

# **Naïve Bayes classifiers**

Terence Parr  
MSDS program  
**University of San Francisco**



# Common problem: detect spam tweets

- Q. Is the following tweet spam or ham (not spam)?

[REDACTED]

- Without knowledge of content, what's the best we can do?



SPAM at big package store

# Common problem: detect spam tweets

- Q. Is the following tweet spam or ham (not spam)?



- Without knowledge of content, what's the best we can do?
- Use prior knowledge about relative likelihoods of spam/ham
- If *a priori*, we know 75% of tweets are spam, always guess spam
- (Note: this is solving same problem as, say, article topic classification not spam detection)

# Our base model

- If 75% of tweets are spam, always guessing spam gives us a lower bound of 75% accuracy
- Accuracy has formal definition: % correctly-identified tweets
- A superior model must do better than 75% accuracy
- What if a priori spam rate was 99%? Our model has 99% accuracy
- That hints that accuracy can be very misleading in isolation (more later)

# Better model using knowledge of content

- If we can see tweet words, we have more to go on; e.g.,  


Viagra sale

Buy catfood
- Given “Viagra sale”, how do you know it’s spam?
- Because among spam emails you’ve received, those words are highly likely to appear
- Words like “Buy catfood” are unlikely to occur in spam email
- Vice versa: “Viagra sale” low likelihood in non-spam
- $P(\text{Viagra} \cap \text{sale} | \text{spam})$  is high,  $P(\text{Viagra} \cap \text{sale} | \text{ham})$  is low

# Model based upon tweet likelihoods

- Predict spam if:

$$P(\text{Viagra} \cap \text{sale} | \text{spam}) > P(\text{Viagra} \cap \text{sale} | \text{ham})$$

- Predict ham if:

$$P(\text{Viagra} \cap \text{sale} | \text{spam}) < P(\text{Viagra} \cap \text{sale} | \text{ham})$$

- This model works great but makes an assumption by not taking into consideration what knowledge? The *a priori* probabilities; assumes equal priors

# Model combining priors and content info

- Predict spam if:

$$P(\text{spam})P(\text{Viagra} \cap \text{sale} | \text{spam}) > P(\text{ham})P(\text{Viagra} \cap \text{sale} | \text{ham})$$

- Predict ham if:

$$P(\text{spam})P(\text{Viagra} \cap \text{sale} | \text{spam}) < P(\text{ham})P(\text{Viagra} \cap \text{sale} | \text{ham})$$

- We are weighting the content likelihoods by prior overall spam rate
- If spam-to-ham priors are .5-to-.5 the prior terms cancel out

# Are these computations probabilities?

- Are these probabilities after weighting?
  - $P(\text{spam})P(\text{Viagra} \cap \text{sale} | \text{spam}) + P(\text{ham})P(\text{Viagra} \cap \text{sale} | \text{ham}) \neq 0$
- Nope. Must normalize term by probability of ever seeing that specific word sequence (the marginal probability):

$$P(\text{Viagra} \cap \text{sale})$$

- Dividing by the marginal probability makes the terms probabilities
- Called likelihood ratio:
$$\frac{P(\text{Viagra} \cap \text{sale} | \text{spam})}{P(\text{Viagra} \cap \text{sale})}$$
- Answers “How much of the marginal does the conditional cover?”

# Yay! You've just reinvented *Bayes Theorem*

- Normalized likelihood:

$$\frac{P(\text{spam})P(\text{Viagra} \cap \text{sale} \mid \text{spam})}{P(\text{Viagra} \cap \text{sale})} \quad \textcolor{brown}{\approx} \quad \frac{P(\text{ham})P(\text{Viagra} \cap \text{sale} \mid \text{ham})}{P(\text{Viagra} \cap \text{sale})}$$

- Says how to adjust *a priori* knowledge of spam rate with tweet content evidence

# Maximum *a posteriori* classifier

- Choose class/category for document  $d$  with max likelihood:

$$c^* = \underset{c}{\operatorname{argmax}} P(c|d)$$

- Substitute Bayes' theorem:

$$c^* = \underset{c}{\operatorname{argmax}} \frac{P(c)P(d|c)}{P(d)}$$

Bayes' theorem

$$P(c|d) = \frac{P(c)P(d|c)}{P(d)}$$

- You'll often see the classification decision rule written as the *Bayes test for minimum error*:

$$P(c_1 | d) \geq P(c_2 | d)$$

# Simplifying the classifier

- $P(d)$  is constant on both sides so we can drop it for classification:

$$c^* = \underset{c}{\operatorname{argmax}} P(c)P(d|c)$$

- If  $P(c)$  is same for all  $c$  OR we don't know  $P(c)$ , we drop that too

# Training the classifier

- We need to estimate  $P(c)$  and  $P(d | c)$  for all  $c$  and  $d$
- Estimating  $P(c)$ ? The number of documents in class  $c$  divided by the total number of documents
- Estimating  $P(d | c)$ ? E.g., we need  $P(\text{Viagra} \cap \text{sale} | \text{spam})$
- That means considering all 2-word combinations (*bigrams*), which grows in size with the square!

# The naïve assumption

- Naïve assumption: conditional independence; Estimate  $P(\text{Viagra} \cap \text{sale} | \text{ham})$  as  $P(\text{Viagra} | \text{spam}) \times P(\text{sale} | \text{spam})$

$$P(d|c) = \prod_{w \in d} P(w|c)$$

- So, our classifier becomes

$$c^* = \operatorname{argmax}_c P(c) \prod_{w \in d} P(w|c)$$

- where  $w$  is each word in  $d$  with repeats, not  $V$  (vocabulary words)

# Fixed-length word-count vectors

- Rather than arbitrary-length word vectors for each document  $d$ , it's much easier to use fixed-length vectors of size  $|V|$  w/word counts:

$$c^* = \underset{c}{\operatorname{argmax}} P(c) \prod_{w \in d} P(w|c)$$

becomes:

$$c^* = \underset{c}{\operatorname{argmax}} P(c) \prod_{\text{unique}(w) \in d} P(w|c)^{n_w(d)}$$

# Estimating $P(w | c)$

- We have  $P(c)$  but we still need to estimate  $P(w | c)$
- It's number of times  $w$  appears in all documents from class  $c$  divided by the total number of words (including repeats) in all documents from class  $c$ :

$$P(w|c) = \frac{\text{wordcount}(w, c)}{\text{wordcount}(c)}$$

# Laplace smoothing

- If  $P(w | c) = 0$  then the entire product goes to zero. Ooops

$$c^* = \underset{c}{\operatorname{argmax}} P(c) \prod_{w \in d} P(w|c)$$

- To avoid, add 1 to each word count in numerator and compensate by adding  $|V|$  to denominator (to keep a probability)

$$P(w|c) = \frac{\text{wordcount}(w, c) + 1}{\text{wordcount}(c) + |V|}$$

- (We have added +1 to each word count and there are  $|V|$  words in each word-count vector)

# Dealing with “mispelled” or unknown words

- Laplace smoothing deals with  $w$  that are in the vocabulary  $V$  but not in class  $c$ , which gives  $P(w | c) = 0$
- What should  $\text{wordcount}(w, c)$  be for a word not in  $V$ ? Zero doesn't seem right as we have no data; OTOH, if  $\text{wordcount}(w, c)=0$  for all classes, classifier is not biased
- Map all unknown  $w$  to a wildcard word in  $V$  so then  $\text{wordcount}(\text{unknown}, c)=0$  is ok but must bump  $|V|$  by 1
- Likely not a huge factor...

$$P(w|c) = \frac{\text{wordcount}(w, c) + 1}{\text{wordcount}(c) + |V| + 1}$$

# Avoiding floating point underflow

- In practice, multiplying lots of probabilities in [0,1] range tends to get too small to represent with finite floating-point numbers
- Take log (a monotonic function) and product becomes summation; also, since  $P(w|c)$  can't be 0 due to smoothing, we can use  $w \in V$  rather than iterating through unique doc words

$$c^* = \underset{c}{\operatorname{argmax}} P(c) \prod_{w \in V} P(w|c)^{n_w(d)}$$

$$c^* = \underset{c}{\operatorname{argmax}} \left\{ \log(P(c)) + \sum_{w \in V} n_w(d) \times \log(P(w|c)) \right\}$$

# An example

# Documents as word-count vectors

- One column per vocab word, one row per document

column for  
unknown  
words

d1 = "sale viagra sale"  
d2 = "free viagra free viagra free viagra free"  
d3 = "buy catfood and buy eggs"  
d4 = "buy eggs"

	unknown	buy	catfood	eggs	free	sale	viagra	spam
0	0	0	0	0	0	2	1	1
1	0	0	0	0	4	0	3	1
2	0	2	1	1	0	0	0	0
3	0	1	0	1	0	0	0	0

Priors:  
 $P(\text{spam}) = 2/4$   
 $P(\text{ham}) = 2/4$

spam or ham

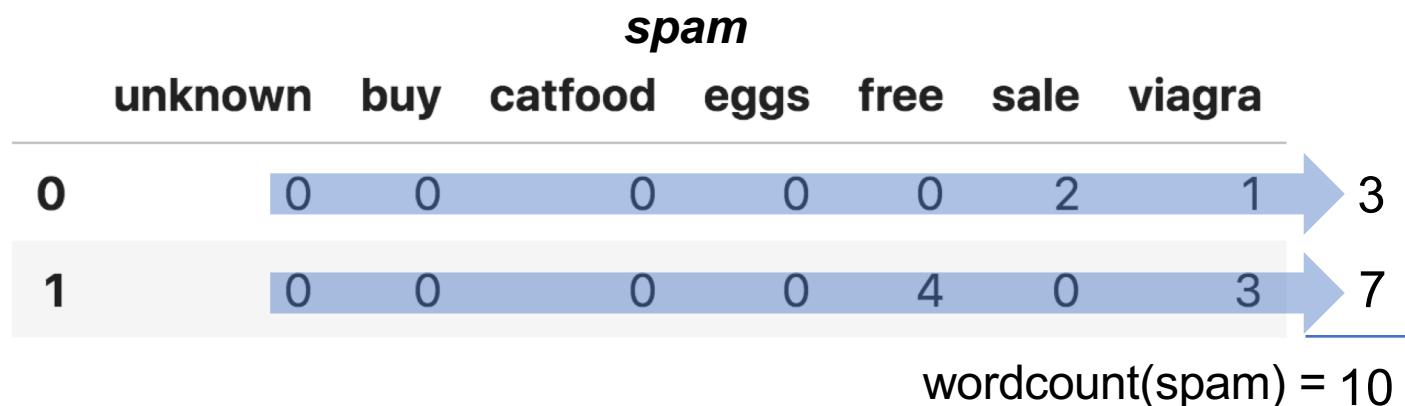
Note:  
 $|V|=6$   
+1 for unknown



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# Estimating probabilities: $P(w | \text{spam})$

- 1<sup>st</sup>, get total word count in spam category: sum across rows or cols then sum that result
- `count(spam) = spam.sum(axis=1).sum()`



$$P(w|c) = \frac{\text{wordcount}(w, c) + 1}{\text{wordcount}(c) + |V| + 1}$$

# Estimating probabilities: $P(w | \text{spam})$

- 2<sup>nd</sup>, get total count for each word in spam docs,  $\text{wordcount}(w, \text{spam})$

	<i>spam</i>						
	unknown	buy	catfood	eggs	free	sale	viagra
0	0	0	0	0	0	2	1
1	0	0	0	0	4	0	3
	0	0	0	0	4	2	4
	—	—	—	—	—	—	—
	10	10	10	10	10	10	10

$$P(w|c) = \frac{\text{wordcount}(w, c) + 1}{\text{wordcount}(c) + |V| + 1}$$

# Estimating probabilities: $P(w | \text{spam})$

- 3<sup>rd</sup>, compute  $P(w|\text{spam})$  w/smoothing & unknown word adjustment
  - wordcount + 1 = 

unknown	buy	catfood	eggs	free	sale	viagra
1	1	1	1	5	3	5

- $\text{wordcount}(\text{spam}) + |\mathcal{V}| + 1 = 10 + 6 + 1 = 17$

$P(w   \text{spam}) \rightarrow$	unknown	buy	catfood	eggs	free	sale	viagra
	0.058824	0.058824	0.058824	0.058824	0.294118	0.176471	0.294118

$$P(w|c) = \frac{\text{wordcount}(w, c) + 1}{\text{wordcount}(c) + |\mathcal{V}| + 1}$$

# Estimate $P(c|w)$ w/o $P(d)$ normalization

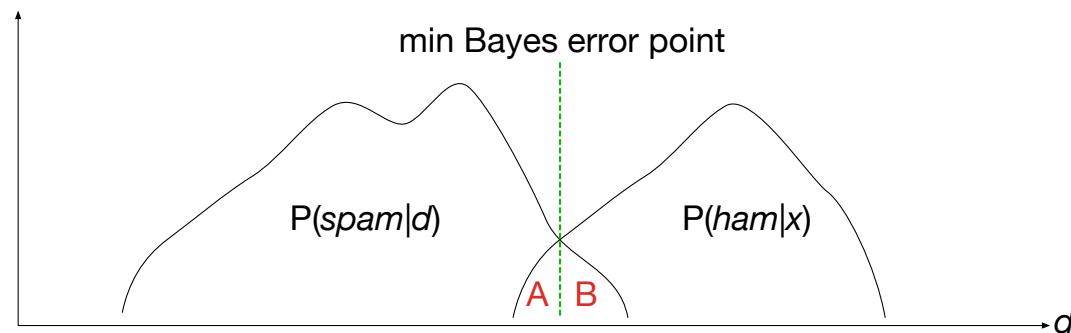
- Dot product of X matrix with log of  $P(w|c)$  vector, add  $\log(P(c))$

	$P(\text{spam} d)$	$P(\text{ham} d)$
d1 = "sale viagra sale"	-5.386125	-8.387995
d2 = "free viagra free viagra free viagra free"	-9.259575	-18.647793
d3 = "buy catfood and buy eggs"	-12.026001	-6.388596
d4 = "buy eggs"	-6.359574	-3.338139

$$c^* = \underset{c}{\operatorname{argmax}} \left\{ \log(P(c)) + \sum_{w \in V} n_w(d) \times \log(P(w|c)) \right\}$$

# Quick idea on Bayes error

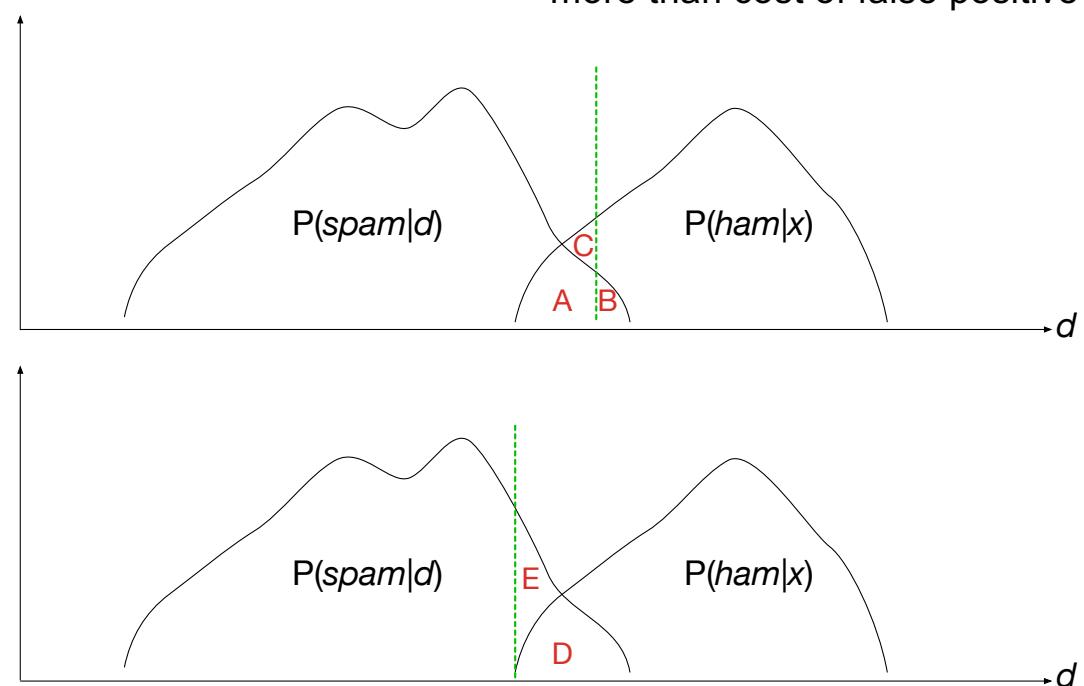
- Bayes error is the lowest possible total error from all classes
- Imagine squashing doc  $d$  down to single dimension
- Bayes error is area under  $P(\text{spam}|d)$  to right and  $P(\text{ham}|d)$  to left of decision point (**A** and **B** here)



# Preview AUC ROC / PR curves

- If our "utility function" values spam or ham differently, can shift boundary to left/right
- Later we'll examine ROC/PR curves that summarize effect of shifting boundary all over
- Tradeoff: to get 0 ham error, must accept many spam, area under  $P(\text{spam}|d)$  to right of threshold: D+E error

E.g., in disease testing, might value catching disease more than cost of false positive



# Key takeaways

- Naïve bayes is classifier often applied to text classification; e.g., spam/ham, topic labeling, etc...
- Less often used these days with rise of deep learning
- Fixed-length word-count vectors are the feature vector per doc
- Bayes theorem gives formula for  $P(c|d)$
- Naïve assumption is conditional independence  $\longrightarrow P(d|c) = \prod_{w \in d} P(w|c)$
- Training estimates  $P(c)$ ,  $P(w|c)$  for each  $w$  and  $c$
- $P(c)$  is ratio of docs in  $c$  to overall number of docs
- $P(w|c)$  is ratio of word count of  $w$  in  $c$  to total word count in  $c$
- Classifier:  $c^* = \operatorname{argmax}_c P(c) \prod_{w \in V} P(w|c)^{n_w(d)}$

# Implementation takeaways

- Avoid vanishing floating-point values from product

$$c^* = \underset{c}{\operatorname{argmax}} \left\{ \log(P(c)) + \sum_{w \in V} n_w(d) \times \log(P(w|c)) \right\}$$

- Avoid  $P(w|c)=0$  via Laplace smoothing
  - add 1 to all word counts
  - adjust  $P(w|c)$  denominator with extra  $|V|$  since every doc now has every word
  - this is for missing words where  $w$  not in  $d$  but in  $V$
- Treat unknown words as  $1 / (\text{count}(c) + |V| + 1)$