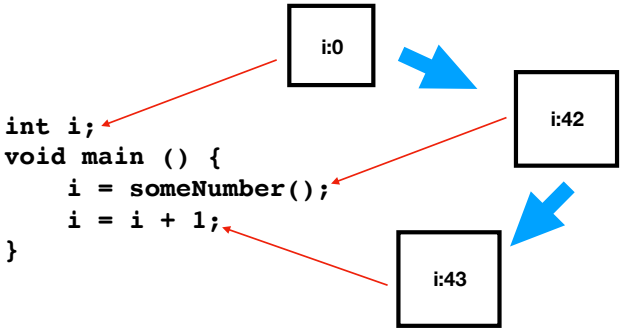


So we know that for the state **[i:42]**, the only next possible state is **[i:43]**.

A small C program - example behaviour

Assume that for this particular *behaviour,* **`someNumber()`** returns **42**.



So we know that for the state **[i:42]**, the only next possible state is **[i:43]**.
And for the state **[i:43]**, the only next state is “program termination”.

A small C program - example behaviour

Now let's look at a behaviour where **`someNumber()`** returned **43**.

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
}
```


A small C program - example behaviour

Now let's look at a behaviour where **`someNumber()`** returned **43**.


```
int i;
void main () {
    i = someNumber();
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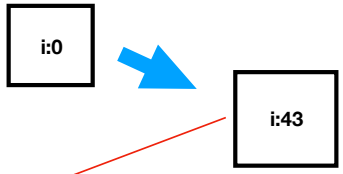
}
```


A small C program - example behaviour

Now let's look at a behaviour where **`someNumber()`** returned **43**.




```
int i;
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}
```


A small C program - example behaviour

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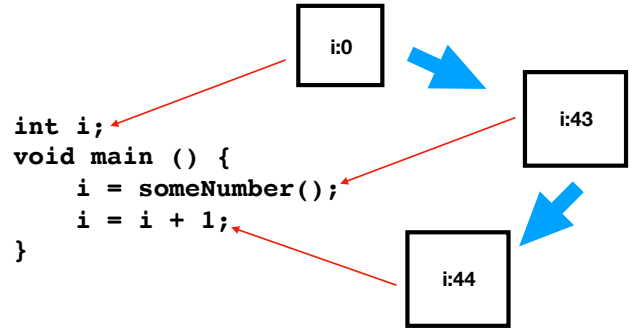


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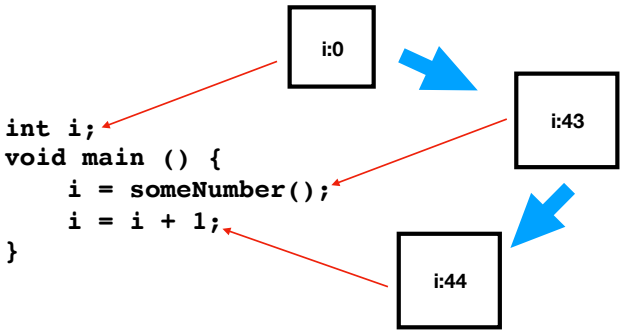
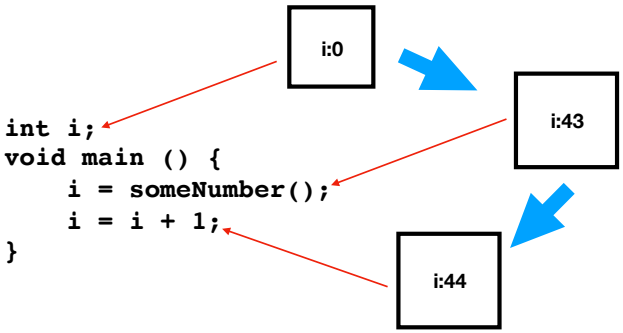


In this case, a state of **[i:43]** has a next state of **[i:44]**.

A small C program - example behaviour

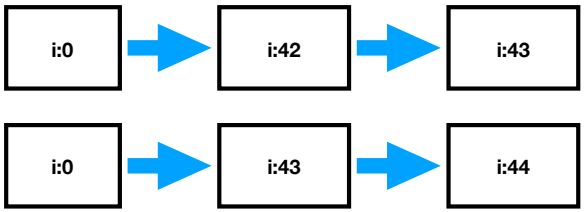
Now let's look at a behaviour where **`someNumber()`** returned **43**.



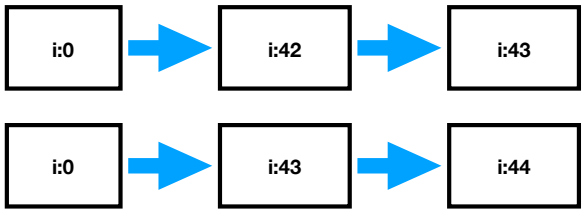
In this case, a state of **[i:43]** has a next state of **[i:44]**.

But that's different than the next state for **[i:43]** in the last behaviour!

43

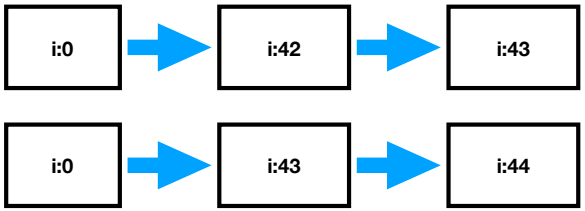


43



Thus representing the state of the program with just the variable **i** is not sufficient.

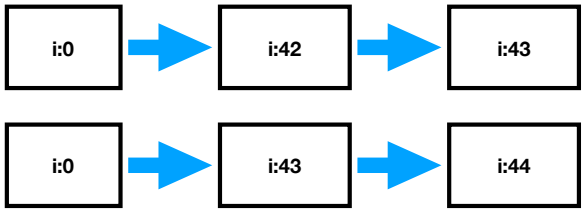
43



Thus representing the state of the program with just the variable **i** is not sufficient.

The two behaviours require different **next values** for **i** for the same **current value** of 43

43



Thus representing the state of the program with just the variable **i** is not sufficient.

The two behaviours require different **next values** for **i** for the same **current value** of 43

We must be able to exactly represent the program using the variables of our state machine.

A small C program - example behaviour

We can't represent this simple program using just the variable **i**

We need a second variable, **pc**, to represent "control state", i.e. where in the program we are

```
int i;
void main () {
    i = someNumber();
    i = i + 1;
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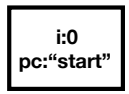


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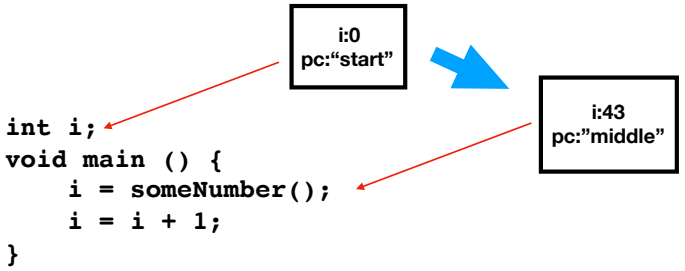

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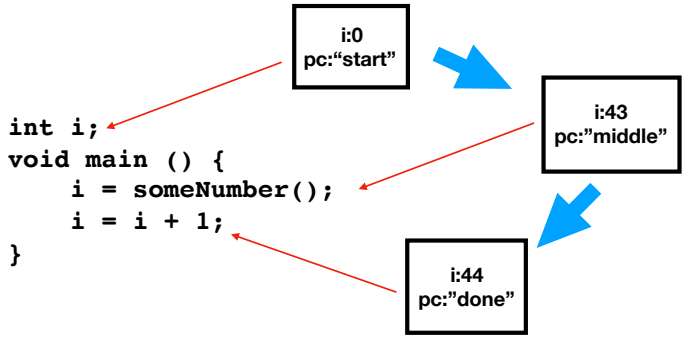

A small C program

Let's make this a bit more formal

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A small C program

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1. Variables: **i, pc**

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1. Variables: **i, pc**
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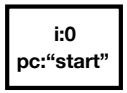

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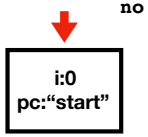

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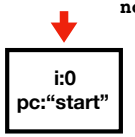

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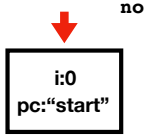

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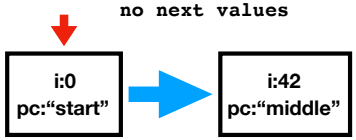

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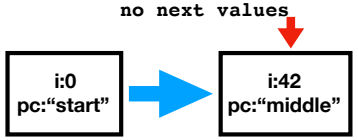

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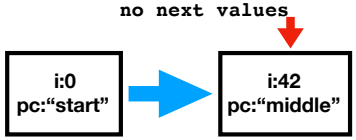

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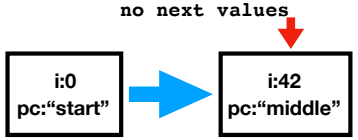

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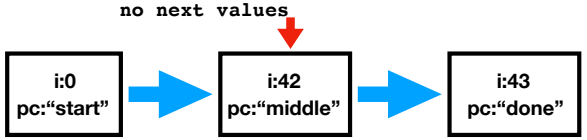

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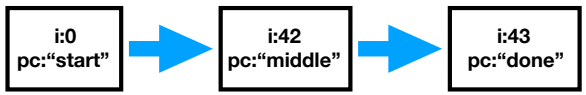

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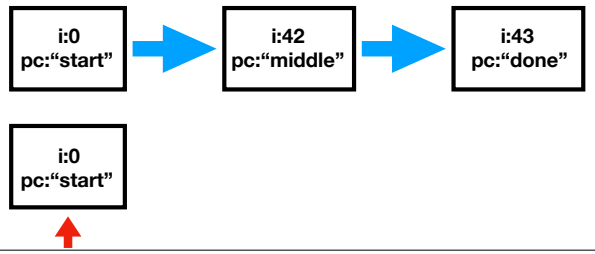

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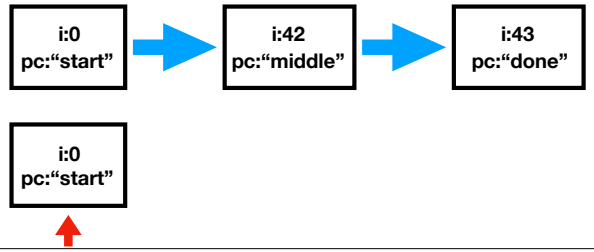

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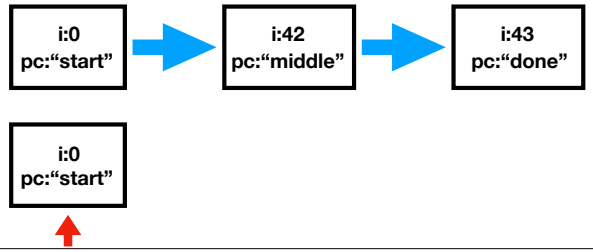

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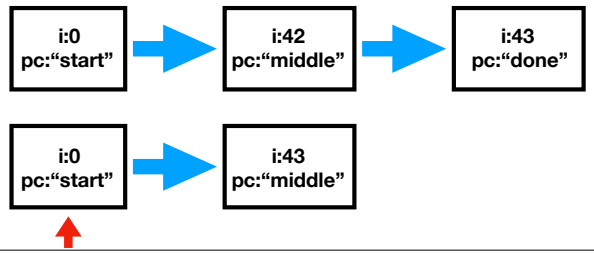

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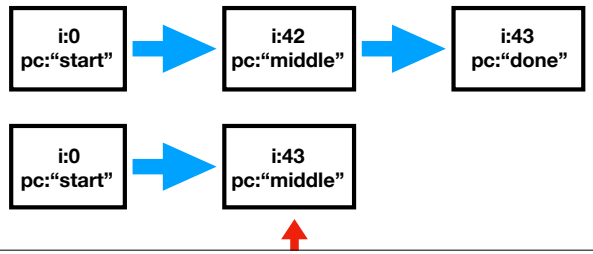

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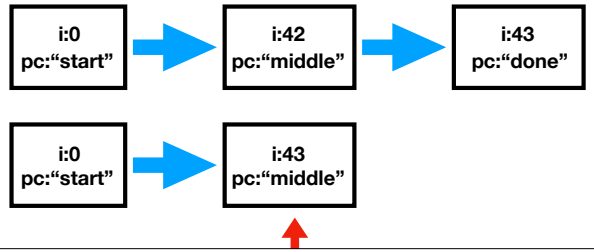

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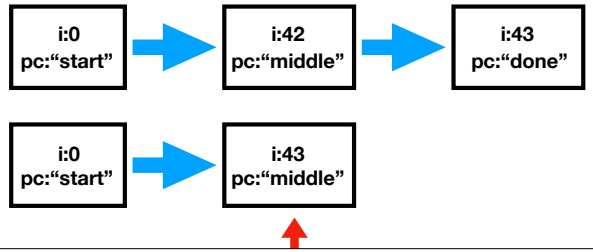

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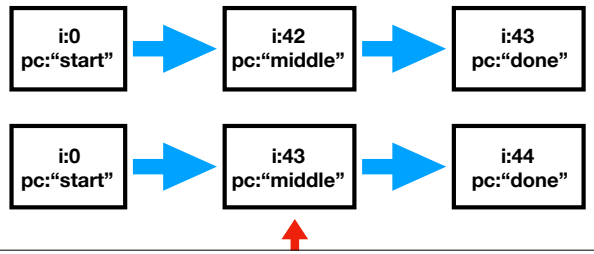

Improving the next state formula

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pc replaces current value of pc

Improving the next state formula

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pc replaces current value of pc

i replaces current value of i

Improving the next state formula

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i′ replaces next value of i

Improving the next state formula

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```

i′ replaces next value of i

pc′ replaces next value of pc

Improving the next state formula

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      no next values
```

`i'` **replaces** `next value of i`

`pc'` **replaces** `next value of pc`

These are called "**primed**" variables, and represent the value in the next state

Improving the next state formula

```
if pc = "start"
  then
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      i' = i + 1
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= doesn't mean assignment!!!

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This means:

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Again, NOT assignment, we are specifying the next possible states in a state machine.

State pairs

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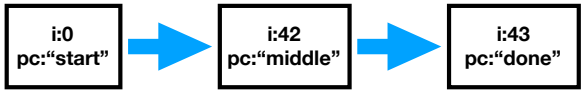

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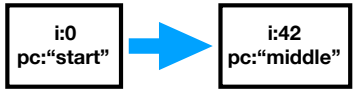

Current state Next state

State pairs

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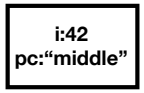

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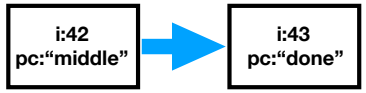

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State pairs

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- That third point is really a **formula** that defines all possible current<->next state pairs (See your cheat sheet for a reminder of what a **formula** is).
- This is SUPER important. Please stop me now with questions if you don't understand!

A little bit more formal

```
if pc = "start"
  then
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This is pretty formal, but to turn it into real TLA+, we need to go a bit further. This is a formula, it must always return a TRUE or FALSE

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The "**then**" and "**else**" clauses must return TRUE or FALSE

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      no next values
```

`(i' ∈ 1..1000) /\ (pc' = "middle")` **replaces** `i' ∈ 1..1000` `pc' = "middle"`

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```

`(i' = i + 1) /\ (pc' = "done")` replaces `i' = i + 1`
`pc' = "done"`

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`FALSE` replaces `no next values`

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```


Looking pretty good…

But we're not there just yet. Our formula is good, but we can make it better.

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```

When does this formula return TRUE?

There are two cases

Case 1

```
if pc = "start"
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Or in other words:

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When **pc="middle"** in the current state

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101-7

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101-8

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101-9

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(We'll turn this into TLA syntax on the next slide.)

101-10

TLA+ Syntax Replacement

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TLA+ Syntax Replacement

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- Think about how `someNumber()` would actually have to be implemented.

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- `someNumber()` returning a value from 1 to 1000 is very hard to specify in C. It represents non-determinism. Almost no programming languages were designed to express non-determinism.
- Think about how `someNumber()` would actually have to be implemented.
- Non-determinism in math is easy. i’ $\in$ 1..1000 is non-determinism. “i can be any value from 1 to 1000 in the next state”.

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- Simple code like this is almost never specified in TLA+.
- It **can** be specified, but it's not where the power lies.
- The point of this exercise has been to go step by step from something you already understand to full TLA+ syntax.
- This spec is 6 lines long. The TLA+ spec is longer. Usually, TLA+ specs are orders of magnitude **shorter** than the systems they specify.

Next State

Next State

```
\/ /\ pc = “start”
   /\ i’ ∈ 1..1000
   /\ pc’ = “middle”
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- This formula defines all the valid state pairs.

Next State

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```

- This formula defines all the valid state pairs.
- It does NOT say “**if pc=‘middle’ then set i to i + 1 and set pc to ‘done’** ”.

Next State

```
\/ /\ pc = "start"
   /\ i' ∈ 1..1000
   /\ pc' = "middle"
\/ /\ pc = "middle"
   /\ i' = i + 1
   /\ pc' = "done
```

- This formula defines all the valid state pairs.
- It does NOT say "**if pc='middle' then set i to i + 1 and set pc to 'done'** ".
- So… What's actually going on? What is this really saying?

Behaviours

Behaviours

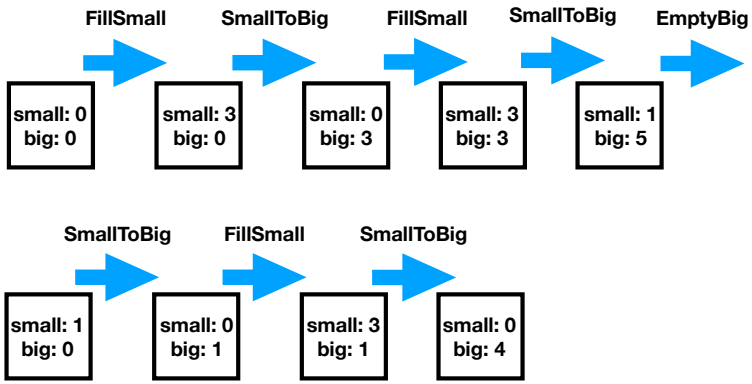
- Think back to our Die Hard example

Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours

Behaviours

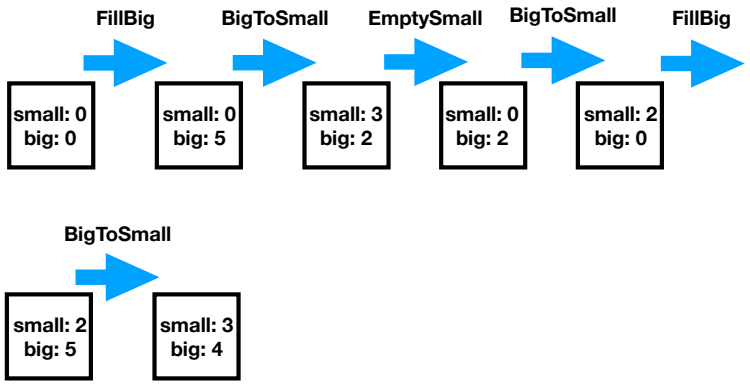
- Think back to our Die Hard example
- We showed three possible behaviours



107-4

Behaviours

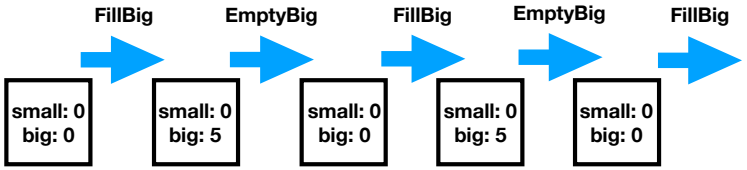
- Think back to our Die Hard example
- We showed three possible behaviours



107-5

Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours



107-6

Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours

- An INFINITE number of behaviours are possible

107-7

Behaviours

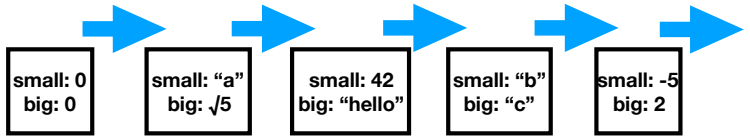
- Think back to our Die Hard example
- We showed three possible behaviours

- An INFINITE number of behaviours are possible
- Not only behaviours that Fill/Empty/Pour, but an infinite number of values can be assigned to **big** and **small**

Behaviours

- Think back to our Die Hard example
- We showed three possible behaviours

- An INFINITE number of behaviours are possible
- Not only behaviours that Fill/Empty/Pour, but an infinite number of values can be assigned to **big** and **small**



Behaviours

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- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.

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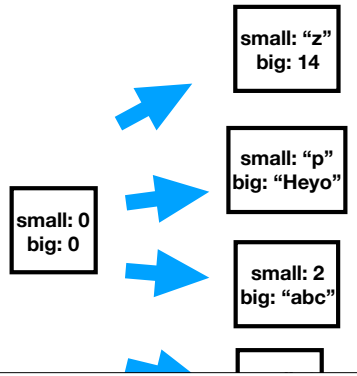

Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.
- From any **state**, an infinite number of **next states** are possible.
- The point of our **formula** is to restrict which of those **state pairs** are permitted. All the combinations of allowed state pairs make up our possible behaviours.

Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.
- From any **state**, an infinite number of **next states** are possible.
- The point of our **formula** is to restrict which of those **state pairs** are permitted. All the combinations of allowed state pairs make up our possible behaviours.
- This is the foundational concept of TLA+.

Behaviours

- The universe presents an infinite combination of behaviours, where an infinite number of **values** can be assigned to our **variables**.
- From any **state**, an infinite number of **next states** are possible.
- The point of our **formula** is to restrict which of those **state pairs** are permitted. All the combinations of allowed state pairs make up our possible behaviours.
- This is the foundational concept of TLA+.
- The formula restricts which subset of infinite behaviours is allowed, by specifying the allowed transitions from one state to another.

Behaviours

```
\/ /\ pc = "start"
   /\ i' ∈ 1..1000
   /\ pc' = "middle"
\/ /\ pc = "middle"
   /\ i' = i + 1
   /\ pc' = "done
```

Behaviours

```
\/ /\ pc = "start"
   /\ i' ∈ 1..1000
   /\ pc' = "middle"
\/ /\ pc = "middle"
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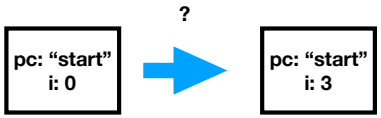

Behaviours

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\/ /\ pc = "start"
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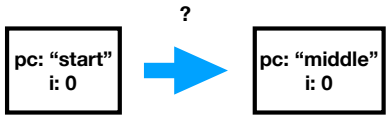

Behaviours

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\/ /\ pc = "start"
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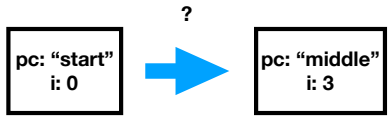

Behaviours

```
\/ /\ pc = "start"
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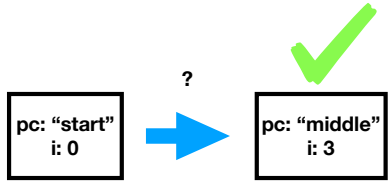
   /\ pc' = "middle"
\/ /\ pc = "middle"
   /\ i' = i + 1
   /\ pc' = "done
```



Behaviours

```
\/ /\ pc = "start"
   /\ i' ∈ 1..1000
   /\ pc' = "middle"
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   /\ i' = i + 1
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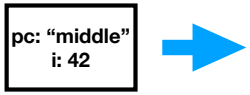
```



Behaviours

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\/ /\ pc = "start"
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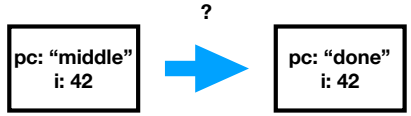

Behaviours

```
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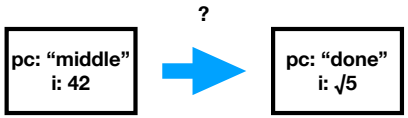
```



Behaviours

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\/ /\ pc = "start"
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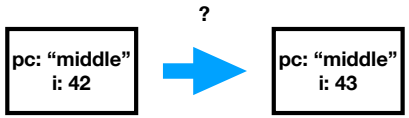

Behaviours

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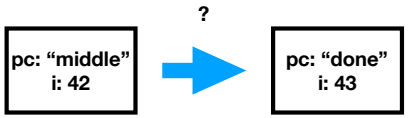

Behaviours

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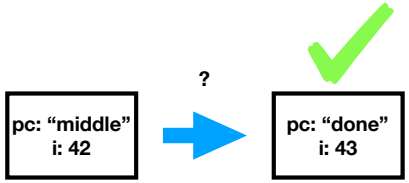


Final Spec for our C Program

111-1

Final Spec for our C Program

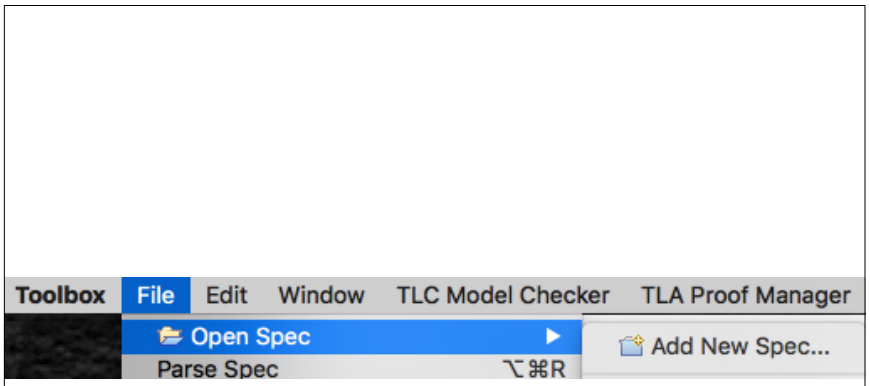
```
---- MODULE SimpleProgram ----

EXTENDS Integers
VARIABLES i, pc

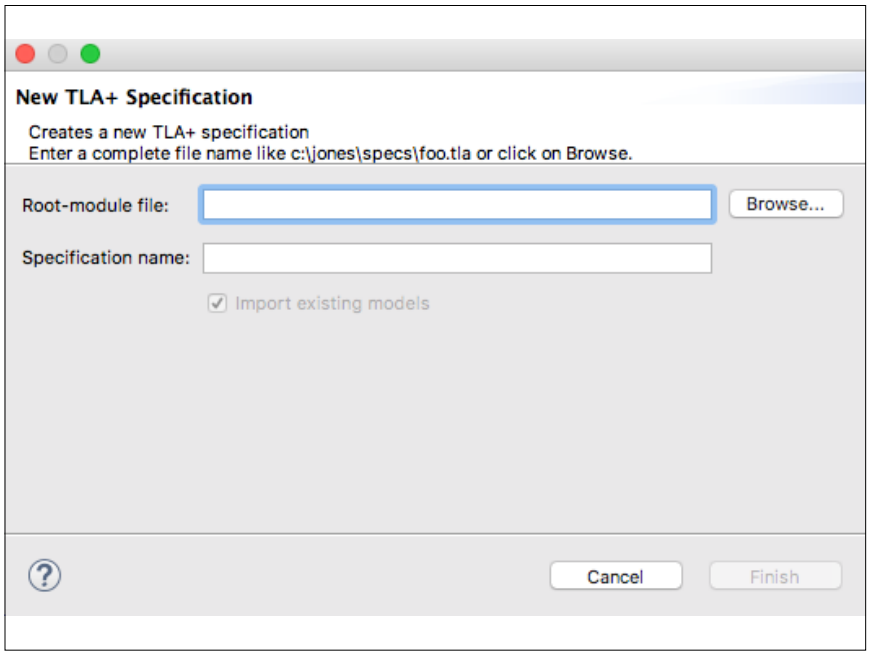
Init == (pc="start") /\ (i=0)

Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done
```

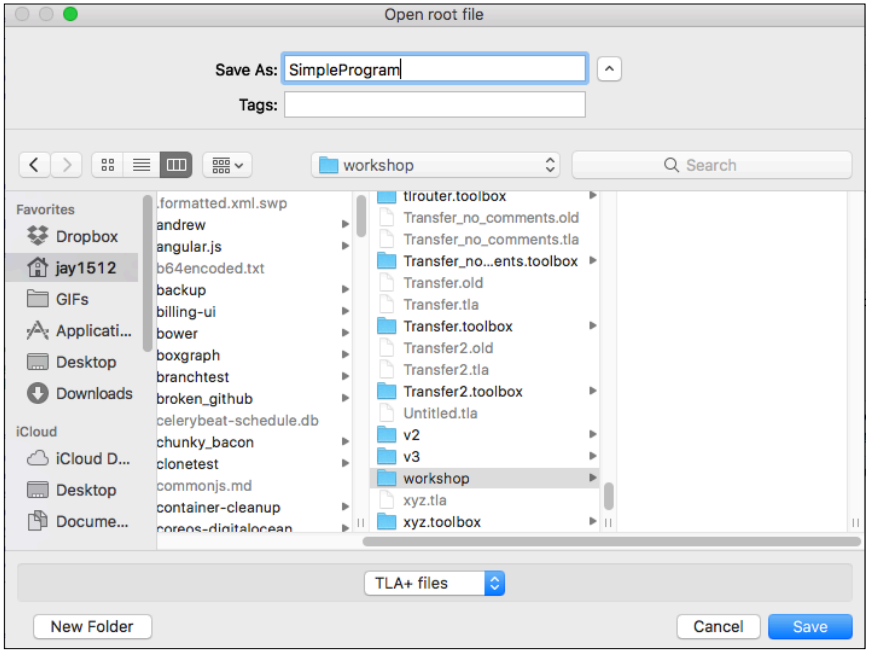
111-2



112



113



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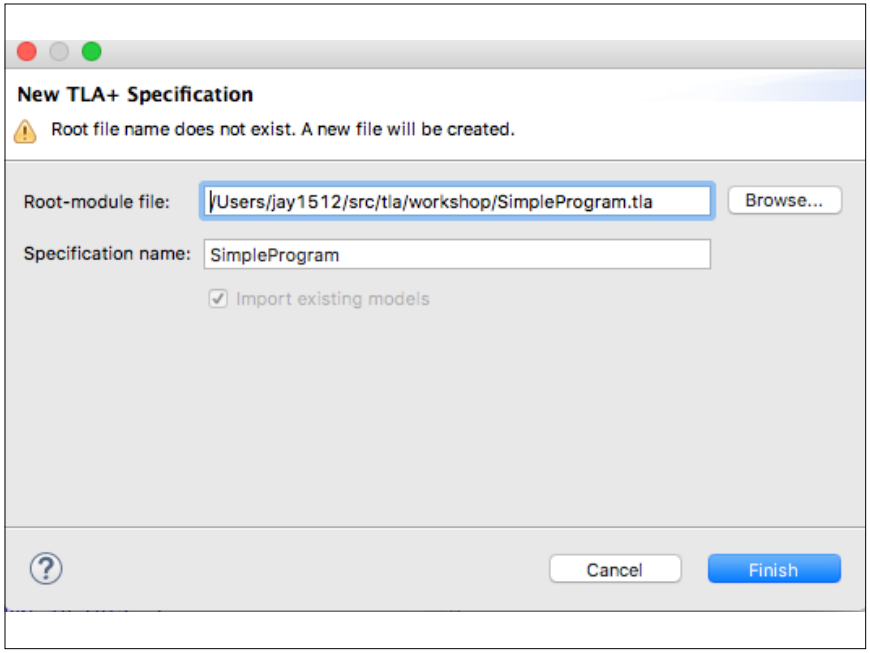

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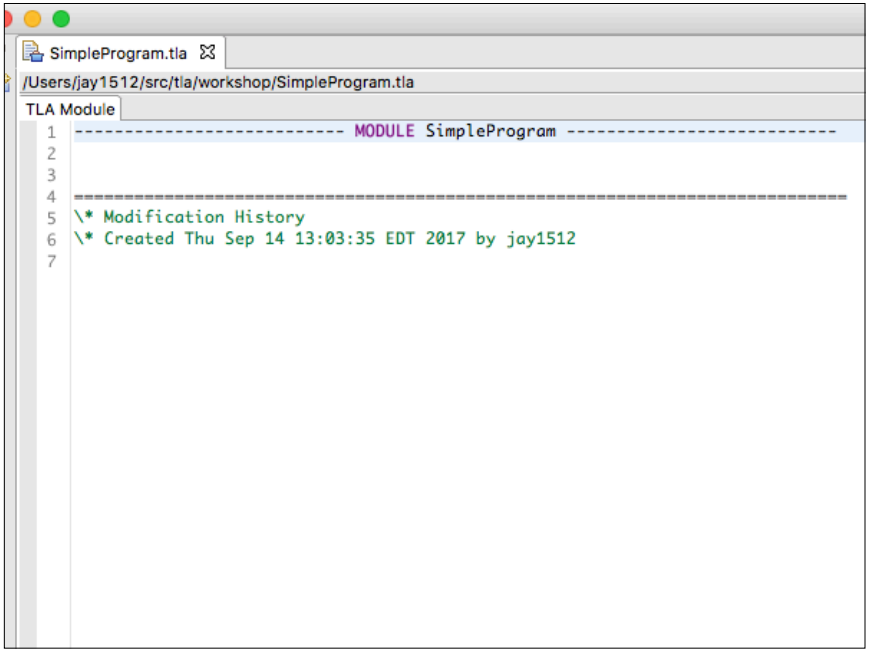
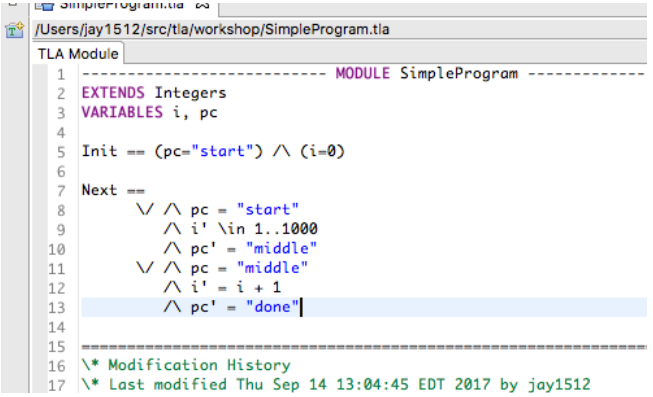
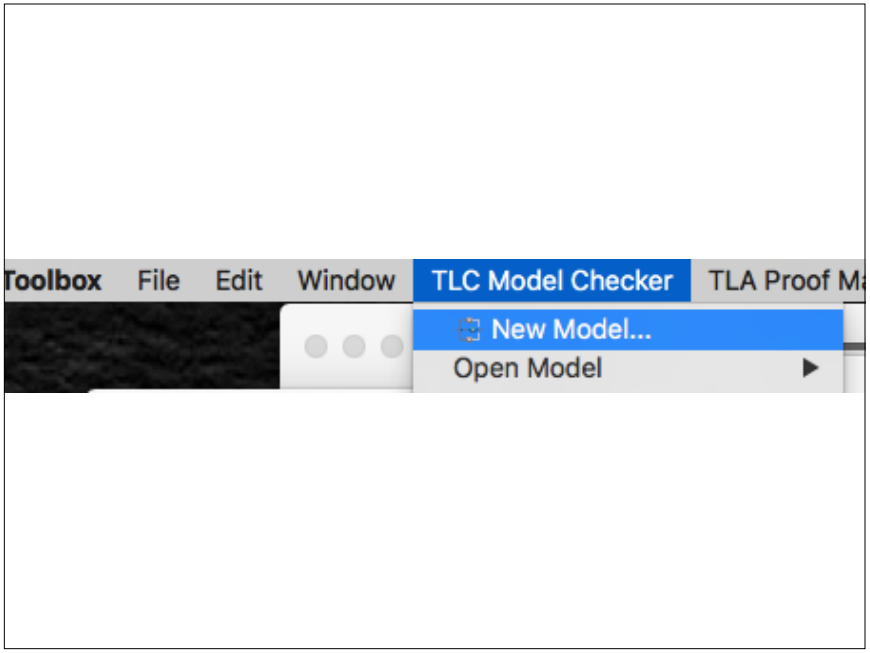


116

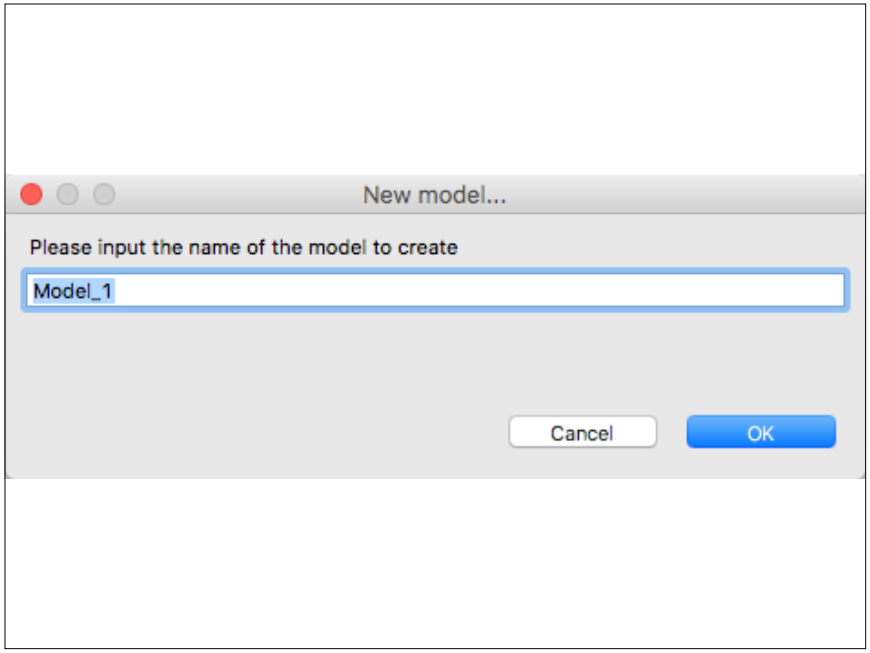


In GitHub at `specs/SimpleProgram.tla`

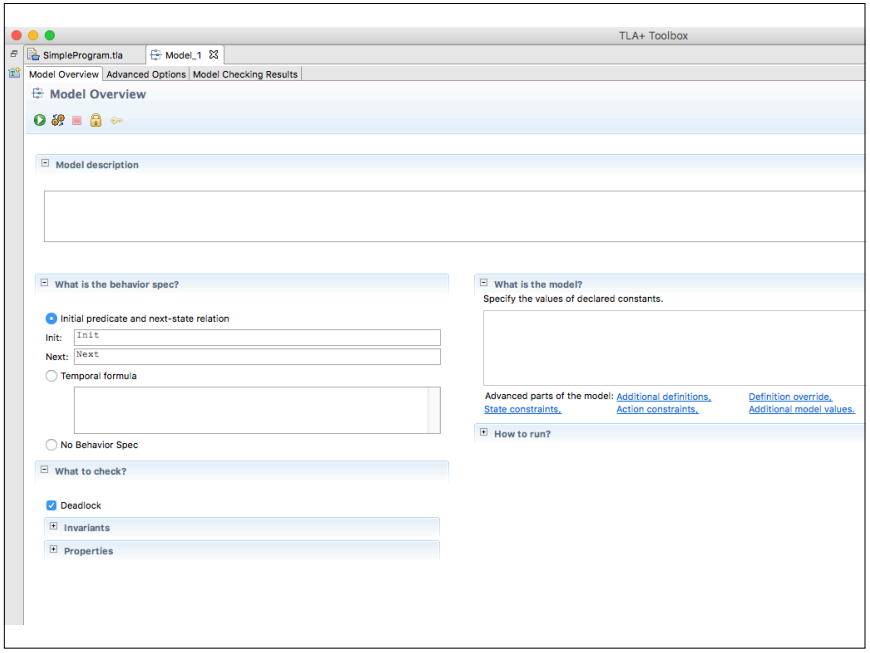
117



118



119



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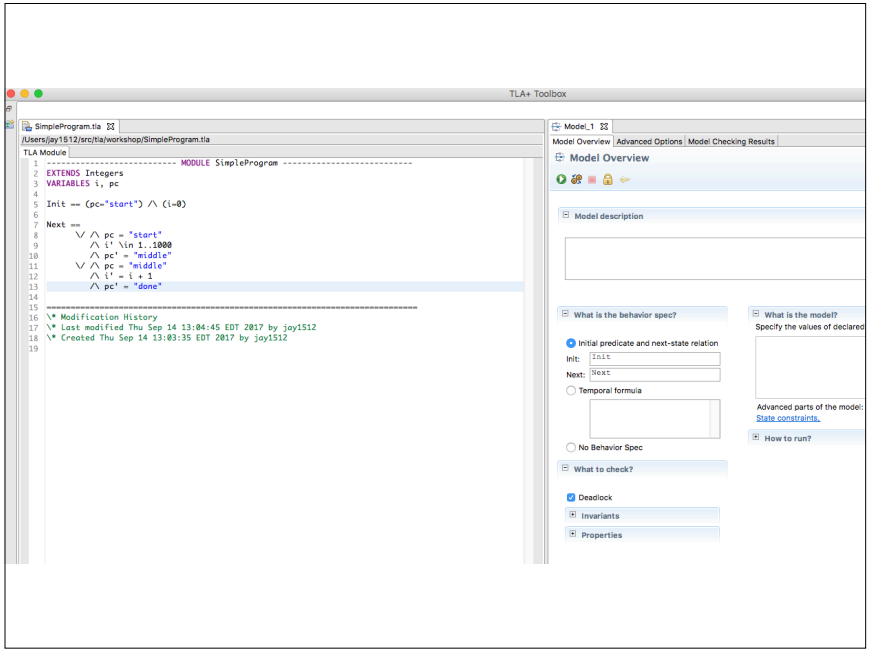
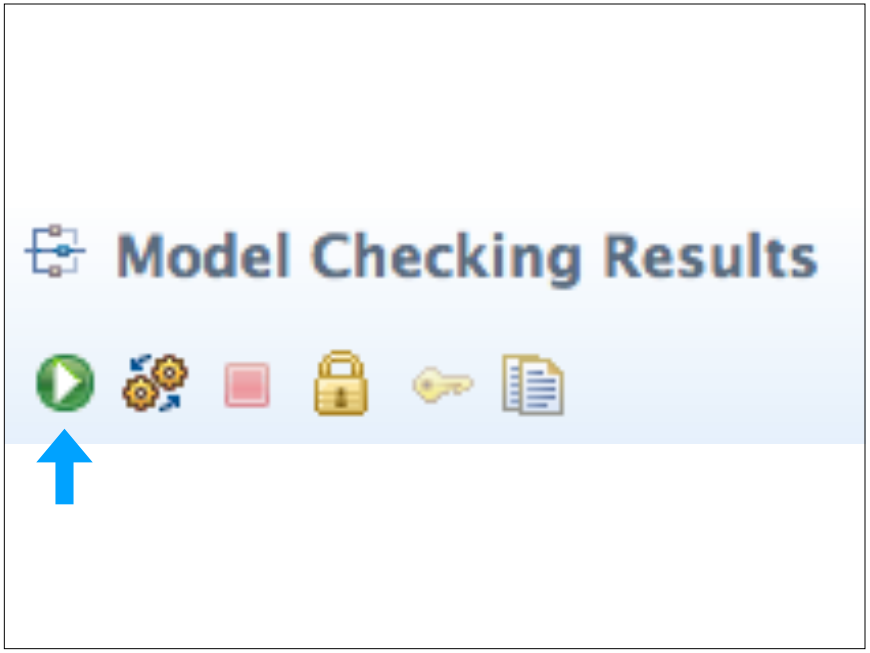
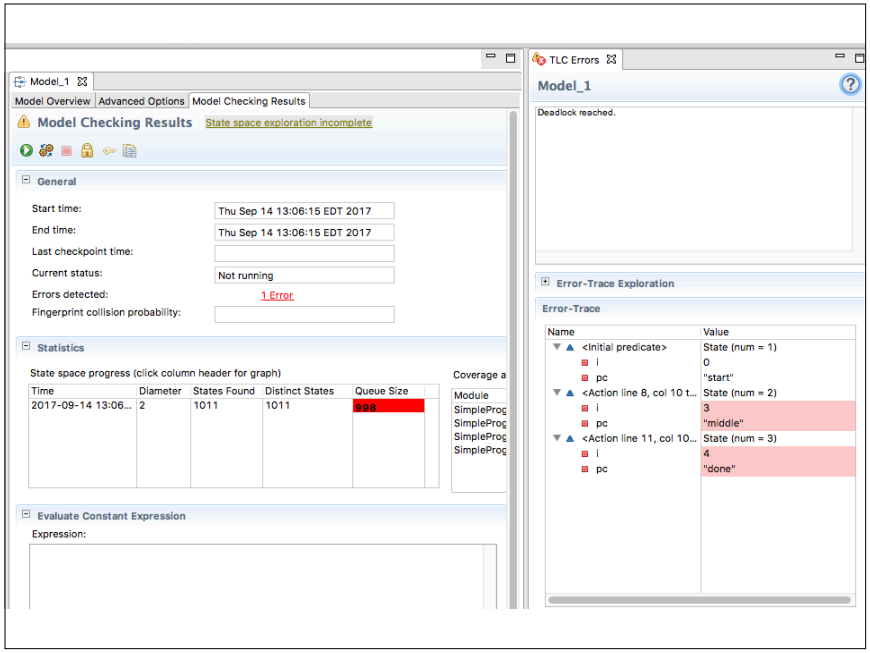


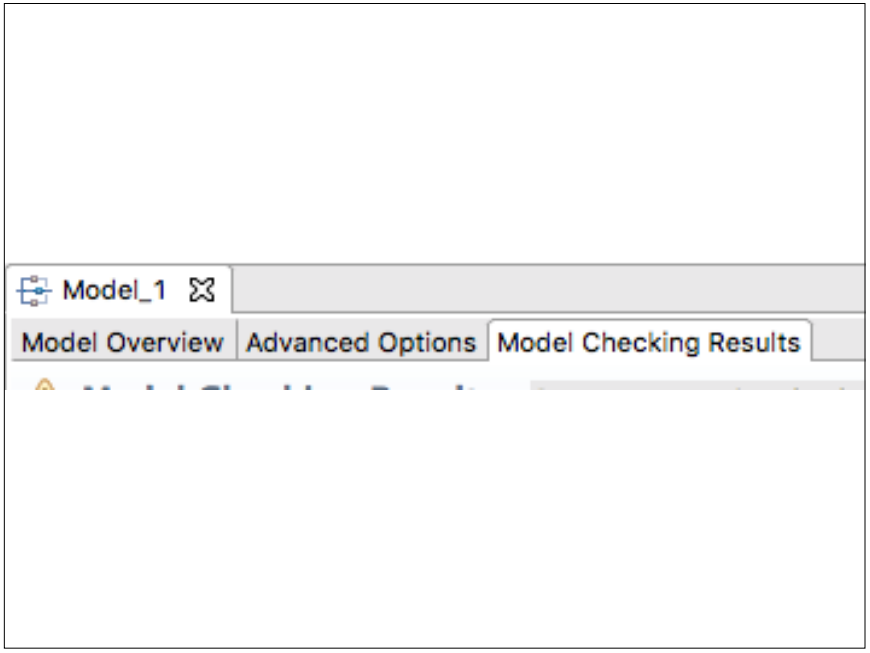
121



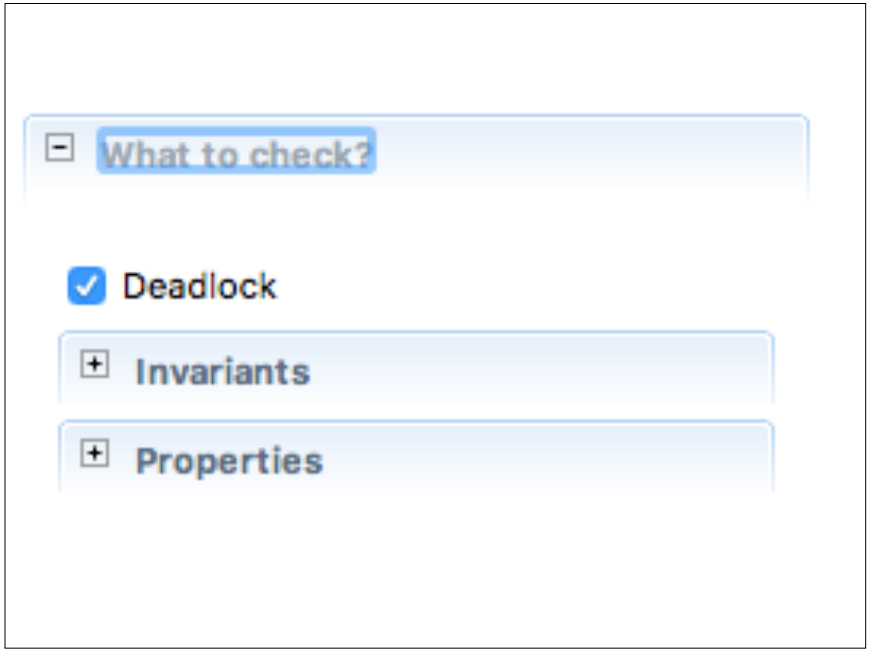
122



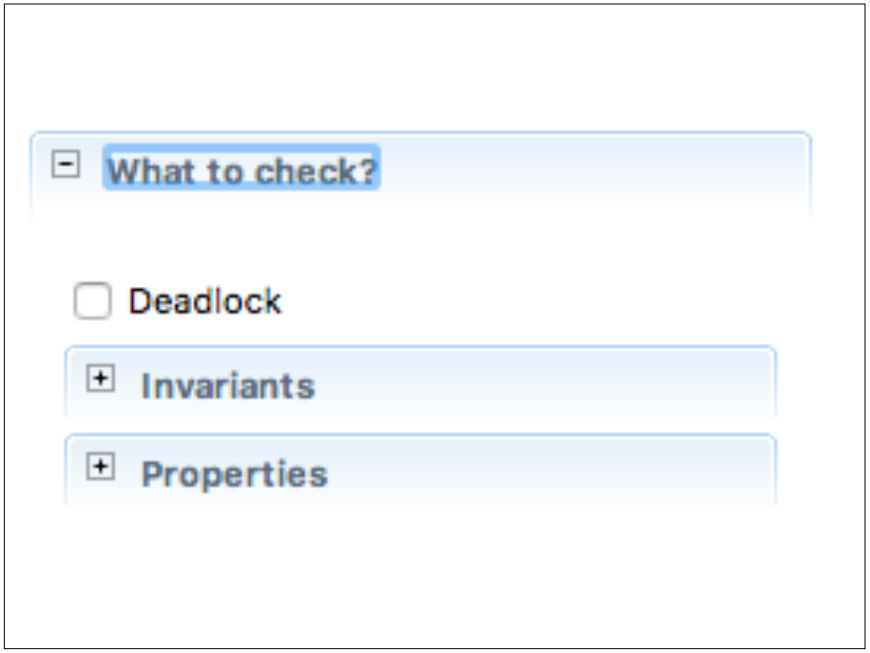
123



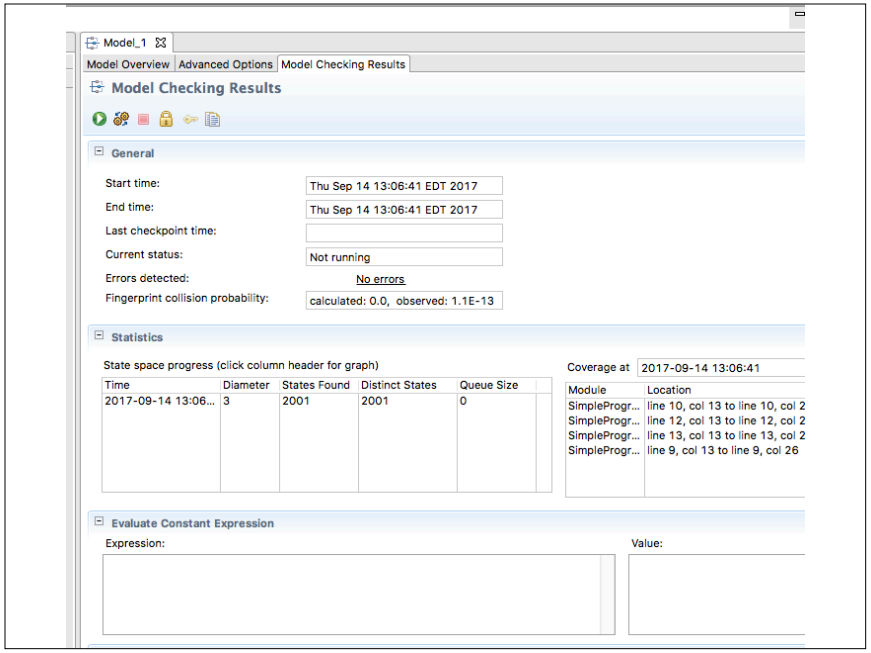
124



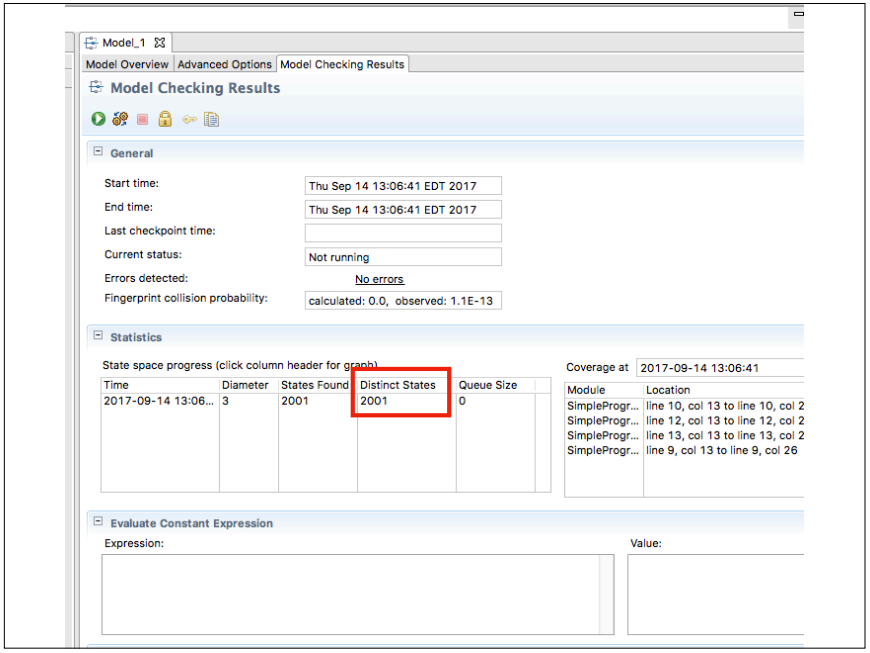
125



126



127-1



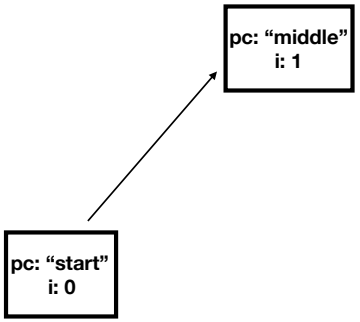
127-2

All Possible State Transitions

pc: "start"
i: 0

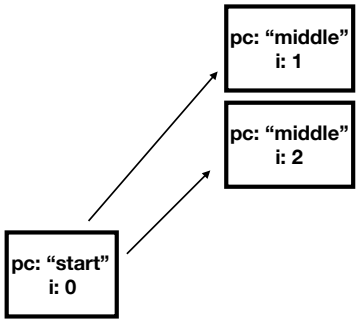
128-1

All Possible State Transitions



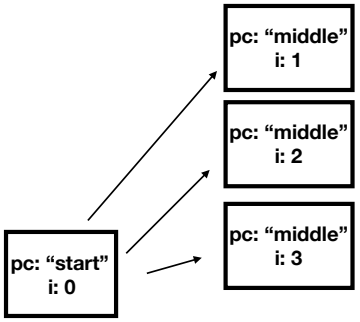
128-2

All Possible State Transitions



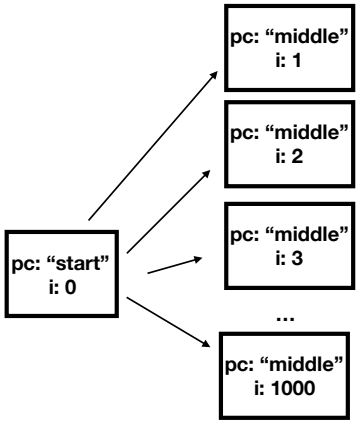
128-3

All Possible State Transitions



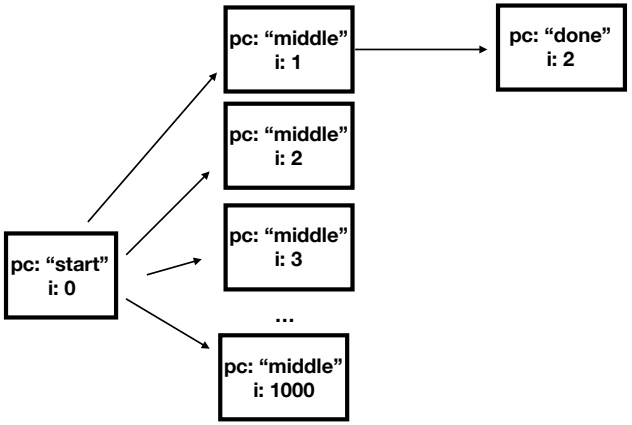
128-4

All Possible State Transitions

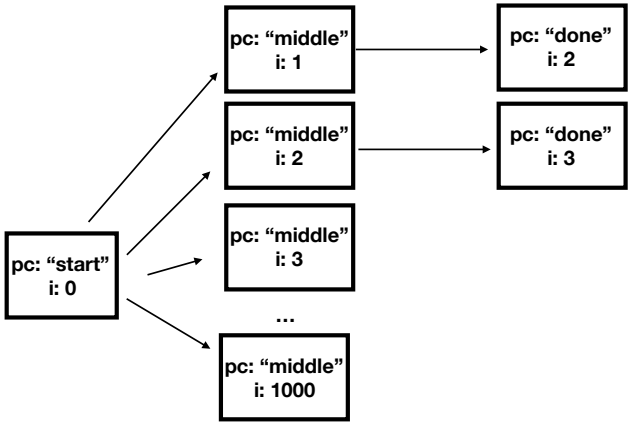


128-5

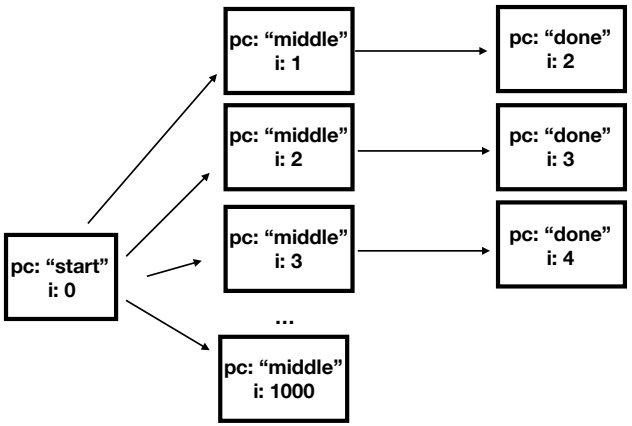
All Possible State Transitions



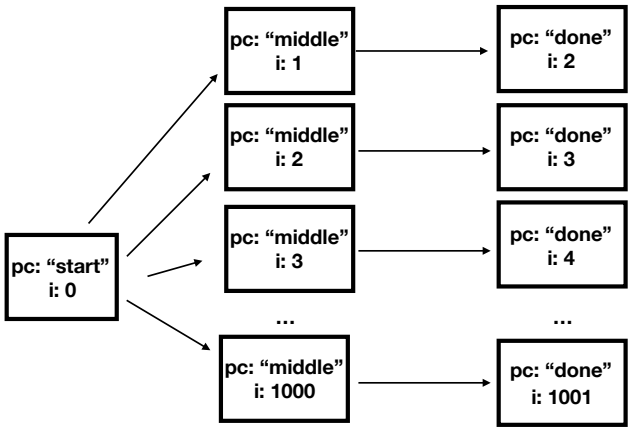
All Possible State Transitions



All Possible State Transitions



All Possible State Transitions



Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up! TLA+ allows for the same thing.

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
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         /\ pc' = "done
```


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Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done
```


Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up! TLA+ allows for the same thing.

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done
```


Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up! TLA+ allows for the same thing.

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done
```

```
Pick ==
      /\ pc = "start"
      /\ i' \in 1..1000
      /\ pc' = "middle"
```

Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up! TLA+ allows for the same thing.

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done"

Pick ==
      /\ pc = "start"
      /\ i' \in 1..1000
      /\ pc' = "middle"

Add ==
      /\ pc = "middle"
      /\ i' = i + 1
      /\ pc' = "done"
```


Sub-definitions

When programming, we don't stuff all our code into a single subroutine. We split it up! TLA+ allows for the same thing.

```
Next ==
      \/ /\ pc = "start"
         /\ i' \in 1..1000
         /\ pc' = "middle"
      \/ /\ pc = "middle"
         /\ i' = i + 1
         /\ pc' = "done"

Pick ==
      /\ pc = "start"
      /\ i' \in 1..1000
      /\ pc' = "middle"

Add ==
      /\ pc = "middle"
      /\ i' = i + 1
      /\ pc' = "done"

Next == Pick \/ Add
```



```
EXTENDS Integers
VARIABLES i, pc

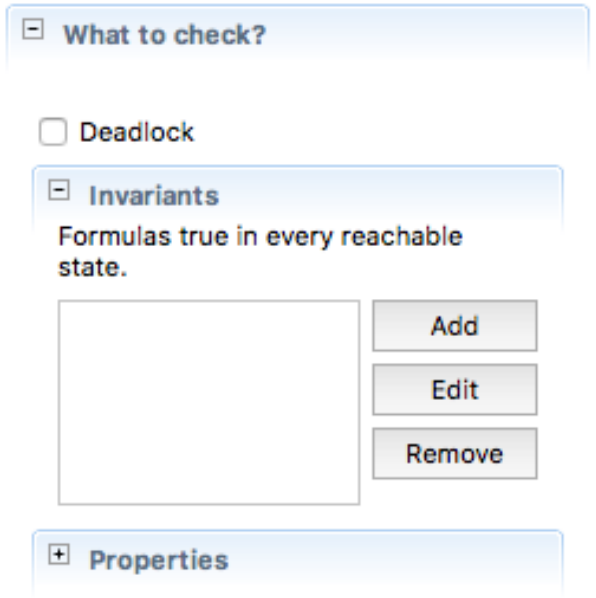
Init == (pc="start") /\ (i=0)

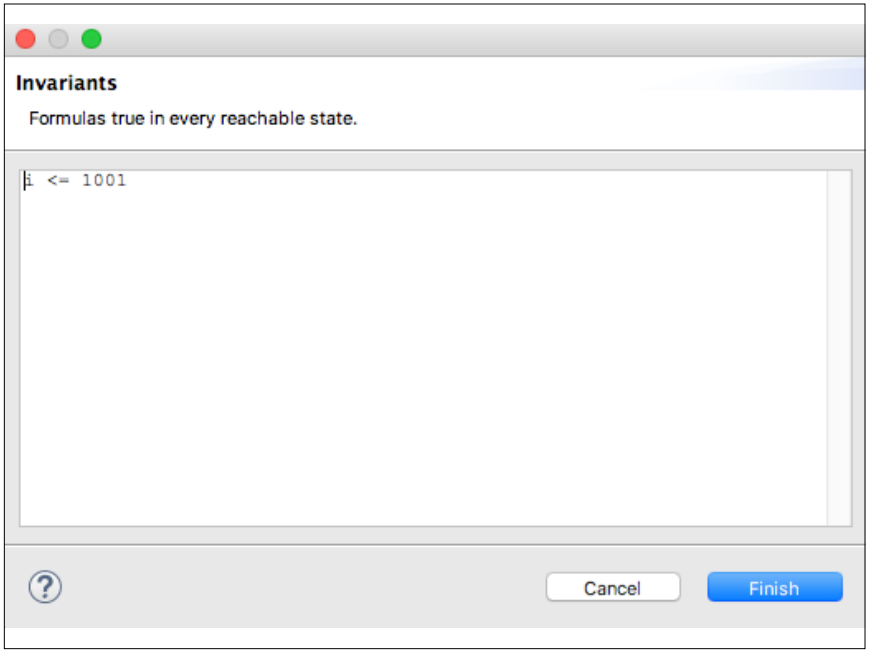
Pick ==
   /\ pc = "start"
   /\ i' \in 1..1000
   /\ pc' = "middle"

Add ==
   /\ pc = "middle"
   /\ i' = i + 1
   /\ pc' = "done"

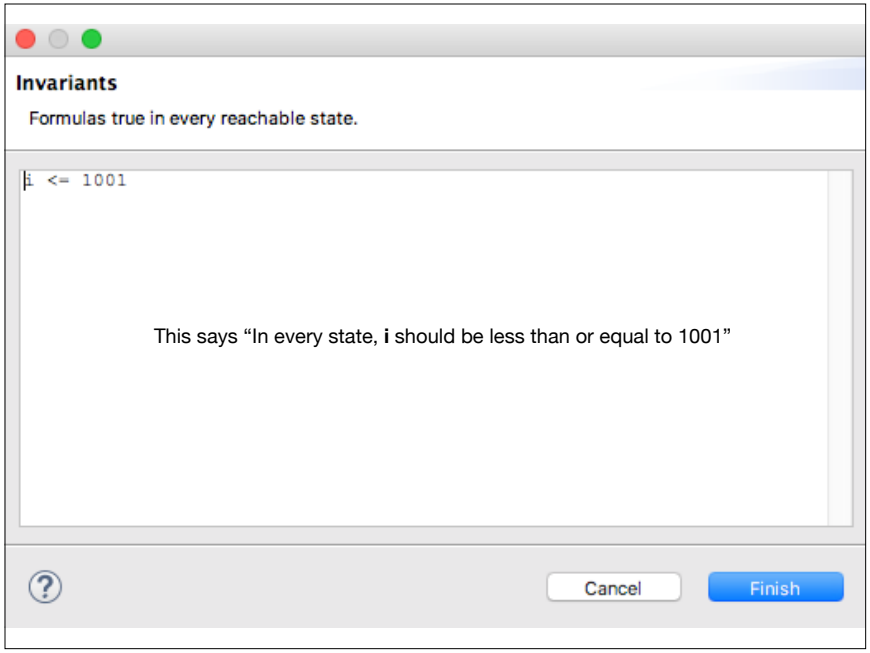
Next == Pick \/ Add
```

specs/SimpleProgram2.tla





132-1



132-2

Run the model again, everything should still be fine

133

Change the Add action

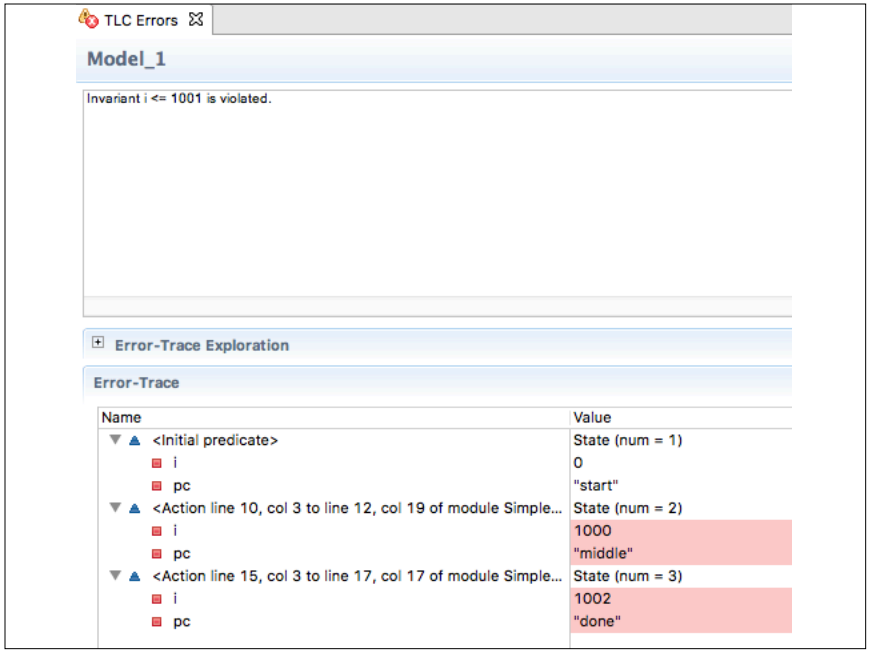
```
Add ==
  /\ pc = "middle"
  /\ i' = i + 2
  /\ pc' = "done"
```

134-1

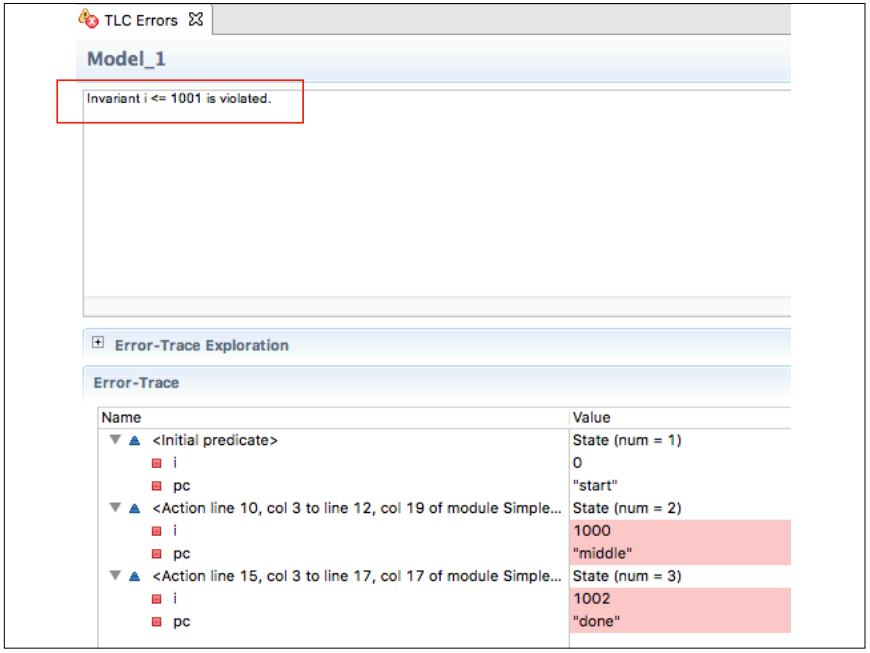
Change the Add action

```
Add ==
   /\ pc = "middle"
   /\ i' = i + 2
   /\ pc' = "done"
```

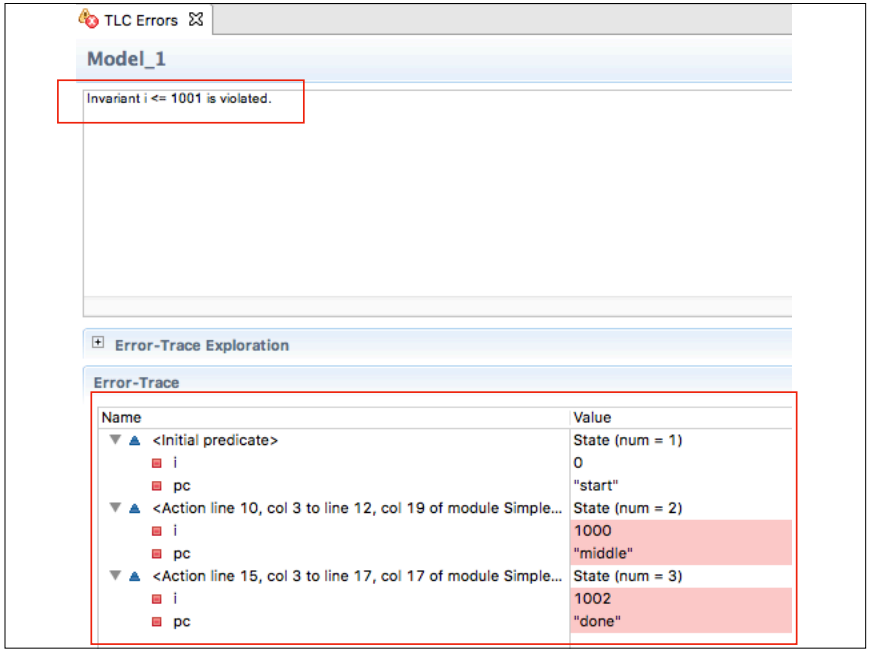
134-2



135-1

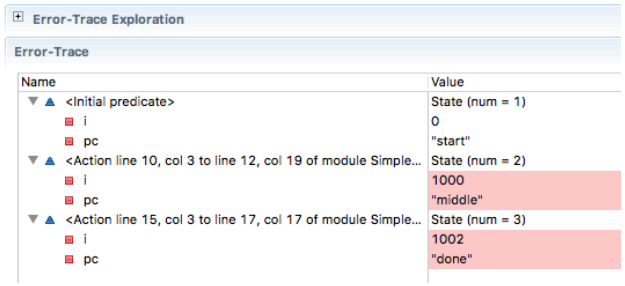


135-2



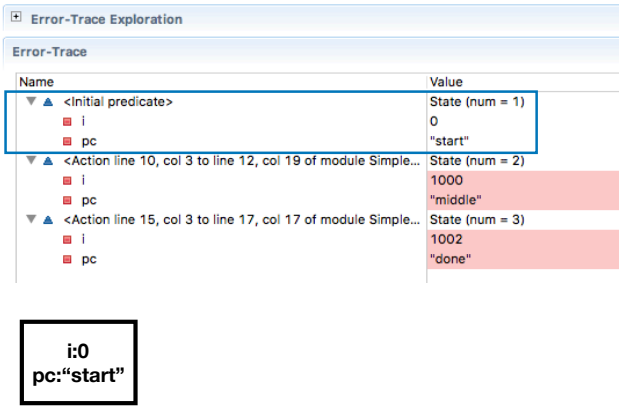
135-3

Error Trace



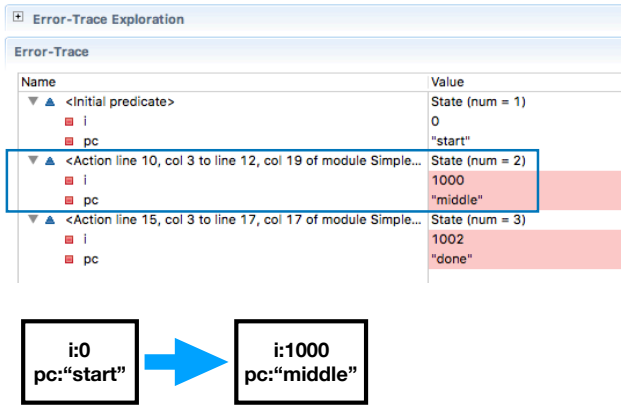
136-1

Error Trace



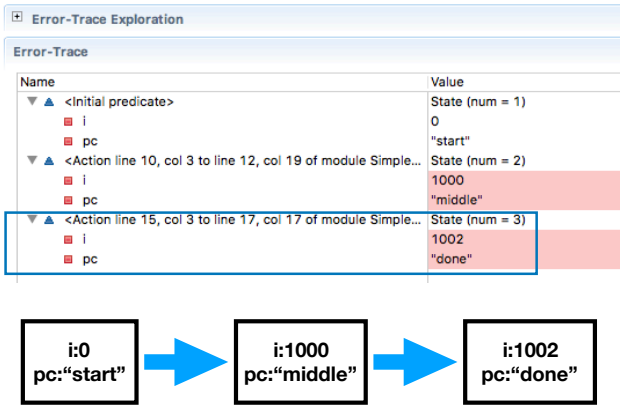
136-2

Error Trace



136-3

Error Trace



136-4

137

Die Hard Variables

```
VARIABLES big, small
```

138

Die Hard Initial States

```
Init == (big = 0) /\ (small = 0)
```

139

Die Hard Next States

140-1

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

3.We can always empty **small**, i.e. in the next state **small=0**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

3.We can always empty **small**, i.e. in the next state **small=0**

4.We can always empty **big**, i.e. in the next state **big=0**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

3.We can always empty **small**, i.e. in the next state **small=0**

4.We can always empty **big**, i.e. in the next state **big=0**

5.We can always pour **small** into **big**

Die Hard Next States

1.We can always fill **small**, i.e. in the next state **small=3**

2.We can always fill **big**, i.e. in the next state **big = 5**

3.We can always empty **small**, i.e. in the next state **small=0**

4.We can always empty **big**, i.e. in the next state **big=0**

5.We can always pour **small** into **big**

6. We can always pour **big** into **small**

Die Hard Next States

```
FillSmall
FillBig
EmptySmall
EmptyBig
SmallToBig
BigToSmall
```


FillSmall

FillSmall

```
FillSmall == /\ small' = 3
```


FillSmall

```
FillSmall == /\ small' = 3
             /\ big' = big
```


EmptySmall

EmptySmall

```
EmptySmall == /\ small' = 0
```

EmptySmall

```
EmptySmall == /\ small' = 0
              /\ big' = big
```


SmallToBig

- Two possible cases:
  1. There **is** room in **big** for all the water in **small**
  2. There **is not** room in **big** for all the water in **small**

SmallToBig

Case 1: Enough room in **big**

We put all the water from the **small** jug into the **big** jug, which has the effect of emptying the **small** jug

SmallToBig

Case 1: Enough room in **big**

We put all the water from the **small** jug into the **big** jug, which has the effect of emptying the **small** jug

```
/\ big' = big + small
```

SmallToBig

Case 1: Enough room in **big**

We put all the water from the **small** jug into the **big** jug, which has the effect of emptying the **small** jug

```
/\ big' = big + small
/\ small' = 0
```


SmallToBig

Case 2: NOT enough room in **big**

We pour what we can from **small** into **big**. **big** gets filled by this action. **small** is emptied by however much was poured out

SmallToBig

Case 2: NOT enough room in **big**

We pour what we can from **small** into **big**. **big** gets filled by this action. **small** is emptied by however much was poured out

```
/\ big' = 5
```


SmallToBig

Case 2: NOT enough room in **big**

We pour what we can from **small** into **big**. **big** gets filled by this action. **small** is emptied by however much was poured out

```
/\ big' = 5
/\ small' = small - (5 - big)
```

SmallToBig

Putting it all together

SmallToBig

Putting it all together

```
SmallToBig == IF big + small < 5
```


SmallToBig

Putting it all together

```
SmallToBig == IF big + small < 5
              THEN /\ big' = big + small
```


SmallToBig

Putting it all together

```
SmallToBig == IF big + small < 5
              THEN /\ big' = big + small
                   /\ small' = 0
```

SmallToBig

Putting it all together

```
SmallToBig == IF big + small < 5
              THEN /\ big' = big + small
                   /\ small' = 0
              ELSE /\ small' = small - (5 - big)
```

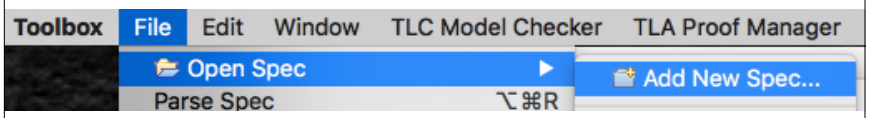
147-5

SmallToBig

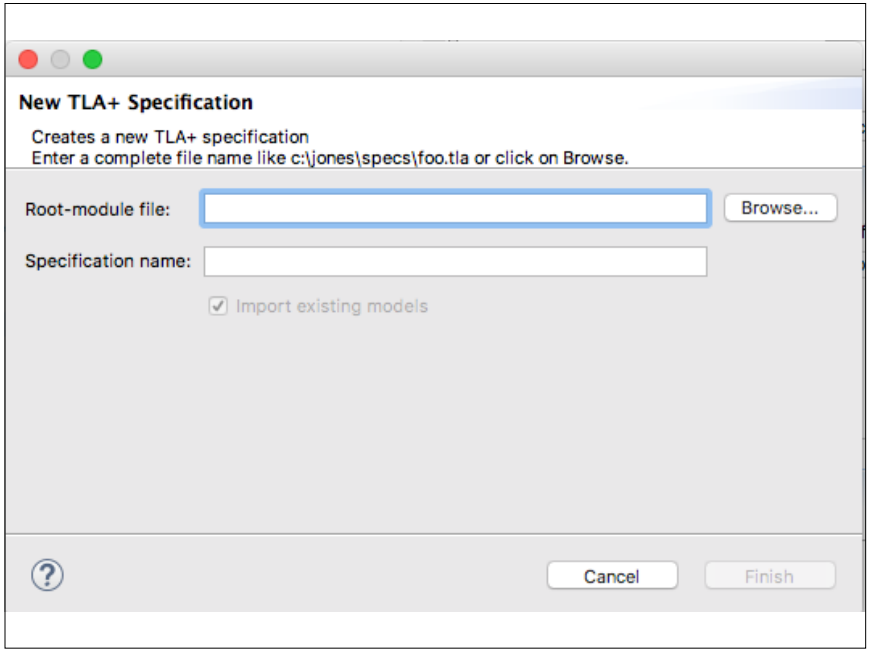
Putting it all together

```
SmallToBig == IF big + small < 5
              THEN /\ big' = big + small
                   /\ small' = 0
              ELSE /\ small' = small - (5 - big)
                   /\ big' = 5
```

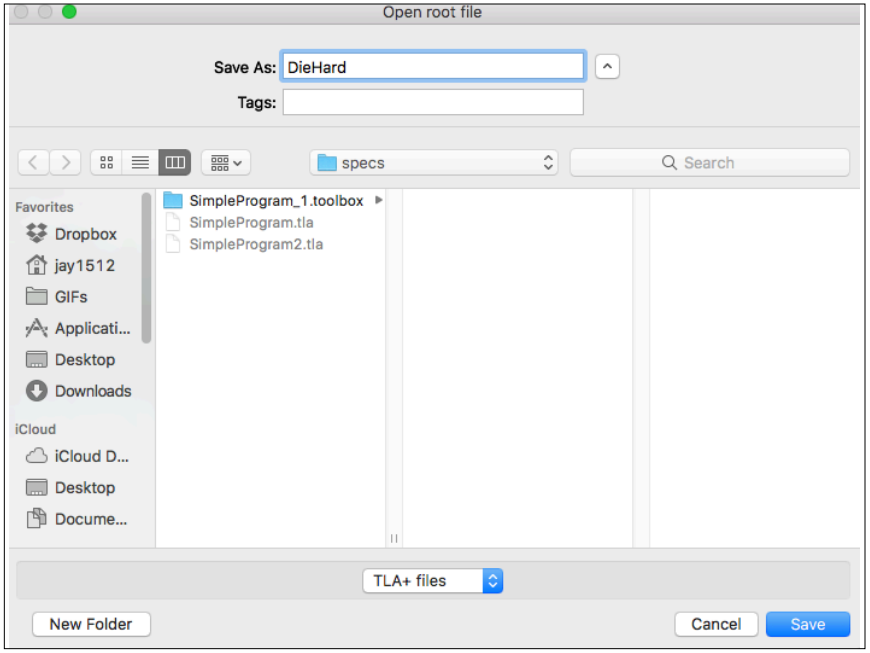
147-6



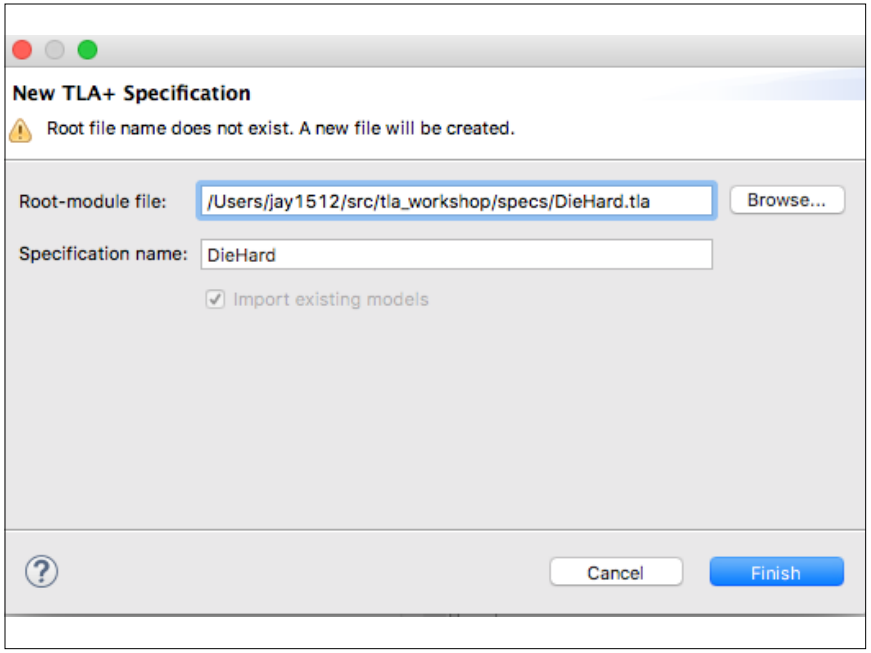
148



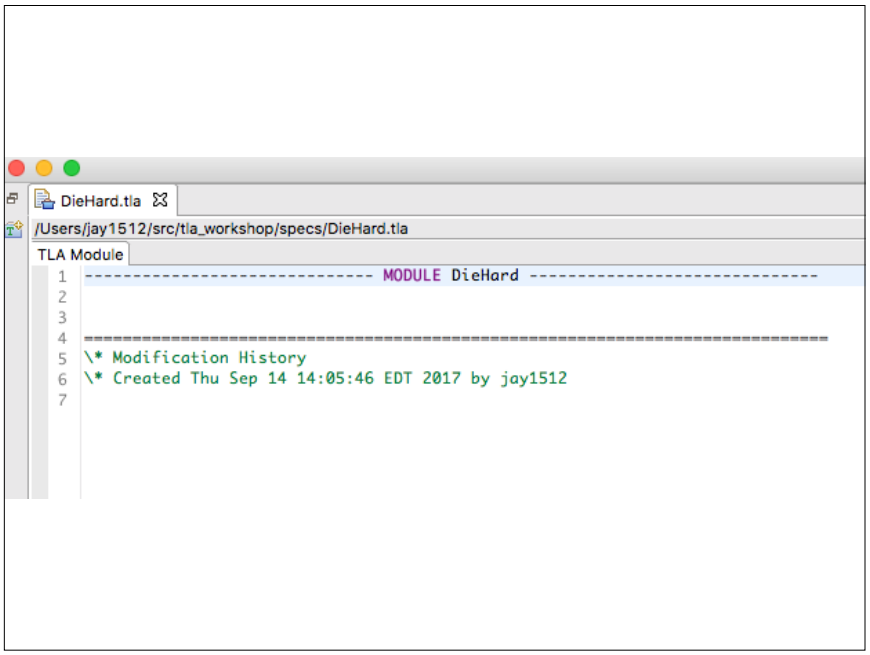
149



150



151



152

```
---------- MODULE DieHard ----------
EXTENDS Integers

VARIABLES small, big

TypeOK == /\ small \in 0..3
          /\ big   \in 0..5

Init == /\ big   = 0
        /\ small = 0

FillSmall == /\ small' = 3
             /\ big'   = big

FillBig ==

EmptySmall == /\ small' = 0
              /\ big'   = big

EmptyBig ==

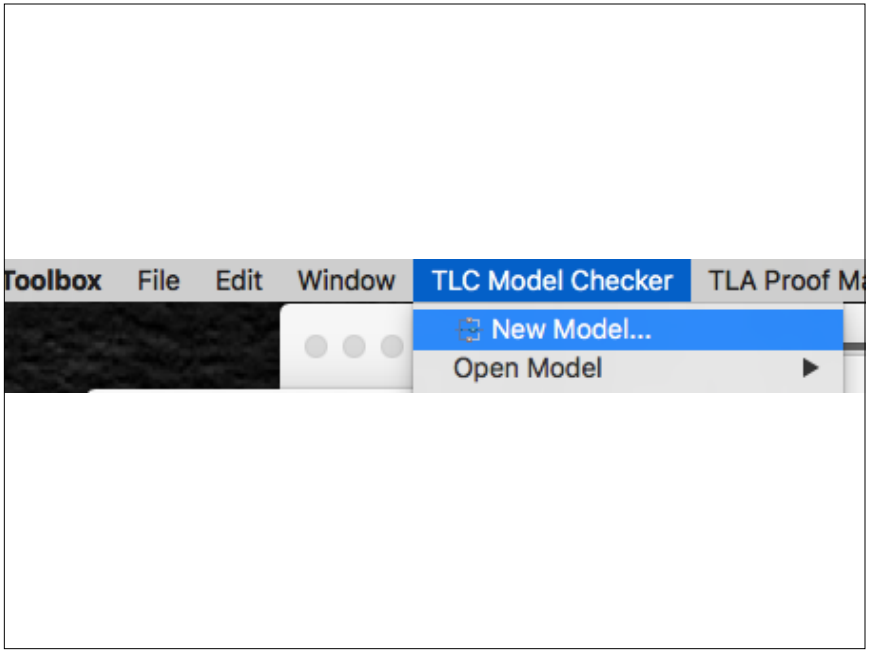
SmallToBig == IF big + small =< 5
              THEN /\ big'   = big + small
                   /\ small' = 0
              ELSE /\ big'   = 5
                   /\ small' = small - (5 - big)

BigToSmall ==

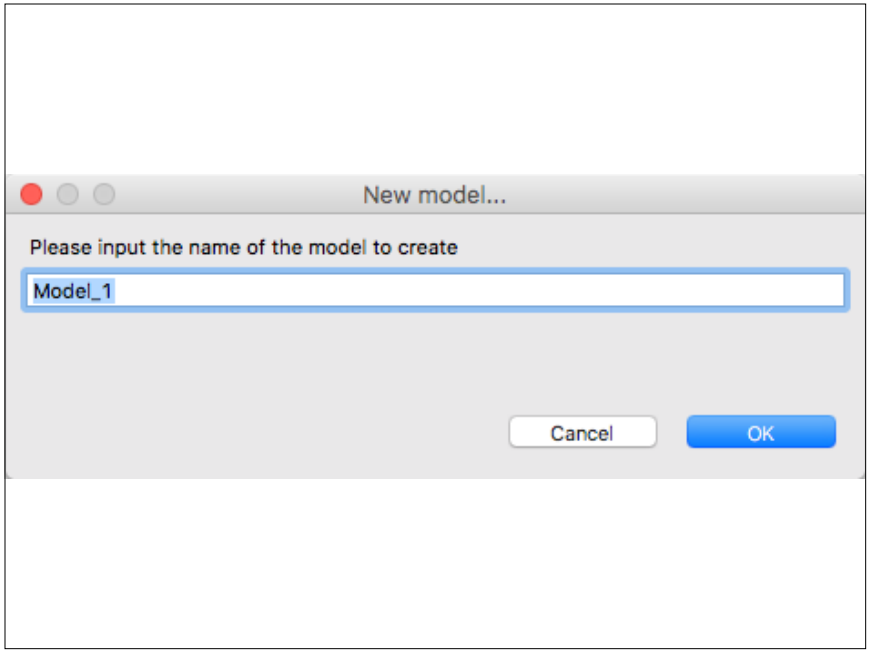
Next == \/ FillSmall
        \/ FillBig
        \/ EmptySmall
        \/ EmptyBig
        \/ SmallToBig
        \/ BigToSmall
```

specs/DieHard_incomplete.tla

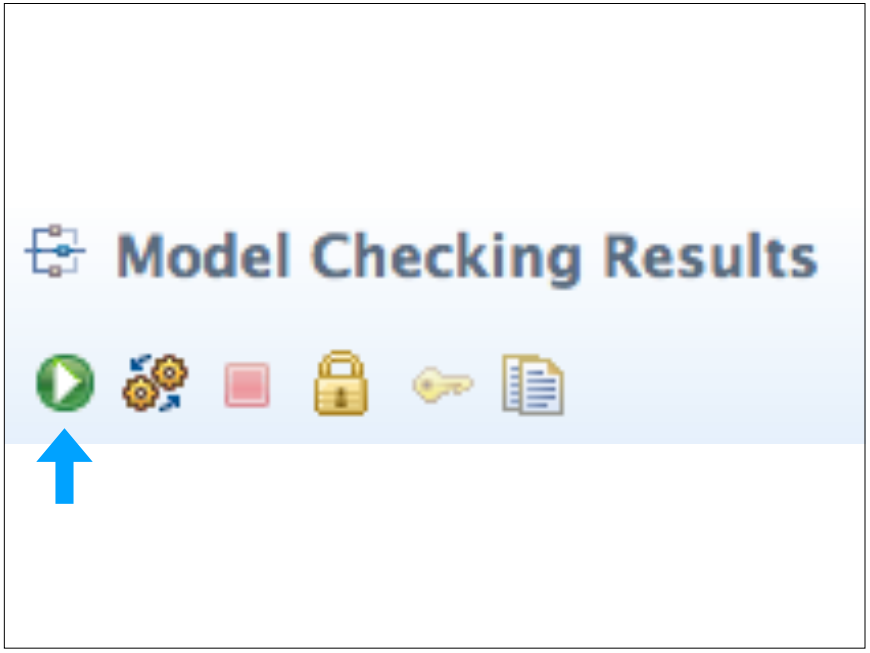
153



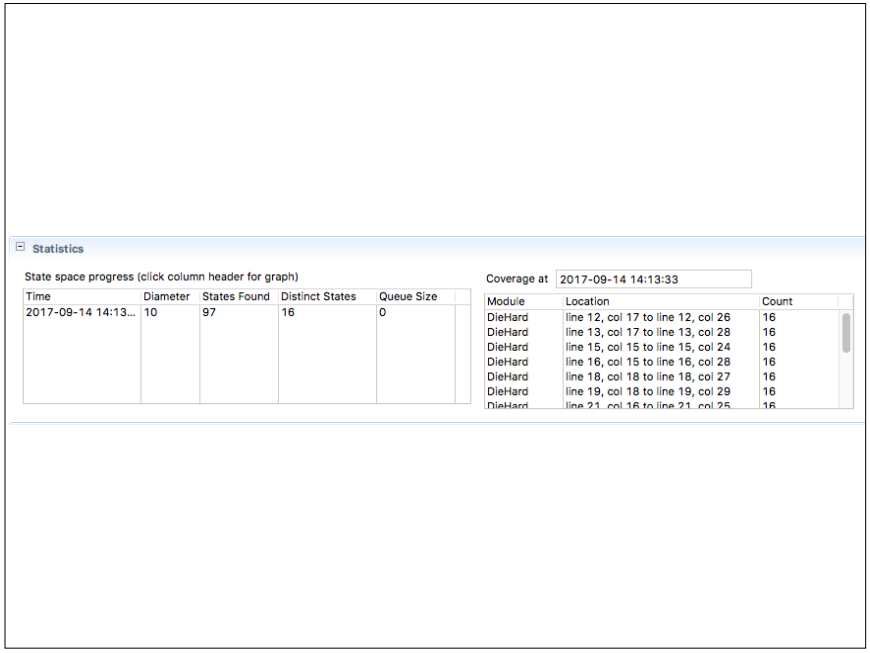
154



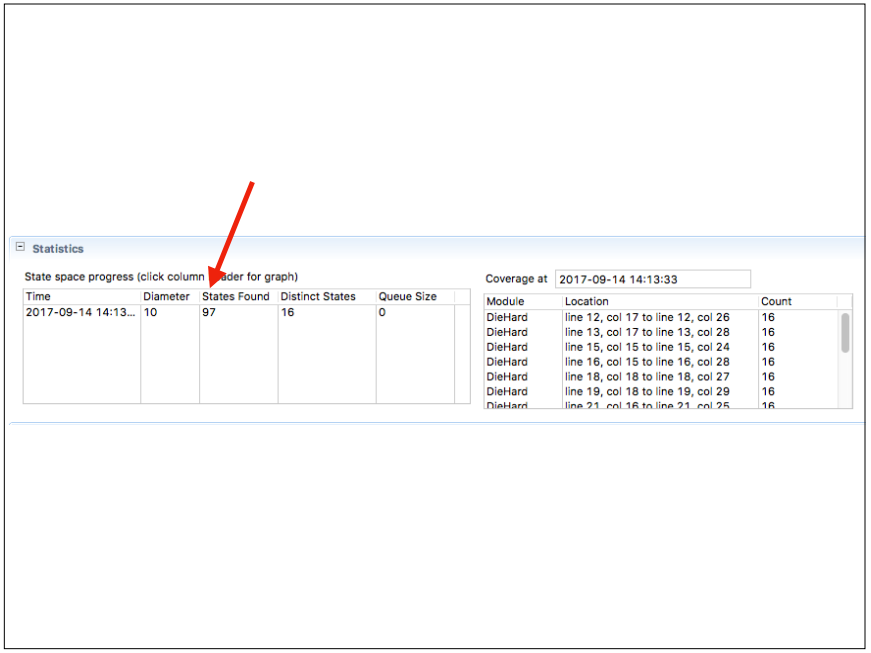
155



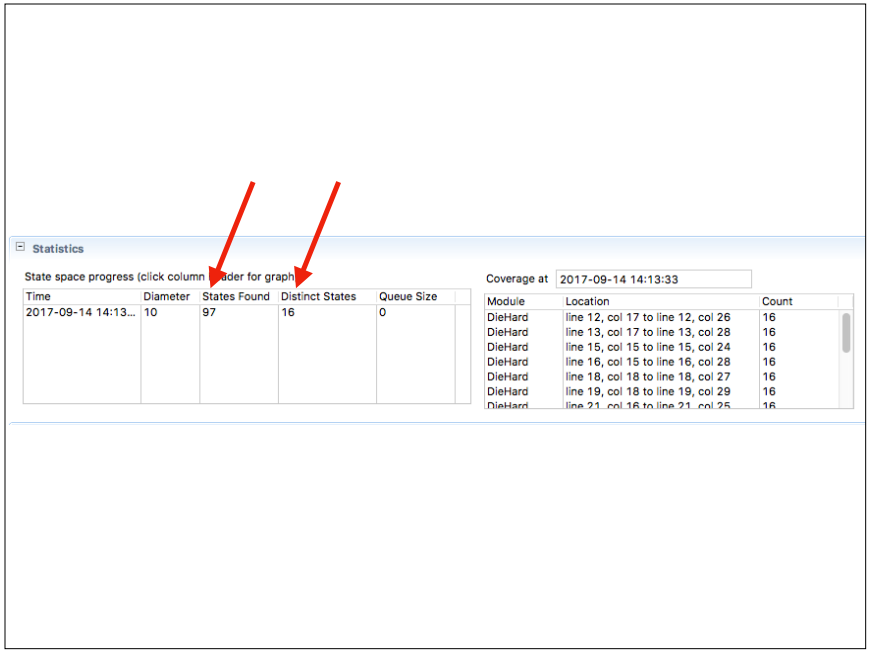
156



157-1



157-2



157-3

```
EXTENDS Integers
VARIABLES small, big

TypeOK == /\ small \in 0..3
          /\ big   \in 0..5

Init == /\ big   = 0
        /\ small = 0

FillSmall == /\ small' = 3
             /\ big'   = big

FillBig == /\ big'   = 5
           /\ small' = small

EmptySmall == /\ small' = 0
              /\ big'   = big

EmptyBig == /\ big'   = 0
            /\ small' = small

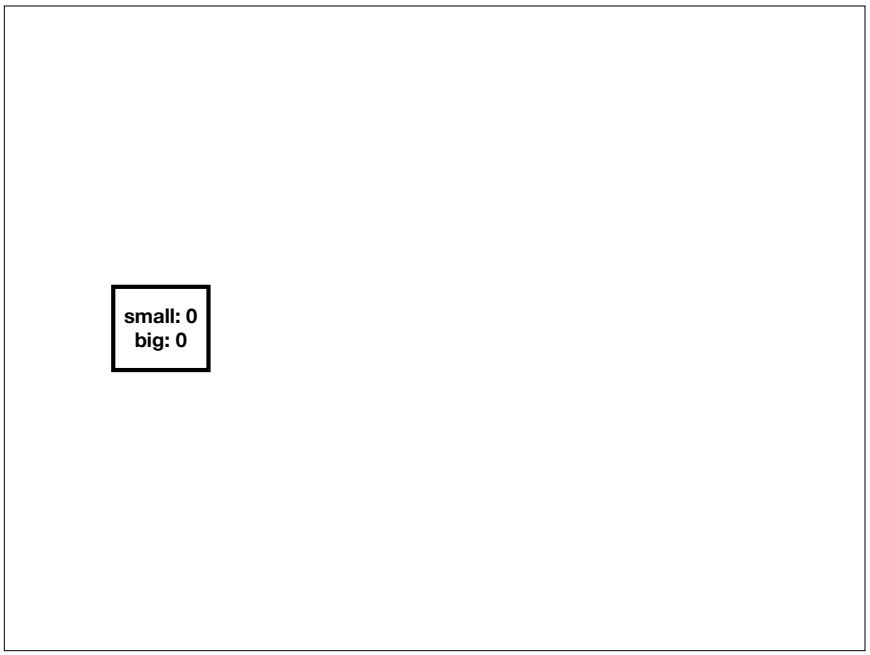
SmallToBig == IF big + small =< 5
              THEN /\ big'   = big + small
                   /\ small' = 0
              ELSE /\ big'   = 5
                   /\ small' = small - (5 - big)

BigToSmall == IF big + small =< 3
              THEN /\ big'   = 0
                   /\ small' = big + small
              ELSE /\ big'   = small - (3 - big)
                   /\ small' = 3

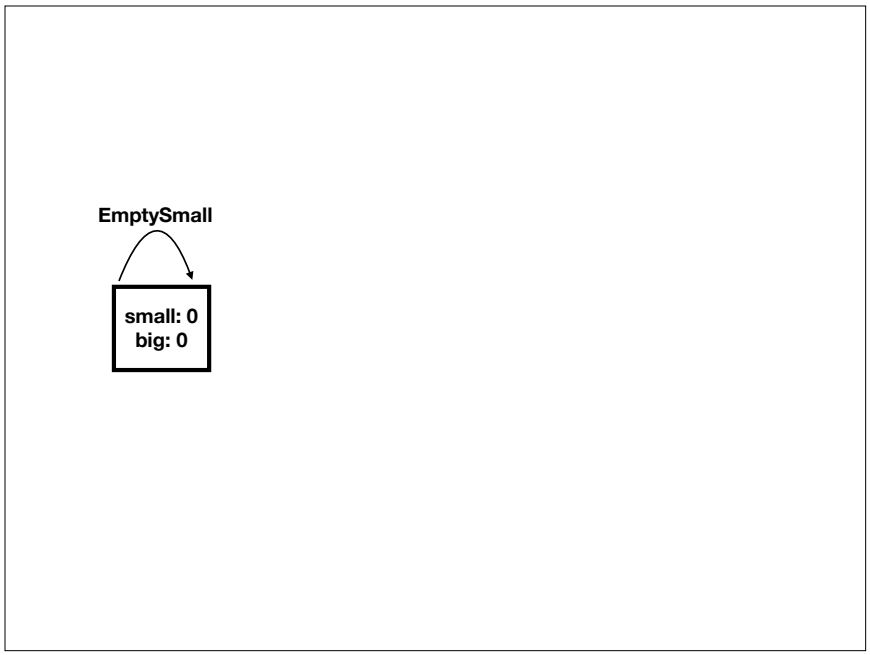
Next == \/ FillSmall
        \/ FillBig
        \/ EmptySmall
        \/ EmptyBig
        \/ SmallToBig
        \/ BigToSmall
```

158

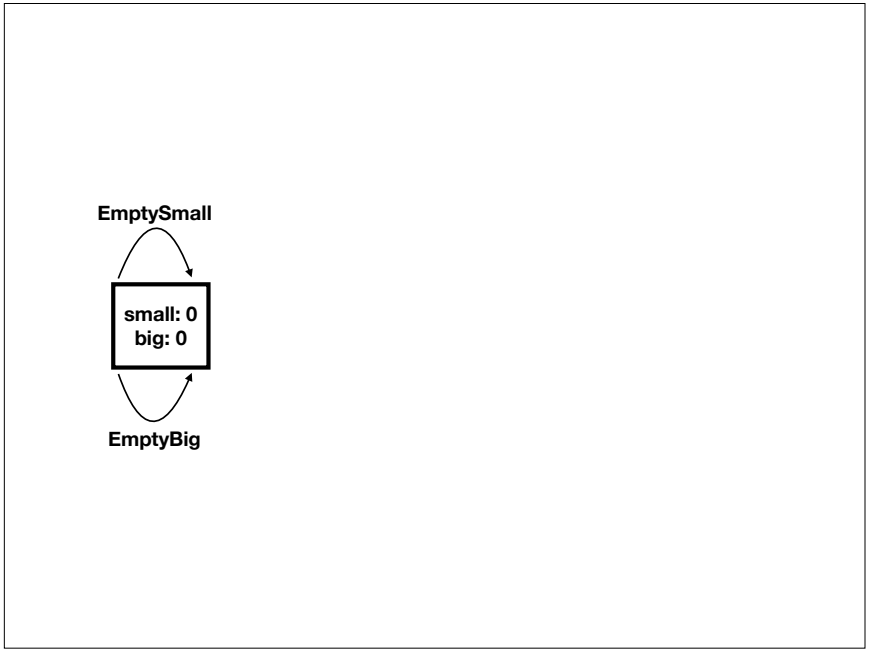
159



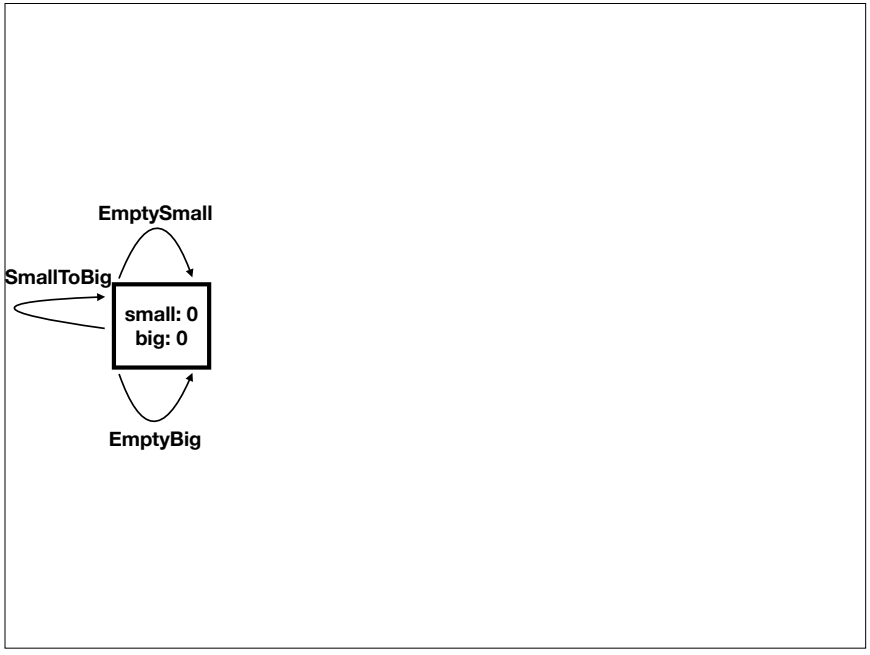
160-1



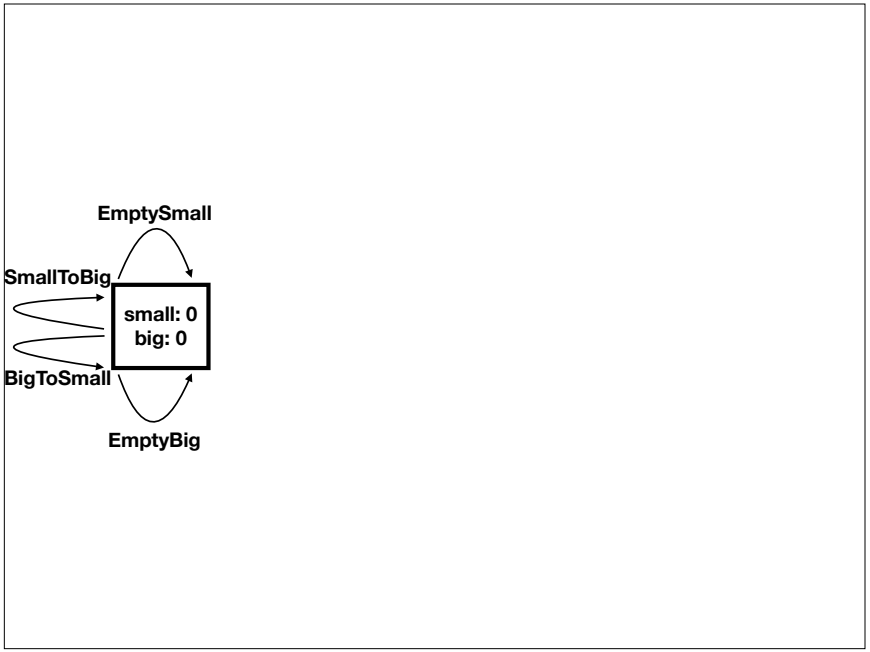
160-2



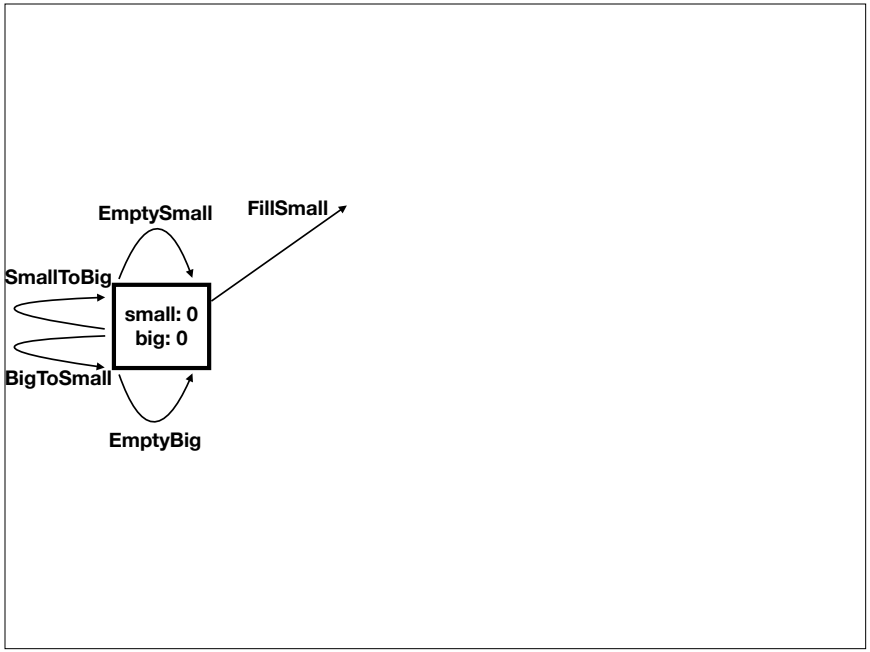
160-3



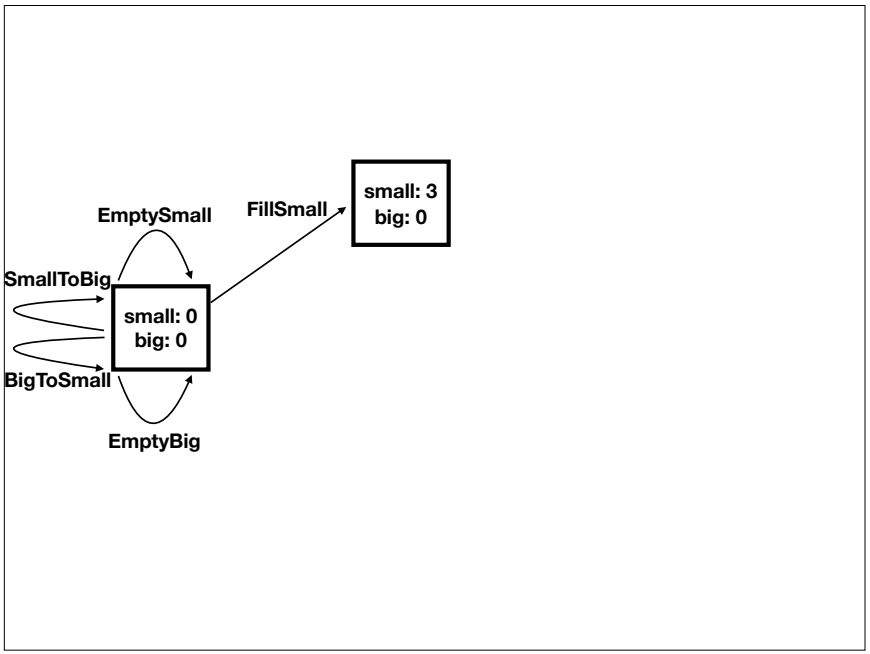
160-4



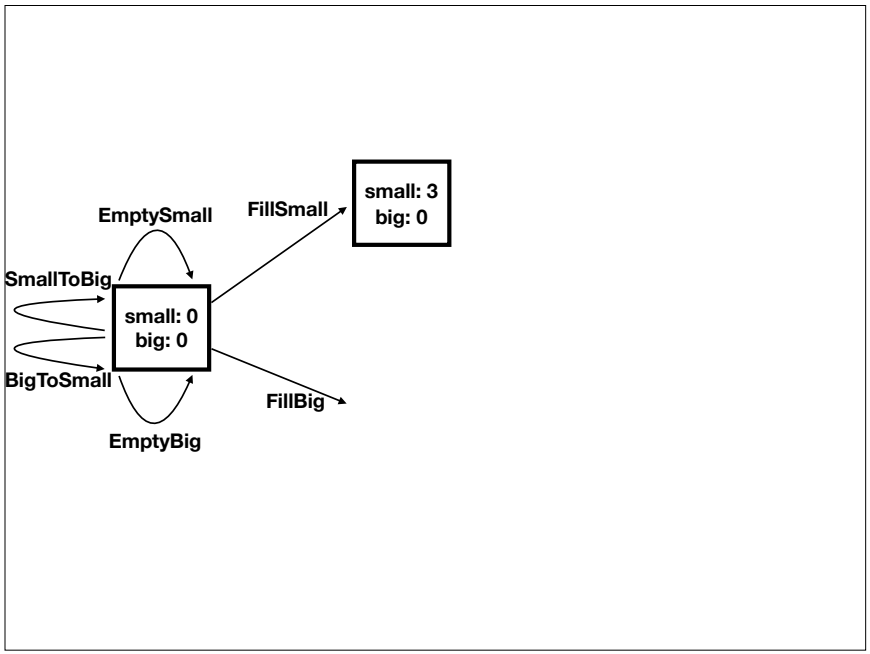
160-5



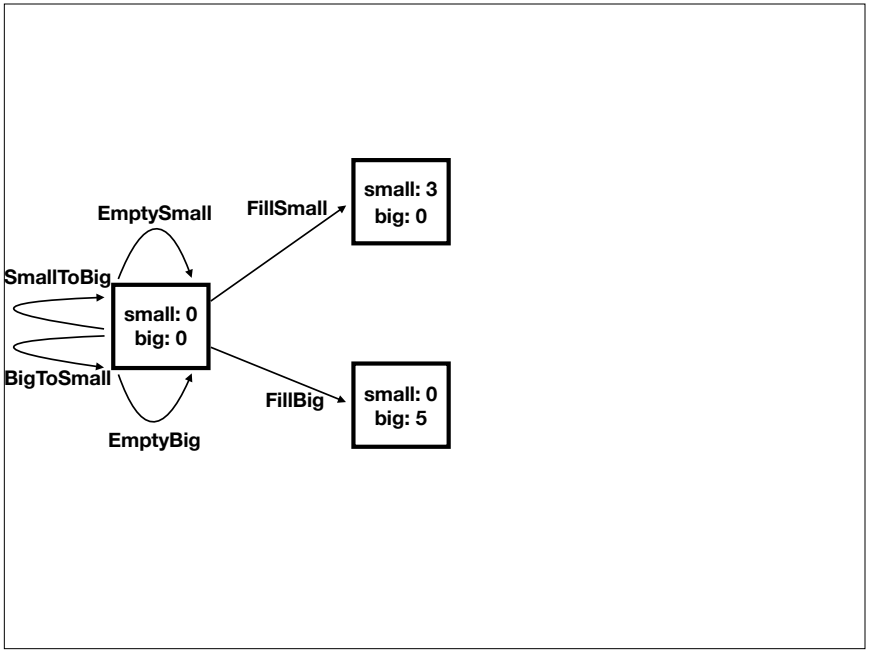
160-6



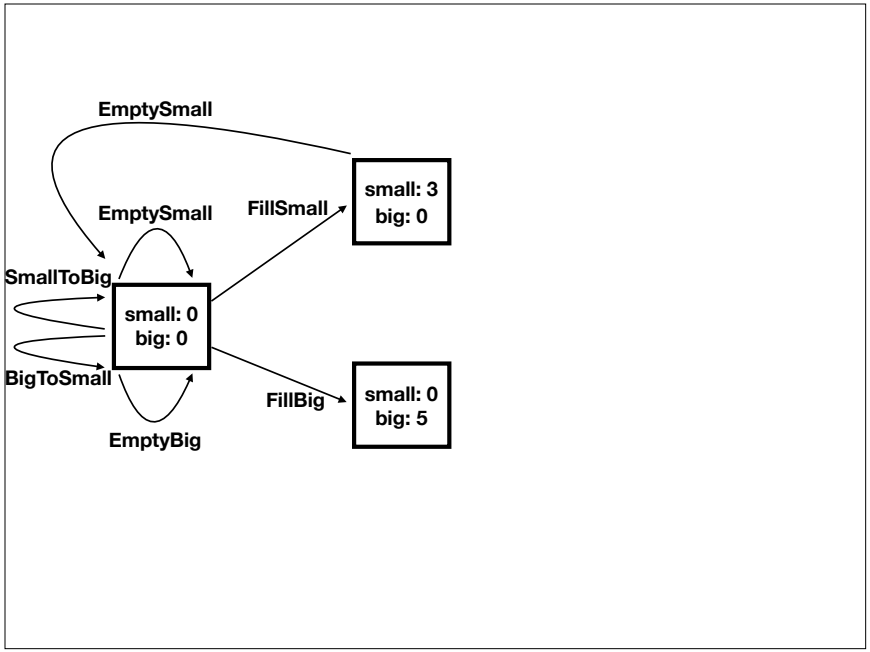
160-7



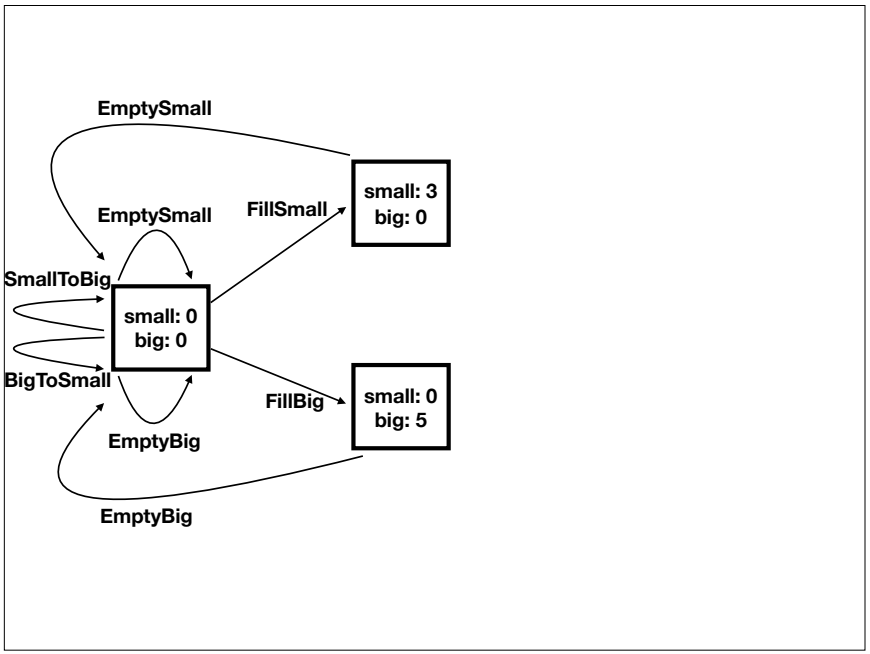
160-8



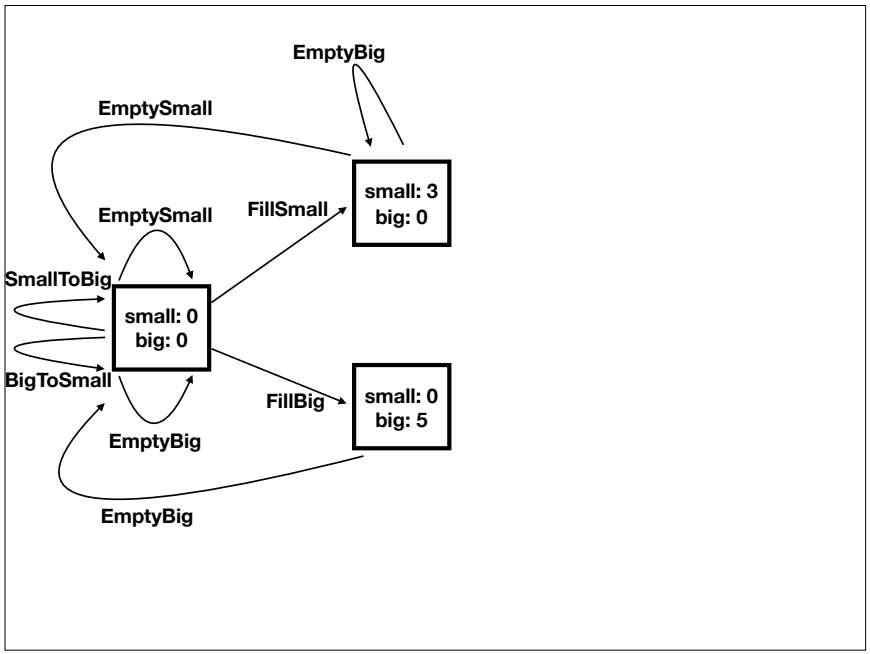
160-9



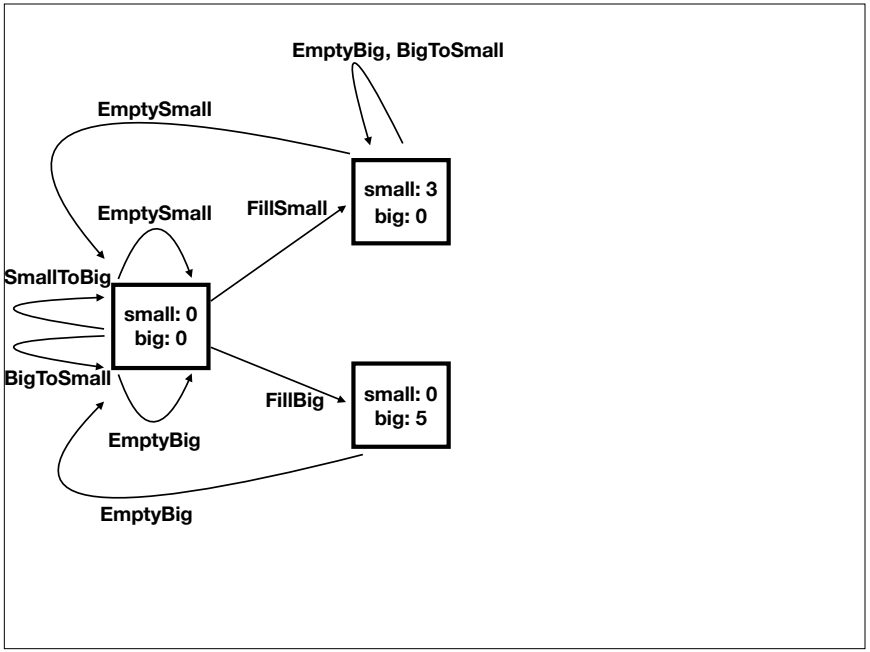
160-10



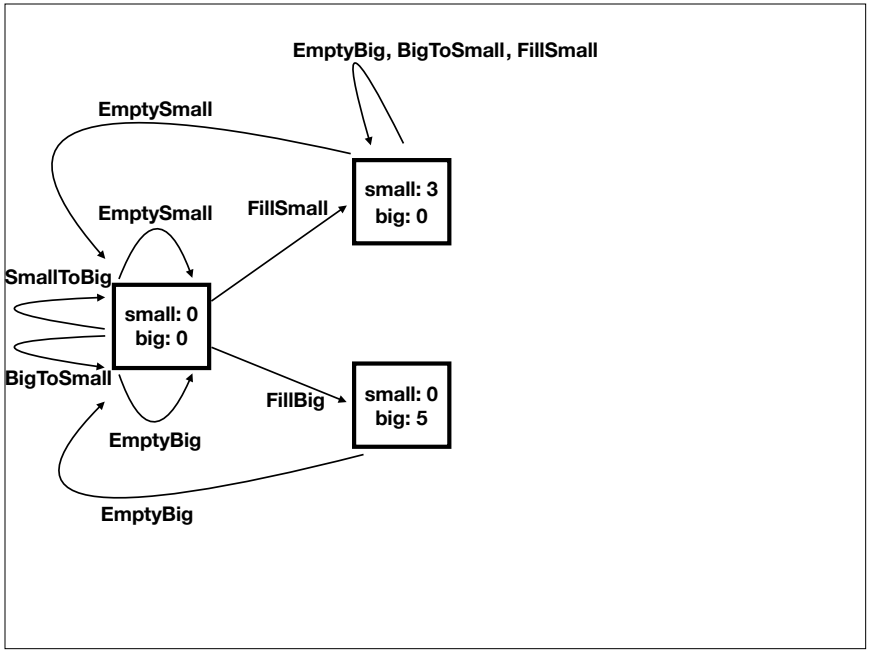
160-11



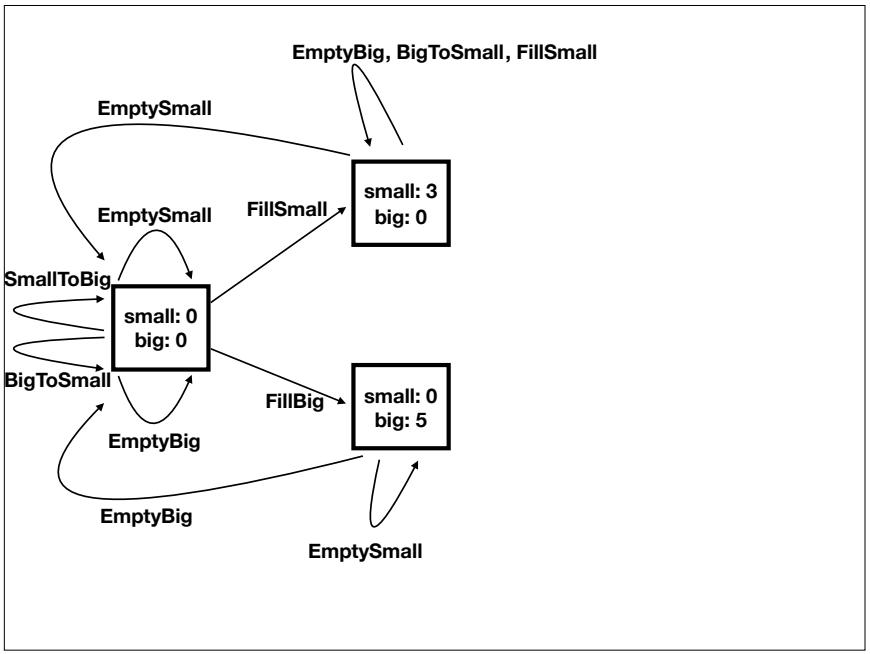
160-12



160-13



160-14



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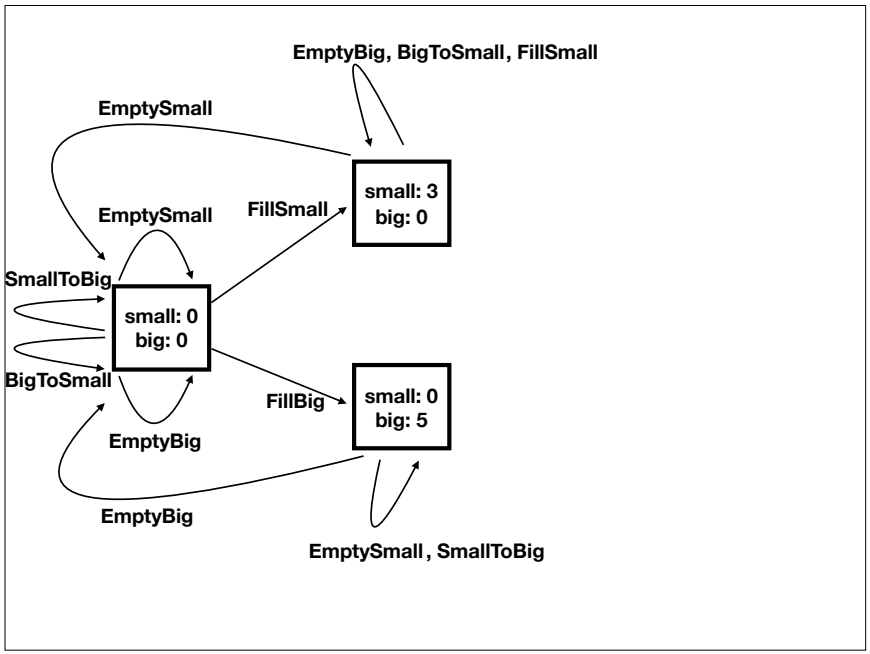

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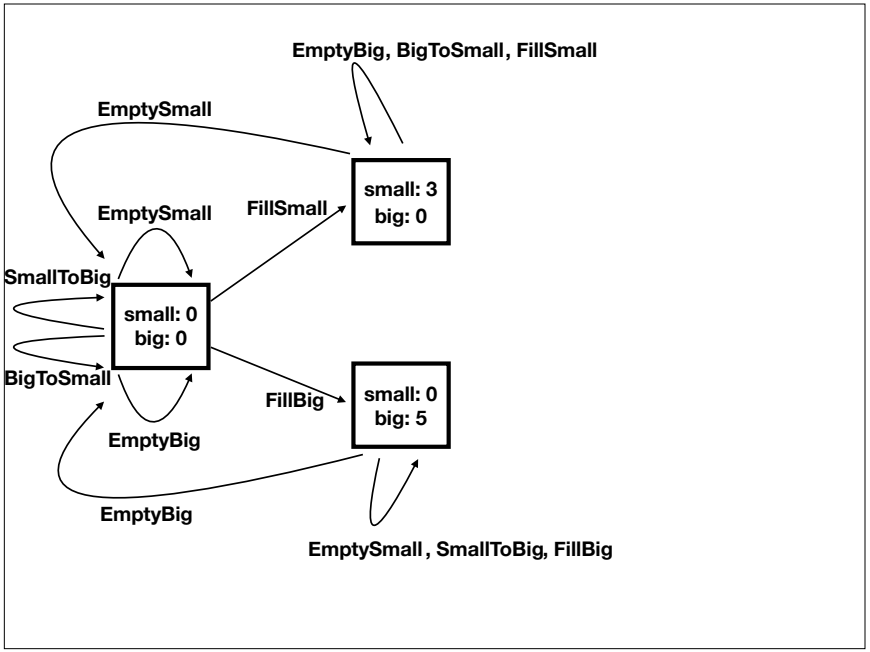

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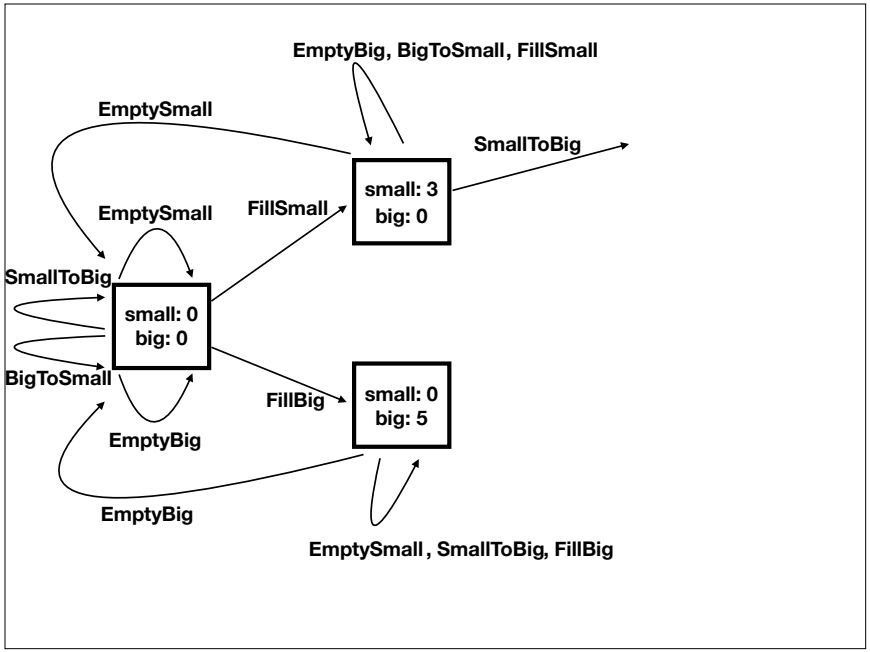
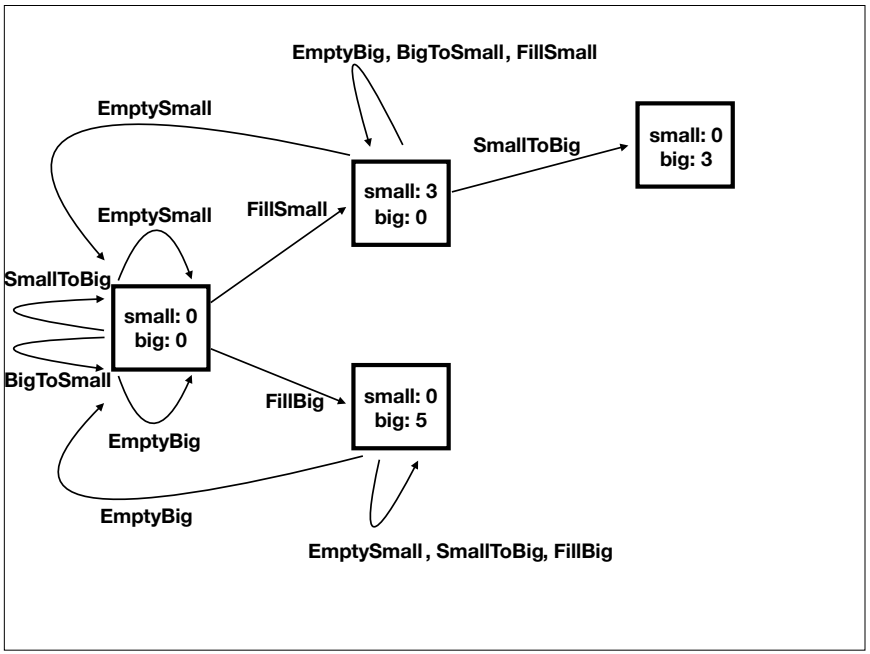


160-18



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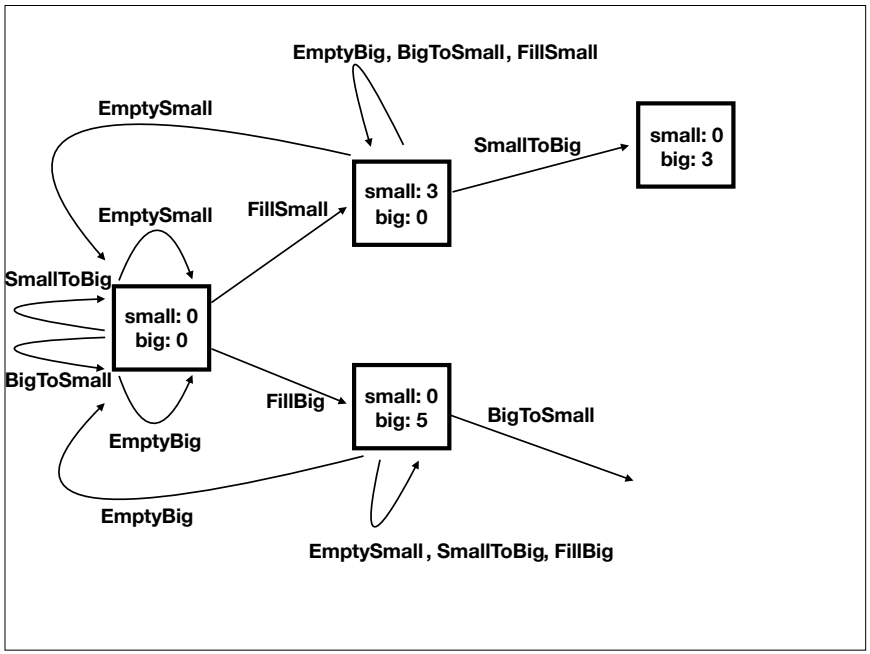

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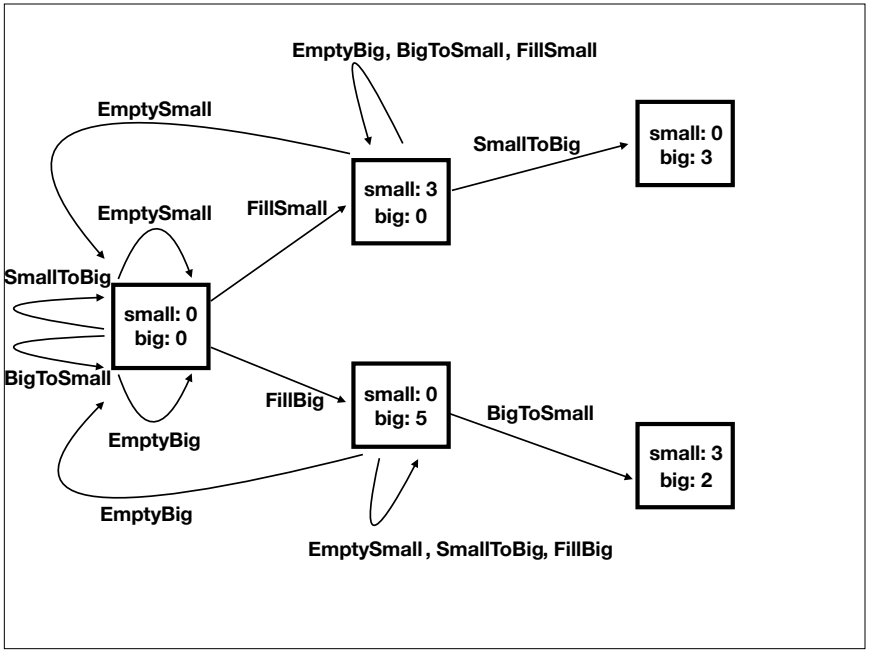

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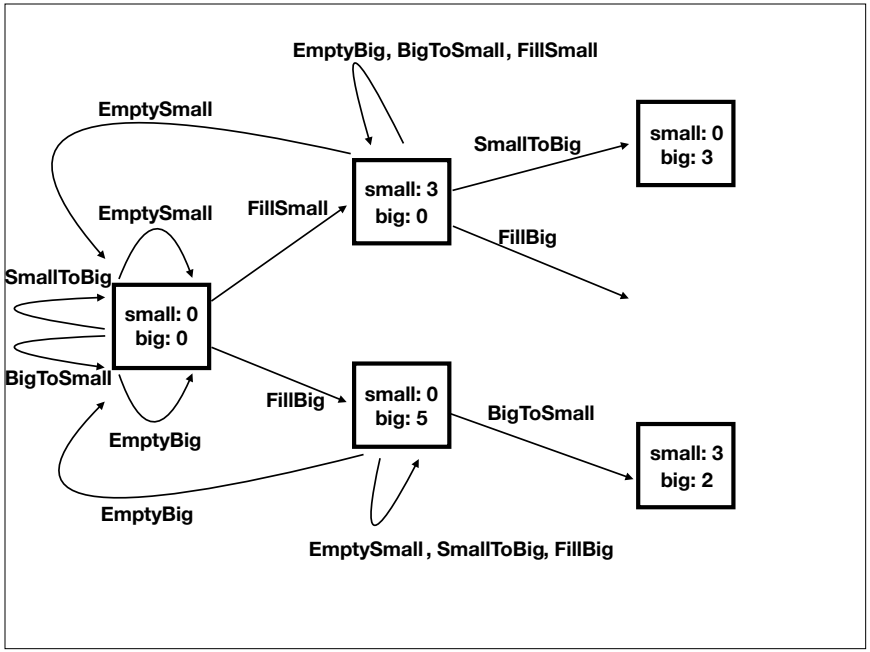

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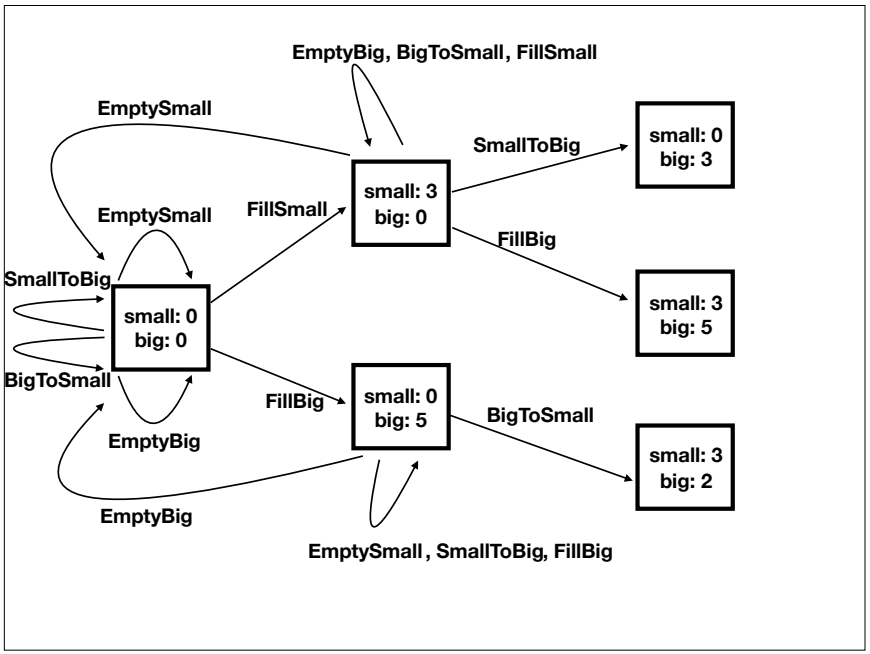

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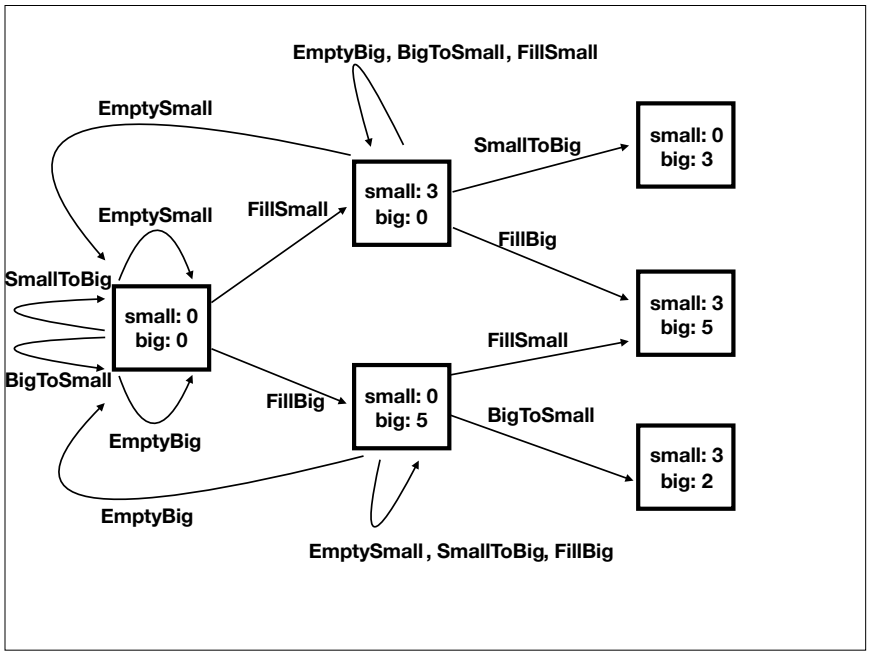

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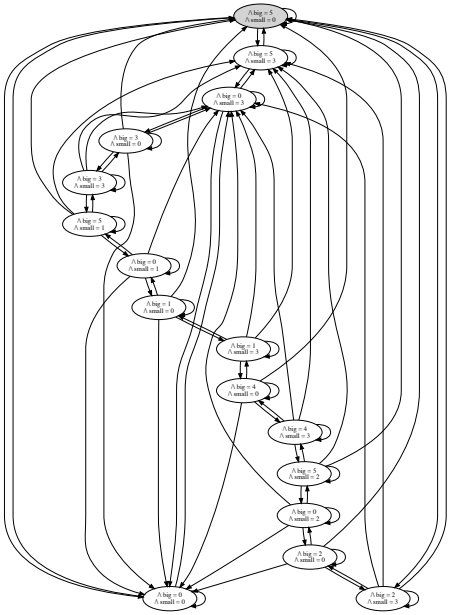

161

Die Hard

162-1

Die Hard

- Given the restrictions determined by our formula, TLC has found a total of 16 **unique** states.

162-2

Die Hard

- Given the restrictions determined by our formula, TLC has found a total of 16 **unique** states.
- Remember that a state is a particular assignment of **values** to our **variables**. Thus, 16 unique and valid combinations of values assigned to **big** and **small**.

162-3

Die Hard

- Given the restrictions determined by our formula, TLC has found a total of 16 **unique** states.
- Remember that a state is a particular assignment of **values** to our **variables**. Thus, 16 unique and valid combinations of values assigned to **big** and **small**.
- …. and so what? What does this get us? Who cares?

Invariants

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- It's time to explore the true power of TLA+ and TLC

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- An invariant is "**something that should always be true**"

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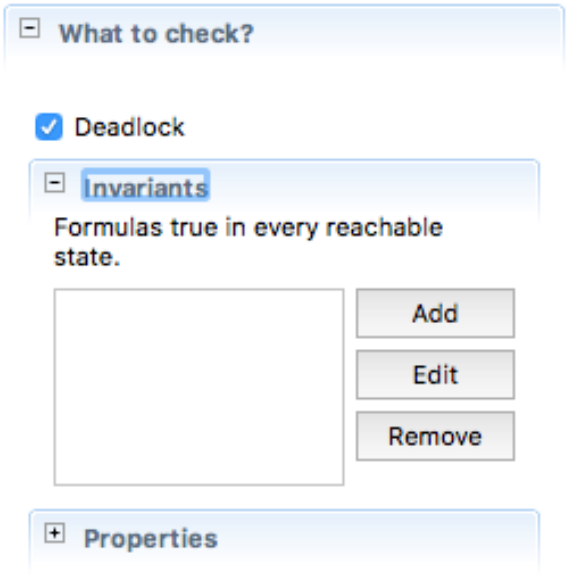
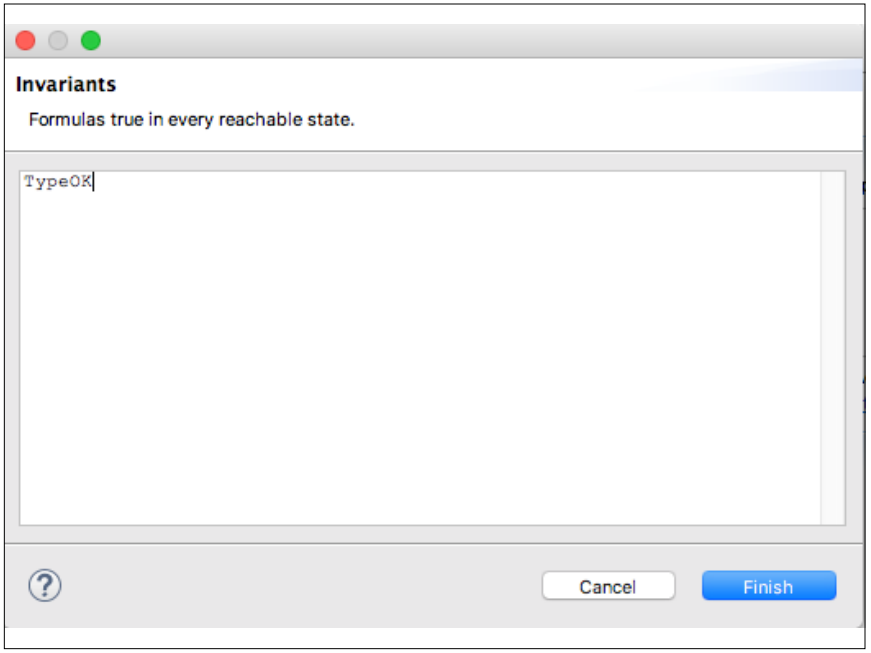
- An invariant is "**something that should always be true**"
  - Or "**something that should never occur**"
- TLC excels at generating every possible state that can occur given your formula
- You can also tell it to check that a certain thing is **always** true (or **never** true) in any of those states, i.e. check invariants





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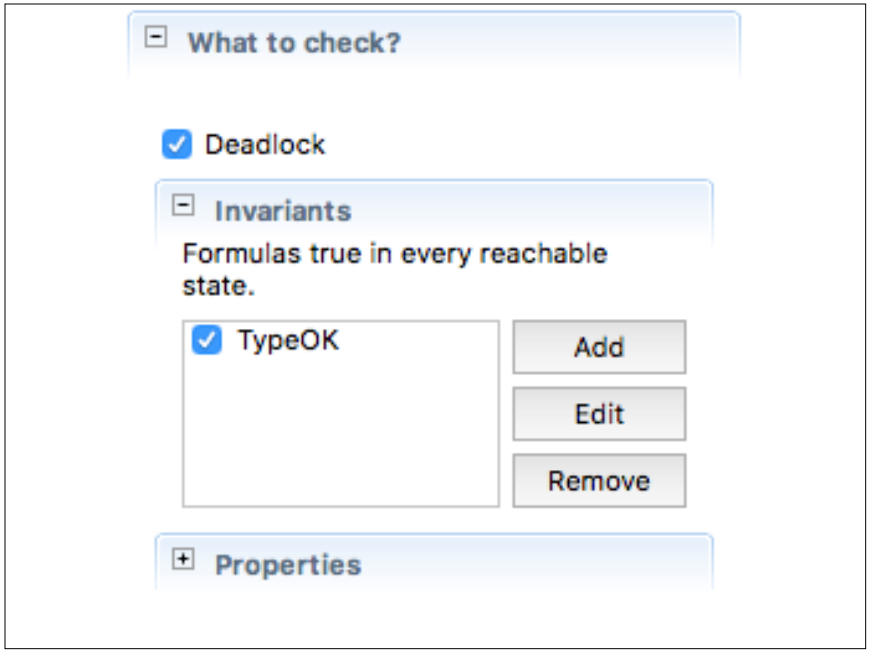

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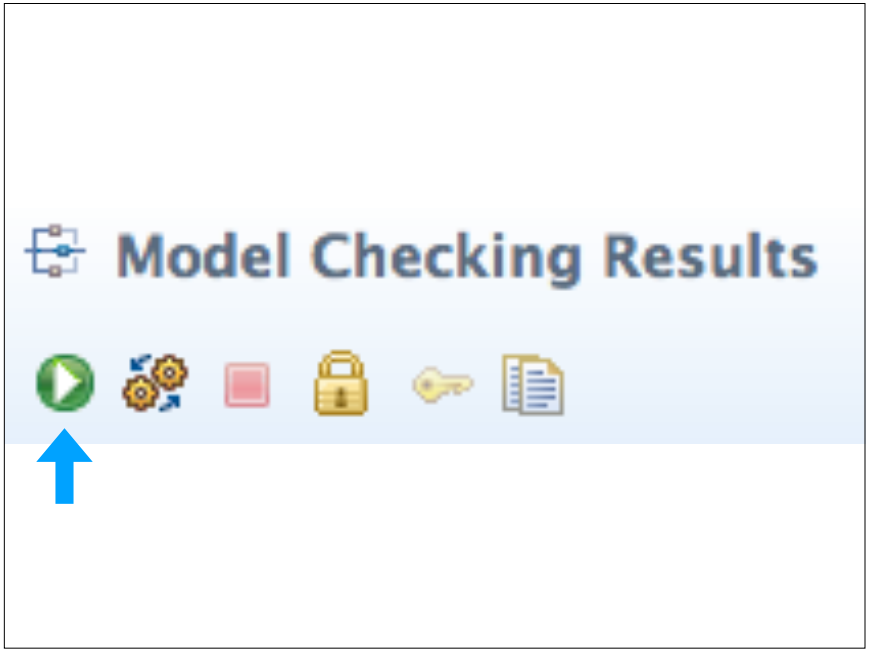

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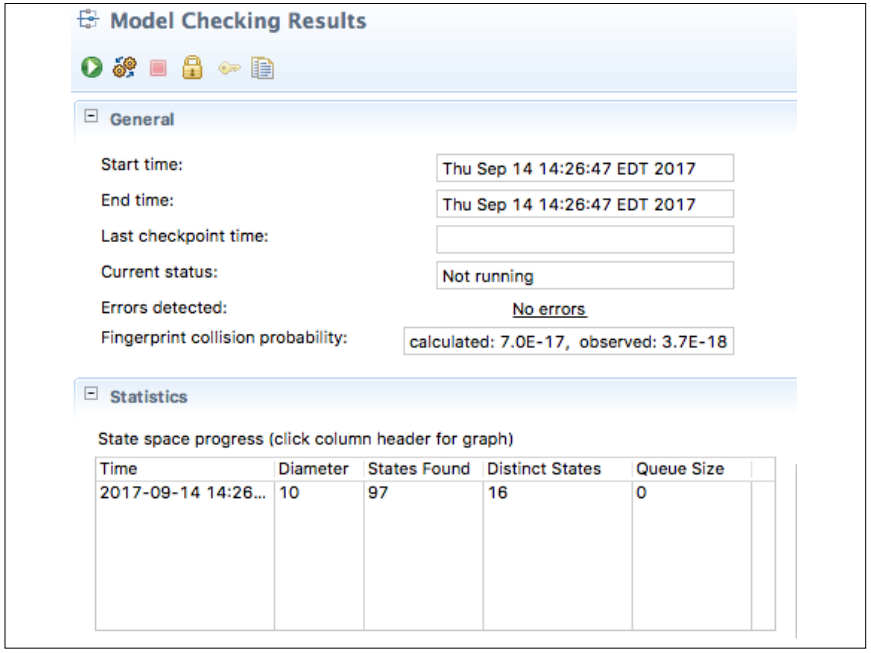
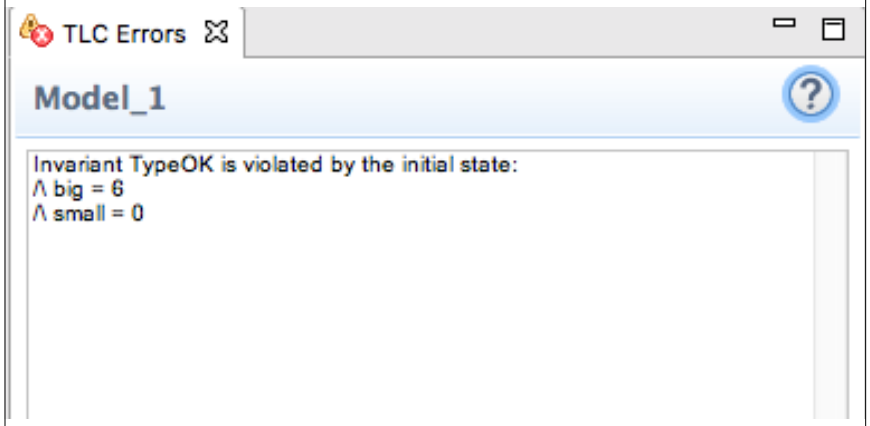


168

```
Init == /\ big   = 6
        /\ small = 0
```

169



170

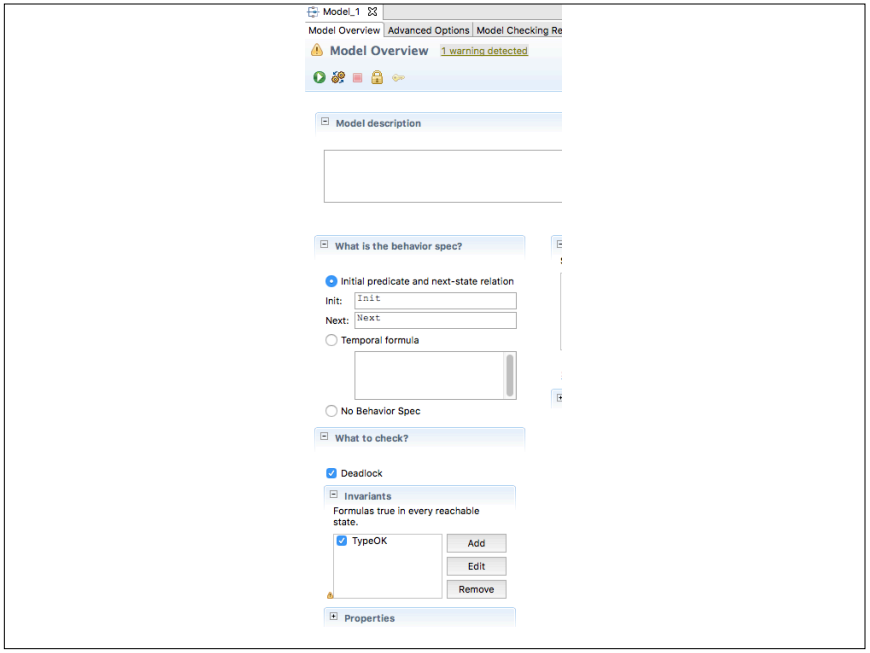
TypeOK

```
TypeOK == /\ small \in 0..3
          /\ big   \in 0..5
```

171

```
Init == /\ big   = 5
        /\ small = 0
```

172



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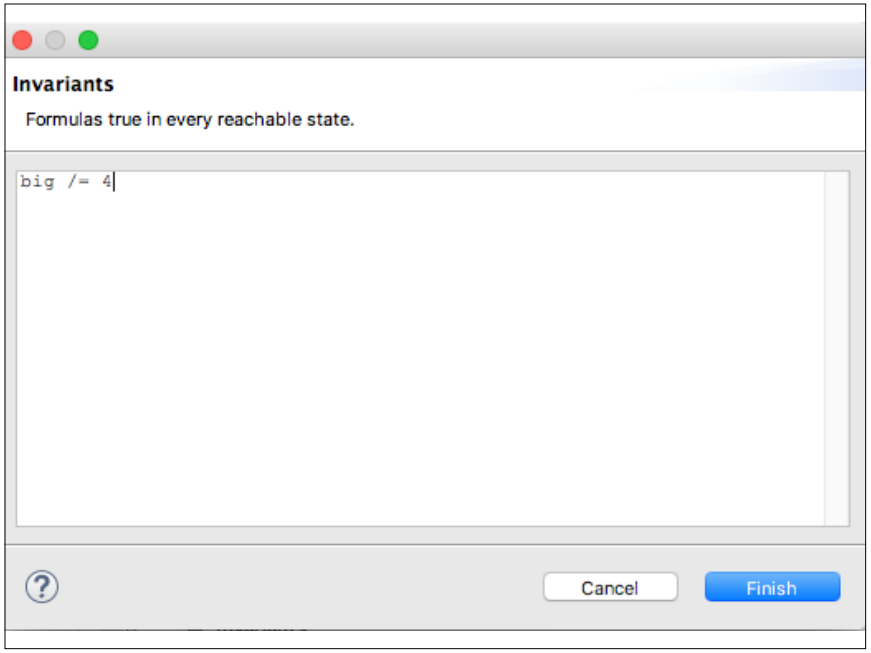

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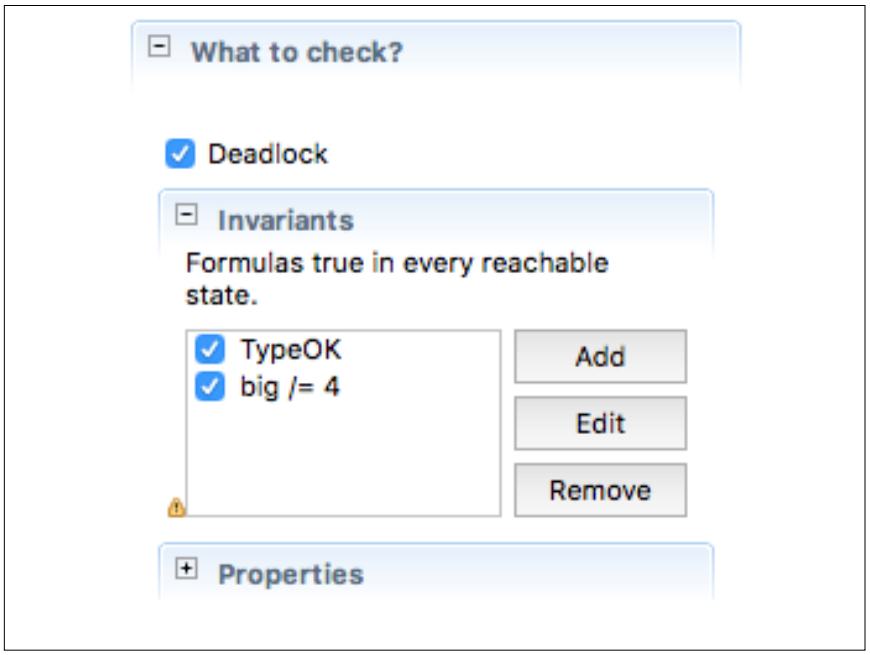

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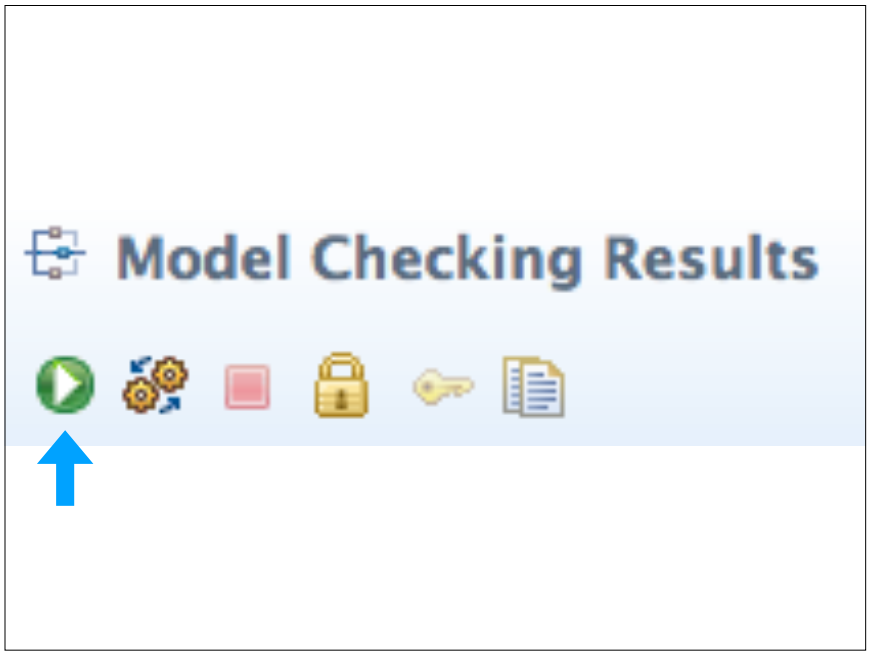

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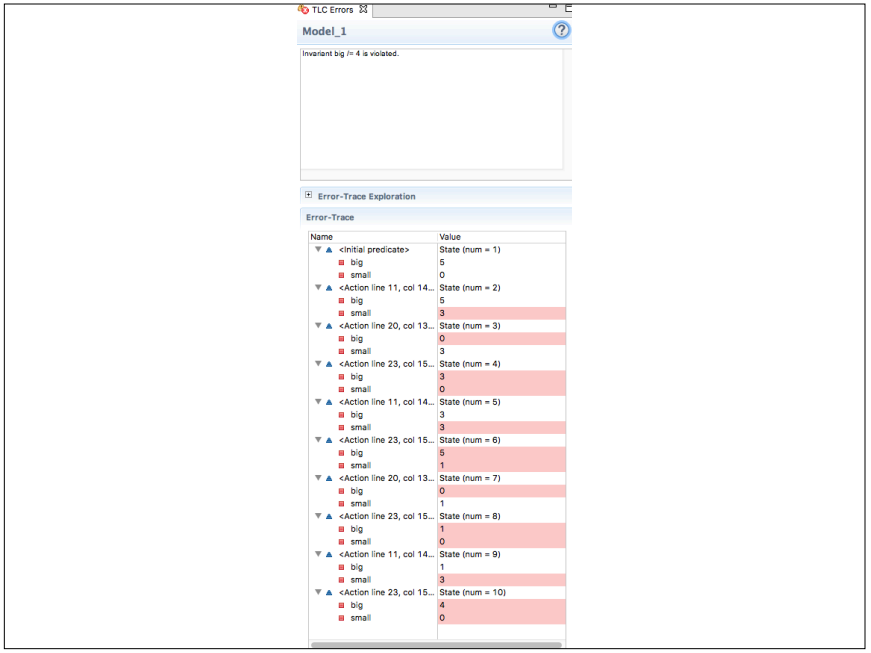
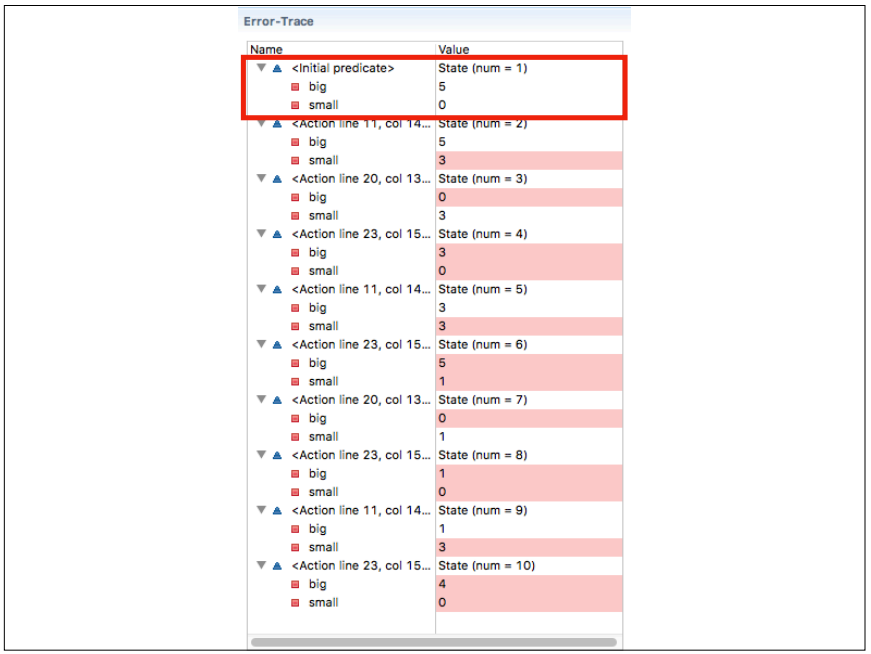
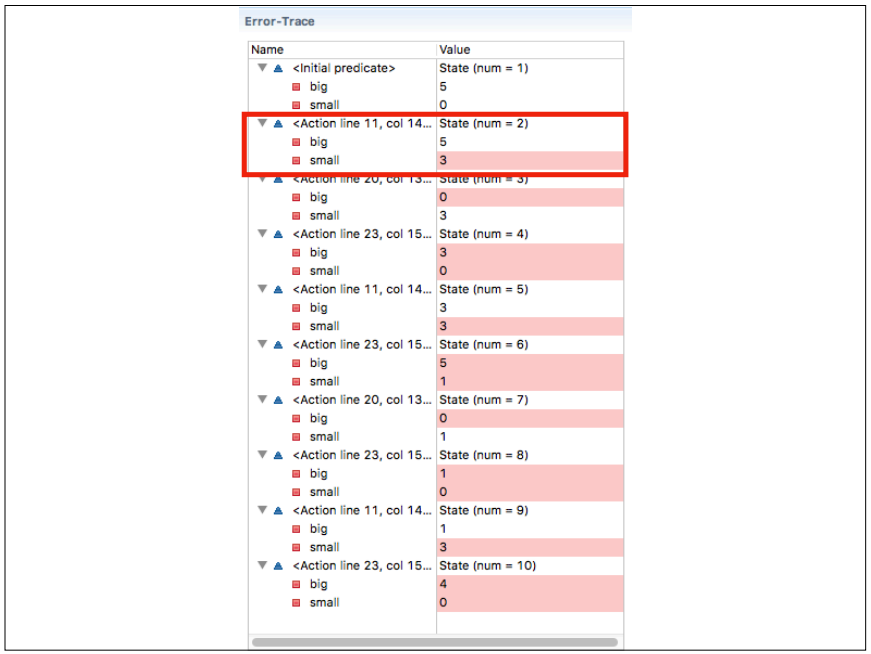


177

178-1



178-2



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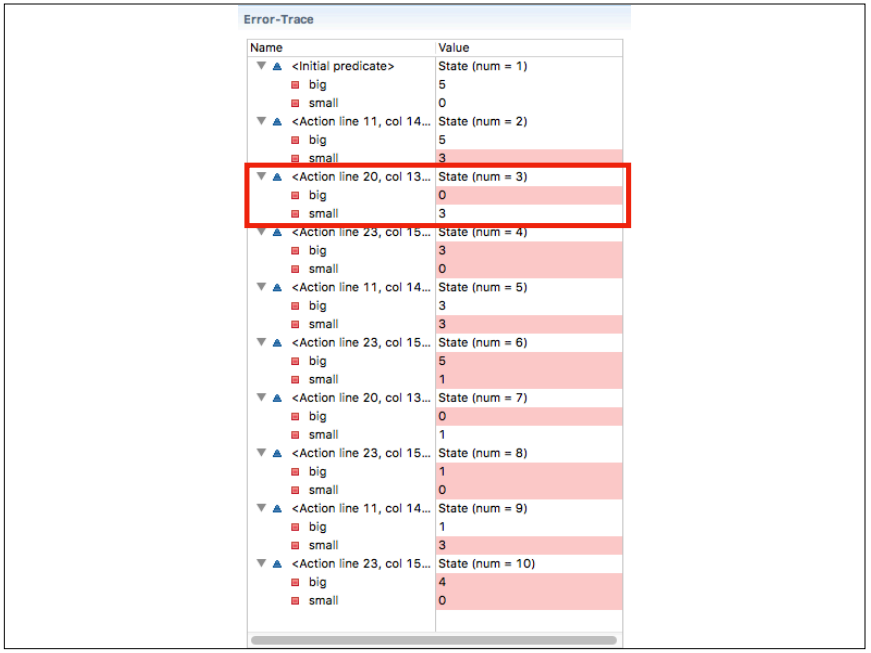

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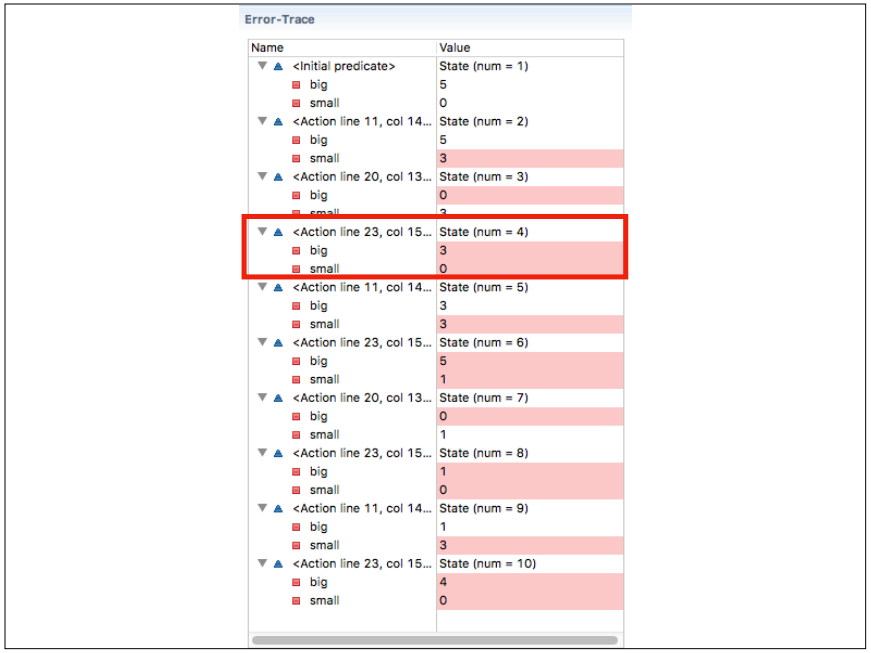

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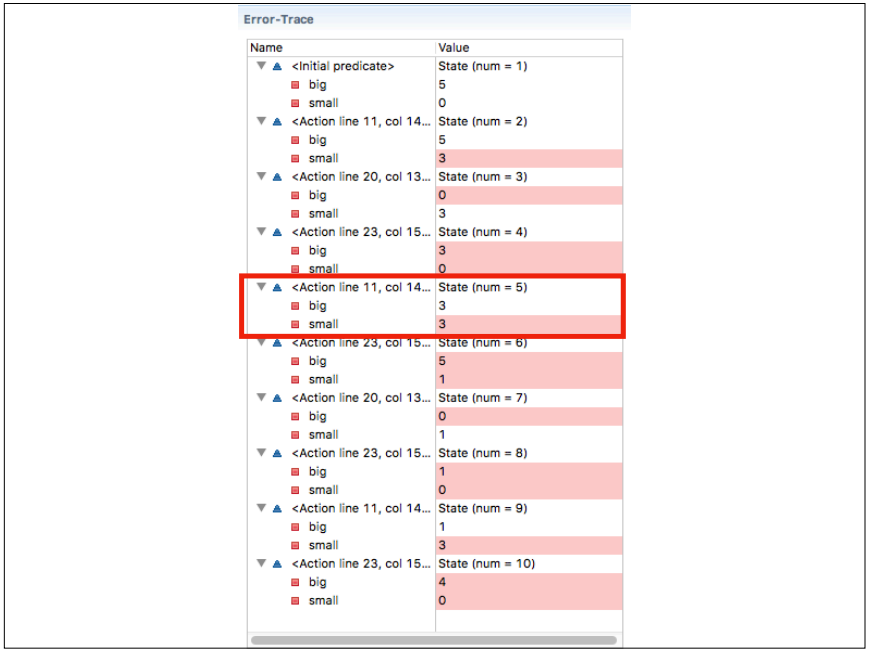

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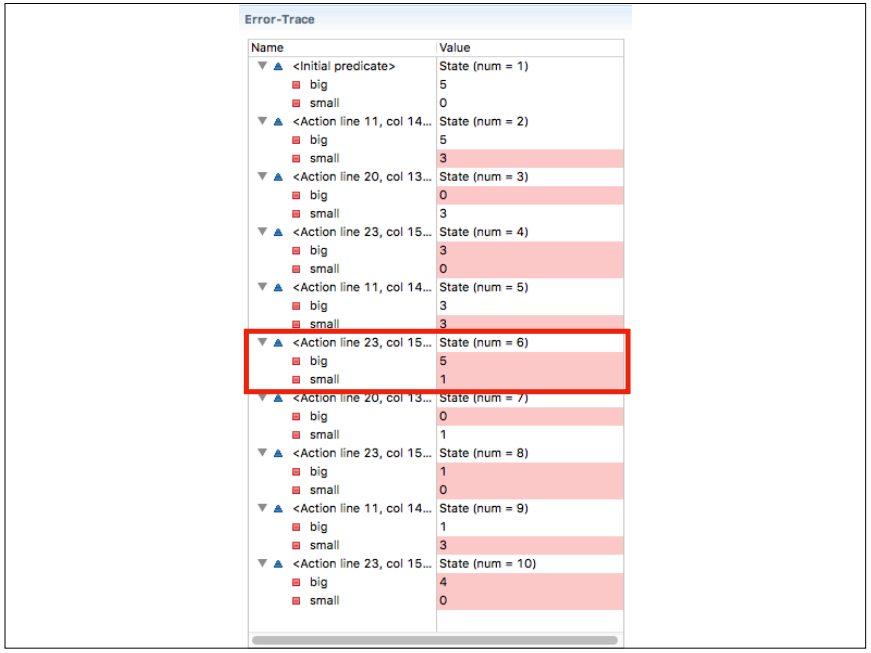

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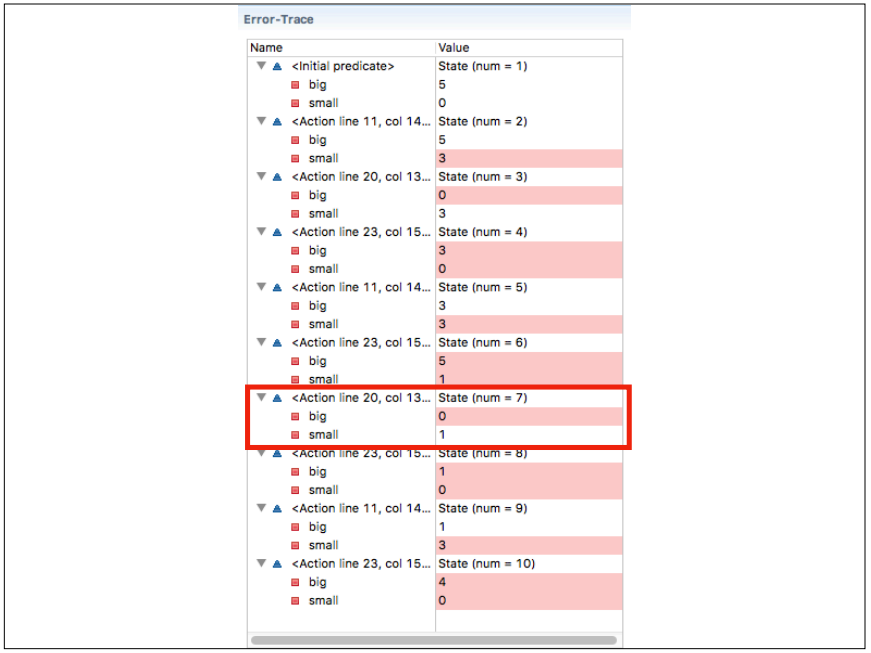
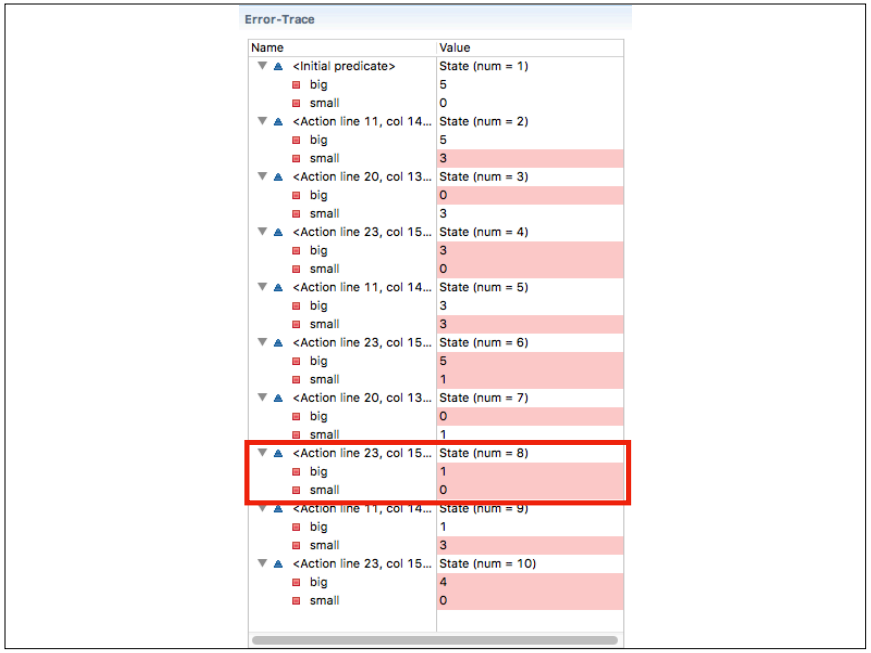


178-8



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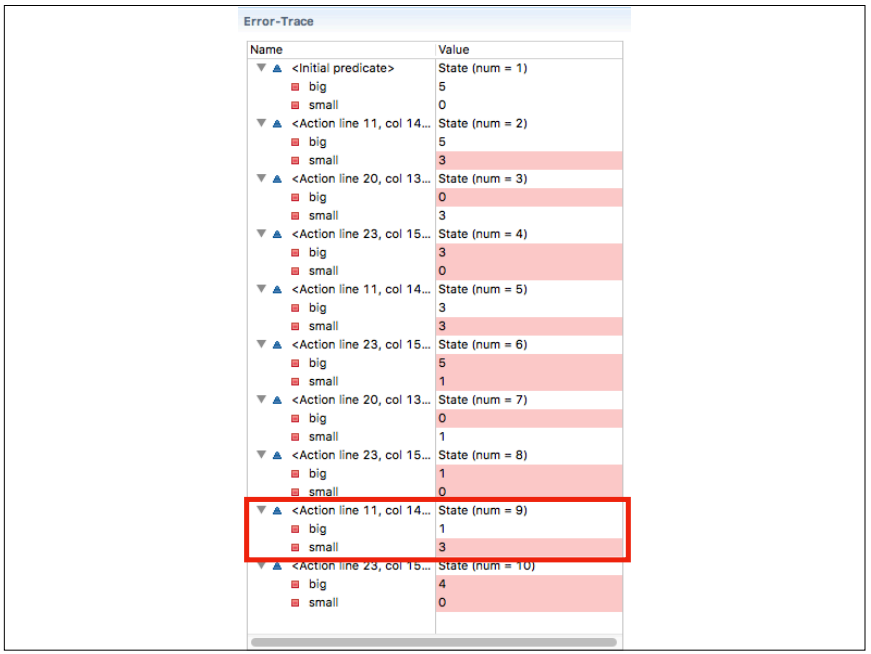

178-10

178-11

Woah

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- This is amazingly powerful.

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- TLC tries every single possible combination of states (i.e. behaviour/execution), and checks invariants for every state in every behaviour.

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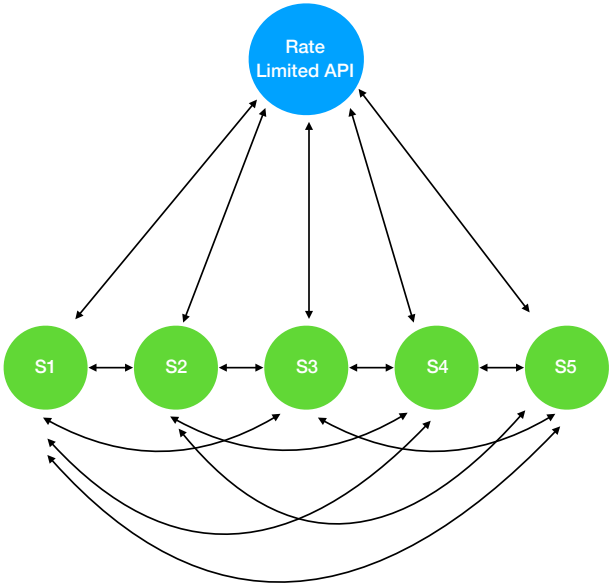
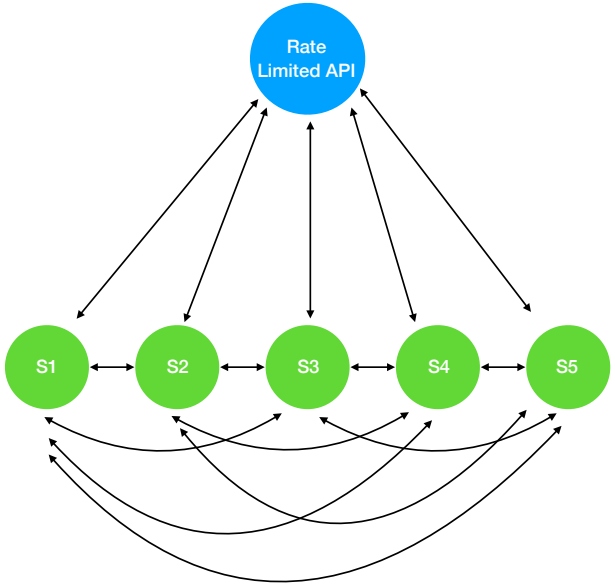
- The entire state history is kept, providing the best “stack trace” you’ve ever seen.

Woah

- This is amazingly powerful.
- TLC tries every single possible combination of states (i.e. behaviour/execution), and checks invariants for every state in every behaviour.
- The entire state history is kept, providing the best “stack trace” you’ve ever seen.
- For concurrent and distributed systems, this allows for detection of incredibly subtle interactions, race conditions, etc.





https://learntla.com/concurrency/example/

AWS Fault-Tolerant Replication Algorithm

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- Engineer took an existing designed and implemented algorithm at AWS, which had been analyzed for months with dozens of pages of informal proofs.

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- Wrote a TLA+ specification for the algorithm.

AWS Fault-Tolerant Replication Algorithm

- Engineer took an existing designed and implemented algorithm at AWS, which had been analyzed for months with dozens of pages of informal proofs.
- Wrote a TLA+ specification for the algorithm.
- TLC found three serious bugs, one of which had a 35+-step trace.

Transaction Commit

Transaction Commit

- Consider a replicated database.

Transaction Commit

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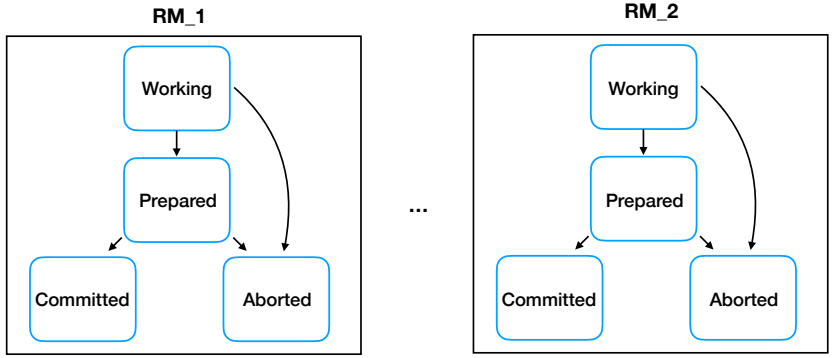
- Multiple “resource managers,” each carrying a full replica.

Transaction Commit

- Consider a replicated database.
- Multiple “resource managers,” each carrying a full replica.
- For any transaction sent to the database, all resource managers **must come to a consensus** on whether the transaction should be committed or aborted.

Resource Managers

Resource Managers



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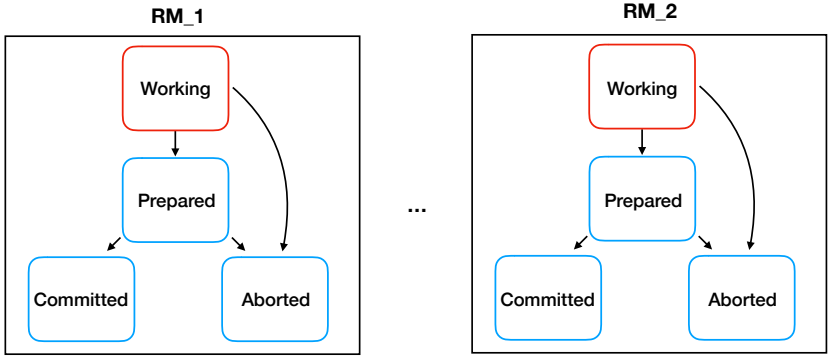
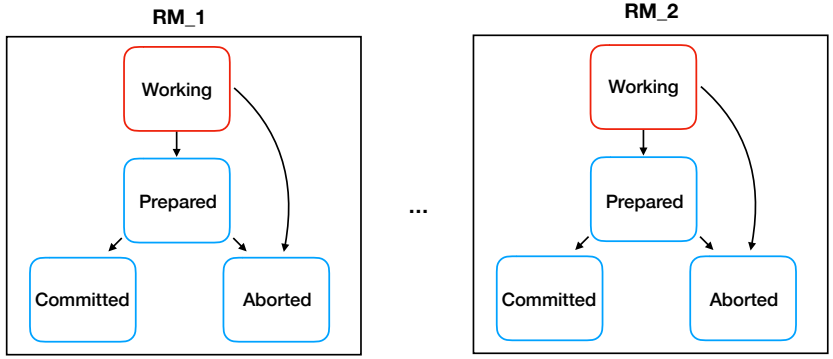
Resource Managers



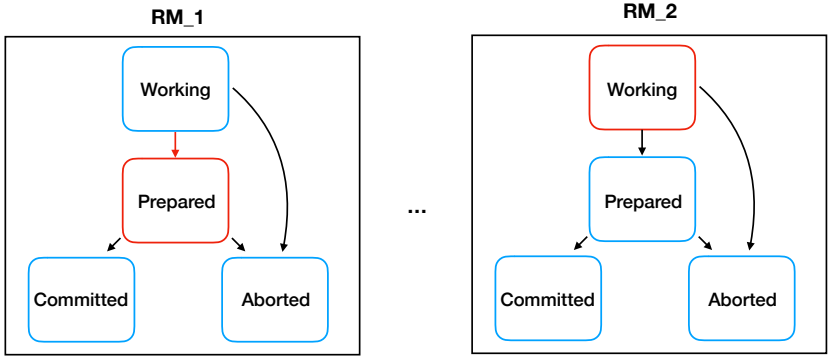
185

Resource Managers



186

Resource Managers



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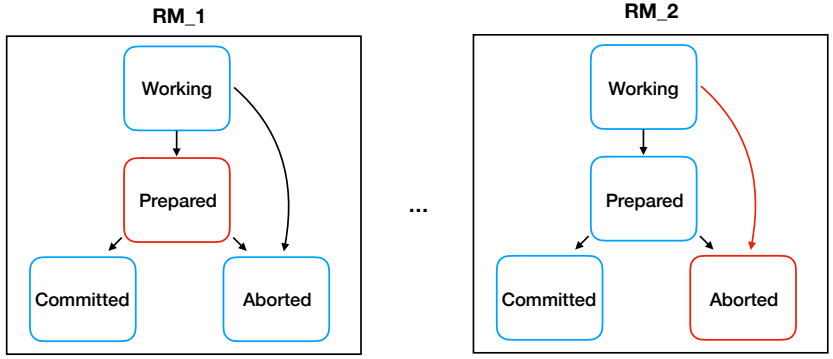
Resource Managers



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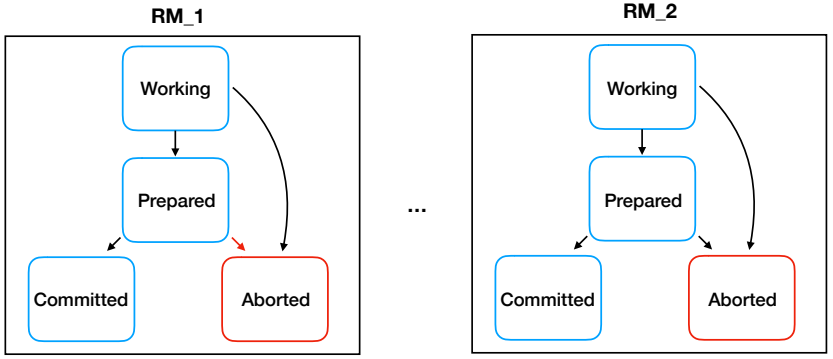
Resource Managers



189

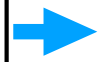
Resource Managers

rmState["rm1"]: "working"
rmState["rm2"]: "working"

190-1

Resource Managers

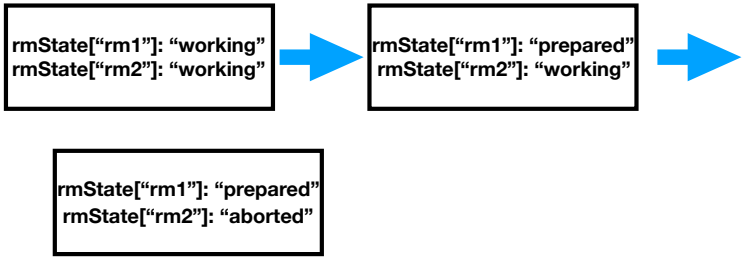
rmState["rm1"]: "working"
rmState["rm2"]: "working"



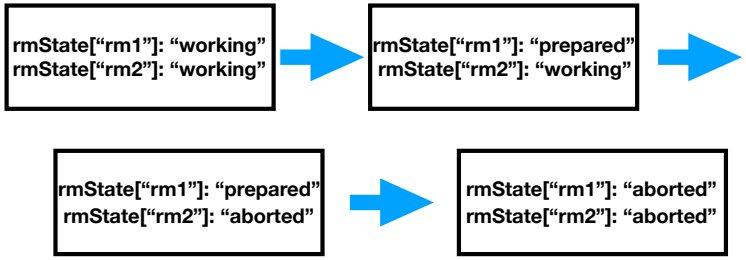
rmState["rm1"]: "prepared"
rmState["rm2"]: "working"

190-2

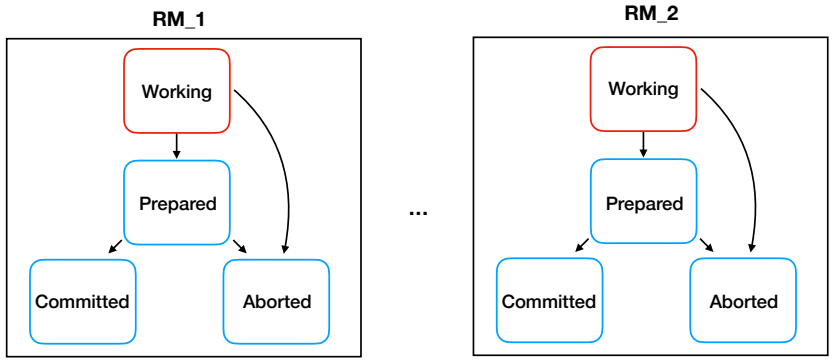
Resource Managers



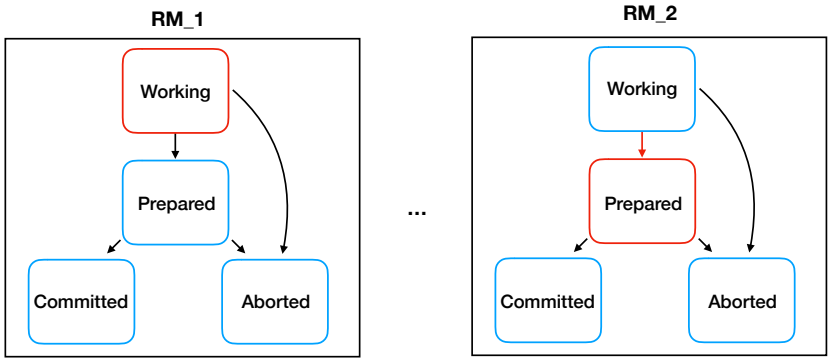
Resource Managers



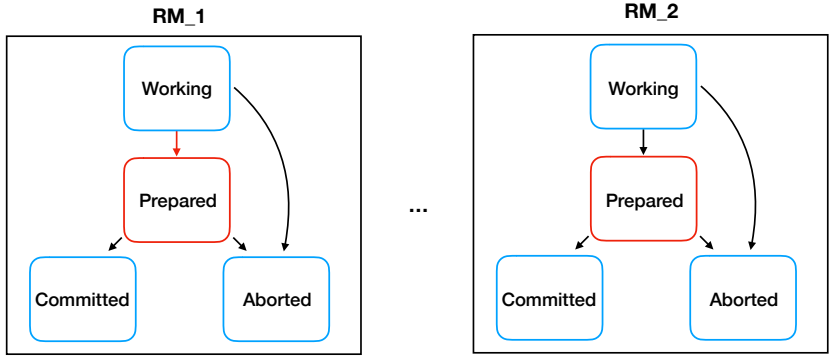
Resource Managers



Resource Managers

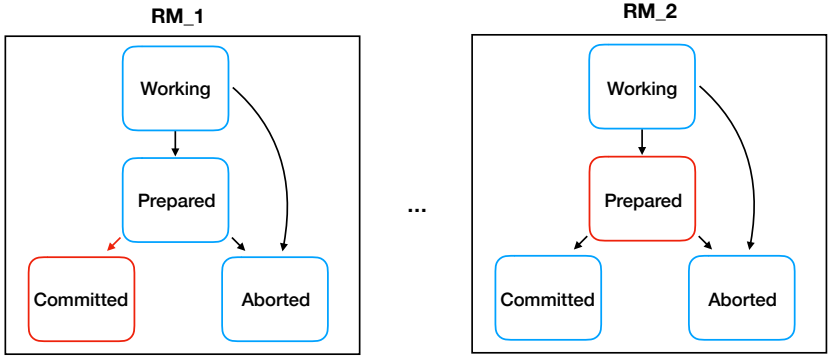


Resource Managers



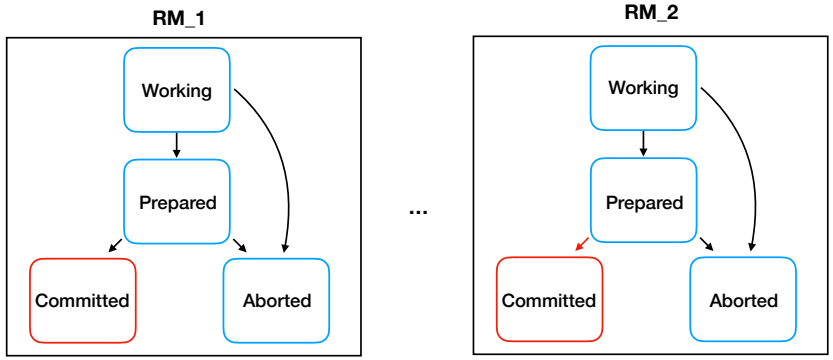
193

Resource Managers



194

Resource Managers



195

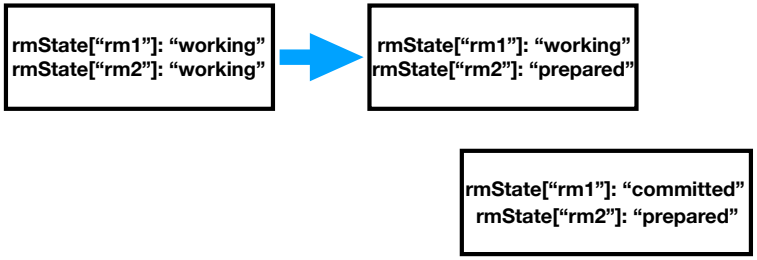
Resource Managers

rmState["rm1"]: "working"
rmState["rm2"]: "working"

rmState["rm1"]: "committed"
rmState["rm2"]: "prepared"

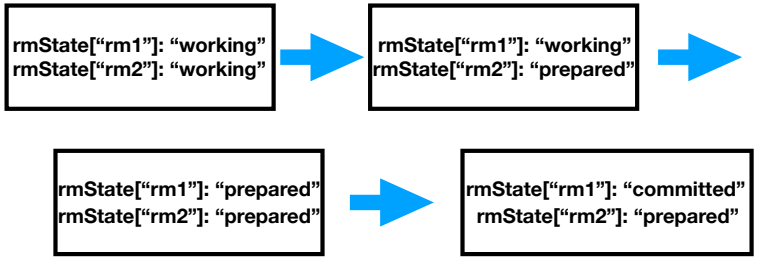
196-1

Resource Managers



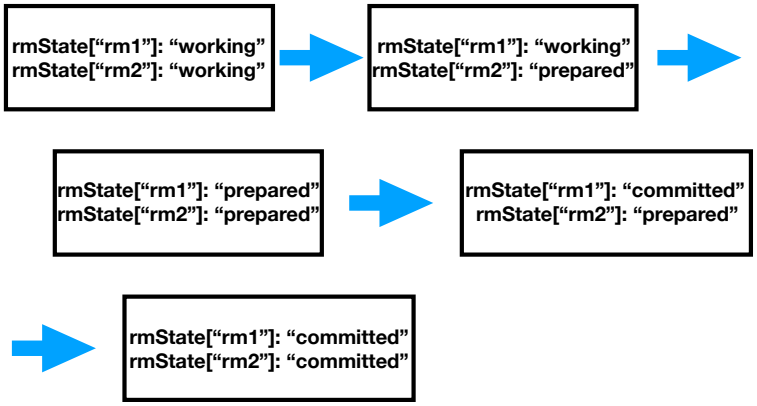
196-2

Resource Managers



196-3

Resource Managers



196-4

Resource Managers

197-1

Resource Managers

- We just looked at two possible behaviours.

197-2

Resource Managers

- We just looked at two possible behaviours.
- How many in total are possible?

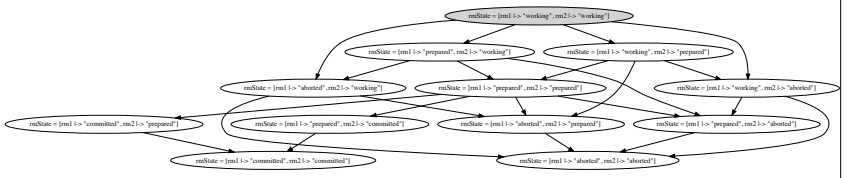
197-3

Resource Managers

- We just looked at two possible behaviours.
- How many in total are possible?
- Lots and lots! And with every additional Resource Manager added, it grows and grows.

197-4

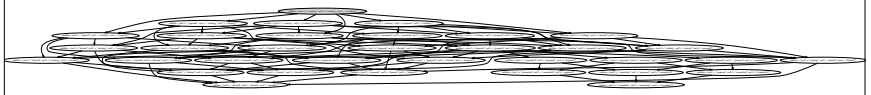
2 Resource Managers



12 Distinct States

198

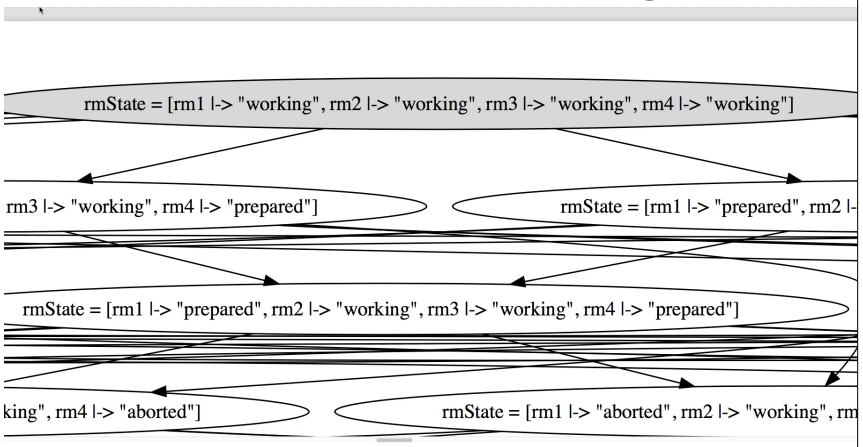
3 Resource Managers



34 Distinct States

199

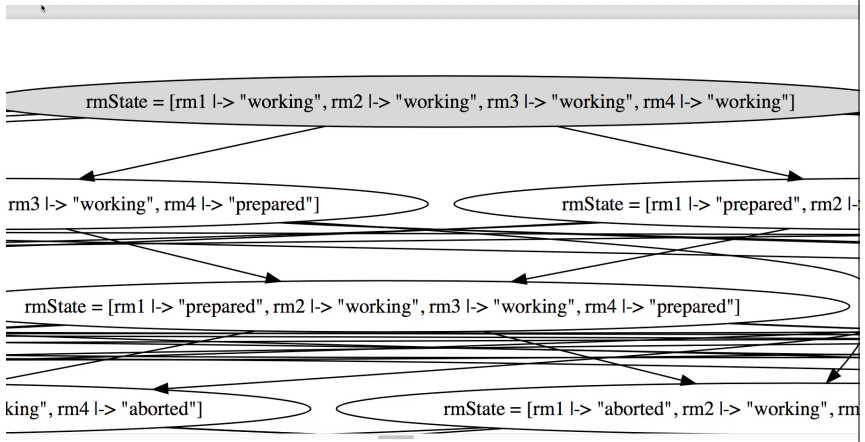
4 Resource Managers



96 Distinct States

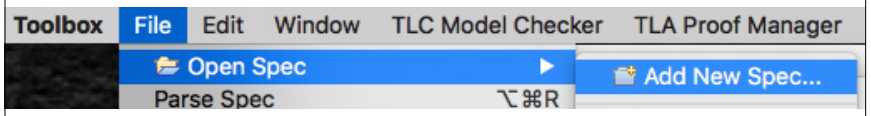
200-1

4 Resource Managers

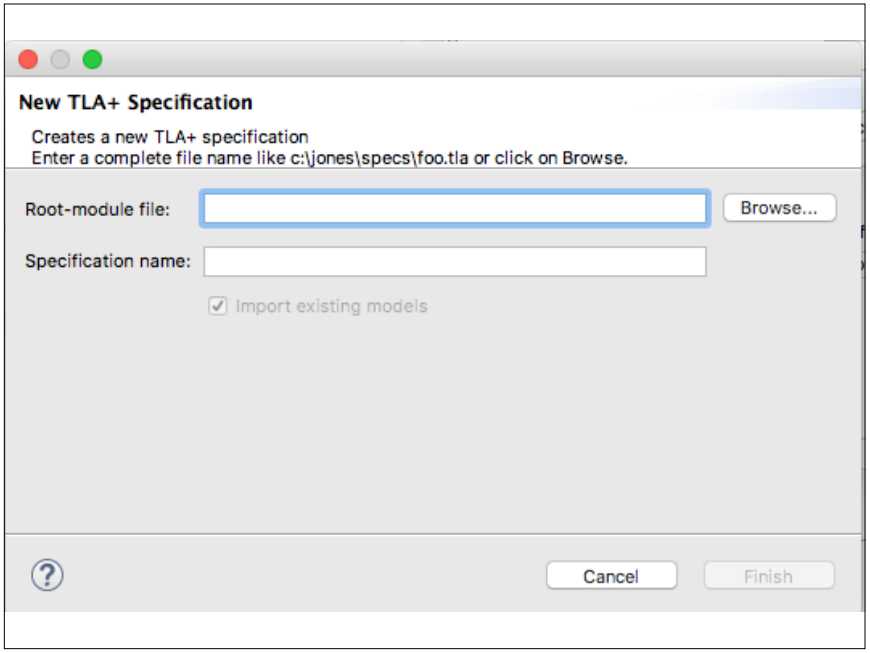


96 Distinct States

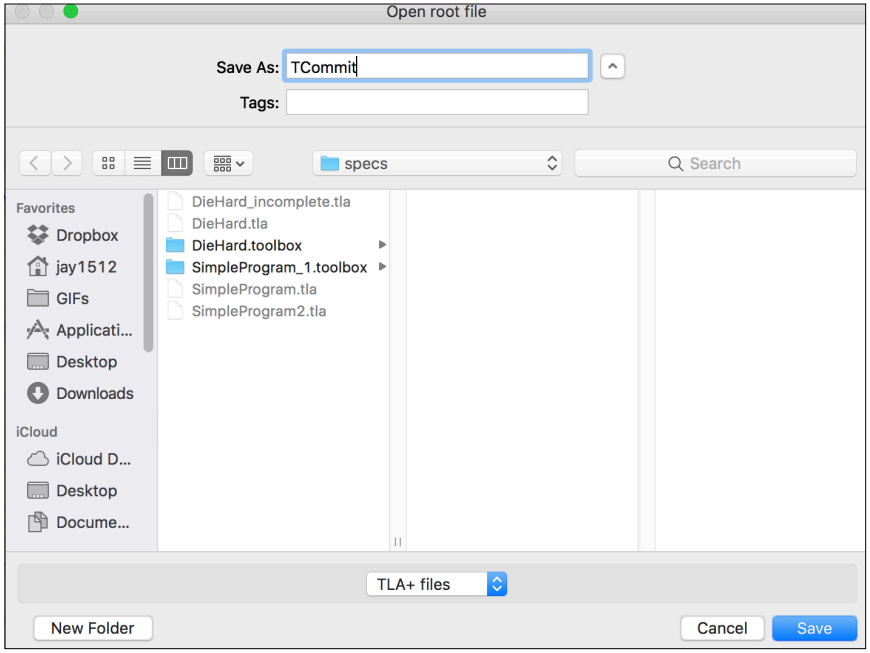
200-2



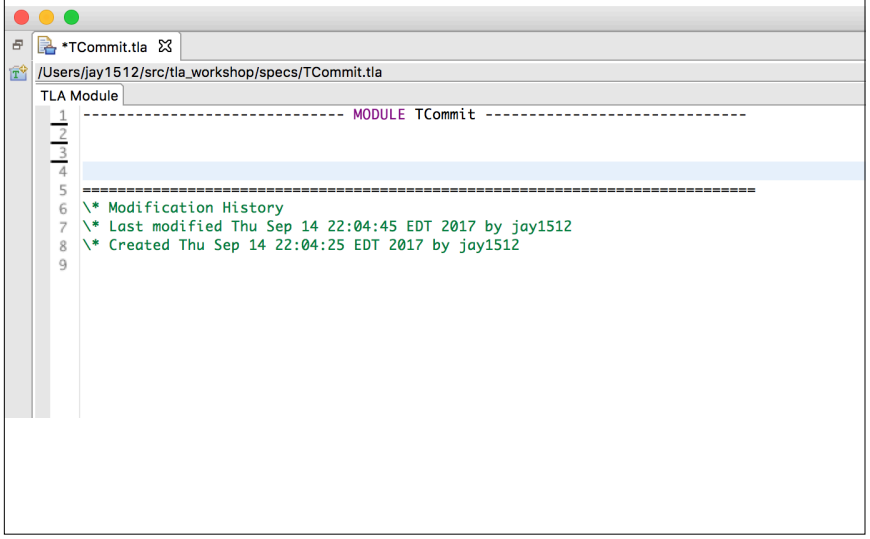
201



202



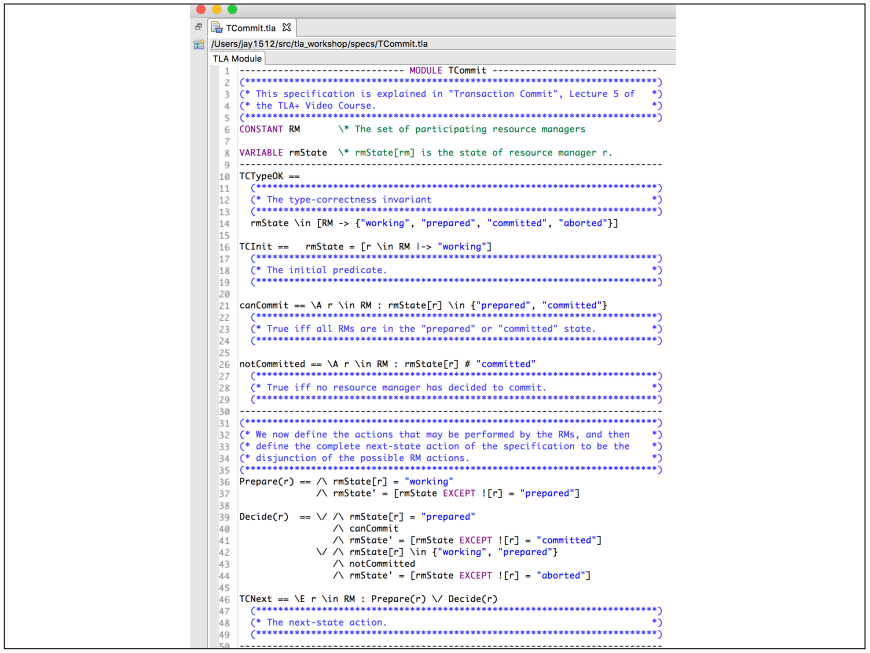
203



204

specs/TCommit.tla

205



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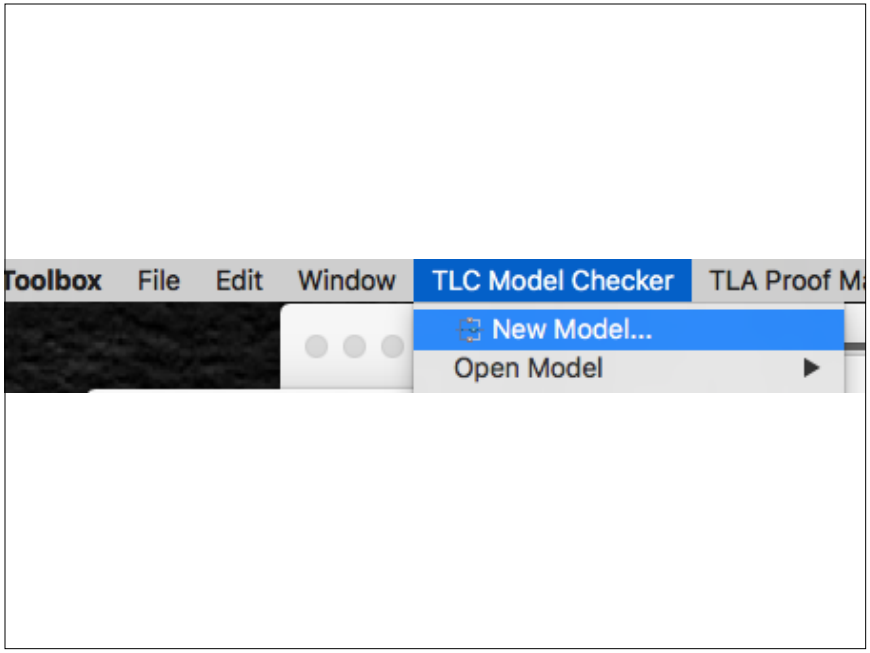
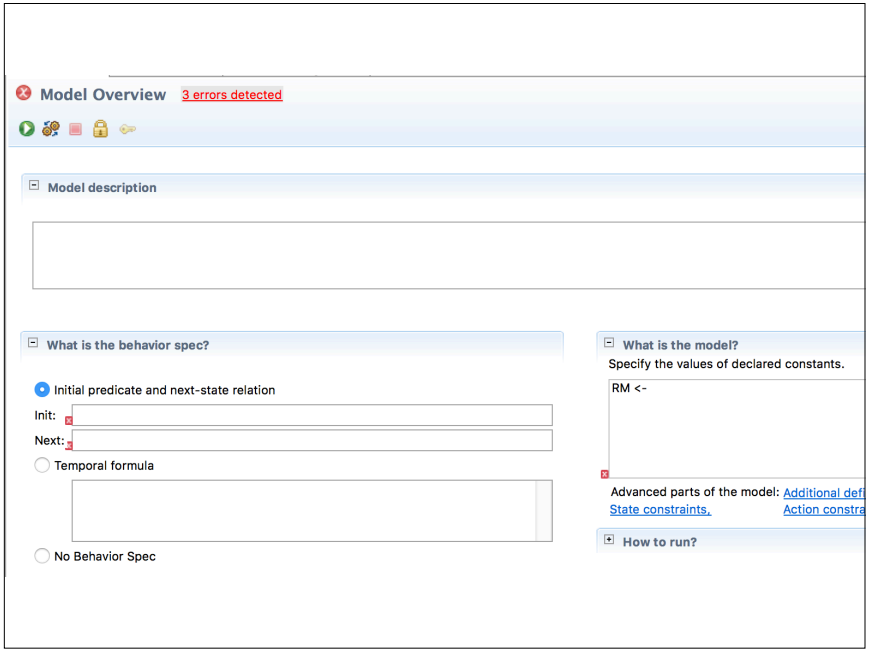
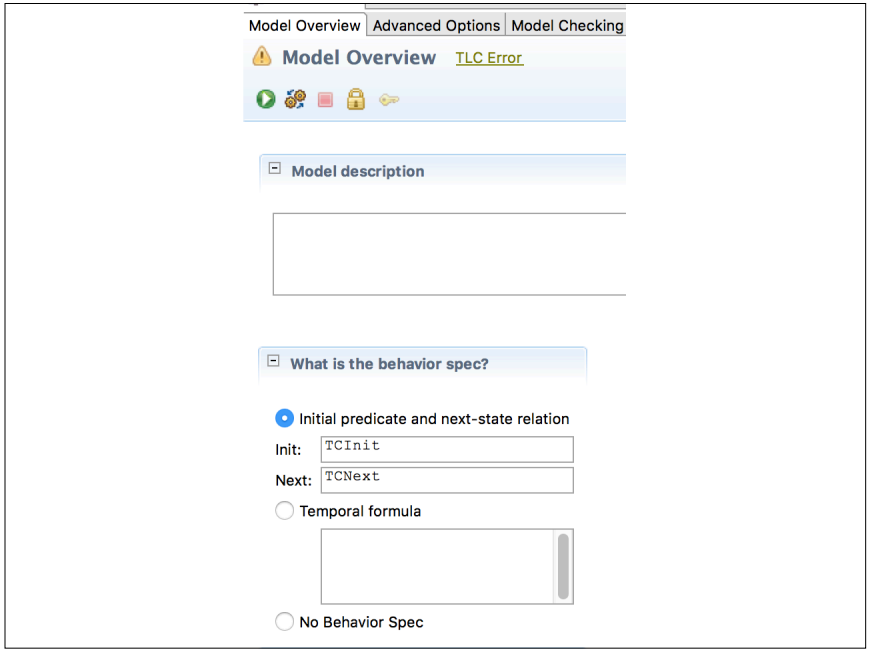


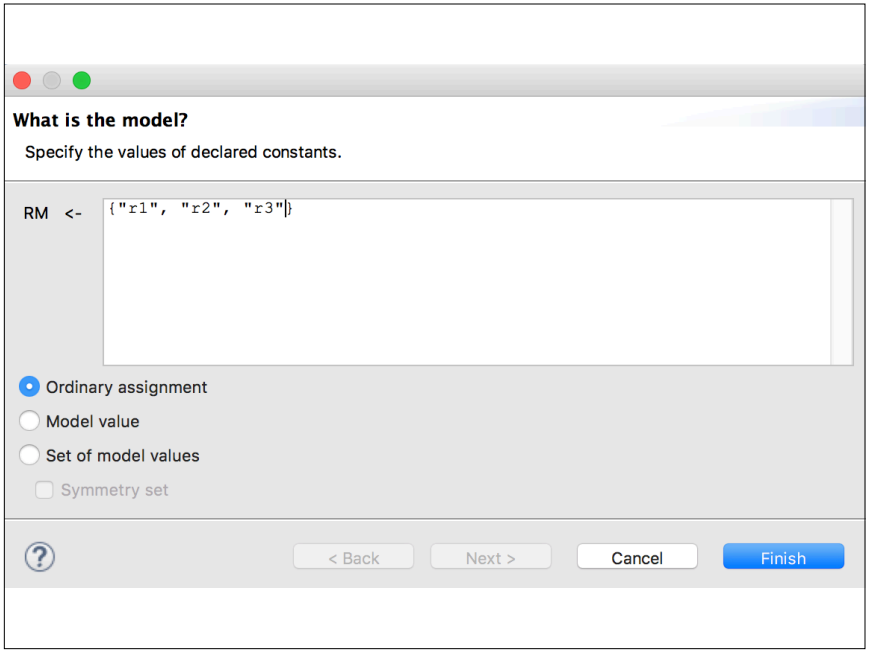
207



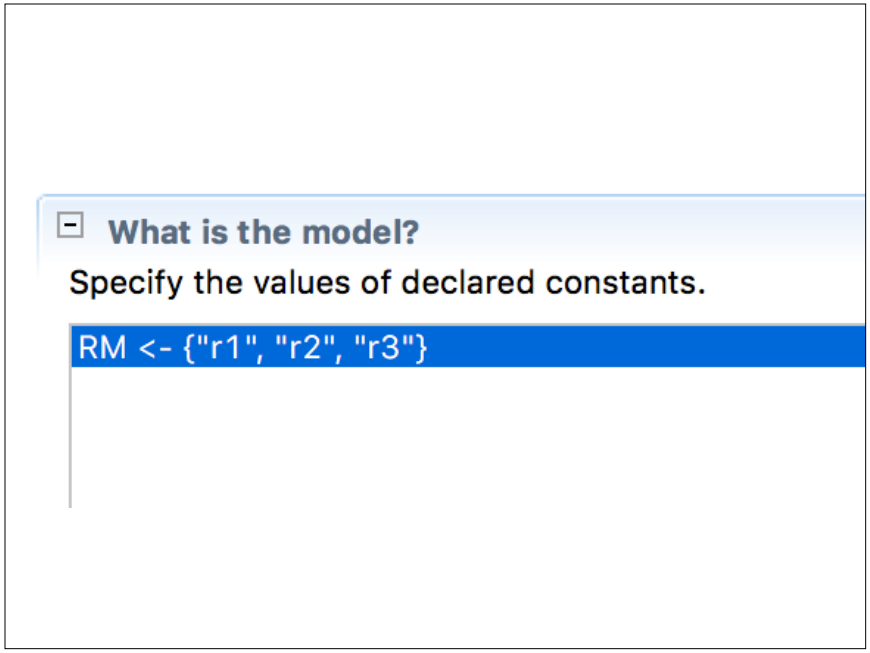
208



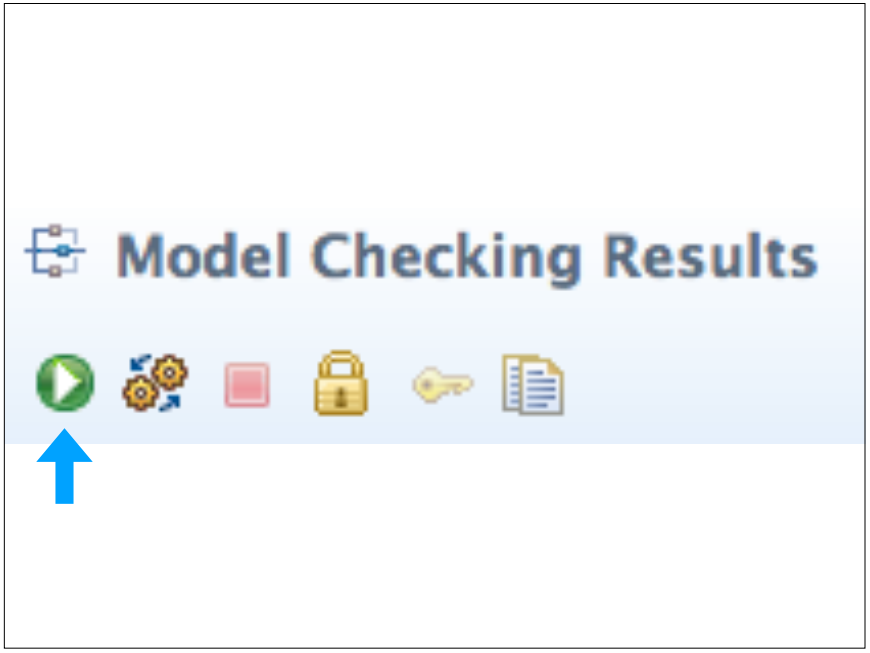
209



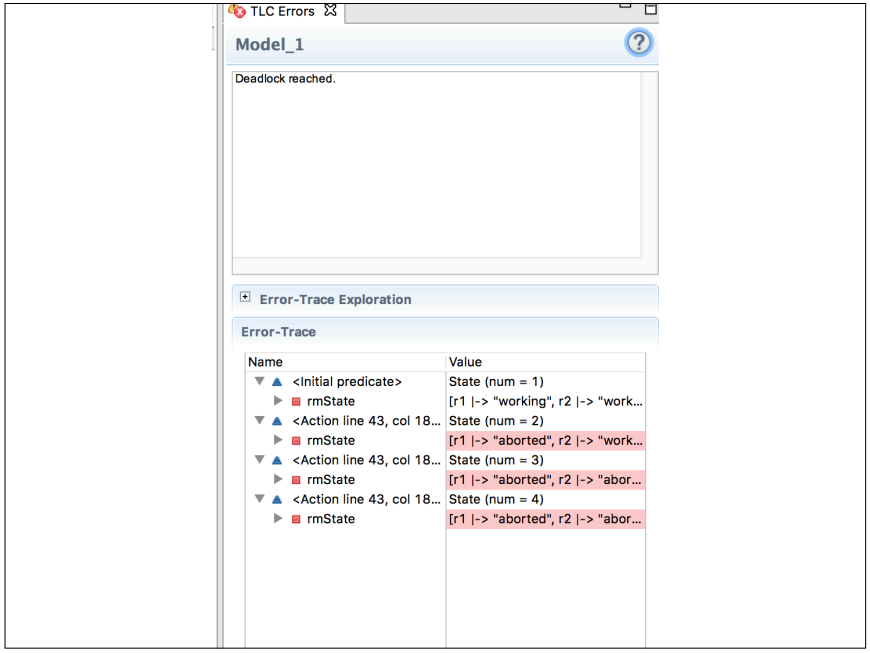
210



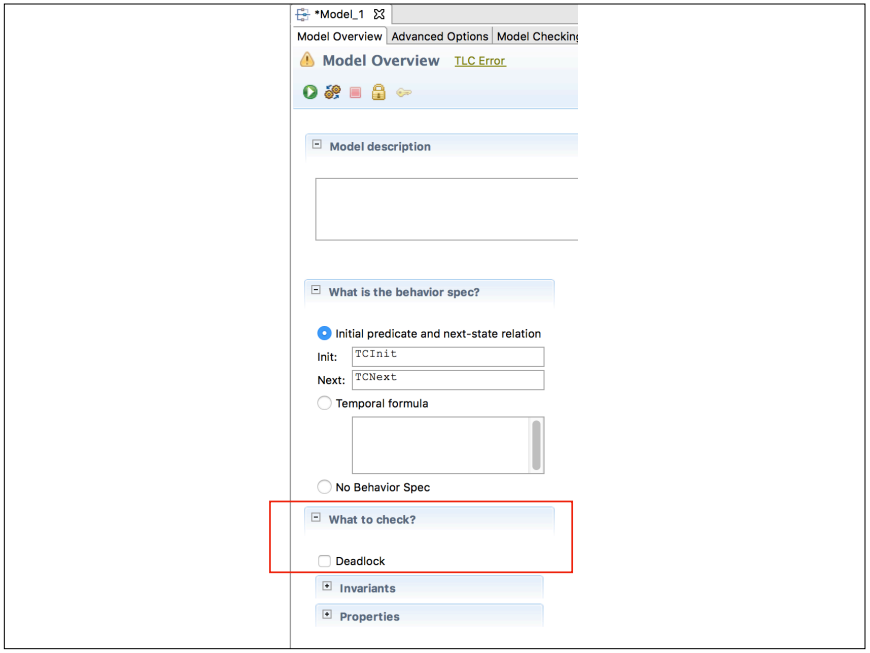
211



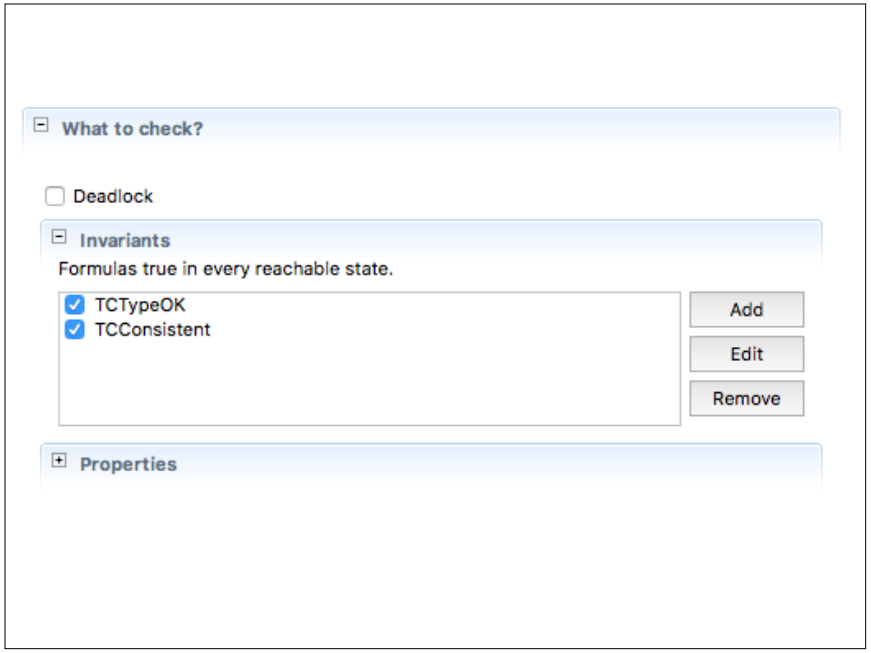
212



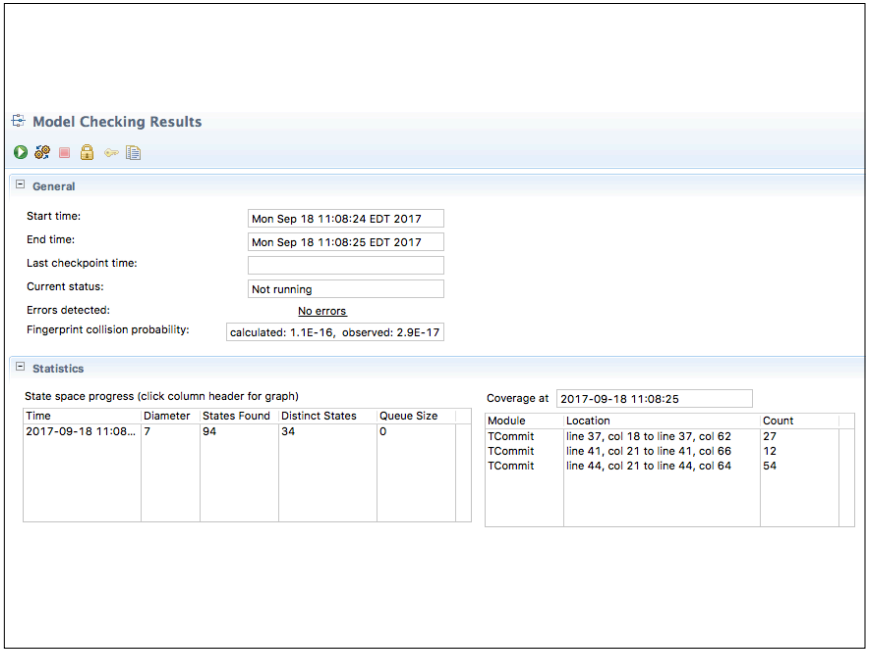
213



214



215



216

Let's try breaking something!

217

```
Decide(r)  == \/ /\ rmState[r] = "prepared"
                 \* /\ canCommit
                 /\ rmState' = [rmState EXCEPT ![r] = "committed"]
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



```
Decide(r)  == \/ /\ rmState[r] = "prepared"
                 \* /\ canCommit
                 /\ rmState' = [rmState EXCEPT ![r] = "committed"]
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```

- The ***** is a TLA+ single-line comment, we're temporarily removing the **canCommit** requirement.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- The ***** is a TLA+ single-line comment, we're temporarily removing the **canCommit** requirement.
- This means that **canCommit** no longer needs to be true to allow a state transition where some RM **r** goes from "prepared" to "committed."


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Decide(r)  == \/ /\ rmState[r] = "prepared"
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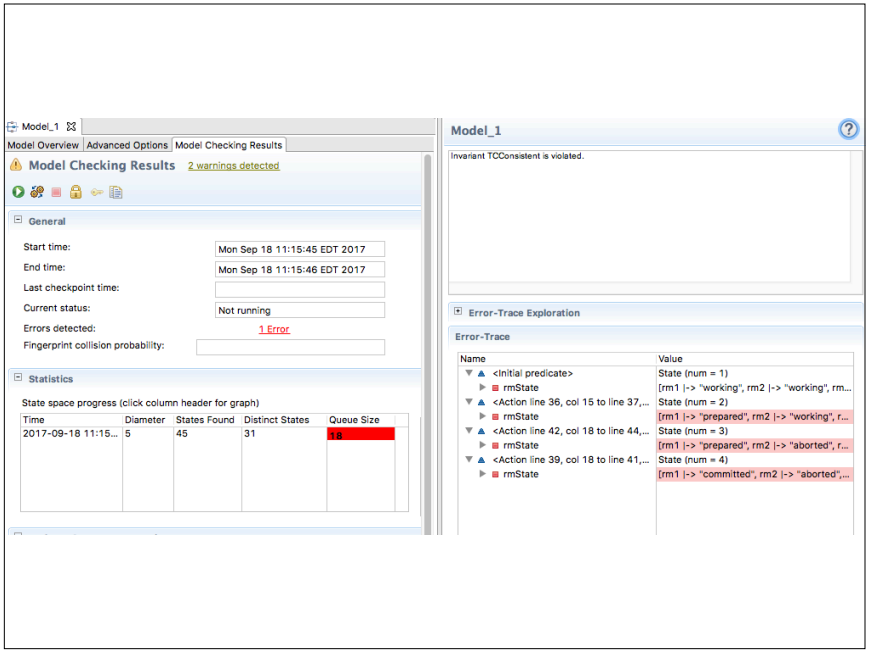
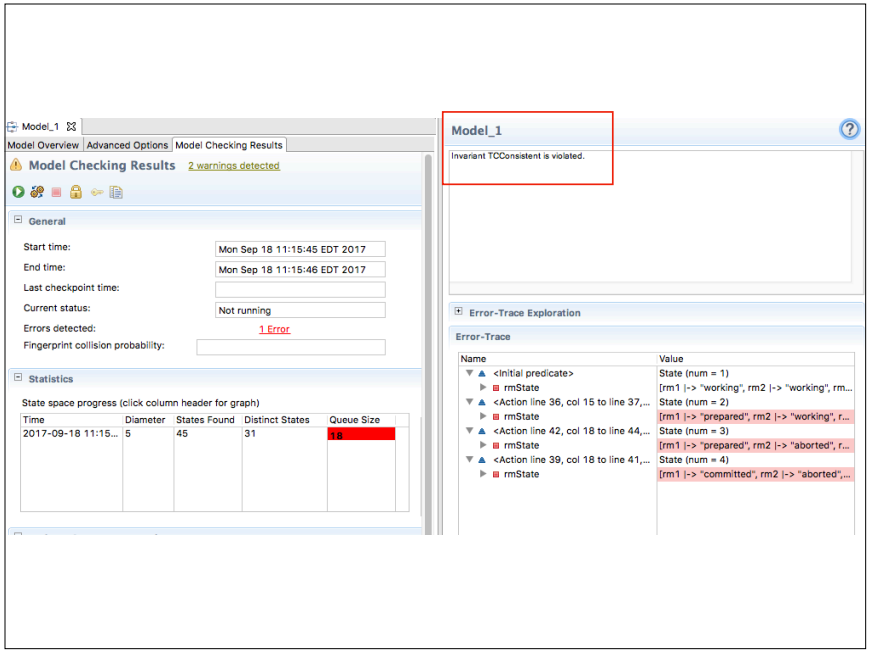
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```

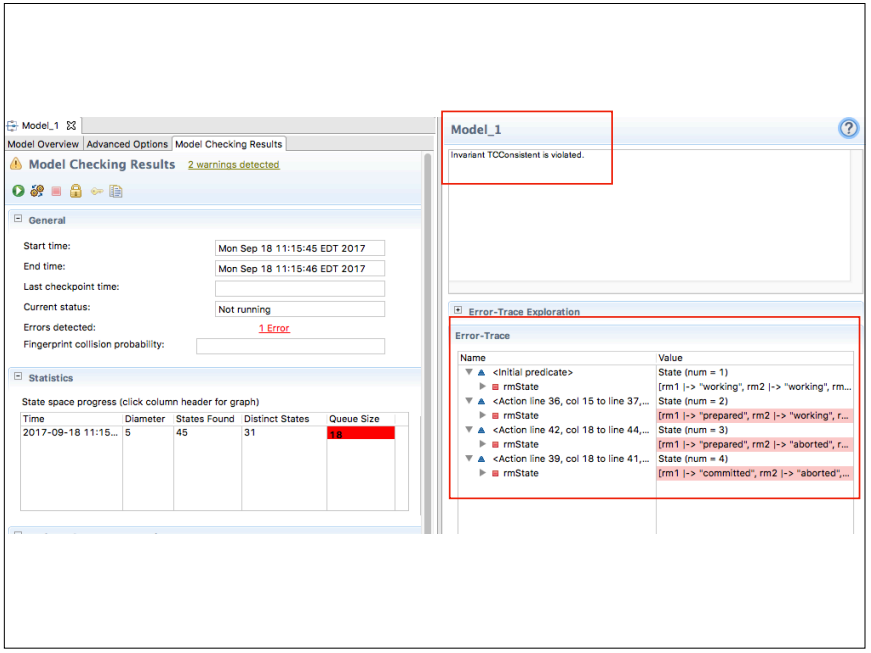
- The ***** is a TLA+ single-line comment, we're temporarily removing the **canCommit** requirement.
- This means that **canCommit** no longer needs to be true to allow a state transition where some RM **r** goes from "prepared" to "committed."
- There could now be some other RM in an "aborted" state, and the specification would permit this transition.



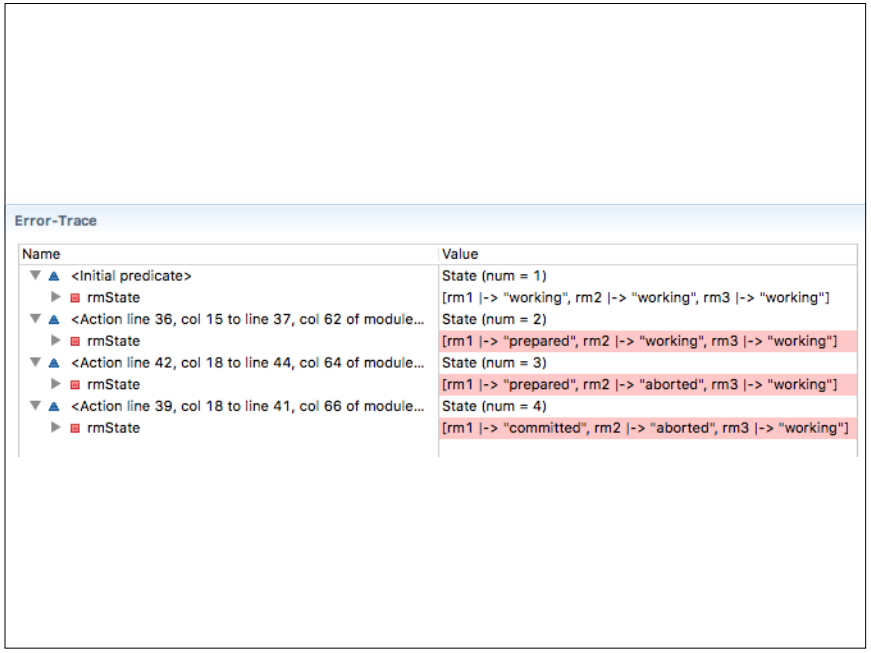
219-1



219-2



219-3



220

Error-Trace

Name	Value
<Initial predicate>	State (num = 1)
rmState	[rm1 |-> "working", rm2 |-> "working", rm3 |-> "working"]
<Action line 36, col 15 to line 37, col 62 of module...	State (num = 2)
rmState	[rm1 |-> "prepared", rm2 |-> "working", rm3 |-> "working"]
<Action line 42, col 18 to line 44, col 64 of module...	State (num = 3)
rmState	[rm1 |-> "prepared", rm2 |-> "aborted", rm3 |-> "working"]
<Action line 39, col 18 to line 41, col 66 of module...	State (num = 4)
rmState	[rm1 |-> "committed", rm2 |-> "aborted", rm3 |-> "working"]

Double-click this row

221

```
30 ----------------------------------------------------------------------------
31 (****************************************************************************)
32 (* We now define the actions that may be performed by the RMs, and then    *)
33 (* define the complete next-state action of the specification to be the     *)
34 (* disjunction of the possible RM actions.                                  *)
35 (****************************************************************************)
36 Prepare(r) == /\ rmState[r] = "working"
37              /\ rmState' = [rmState EXCEPT ![r] = "prepared"]
38
39 Decide(r)  == \/ /\ rmState[r] = "prepared"
40                  \* /\ canCommit
41                  /\ rmState' = [rmState EXCEPT ![r] = "committed"]
42               \/ /\ rmState[r] \in {"working", "prepared"}
43                  /\ notCommitted
44                  /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
45
```

222

Resource Managers

rmState[“rm1”]: “working”
rmState[“rm2”]: “working”
rmState[“rm3”]: “working”

223-1

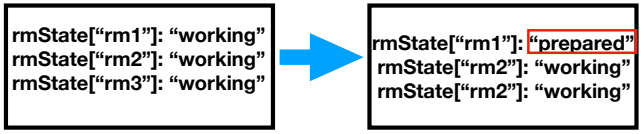
Resource Managers

rmState[“rm1”]: “working”
rmState[“rm2”]: “working”
rmState[“rm3”]: “working”

rmState[“rm1”]: “prepared”
rmState[“rm2”]: “working”
rmState[“rm2”]: “working”

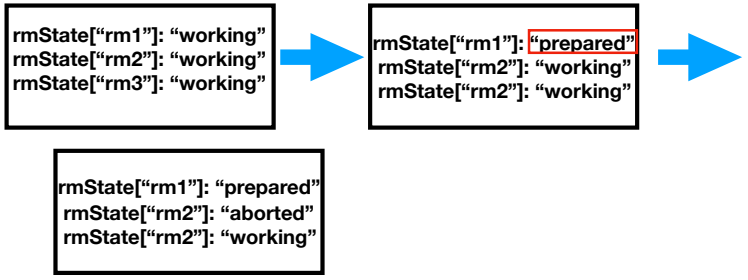
223-2

Resource Managers



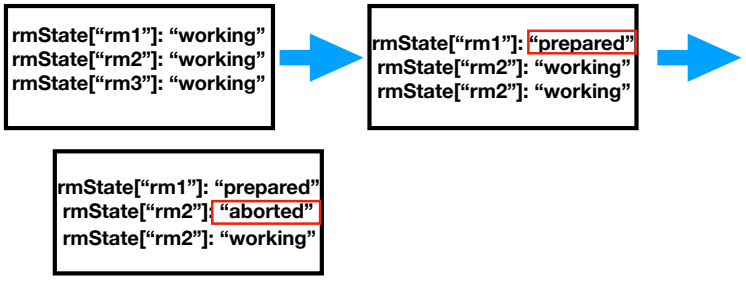
223-3

Resource Managers



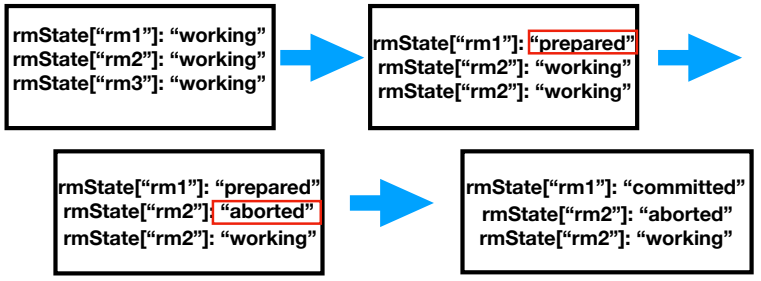
223-4

Resource Managers



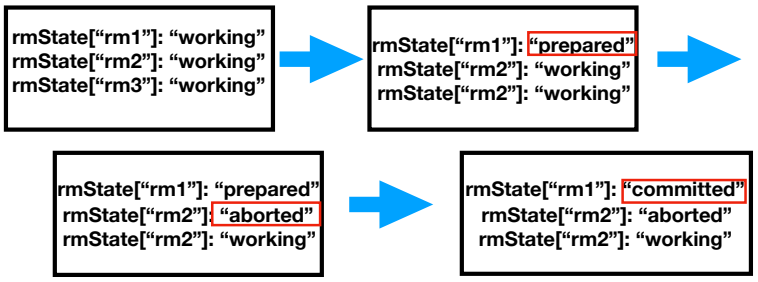
223-5

Resource Managers



223-6

Resource Managers



Syntax time

Functions

Functions

- A TLA+ function is a *true* mathematical function. The term “function” we use in most programming languages is completely wrong. Those things should generally be called “sub-routines”

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- The **values** can be of mixed types
- The set of **keys** are called the DOMAIN, the set of **values** are the RANGE or IMAGE

Functions

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- Read this as: "Take the set to the left of **|->** and use that as the keys. For each key, evaluate the expression to the right of **|->**, and use that as the value"

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- Read this as: "Take the set to the left of **|->** and use that as the keys. For each key, evaluate the expression to the right of **|->**, and use that as the value"
- That syntax on the right side of the = is rarely used. But it's equivalent to the following Python dict: {2:4, 3:6, 4:8}

Functions

- TLA+ provides lots of ways to create functions. We'll only look at the commonly used ones
- [i ∈ S |-> e] This creates a function whose domain is all the elements of **S** (i.e. the keys are the elements of **S**), and the corresponding values are created by evaluating the expression **e**

  [i ∈ {2,3,4} |-> i*2] = (2 :> 4 @@ 3 :> 6 @@ 4 :> 8)

- Read this as: "Take the set to the left of **|->** and use that as the keys. For each key, evaluate the expression to the right of **|->**, and use that as the value"
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```
{i: i*2 for i in [2,3,4]}
```

Functions

- The value for some key can be accessed using the same syntax as array/dictionary access in Python

```
f == [ x \in {1,2,3} |-> x * 5 ]
```

```
f[1] = 5
f[2] = 10
f[3] = 15
```


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Exercise:

If we have the following function **f**

```
f == [x \in {1,2,3} |-> x*5]
```

How could you find the range (i.e. the set of values) of this function?

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Answer:

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Answer:

```
{ f[i] : i \in DOMAIN f }
```


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- Huh?

Sets of functions

```
[{2,3} -> {"a", “b”, “c”}] =   { (2 :> "a" @@ 3 :> "a"),
                                 (2 :> "a" @@ 3 :> "b"),
                                 (2 :> "a" @@ 3 :> "c"),
                                 (2 :> "b" @@ 3 :> "a"),
                                 (2 :> "b" @@ 3 :> "b"),
                                 (2 :> "b" @@ 3 :> "c"),
                                 (2 :> "c" @@ 3 :> "a"),
                                 (2 :> "c" @@ 3 :> "b"),
                                 (2 :> "c" @@ 3 :> "c") }
```

That output is **roughly** equivalent to the following Python set of dictionaries:

```
{ {2:”a”, 3:“a"},
  {2:”a", 3:“b"},
  {2:”a", 3:“c"},
  {2:”b", 3:“a”},
  {2:”b", 3:“b"},
  {2:”b", 3:“c"},
  {2:”c", 3:“a"},
  {2:”c", 3:“b"},
  {2:”c", 3:“c”} }
```

:> and @@

```
[{2,3} -> {"a", "b"}] =  { (2 :> "a" @@ 3 :> "a"),
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```

:> is used as the as the separator between **keys** and **values**
@@ is used to concatenate together two functions (remember, a function is a set of key/value pairs)

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- Tuples are just syntactic sugar for functions! The keys are the integers from `1..N`, and the values are whatever you write inside the `<< >>`

- Equivalent to the Python tuple `("a","b","c")`, except TLA+ is 1-index

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t = [f EXCEPT ![4] = 30]
```

- You can read that as:
  - **t** is the same as the function as **f**, except with the **value** for the **key 4** being **30**

EXCEPT

```
f == [x \in {2,4,6} |-> x*4]
t == [f EXCEPT ![4] = 30]
```

A Python equivalent

```
import copy
f = {2: 8, 4: 16, 6: 24}
t = copy.deepcopy(f)
t[4] = 30
```

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- A **tuple** is a function whose keys are integers from 1..N, like an array in Python/C/etc.
- **Tuples** are syntactic sugar over functions.


```
CONSTANT RM       \* The set of participating resource managers

VARIABLE rmState  \* rmState[rm] is the state of resource manager r.
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Let's assume that RM will be the set `{“r1”, “r2”}`


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  [r1 |-> "prepared", r2 |-> "waiting"],
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  [r1 |-> "aborted", r2 |-> "prepared"],
  [r1 |-> "aborted", r2 |-> "aborted"],
  [r1 |-> "aborted", r2 |-> "committed"],
  [r1 |-> "committed", r2 |-> "waiting"],
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  [r1 |-> "committed", r2 |-> "aborted"],
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  - Where `Integers` is the set of all integer values

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TCInit ==   rmState = [r \in RM |-> "working"]
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241-1


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241-2


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- Thus only one possible initial state is permitted

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  - At every state, an infinite number of next states are theoretically possible. Our formula identifies which next states are **actually** possible, by applying the infinite set of next states to the formula and seeing which ones come out as TRUE

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- This says:
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  - And if in the **next** state, the state of resource manager **r** is "prepared", and all the other resource managers have the same state as they do right now

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              /\ rmState' = [rmState EXCEPT ![r] = "prepared"]
```

rmState: [r1 |-> "prepared",
r2 |-> "working"] ⟶ ? rmState: [r1 |-> "committed",
r2 |-> "working"]

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Prepare(r) == /\ rmState[r] = "working"
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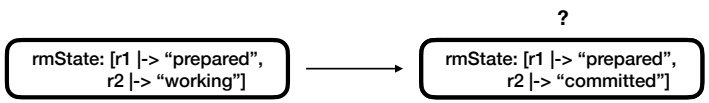

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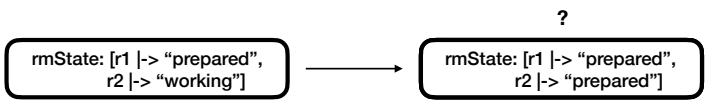

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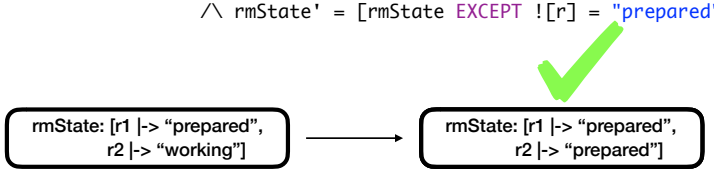

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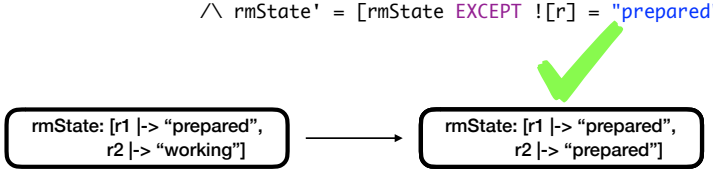

- The parts of the formula with no primed variables are called **enabling** conditions

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

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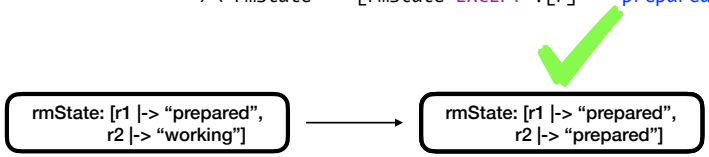

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TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

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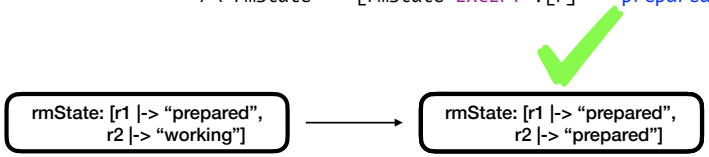

- The parts of the formula with no primed variables are called **enabling** conditions
- They describe what the **current state** must look like to be able to use this action

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

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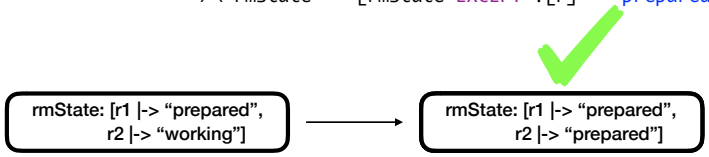

- The parts of the formula with no primed variables are called **enabling** conditions
- They describe what the **current state** must look like to be able to use this action
- Any formula with **primed variables** is called an **action**

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
                 /\ canCommit
                 /\ rmState' = [rmState EXCEPT ![r] = "committed"]
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
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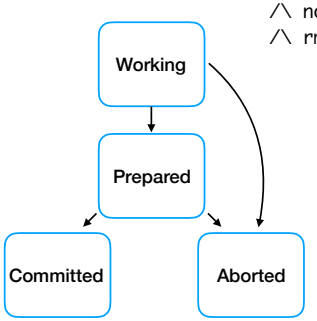

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TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

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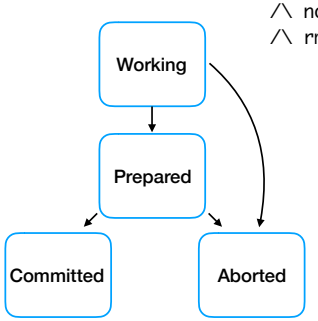

- This formula is true when either

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

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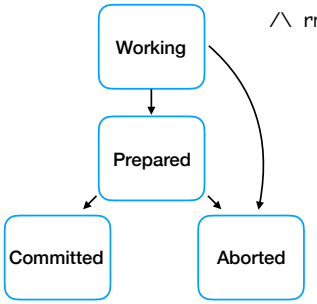
                 /\ canCommit
                 /\ rmState' = [rmState EXCEPT ![r] = "committed"]
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



- This formula is true when either

Next state formula

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TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
                 /\ canCommit
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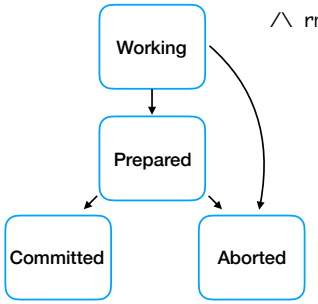
              \/ /\ rmState[r] \in {"working", "prepared"}
                 /\ notCommitted
                 /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```

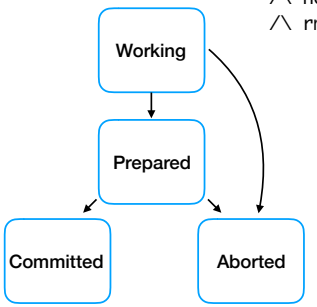


- This formula is true when either
  - RM **r** is in state “prepared”

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
              /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



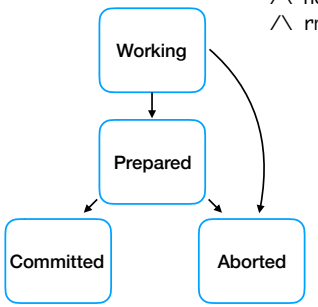
- This formula is true when either
  - RM **r** is in state “prepared”

246-6

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
              /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



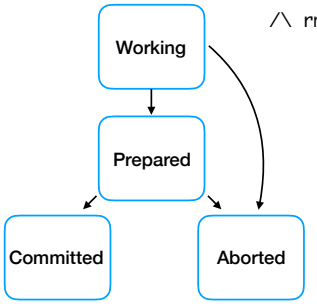
- This formula is true when either
  - RM **r** is in state “prepared”
  - And **canCommit** is true

246-7

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
              /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



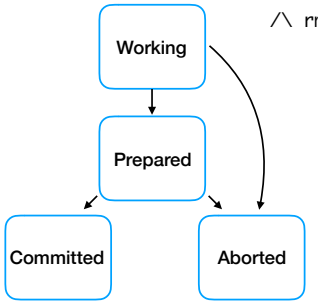
- This formula is true when either
  - RM **r** is in state “prepared”
  - And **canCommit** is true

246-8

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
              /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



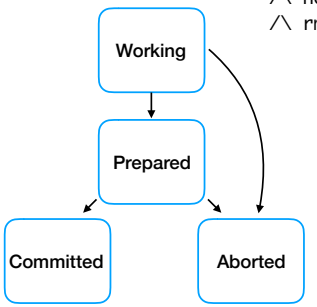
- This formula is true when either
  - RM **r** is in state “prepared”
  - And **canCommit** is true
  - And that RM **r** is “committed” in the next state

246-9

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
              /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



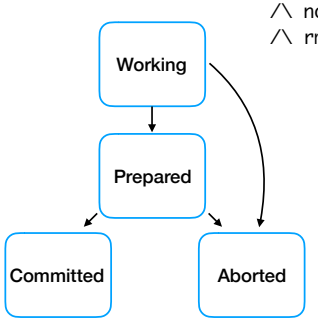
- This formula is true when either
  - RM **r** is in state "prepared"
  - And **canCommit** is true
  - And that RM **r** is "committed" in the next state

246-10

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
              /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



247-1

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
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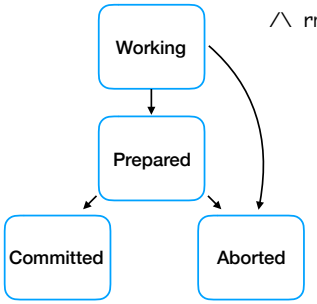
```



- or when

247-2

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
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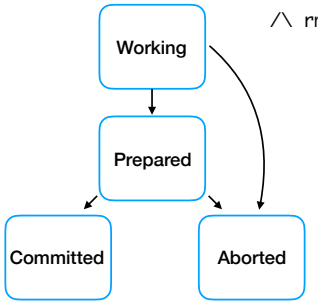

- or when

247-3

Next state formula

```
TCNext == \E r \in RM : Prepare(r) \/ Decide(r)

Decide(r)  == \/ /\ rmState[r] = "prepared"
              /\ canCommit
              /\ rmState' = [rmState EXCEPT ![r] = "committed"]
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
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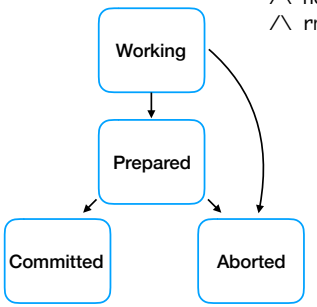
```



- or when
  - RM **r** is currently “working” or “prepared”

Next state formula

```
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Decide(r)  == \/ /\ rmState[r] = "prepared"
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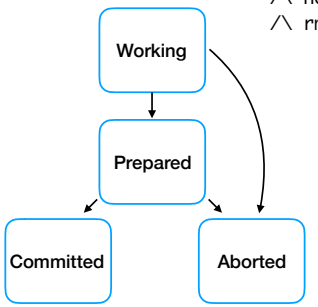
           \/ /\ rmState[r] \in {"working", "prepared"}
              /\ notCommitted
              /\ rmState' = [rmState EXCEPT ![r] = "aborted"]
```



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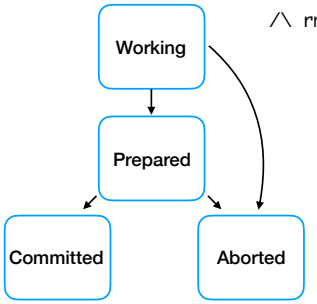

- or when
  - RM **r** is currently “working” or “prepared”
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Next state formula

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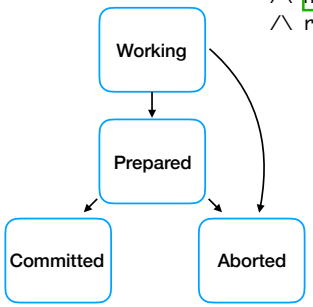

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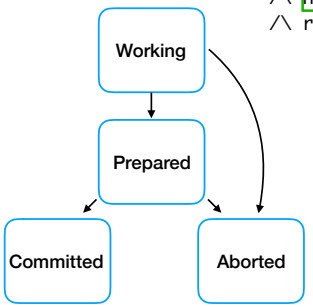

- or when
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Next state formula

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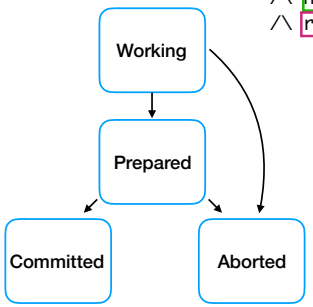

- or when
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canCommit

```
canCommit == \A r \in RM : rmState[r] \in {"prepared", "committed"}
```


canCommit

```
canCommit == \A r \in RM : rmState[r] \in {"prepared", "committed"}
```

Python Equivalents

```
RM = [“rm1”, “rm2”] # a constant
rmState = {“rm1”: “working”, “rm2”: “working”}
canCommitStates = [“prepared”, “committed”] # a constant

...

def canCommit():
    for r in RM:
        if rmState[r] not in canCommitStates:
            return False
    return True

def canCommit():
    return all(map(lambda r: rmState[r] in canCommitStates, RM))
```

canCommit

```
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...

def canCommit():
    for r in RM:
        if rmState[r] not in canCommitStates:
            return False
    return True

def canCommit():
    return all(map(lambda r: rmState[r] in canCommitStates, RM))
```


notCommitted

```
notCommitted == \A r \in RM : rmState[r] # "committed"
```

“Not equal.” You can also use **/=**

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Python Equivalents

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- TLC always explores every possible state

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Python equivalent

```
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                return False
    return True
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- Now we can write a new spec which presents a solution to this problem
  - (There are **LOTS** of algorithms that solve this problem, we’ll just look at one)

Two Phase Commit

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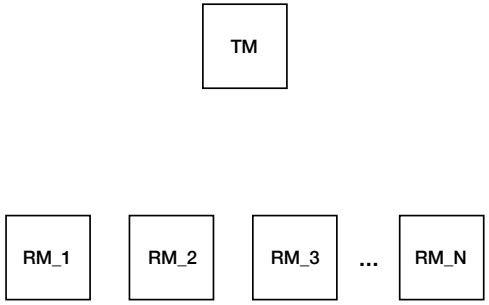
- The TM becomes the “source of truth” on whether a transaction should commit or abort

Transaction Manager

Transaction Manager



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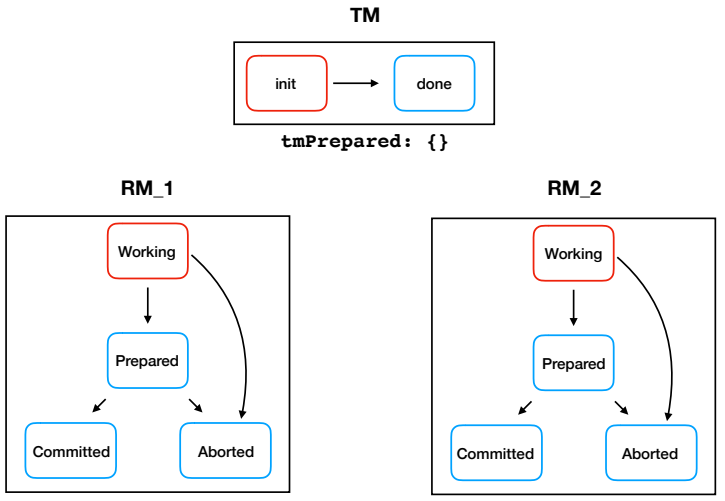

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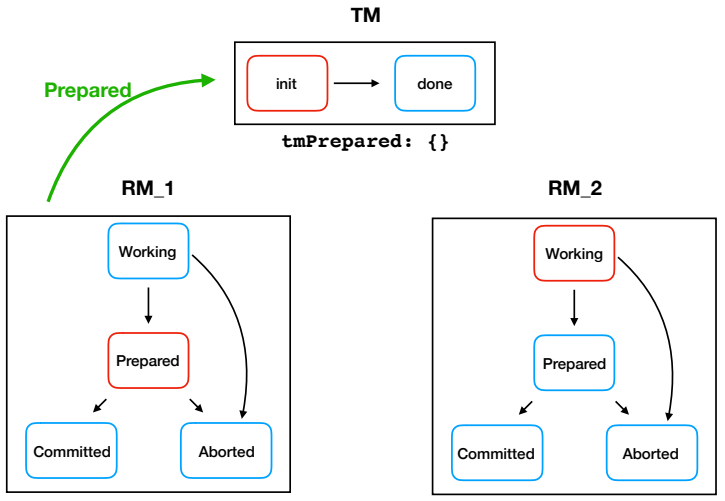

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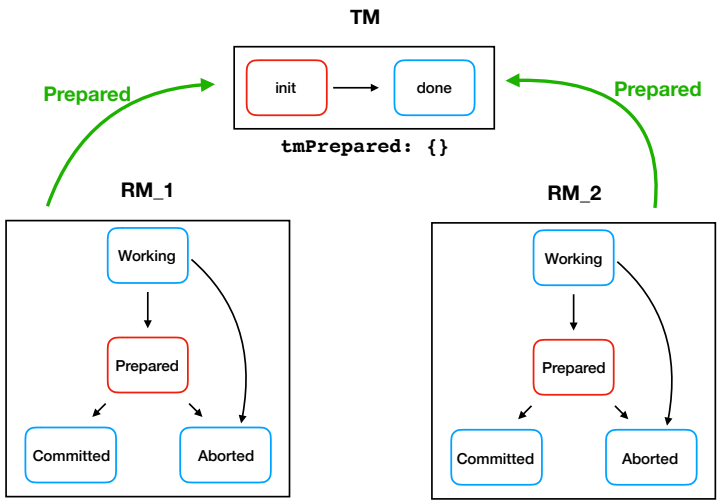

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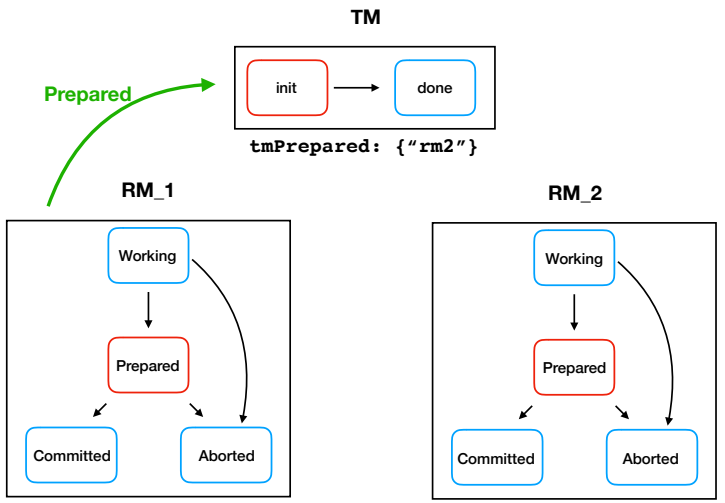

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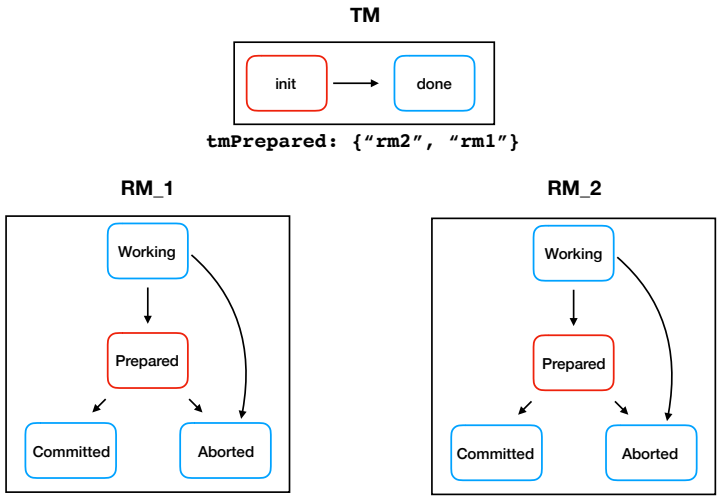

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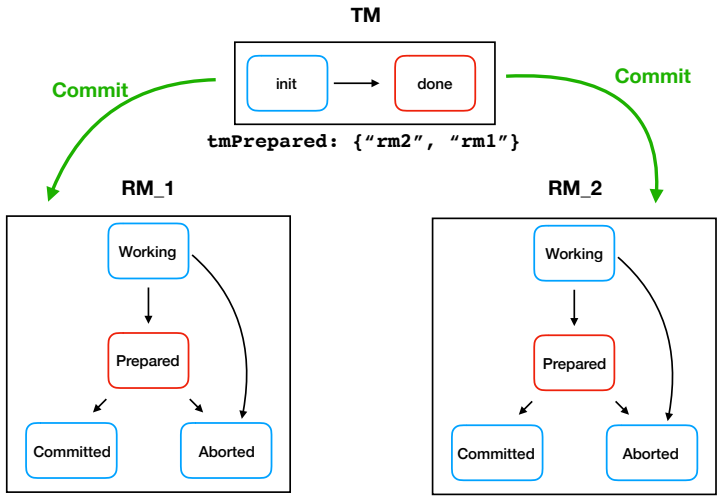

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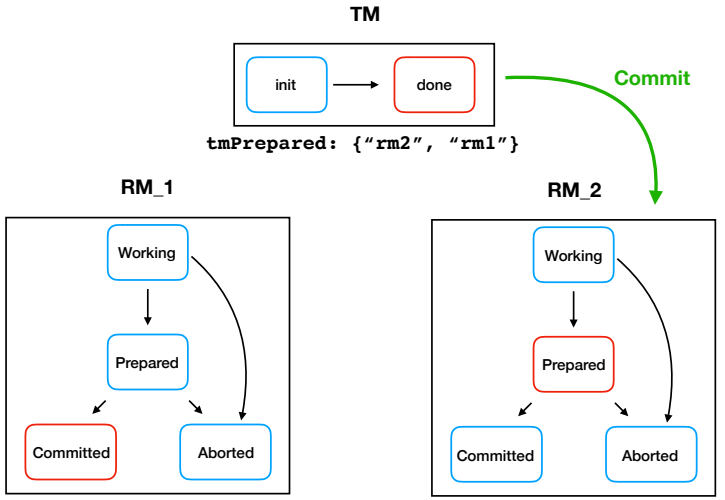

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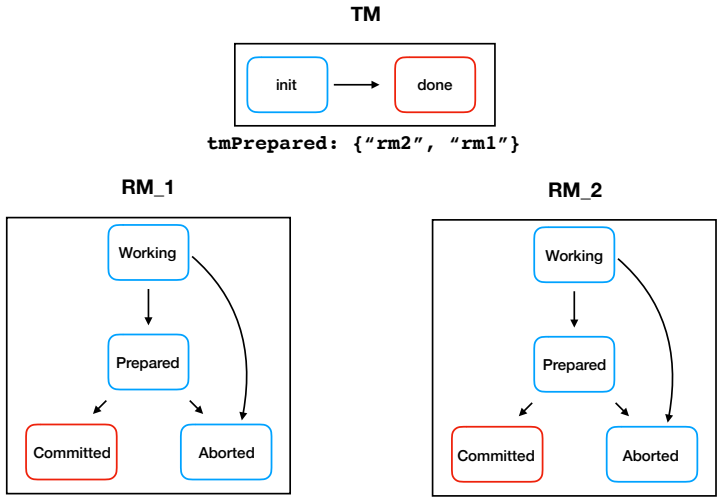

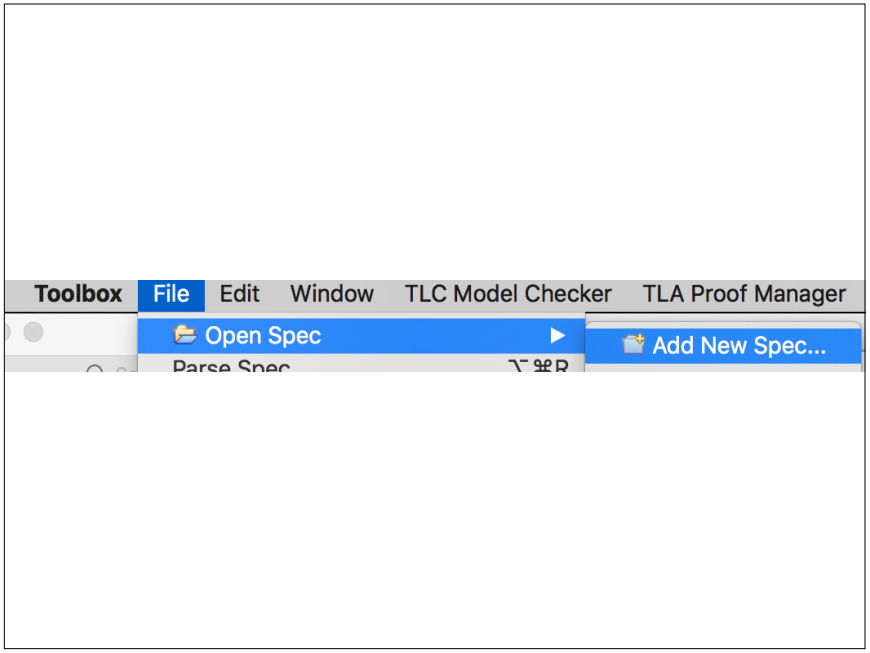

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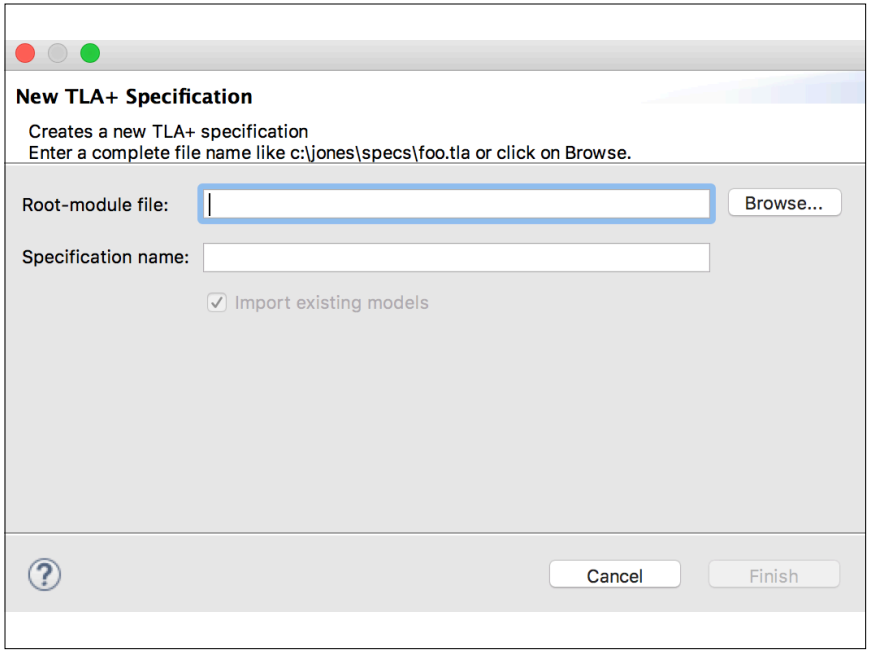

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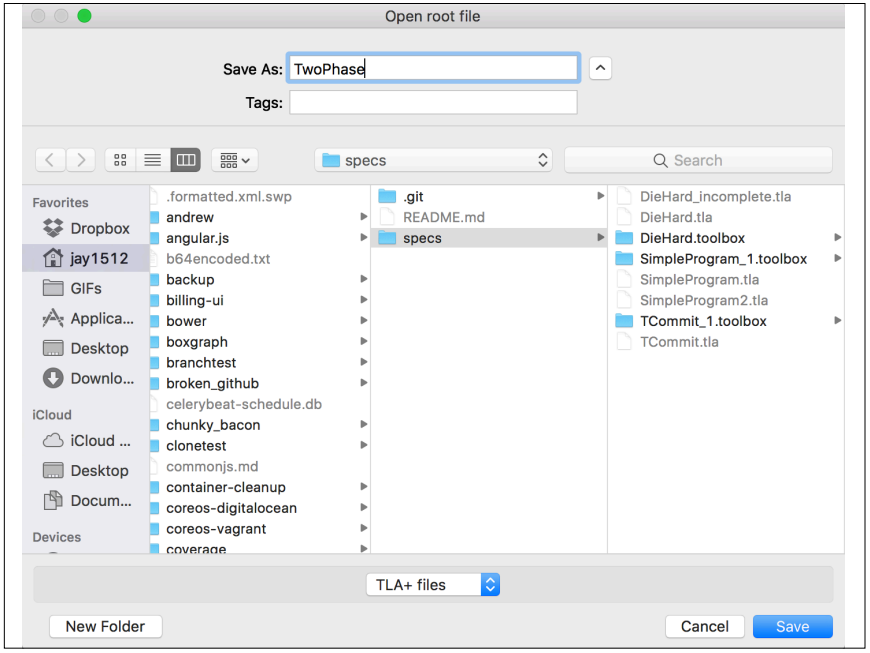

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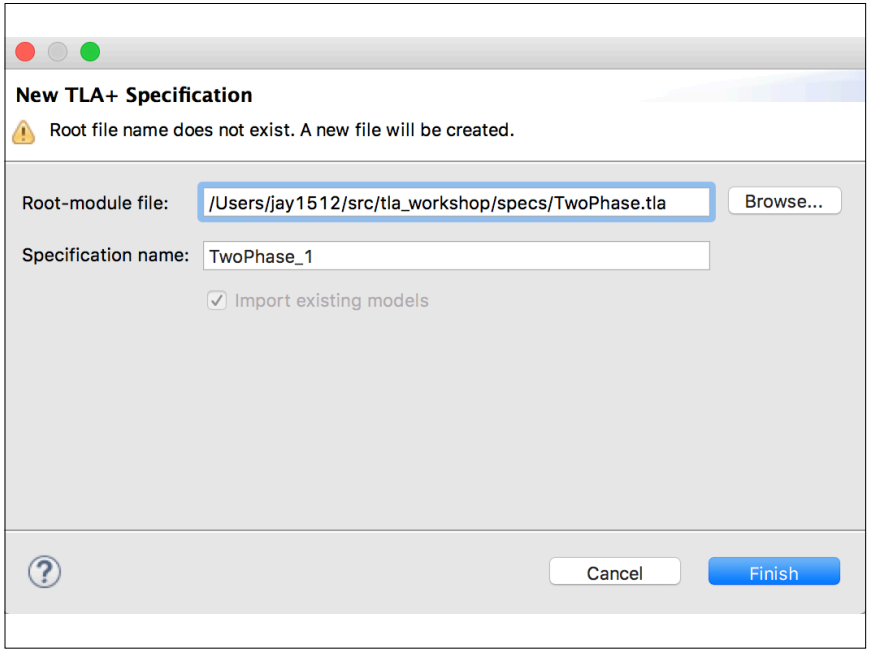

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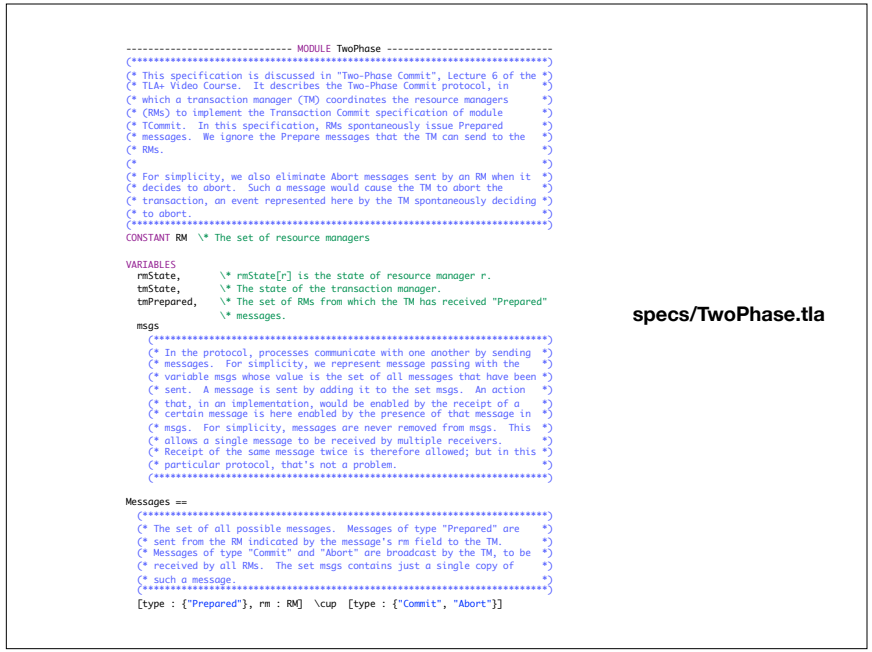

```
---------------------------- MODULE TwoPhase ----------------------------
(***************************************************************************)
(* This specification is discussed in "Two-Phase Commit", Lecture 6 of the *)
(* TLA+ Video Course.  It describes the Two-Phase Commit protocol, in      *)
(* which a transaction manager (TM) coordinates the resource managers      *)
(* (RMs) to implement the Transaction Commit specification of module       *)
(* TCommit.  In this specification, RMs spontaneously issue Prepared       *)
(* messages.  We ignore the Prepare messages that the TM can send to the   *)
(* RMs.                                                                    *)
(*                                                                         *)
(* For simplicity, we also eliminate Abort messages sent by an RM when it  *)
(* decides to abort.  Such a message would cause the TM to abort the       *)
(* transaction, an event represented here by the TM spontaneously deciding *)
(* to abort.                                                               *)
(***************************************************************************)
CONSTANT RM  \* The set of resource managers

VARIABLES
  rmState,       \* rmState[r] is the state of resource manager r.
  tmState,       \* The state of the transaction manager.
  tmPrepared,    \* The set of RMs from which the TM has received "Prepared"
                 \* messages.
  msgs
    (***********************************************************************)
    (* In the protocol, processes communicate with one another by sending  *)
    (* messages.  For simplicity, we represent message passing with the    *)
    (* variable msgs whose value is the set of all messages that have been *)
    (* sent.  A message is sent by adding it to the set msgs.  An action   *)
    (* that, in an implementation, would be enabled by the receipt of a    *)
    (* certain message is here enabled by the presence of that message in  *)
    (* msgs.  For simplicity, messages are never removed from msgs.  This  *)
    (* allows a single message to be received by multiple receivers.       *)
    (* Receipt of the same message twice is therefore allowed; but in this *)
    (* particular protocol, that's not a problem.                          *)
    (***********************************************************************)

Messages ==
    (***********************************************************************)
    (* The set of all possible messages.  Messages of type "Prepared" are  *)
    (* sent from the RM indicated by the message's rm field to the TM.     *)
    (* Messages of type "Commit" and "Abort" are broadcast by the TM, to be *)
    (* received by all RMs.  The set msgs contains just a single copy of   *)
    (* such a message.                                                     *)
    (***********************************************************************)
    [type : {"Prepared"}, rm : RM]  \cup  [type : {"Commit", "Abort"}]
```

specs/TwoPhase.tla

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  [nodes |-> 1, edges |-> "b"],
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   {“nodes”: 2, ”edges”: “a”},
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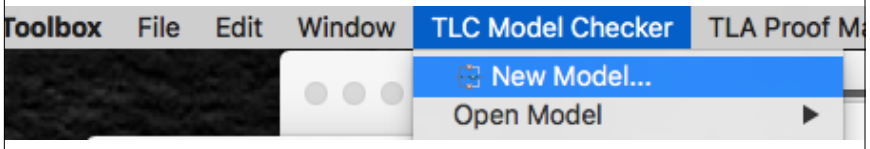
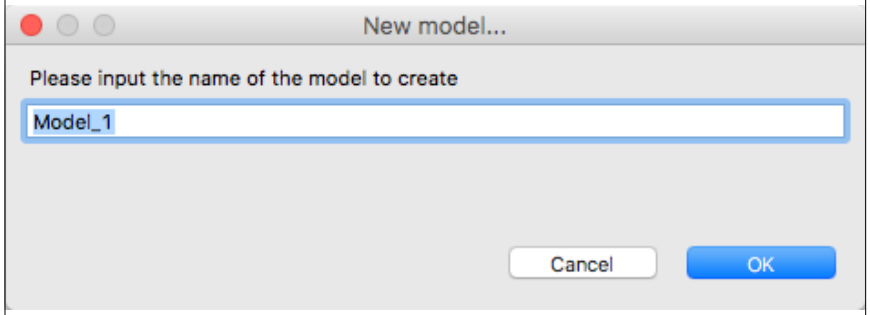
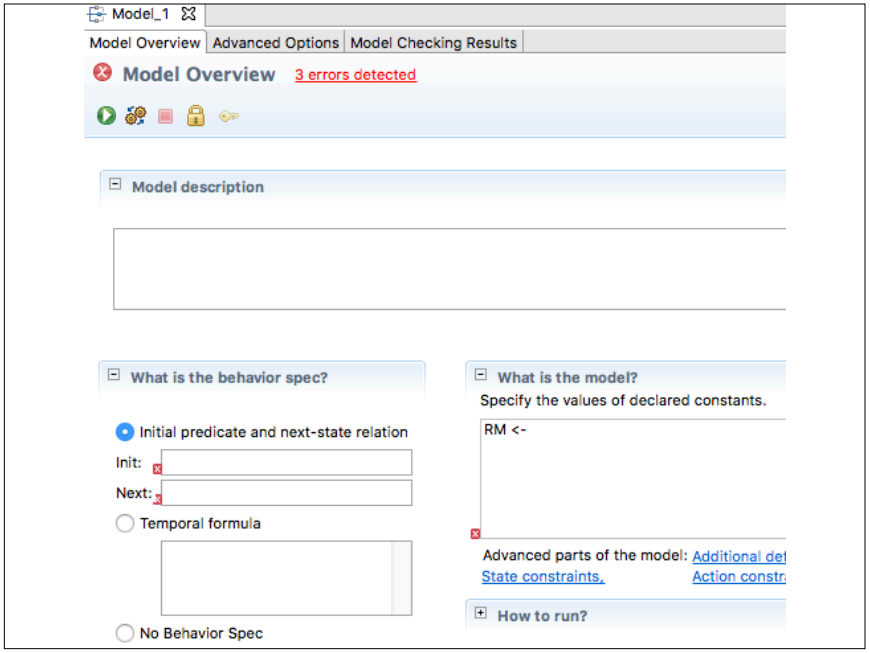
```





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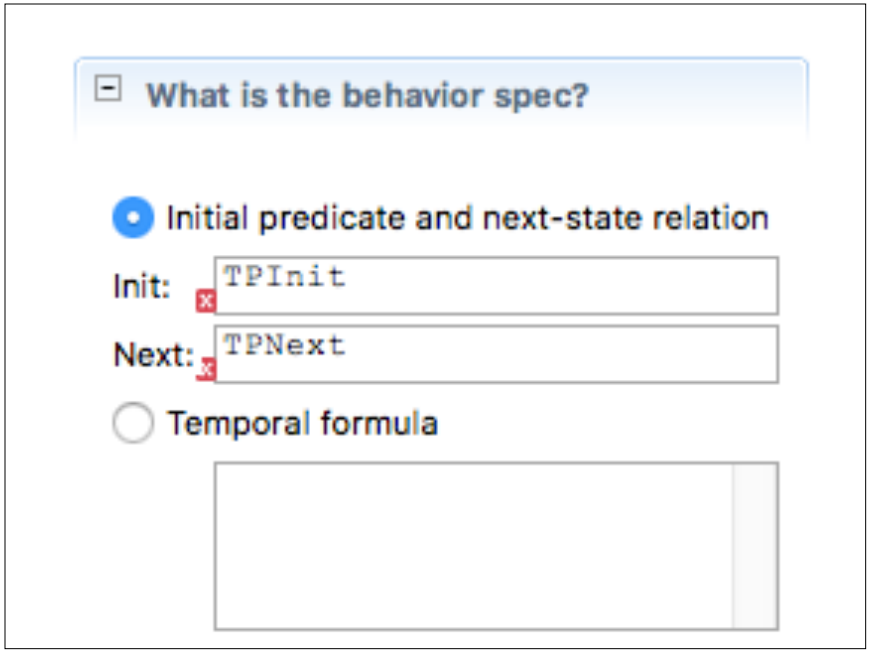

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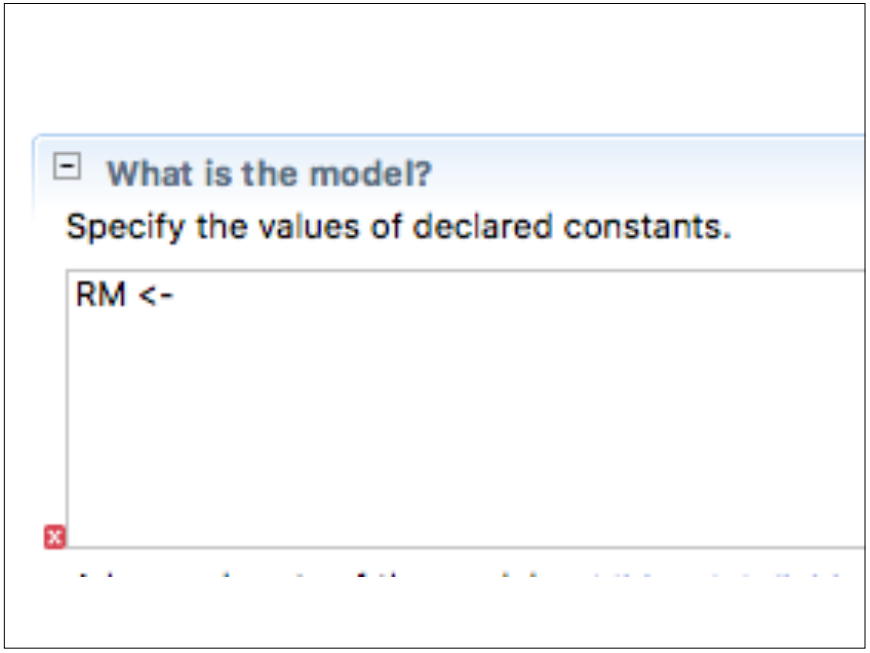

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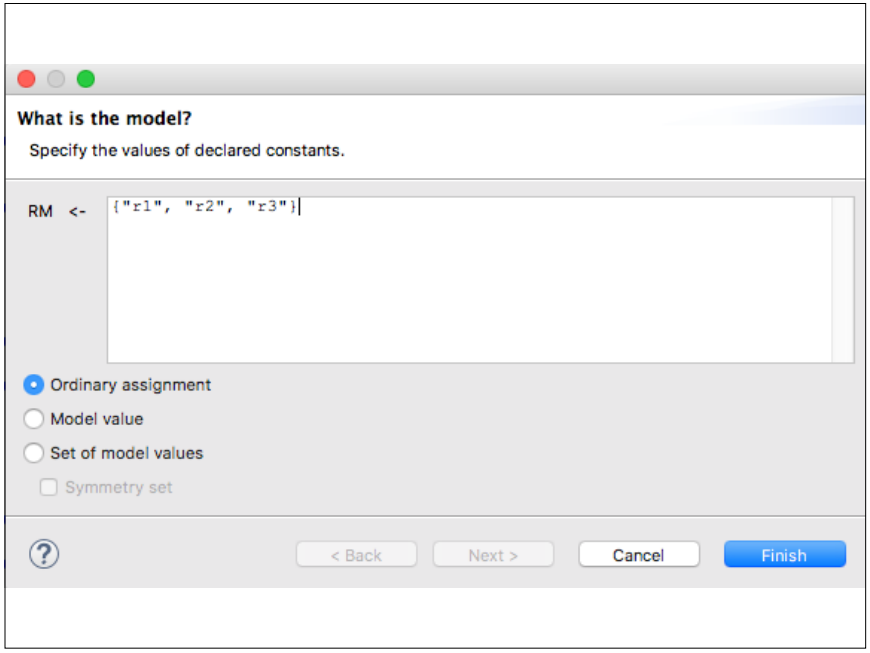

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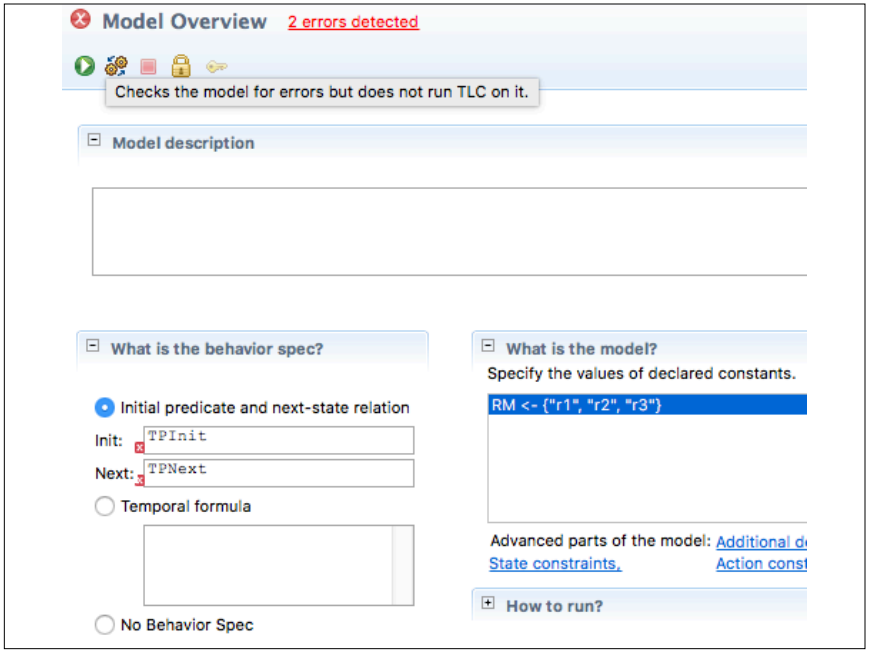

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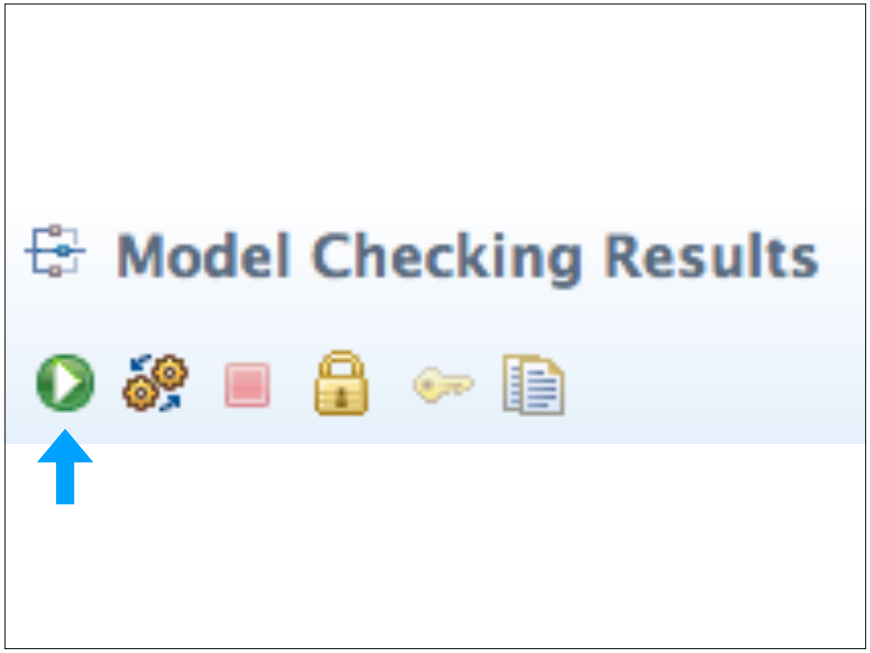

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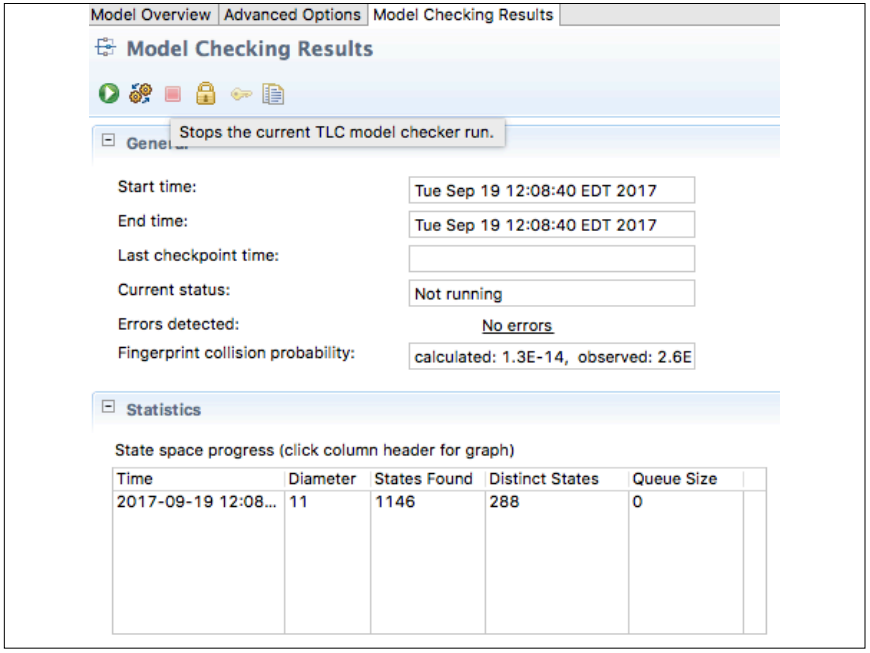
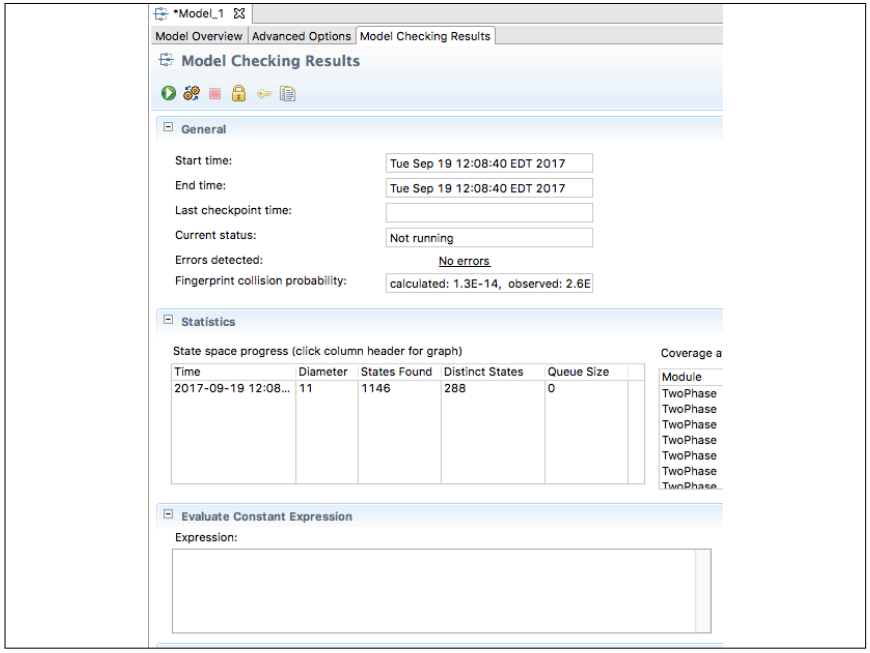
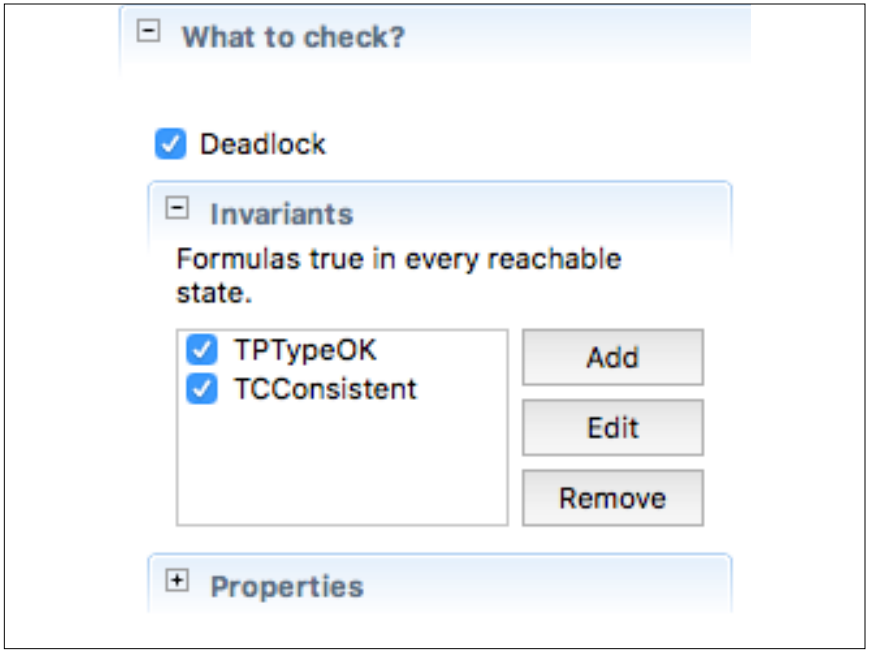


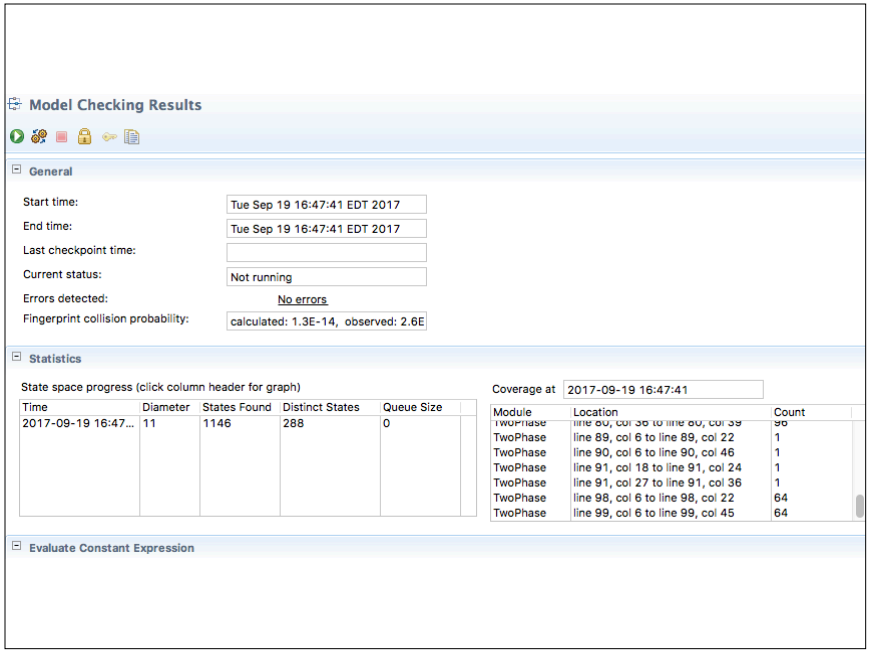
280



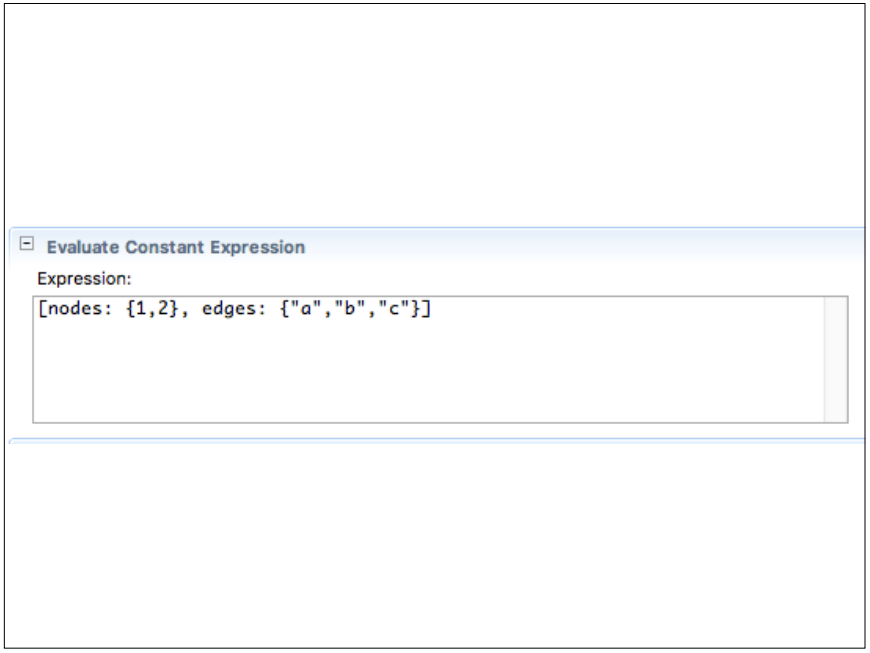
281



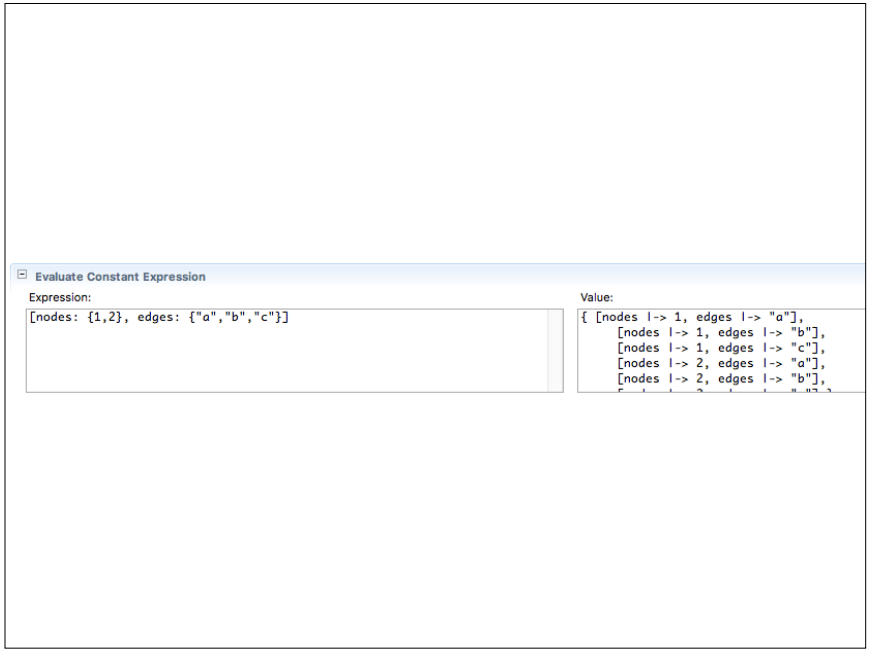
282



283



284



285

TwoPhase

```
CONSTANT RM  \* The set of resource managers

VARIABLES
  rmState,       \* rmState[r] is the state of resource manager r.
  tmState,       \* The state of the transaction manager.
  tmPrepared,    \* The set of RMs from which the TM has received "Prepared"
                 \* messages.
  msgs
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- **RM** and **rmState** are the same as before
- **tmState** is either “init” or “done”
- **tmPrepared** is either the empty set {}, or contains some combination of RMs (ex. {“rm1”, “rm3”})
- **msgs** starts as an empty set

TwoPhase

```
Messages ==
  (***************************************************************************)
  (* The set of all possible messages.  Messages of type "Prepared" are    *)
  (* sent from the RM indicated by the message's rm field to the TM.       *)
  (* Messages of type "Commit" and "Abort" are broadcast by the TM, to be  *)
  (* received by all RMs.  The set msgs contains just a single copy of     *)
  (* such a message.                                                       *)
  (***************************************************************************)
  [type : {"Prepared"}, rm : RM]  \union  [type : {"Commit", "Abort"}]
```


TwoPhase

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  (* such a message.                                                       *)
  (***************************************************************************)
  [type : {"Prepared"}, rm : RM]  \union  [type : {"Commit", "Abort"}]
```

Evaluate Constant Expression

Expression:

```
[type: {"Prepared"}, rm: RM]
```

Value:

```
{ [type |-> "Prepared", rm |-> "r1"],
    [type |-> "Prepared", rm |-> "r2"],
    [type |-> "Prepared", rm |-> "r3"] }
```

287-4

TwoPhase

```
Messages ==
  (***************************************************************************)
  (* The set of all possible messages.  Messages of type "Prepared" are    *)
  (* sent from the RM indicated by the message's rm field to the TM.       *)
  (* Messages of type "Commit" and "Abort" are broadcast by the TM, to be  *)
  (* received by all RMs.  The set msgs contains just a single copy of     *)
  (* such a message.                                                       *)
  (***************************************************************************)
  [type : {"Prepared"}, rm : RM]  \union  [type : {"Commit", "Abort"}]
```

288-1

TwoPhase

```
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  (***************************************************************************)
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  (* such a message.                                                       *)
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  [type : {"Prepared"}, rm : RM]  \union  [type : {"Commit", "Abort"}]
```

288-2

TwoPhase

```
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  (* The set of all possible messages.  Messages of type "Prepared" are    *)
  (* sent from the RM indicated by the message's rm field to the TM.       *)
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  (* such a message.                                                       *)
  (***************************************************************************)
  [type : {"Prepared"}, rm : RM]  \union  [type : {"Commit", "Abort"}]
```

Evaluate Constant Expression

Expression:

```
[type: {"Commit", "Abort"}]
```

Value:

```
{[type |-> "Commit"], [type |-> "Abort"]}
```

288-3

TwoPhase

```
Messages ==
  (***************************************************************************)
  (* The set of all possible messages.  Messages of type "Prepared" are    *)
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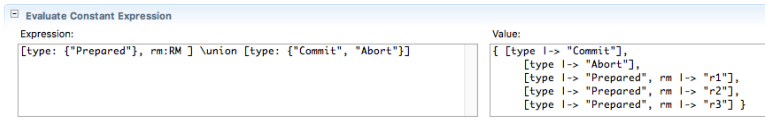

TPTypeOK

```
TPTypeOK ==
  (***************************************************************************)
  (* The type-correctness invariant                                          *)
  (***************************************************************************)
  /\ rmState \in [RM -> {"working", "prepared", "committed", "aborted"}]
  /\ tmState \in {"init", "done"}
  /\ tmPrepared \subseteq RM
  /\ msgs \subseteq Messages
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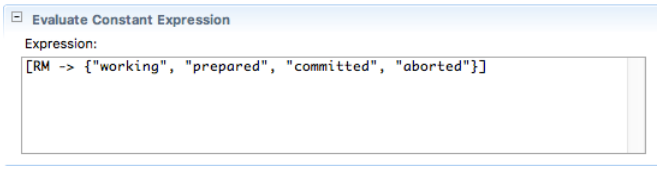

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```

```
{ [r1 |-> "working", r2 |-> "working", r3 |-> "working"],
     [r1 |-> "working", r2 |-> "working", r3 |-> "prepared"],
     [r1 |-> "working", r2 |-> "working", r3 |-> "committed"],
     [r1 |-> "working", r2 |-> "working", r3 |-> "aborted"],
     [r1 |-> "working", r2 |-> "prepared", r3 |-> "working"],
     [r1 |-> "working", r2 |-> "prepared", r3 |-> "prepared"],
     [r1 |-> "working", r2 |-> "prepared", r3 |-> "committed"],
     [r1 |-> "working", r2 |-> "prepared", r3 |-> "aborted"],
     [r1 |-> "working", r2 |-> "committed", r3 |-> "working"],
     [r1 |-> "working", r2 |-> "committed", r3 |-> "prepared"],
     [r1 |-> "working", r2 |-> "committed", r3 |-> "committed"],
     [r1 |-> "working", r2 |-> "committed", r3 |-> "aborted"],
     [r1 |-> "aborted", r2 |-> "prepared", r3 |-> "aborted"],
     [r1 |-> "aborted", r2 |-> "committed", r3 |-> "working"],
```


TPTypeOK

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  /\ rmState \in [RM -> {"working", "prepared", "committed", "aborted"}]
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\subseteq means “subset or equal”

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msgs

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- Messages will be added to this set, but never removed
- The protocol behaves correctly in the face of messages never being removed
- This will make more sense after reading through the spec!

TPInit

```
TPInit ==
  (***************************************************************************)
  (* The initial predicate.                                                  *)
  (***************************************************************************)
  /\ rmState = [r \in RM |-> "working"]
  /\ tmState = "init"
  /\ tmPrepared   = {}
  /\ msgs = {}
```


TPInit

```
TPInit ==
  (***************************************************************************)
  (* The initial predicate.                                                  *)
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Evaluate this expression. What do you get?

TPInit

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Value:

```
[r1 |-> "working", r2 |-> "working", r3 |-> "working"]
```

TPInit

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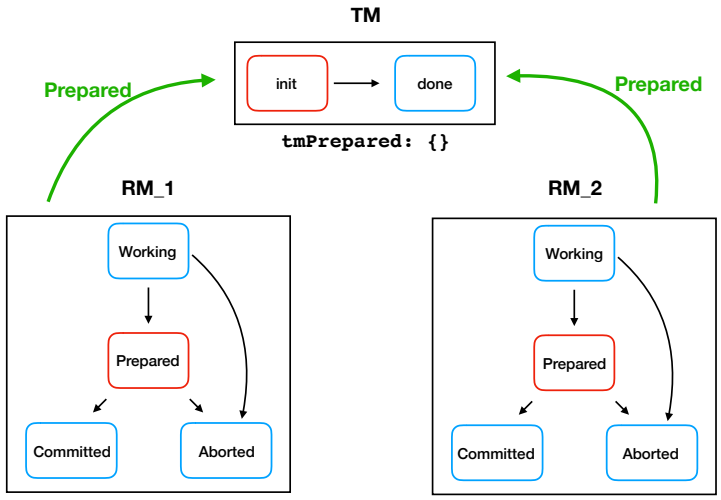
Evaluate this expression. What do you get?

Value:

```
[r1 |-> "working", r2 |-> "working", r3 |-> "working"]
```

This is a single function, where the DOMAIN is made up of resource managers, and the value associated to each is “working”

TMRcvPrepared(r)



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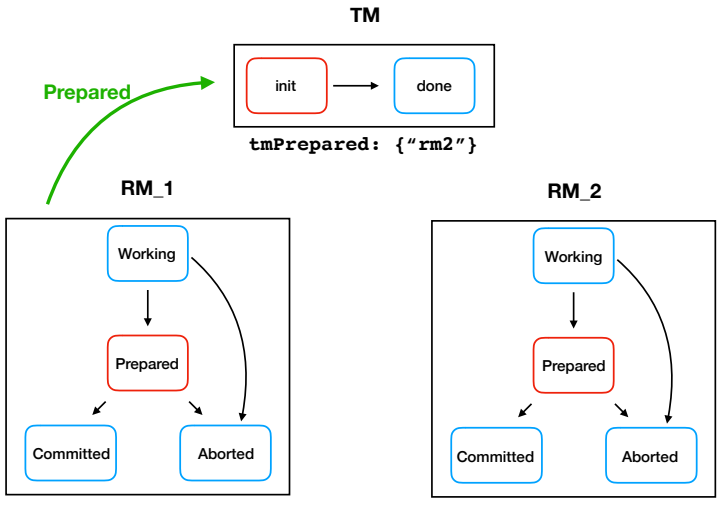

TMRcvPrepared(r)

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TMRcvPrepared(r) ==
  /\ tmState = "init"
  /\ [type |-> "Prepared", rm |-> r] \in msgs
  /\ tmPrepared' = tmPrepared \union {r}
  /\ UNCHANGED <<rmState, tmState, msgs>>
```

This formula describes the TM receiving a “Prepared” message from a RM

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This formula describes the TM receiving a “Prepared” message from a RM

This formula has primed and unprimed variables, and is therefore an “action”

What are the enabling conditions?

`tmState = “init"` means that the TM must be in the “init” state

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```

This formula describes the TM receiving a “Prepared” message from a RM

This formula has primed and unprimed variables, and is therefore an “action”

What are the enabling conditions?

`tmState = “init"` means that the TM must be in the “init” state

`[type |-> "Prepared", rm |-> r] \in msgs` means that a “Prepared” message from RM **r** must be present in the **msgs** set

TMRcvPrepared(r)

```
TMRcvPrepared(r) ==
  /\ tmState = "init"
  /\ [type |-> "Prepared", rm |-> r] \in msgs
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  /\ [type |-> "Prepared", rm |-> r] \in msgs
  /\ tmPrepared' = tmPrepared \union {r}
  /\ UNCHANGED <<rmState, tmState, msgs>>
```

This says that “in the next state, **tmPrepared** will be the result of taking the UNION of the current **tmPrepared** and **r**”

ex. If **r** is **“rm3”**, and **tmPrepared = {“rm1”}**, then in the next state, **tmPrepared** will be **{“rm1”, “rm3”}**

TMRcvPrepared(r)

```
TMRcvPrepared(r) ==
  /\ tmState = "init"
  /\ [type |-> "Prepared", rm |-> r] \in msgs
  /\ tmPrepared' = tmPrepared \union {r}
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This is new

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This is new

It says: “In the next state, **rmState** and **tmState** and **msgs** will be unchanged”

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```

This is new

It says: “In the next state, **rmState** and **tmState** and **msgs** will be unchanged”

It is syntactic sugar for:

```
/\ rmState’ = rmState
/\ tmState’ = tmState
/\ msgs’ = msgs
```


TMRcvPrepared(r)

```
TMRcvPrepared(r) ==
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```

- Remember, a formula describes the valid pairs of states, i.e., the valid next state given the current state

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```
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- Why would that be a valid next state if we didn’t have the **UNCHANGED**?
- Because the possible set of next states is infinite, and the purpose of our formula is to identify the “valid” ones

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- It is valid for an RM **r** whenever that **r** is in a “working” state
- In the new state:
  - **rmState** has that **RM** set to “prepared”
  - And a new message has been added to **msgs**, specifically, the message from RM **r** to the TM, telling it that it is prepared

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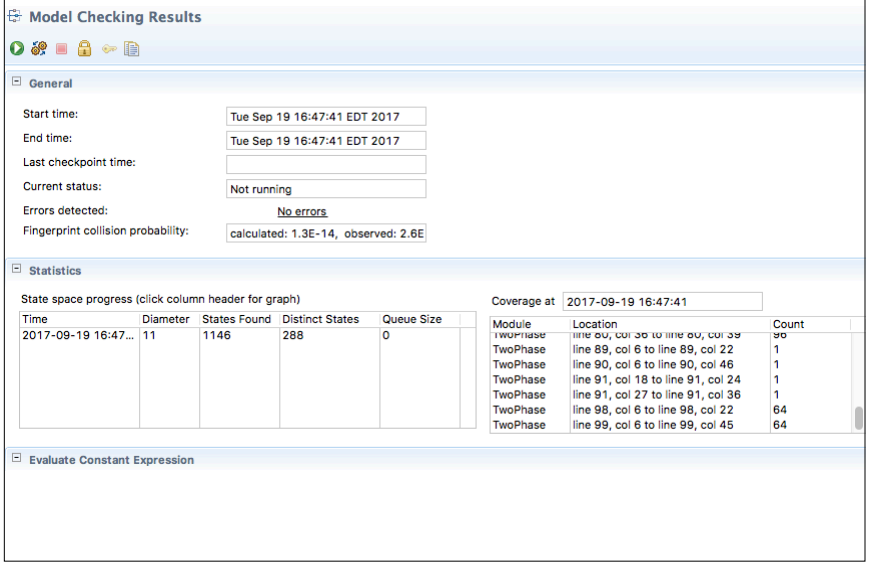
- This is how we got access to the **TCConsistent** invariant
- Because the model checker still passes with the **TCConsistent** invariant enabled, it means that the **TwoPhase** specification "implements" the **TCommit** specification

Exercise

- Replace the contents of your TwoPhase.tla with the contents of TwoPhase_incomplete.tla
- Try to write the correct definitions for
  - TMCommit
  - TMAbort
  - RMChooseToAbort
  - RMRcvCommitMsg
  - RMRcvAbortMsg



The End?

- This brings us to the end of Video 6 in Lamport's series
  - TLA Video Series: https://rax.io/tla_videos
  - http://lamport.azurewebsites.net/video/videos.html
- You now know **most** of the important TLA+ syntax, but not all
  - `LET/IN` is an important one, so is `CHOOSE`
- I recommend watching Lamport's videos, as they'll offer a slightly different perspective and will solidify your understanding
- Thanks!