Report on the possibility of technical energy production from uranium fission (II)

By **W. Heisenberg** **29.2.40**

11.1

The following report contains the exact elaboration of the considerations which were briefly sketched in my earlier report. First of all, it has proved necessary to carry out the theory of resonance capture of neutrons in a mathematically rigorous manner. For the case of neutron braking <by hydrogen> *Fliigge* has already developed the theory. Section 11.2a contains the generalization for arbitrary substances. Furthermore, Mr. *Bothe* had drawn my attention to the fact that, in the case of absorption in uranium plates, scattering by U and O atoms cannot be dispensed with. Section 11.2b deals with absorption in the plates, taking scattering into account. This seems to me to be an exhaustive treatment of the whole question of resonance absorption. The two sections 11.2a and binder contain mainly mathematical developments which can claim a certain interest apart from the uranium problem, and I would therefore like to ask whether these two sections could possibly be published, avoiding any reference to the uranium problem.

Section 11.3 contains a critical appraisal of the existing
experimental results on the constants important for the uranium machine. It is shown that the values of these constants taken as a basis in Part I have to be changed, in some cases considerably - which was already foreseen in Part I. Finally, Section 11.4 contains a new calculation of the most important data of the uranium machine on the basis of the new experimental results and the more exact theoretical formulas; in the process, an error which had been made in Part I. Table 1, and which was due to the fact that it was not possible to calculate the data of the uranium machine, is also improved. Table 1 and to which Mr. *Bothe* drew my attention.
had made. The results of this section 4 show that the conditions 1 a for the production of the uranium machine in Part I were probably judged somewhat too favourably. In particular, it has become doubtful whether the uranium machine could be produced with pure coal. The experimental data available so far are still too i n a c c u r a t e to make a final decision on this. Otherwise, there is not too much change in the data of Part I.

11.2 The capture of neutrons in resonance sites 1 b

(a) solutions of the absorbing substance in the decelerating substance

We consider a solution in which *N;* atoms of mass *M; are* present in volume

unit, and first ask for the distribution of neutrons in velocity space if per sec Q neutrons of velocity v_0 are present in

are sent into space. After stol3 of a neutron of mass m and energy E with an atom of mass $M;$ the energy of the neutron takes any value between E and $E[(M;-m)l(M;+m)]^2$ with equal probability. We set

$$\left(\frac{M;-m}{M;+m}\right)^2 = 1-a; \tag{1}$$

we also introduce

$$Y; = N;a;(v) , \tag{2}$$

where $a;$ are the cross sections fi.ir the elastic pole3 of the neutron with the atoms $N;$. The ratios of $a;$ and hence of $y;$ are supposed to be independent of v. For the number $N(v)\ dv$ of neutrons with the velocity between v and $v + dv$ we get the integral equation

$$vN(v)\ LY; = QJ(v-vo) + L\ \_v\ \frac{2}{a;}\ \int_{v}^{v!l,1-a;} N(v')\ y;(v')\ \_v\_\ \frac{d}{v'} , \tag{3}$$

Here $J(v$ - v0) denotes the Dirac's J-function; the term $QJ(v$ - v0) represents the particles delivered by the neutron source. We fi.her now as a new variable $u =$ lg $vivo$ and set

$$P; = 1/2\lg(1-a;) \tag{4}$$

then

$$N(u)\ L\ Y;(u) = \frac{Q}{vo}\ y\ \mathrm{J(u)} + L\ \frac{2}{a;}\ \int_{u}^{u+fl;} N(u')\ y;(u')\ du' \tag{5}$$

This equation can be solved by a Laplace transformation. We set

$$N(u)LY;(u) = \int_{}^{+\infty} e2niuxf(x)dx \tag{6}$$

and use

2 $\mathrm{J(u)} = \int_{-\infty}^{+\infty} \mathrm{e2niux}\ dx$, further write

$$\nu_i/\sum \gamma_i = g_i \ . \tag{7}$$

Then the integral equation becomes (5):

$$f(x) = +\ \frac{L\ 2\_}{vo\quad a;\ 2n1x}\ (\mathrm{e2nifi;x} - 1)/(x) , \quad \text{thus} \tag{8}$$

$$N(u)\sum_i \gamma_i(u) = -\frac{Q}{2}\int_{-\infty}^{\infty} dx \; \frac{nix\, e^{2\pi i x u}}{nix - \sum_i (g_i/a_i)(e^{2\pi i \beta_i x} - 1)} \qquad (9)$$

The integration path is <l determined by the requirement that $N(u)$ must vanish3 for $u > 0$. This means that in the complex x-plane the integration must be carried out above all singular points of the integrand. In
(9), one can also separate the singular part, which originates from the neutron source, and thus obtains

$$N(u)\sum_i \gamma_i(u) = \frac{Q}{v_0^2}\,\delta(u) + \frac{Q}{v_0^2}\int_{-\infty}^{+\infty} dx\, e^{2ixu}\left\{\frac{\sum (g_i/\alpha_i)\,(e^{2\pi\beta_i x} - 1)}{\pi i x - \sum (g_i/\alpha_i)\,(e^{2\pi i\beta_i x} - 1)}\right\} . \qquad (10)$$

The denominator of the integrand in (9) and (10) has a zero at $x = i/n$ and another at $x = 0$. All the zeros (there are infinitely many) lie below the real axis and symmetrically about the imaginary axis (Fig. 1).

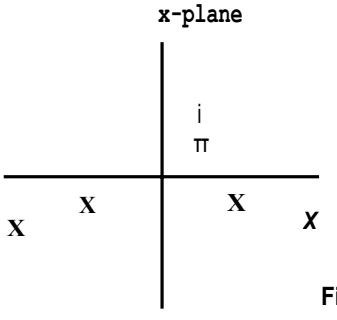


Fig.

From the representation (10) the asymptotic behavior of $N(u)$ for large negative u, i.e. small velocities, can be easily determined. To do this, one deforms the integration path in such a way that it fits into a circle around the singularity $x = ii\,n$
around and I decays one way along the real axis. By far the largest part- 3
The result of (10) then comes for large negative u from the integration around the point i/n. This results in

$$N(u)\sum_i \gamma_i(u) = \frac{Q}{v_0^2}\,\frac{2e^{-2u}}{1 - \sum 2g_i\beta_i(1-\alpha_i)/\alpha_i}$$

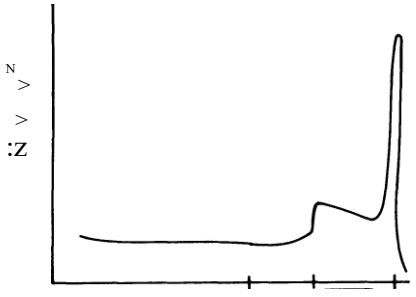
$$\frac{2Q}{v2[1 + \sum_i (gi(1 - aMaJlg(1-\ aJ]} \qquad (11)$$

Or

$$N(v) = 2\,\frac{2Q}{v\,LYd1+(1-ai)laJlg(1-aJ]} \qquad (12)$$

The distribution of neutrons thus follows at low velocities again

the law const./v2; for the effect of the different masses the factor (1 + (1 - *ai)I a;)* lg(1 - *a;)* in the denominator is characteristic. The distribution near v_0 la/3t can be given only qualitatively. If there is only one type of atom in the solution for which $a < 1$, it is represented approximately by Fig. 2:



Vo(1-IX) vo 1-IX Vo -- v Fig. 2

The function behaves like an o-function at *v0*; at *v0Vt-=-a* it is finite, but discontinuous; at v_0 (1- a *) it is* continuous, but the 1st differential quotient is discontinuous, etc.

After this preparation we proceed to the question of absorption of neutrons in the resonance point $v =$ v, a type of atom of which there are zero atoms in the volume unit and for which the cross section of action for capture in the vicinity of the resonance point is given by the formula

$$\sigma_U = \sigma_r \frac{(\Gamma_r/2)^2}{(E-E_r)^2+(\Gamma_r/2)^2} \tag{13}$$

4 is given. If one again introduces the new variable $u = lg(v/v,)$, one can set $E =$ *E,e2u* and obtain in sufficient approximation:

$$\sigma_U = \sigma_r \left(\frac{\Gamma_r}{4E_r}\right)^2 \frac{1}{u^2+(\Gamma_r/4E_r)^2} \tag{14}$$

For the following calculations we assume that/3 the concentration of the absorbing atoms is so large/3 that/3 the equivalence width of the line is large/3 is against $r,.;$ then one can also simply set

$$au = a, \left(\frac{r,}{4E,}\right)^2 \quad u \qquad - \tag{15}$$

We still drove in the designation

$$Nu\ a, \left(\frac{r,}{4E,}\right) = Yu\ . \tag{16}$$

If one assumes that the generation of the neutrons has taken place at very high speeds, then in the region of the resonance line GI.
(5) <by the equation

$$N(u)\left(\sum \gamma_i + \frac{\gamma_u}{u^2}\right) = \sum \frac{2}{\alpha_i} \int_u^{u+\beta_i} N(u')\,\gamma_i(u')\,du' \tag{17}$$

substitute. Since *N(u)* varies like e *-to* at a far distance from the resonance line, we set

$$N(u) = y(u)e\text{-}2u, \tag{18}$$

further we differentiate (17) by *u* and obtain

$$\text{-}2ye\text{-}2u\ [\ Ly\,I + \ldots] + e\text{-}2u\ \ldots\ [L\ Yi + \ldots$$

$$= r2\text{-}_{ai}\ [YYi(u+Pi)e\text{-}2u\text{-}2/Lyyie\text{-}2u]' \tag{19}$$

or according to (4)

$$\frac{d}{du}\left[\left(y\sum \gamma_i + \frac{\gamma_U}{u^2}\right)\right] = \sum \frac{2(1-\alpha_i)}{\alpha_i}\ [y\,\gamma_i(u_i+\beta_i) - y\,\gamma_i(u)]\ . \tag{20}$$

The magnitude *y L Yi* must become constant for both very large positive and very large negative *u.* We now also set the *Yi* constant in the whole region, i.e. we assume that the *Yi* in the close vicinity I of the resonance point are not 5
vary greatly. The difference

$$y + {}_{00}\text{-}y - {}_{00} = L1y$$

gives a mall fi.ir the total amount of neutrons absorbed. By integration of (20) from - oo to + oo one immediately obtains

$$L1y\ L\ Yi = L\ \frac{2(1 - a\text{-}i)}{ai}\ PiL1y + 2\int_{-00}^{+J00} y(yulu^2)du\ , \quad \text{so} \tag{21}$$

$$L1=y\ \frac{2\int_{\ldots}^{+\infty} y(yulu2)du}{L\ Yi\ [\ +\ 1\ ((1 - a\ i\ )I\ ai)\ \lg(1 - a\ i\ )J} \tag{22}$$

From GI. (20) it is immediately clear that in many cases the role *Y(L y1+Yulu2)* undergoes only relatively small changes in the critical region *u* - 0, since the right-hand side remains small and the critical region is relatively narrow. This is true if the total neutron absorption is small and if the *a; are* not too small. Thus, in this case, in the critical region, in the first approximation, one can

$$y(\sum \gamma_i + \gamma_U / u^2) = y_{+\infty} \sum \gamma_i^{5)} \text{and} \quad (23)$$

$$\int_{-\infty}^{+\infty} y\, y\, dU = Y+oolrVYuLYi \quad u \tag{24}$$

set. One receives finally after (16)

w , =L'ly-= 2 AY-Y-u L =Yi-----
Y+oo L Yi [1 +((1- ai)/ai) lg(1- ai)]

= !!_ r, ------ V_N_u_a_, L_Y,_- _ (25)
2 E , L Yi [1 + ((1 - a i)la;) lg (1- ai)l

If a magnitude *ai,* and hence */Ji*, becomes small against the aquivalence width of the line, that is, according to (24) small against *n* V Yu *IL* *Yi,* then the member in question on the right-hand side of (20) becomes magnitude3. Then *y Yi (u* + /Ji) can be placed in a line after */Ji*
and because of (4) applies approximately

_2_(1_-_a,_) [y *Yi (u*+ /Ji) - *Y Yi* (u)] = *_d_ (y Yi)* _2_(1_-_a,_) */Ji* (26)

One can then take this link to the left heri.iber, and the GI. (24) rightly remains, if one takes in the *L* *Yi* the one member *Yi* which belongs to the small-
6 nen *Yi* is multiplied by 1 +((1 - *ai)/ a;)* lg(1 - *ai).* In general

w=-----=-c-- *2n Lr VYurLiYitir* ------ (27)
L; y; [1 +((1 - *ai)/a;)* lg (1- a ;)]

where

e i = r { + ((1 - *ai)I ai)* lg(1 - *a;)* fi.ir */Ji*►*n VYu,IL;Y;ei,*
fi.ir */Ji-<1ii;nVLYu,ILiYieir-*

Also GI. (27) is valid only when *w* is 4ii; 1. Fi.ir values of *w of* the order of magnitude 1 one can probably put in good approximation the following of the results found by *Fliigge* for absorption in hydrogen:

$$1 - w = \exp\left(-\frac{2\pi \sum_r \sqrt{\gamma_{Ur}} \sum_i \gamma_i \varepsilon_{ir}}{\sum_i \gamma_i [1 + ((1-\alpha_i)/\alpha_i)\lg(1-\alpha_i)]}\right). \tag{28}$$

This formula will be used in all further calculations of resonance absorption. If one is interested not only in the total number of neutrons, but also in their energy distribution, one proceeds appropriately from GI. (17), which can be expressed in the form:

N(u) L Yi= -N(u) + *L* - 2 ∫ *N(u')(u')du'* (29)
; u2 ; ai u u+P-

can be written. The comparison with (5) teaches that here the member *-N(u)(yulu2)*

represents in a certain sense a negative neutron source, which then also leads according to (12) to an absorption according to formula (22). The general course of the neutron number *N(v)* is therefore also shown qualitatively in Fig. 3 for a single atomic species *N;(v)* in analogy to Fig. 2.

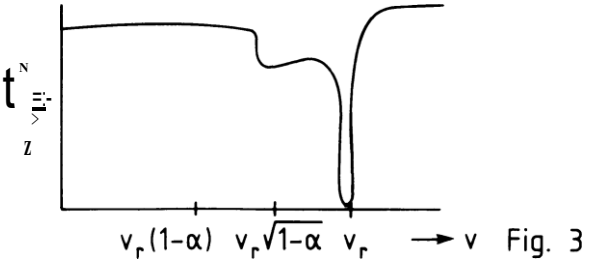


Fig. 3

It can be seen from the figure thatJ3 the total absorption is *not* simply determined by the 7
difference of *N(v)* to the right and left of the resonance point. This difference was calculated by *Flagge* in part II of his investigations and was erroneously equated with the total absorption.

We now compare the result (27) and (28) with the estimation of Part I, Section 2a. There, in GI. (6) for *w* in the case of a single braking substance the formula

$$w = \frac{1}{-\mathrm{Igo}}\,\frac{n\,I'r}{2Er}\sqrt{\frac{arNv}{aN}}$$

is given. Comparison with the exact formula (27) shows (for a single substance *e* must always be 1 because of *w* -c 1) thatJ3 the magnitude - Igo in part I in the more exact theory is to be replaced by 1 + ((1- a*;)la;)* lg(1- a*)*. In the limiting case of large masses *(a* -c 1) the two expressions agree, since *O* = 1-(a/2):

$$-\mathrm{Igo} = -\lg\left(1 - \frac{a)}{2}\right) = \frac{a}{2} = \frac{1}{2} + \frac{1}{a}\lg(1-a). \qquad (30)$$

In the opposite limiting case *a=* 1, i. e. at the deceleration of the neutrons in hydrogen, the two expressions differ as lg 2 and 1, i. e. about 30%, as *Flagge* already indicated in Part I. The deviation of the earlier approximation formula from the exact formula is therefore not very large, but it must be taken into account. Furthermore, *Flagge* has rightly pointed out that the contribution of the oxygen atoms to the braking in H20 and especially D20 must not be neglected. Indeed, formula (27) shows thatJ3 the magnitude *w* in D20 becomes about 20% higher than would result from neglecting O *(w* = probability thatJ3 a fast neutron in U238 is captured by resonance before it reaches thermal energy).

8 **b) Plane plates of uranium-containing substance in a medium serving for neutron deceleration**

We consider an infinitely extended plate of thickness *d* consisting of a mixture of U with other substances. Let the plate be embedded in a medium which itself is mixed from different kinds of atoms. Let the neutron source be uniformly distributed over the space and supply neutrons of such high energy that in the vicinity of the resonance point of uranium it has already set up a uniform neutron distribution according to the law const/ v2, which is then <l> is disturbed by the resonance point. We divide the scattering substances into two groups *a* and *b.* Group *a* includes the lighter atoms which, in the case of a *shock,* extract on average much more energy from a neutron in the region of the resonance energy than the effective width of the resonance line (which in turn depends on *d*); group *b* includes the heavier atoms which, in the case of a shock, extract much less energy from a neutron than the effective width of the resonance line. Atoms for which neither of the two conditions is fulfilled should not occur. The effective width of the resonance line should in any case be very small compared to the resonance energy itself.

The space inside the plate is called area 1, the space outside is called area 2. Let the plate be perpendicular to the z-axis. Then the neutron distribution will depend on the absolute value of the velocity, the z-coordinate and the angle of the velocity relative to the z-axis.

$$N(v, (, z)dv\ d(\ dz \tag{1}$$

let be the number of neutrons per cm3 between *z* and *z+ dz,* with a velocity between *v* and *v* + *dv* and an angle to the z-axis, <lessen cosine between-
9 between (and (+ *d(.* As in [Section] 11.2a, Eq. (2) I we introduce the products of cross section and atomic number:

$$\gamma_i = N_i \cdot \sigma_i \tag{2}$$

and put again, as in [sec.] 11.2a, Eqs. (15) and (16):

$$\text{Nu-}\ av = Yvlu2' \tag{3}$$

where $u =$ lg *vlv0;* finally, let *M;l(M;+ m) = r/;,* thus

$$4\,\eta_i(1-\eta_i) = \alpha_i/2\ . \tag{4}$$

Then the following integral equation results for the neutron distribution (in region 2 Yu = 0):

$$\text{Where} \tag{5}$$

$$Z; = \{(v'^2\text{-}v2)[v2\text{-}\ (1\text{-}2r,;)^2\ v'^2]\text{-}4(1\text{-}\ r,r)\ v2\ v'^2((^{2+}\ ('^2)$$
$$+4((\text{'}(1\text{-}r,;)vv'[v'^2(1\text{-}2r,;)^{+v2}]\}^{112}\cdot \tag{6}$$

The integration limits in (5) result from the real domain of Z;. One can easily convince oneself by substituting that for $Yu = 0$ the equation (5) is solved by the approach $N =$ canst/v 2.

We now take in (5) first the division into the two atomic species *a* and *b* var. For the atoms of the group *b,* 1 - *f/;* <iii; 1, i. e. *I'/;~ 1 can* be assumed. Under this condition and under the assumption that *N(v',* (', *z) does* not vary too rapidly with *v',* the integration over v′ can be carried out on the right-hand side of van (5), since *v′* always remains in the neighbourhood of van *v.* Thus one obtains:

$$\frac{oN}{oz} + (IY; + Y)N = \frac{1}{2b} \int_0^{\infty} I;v;JN(v,z,(')d(' + I; \int_a^{\infty} vv; JN(v',(',z)dv'd(' \qquad (7)$$

We are now interested in the values of *N in* the immediate vicinity of the reso nce $v = v_r$. In this region *N* will vary very much; however, the integrals occurring under I *a will* dart our little, since they will vary over a range of velocities that is very large compared to the effective width of the resonance line. We can thus define these integrals as approximate I constant and obtain for Nin the immediate surrounding 10 of the resonance point the simpler integral equation

$$\frac{ON}{OZ} + (IY; + Y)N = \frac{1}{2b} IY; \int_{-I}^{a} YN(v,z,(')d(' + NoIY;' \qquad (8)$$

where *No* denotes the number *N(v, z,* () just above the resonance point.

A particulate solution van (8) is:

$$N = N_0 \sum \gamma_i \Big/ \left(\sum_i \gamma_i + \frac{\gamma_U}{u^2} \right) . \qquad (9)$$

To obtain the most general solution, we need to find the most general solution of the homogeneous integral equation

$$\frac{oN}{OZ} + (LY; + Y)N = \frac{1}{2b} I\ Y; \int_{-I}^{a} YN(v,z,d')d(' \qquad (10)$$

add. If atoms of the type *b do* not occur, then a simple differential equation arises from (10), the solution of which can be written out immediately. If the elastic scattering at the U-atoms themselves could be omitted, then all other atoms (H, D, 0) must indeed be counted to the group *a for* not too thick plates; the further calculations can be carried out strictly in this case:

Case 1: $Y;b = 0$. (10) now becomes

$$\left(\frac{dN}{oz} + Y_i + \frac{Y_U}{u}\right) N = 0, \qquad (11)$$

$$N = f(\zeta, u) \exp[-(\sum_a \gamma_i + \gamma_U/u^2) z/\zeta] \tag{12}$$

To obtain the complete solution, the boundary conditions at the boundary between area 1 and area 2 must be taken into account; the boundaries should be + *d/2* and - d/2. The distribution must then be symmetric about *z* = 0,

11 i.e. with simultaneous change of signj of *z* and (*N(v, z,* d) must remain unchanged. We distinguish the magnitudes related to the different domains by the indices 1 and 2. Then at the boundary + d/2 holds:

$$N1\ (d/2, 0 = \mathrm{N2}\ (d/2, 0$$

or

$$N_0 \frac{\sum_a \gamma_i^{(1)}}{\sum_a \gamma_i^{(1)} + (\gamma_U/u^2)} + f_1(u, \zeta) \exp\left[-\left(\sum_a \gamma_i^{(1)} + \frac{\gamma_U}{u^2}\right)\frac{d}{2\zeta}\right]$$
$$= N_0 + f_2(u, \zeta) \exp\left[-\sum \gamma_i^{(2)} \frac{d}{2\zeta}\right] . \tag{13}$$

So that for large *z* Nin N_0 transitions, which must be required for physical reasons, it follows that for *z d/2*

$$Ii\ (u, O = 0 \quad \text{for} \quad (< 0 . \tag{14}$$

This results in

$$f1(u, d\_)\text{-No}_2 \quad Yu_{(1)} \quad \exp[(\ Yi^{(t)} + \text{-}Y2u) - \quad \text{fur} \quad (< o \tag{15}$$
$$U La\ Yi + Yu \quad a \quad U \quad 2($$

For reasons of symmetry it follows for any (:

$$f_1(u, \zeta) = N_0 \frac{\gamma_U}{u_2 \sum_a \gamma_i^{(1)} + \gamma_U} \exp\left[-\left(\sum_a \gamma_i^{(1)} + \frac{\gamma_U}{u^2}\right)\frac{d}{2|\zeta|}\right] \tag{16}$$

and finally

$$N1\ (z, (, u) = No\{L \quad \underset{aYi}{\underset{<t>}{La}}\ y\}O \quad \underset{+Yu\ U}{\underline{l}}\ ^{2} + \text{-} \quad \underset{aYi}{Yu_{(l)}} \quad +Yu$$
$$. \quad \exp\left[-\left(E_{a'}\ y<^{1>+}\right)\left(\ +\text{-}d\right)\right]\} \tag{17}$$

The total number of all neutrons absorbed per sec is obtained if *N1* multiplied by *v,(Yulu2)* and integrated over (, *z* and *u*; in *u* of - ex, to + ex, can be integrated:

$$\text{nabs} = \int_{-d/2}^{+d/2} dz \int_{-1}^{+I} d(\int_{-oo\ U}^{+oo\ you} Yuv; N1 \tag{18}$$

If one sets *df.* $_{0}ypi = a$ and bentitzt the designation

$$tP(x)=\frac{2}{\sqrt{n}O}\int_{0}^{x} e^{-t^{2}}\,dt, \tag{19}$$

so it results after a longer calculation:

$$nabs=Nov\,\{rrVa+Vn\,(+!\,a)e-a$$
$$+2\,\underline{Va}\,[tP(Va)-4a\,(1+;\,)0-\,tP(Va'' \tag{20}$$
$$]\}$$

To calculate the probability *w* of capture, we have to divide 12 by the number of neutrons emitted per second. This probability vanishes, if only one plate is placed in an infinitely large
area 2 is present. Therefore, we now assume that the area 2 only contains the
that is, that the distance between two successive plates in the uranium machine is *l,* where $l \blacktriangleright (1/f.\ y)^{2l}$), i.e., is large in relation to the median
Then, according to [Section] 11.2a, Eq. (12), the number of neutrons emitted per sec and cm2 in the plate and in region 2 of extension *l*:

$$N_0\,v\int_{i} ypi\,[1+((1-a;)la;)\,\lg(1-a;)]\,d$$
$$+Nov\int_{l} YFl\,[1+((1-a;)!\,a;)\,\lg(1-a;)] \tag{21}$$

Closely follows:

$$\bar{W}=$$
$$\frac{LVYu,d\,\{rr\,Va+Vn(f+ta)\,e-a+rr/2\,Va\,[tP(Ya)-4a(1+(a/3))(1-tP(Ya))J\}}{\int_{r} ypi\,[1+((1-a;)/a;)\,\lg(1-a;)]\,d+\int_{;} y\,Fl\,[1+((1-a;)/a;)\,lg\,(1-a;)]\,l\,.} \tag{22}$$

In the limiting case $a = 0$, Eq. (22) can be compared with the corresponding results of Part I, Eqs. (15), (60) and (61). The comparison shows that the formulas of part I pass into the exact formula (22), if one again, as in
[Sec.] 11.2a -(lgc5)/,l is replaced by the expression $L;\ y;[1+((1-a;)la;)\,\lg(1-a;)]$.

Unfortunately, Eq. (22) cannot be applied practically without further ado, since the elastic scattering of the neutrons by the uranium atoms possibly plays a considerable role, i.e., atoms of the group *b* also occur. We therefore proceed to the treatment of case 2 Ober:

Case 2: $Yib = 0$. Here the integral equation (10) must be solved under the boundary conditions provided by the physical problem. The integral equation (10) has been treated in detail in the literature by G. *Wick:*
Lincei Rend. **23,** 775, 1936 and *0. Halpern, R. Lueneburg* and *0. Clark:* Phys.

Rev. **53,** 173, 1938. The methods of solution obtained there are, however, too difficult for further calculation. We shall therefore content ourselves with a method of approximation which *E. Fermi,* Ric. Scient. II, 1936. *Fermi* remarks that the diffusion of neutrons in the problem represented by (10) proceeds similarly to the diffusion of neutrons which can move only on a line, but in the latter case (which in a sense represents the projection of the three-dimensional motion on the z-axis) one must reduce the free path lengths by the factor. So instead of the different angles (one distinguishes only $N(v, z, +1) = N+$ and $N(v, z, -1) = N_-$ and replaces the integral equation (10) <by

$$\text{[illegible]} \qquad (23)$$

where $yf = V3^{-} y;$, Yu= $V3^{-}$Yu is set. We further use the abbreviations

$$\text{[illegible]} \qquad (24)$$

Then the two simultaneous differential equations are

$$\begin{aligned} &\frac{\partial N_+}{\partial z} + \left(A + \frac{B}{2}\right) N_+ - \frac{B}{2} N_- = 0 \ , \\ &-\frac{\partial N_-}{\partial z} + \left(A + \frac{B}{2}\right) N_- - \frac{B}{2} N_+ = 0 \end{aligned} \qquad (25)$$

to Jasen. The slogan is

$N+ = C+ e^{az}$, $N_- = C_- e^{az}$, where

$$\begin{vmatrix} a + A + \frac{B}{2} & \frac{B}{2} \\ \frac{B}{2} & -a + A + \frac{B}{2} \end{vmatrix} = 0, \quad \text{i.e.} \quad \alpha^2 = A(A+B) \quad \text{and} \qquad (26)$$

$$\begin{array}{ll} C+ & \underline{B/2} \\ C- & a+A+(B/2) \end{array} \qquad \begin{array}{l} \underline{-a+A+(B/2)} \\ B/2 \end{array} \qquad (27)$$

14 In area 1 the solution must be symmetric about the point $z = 0$. It is then

$$\underline{N+=C[\ e+\text{Ialz}+(\text{Ial}+A+\ )e-\text{Iatz}]'}$$

$$N_- = C\ [(1\ a\ | + A \pm\ )\ e+\text{IaIz} \quad -e\ |a\ |z] \qquad (28)$$

In area 2, however, fi.ir *zd/2* only the solution with negative *a is* needed, i.e.

N+ = C *a l* + *A* +)e-lalz,

(29)

N- -CBe -lalz .
2

Including the particular solution (9) we obtain the boundary condition at the position z = *d/2:*

No *La* ypi + c1 [!!J.... ela1ld/2+ *ii* *a1* l +*A1* +!!J.) e-la1ld/2]
La y\1) + *(yulu2)* 2 \ 2

=*No*+*C2* a2l+A2+ 2)e-lazld/2'

(30)

No *La yp>* + c1 [*ii* *a1* l +*A1* +!!J....) e la1ld/2+.!!J. e-

ia1ld/2]
LaYP>+(yulu2) \ 2 2

$$= N_0 + C_2 \frac{B_2}{2} e^{-|\alpha_2| d/2}$$

Finally, it follows:

C1 {[a1l+A1+1) a21 +*A2*+*2*) -*B*1- t2 }la11d12

+[1- a21 +*A2*+*2*)-2a1 l +*A1*+ 1)]e-la11d12}

= *No* 2 *Y 1)* (l a2l +*A2)* (31)
U LaYi +Yu

Thus the course of the neutron density in the interior of the plate is derived; fi.ir its total number is then given by (18)

$$\int_{-d/2}^{+d/2} dz(N_+ + N_-) = \frac{2C_1}{|\alpha_1|}(|\alpha_1| + A_1 + B_1)(e^{|\alpha_1| d/2} - e^{-|\alpha_1| d/2}) \quad (32)$$

For the rest of the calculation, we will use the variable *t* instead of *u*.
<through the relationship

$$u^2 = \gamma_U d \sqrt{3}\, t^2 \quad \mathbf{(33)}$$

Furthermore we set 15

$$d1_a > 1°0 = aV3 = a', \qquad d1_b > Plif3 = bV3 = b',$$

$$\frac{B2}{2(A2 + |azl)} = e. \tag{34}$$

Then

$$A_1 d = 1/t^2 + a' \qquad B_1 d = b' \quad \text{und} \quad |\alpha_1| d = \eta/t^2 \,, \quad \text{wobei} \tag{35}$$

$$\eta = \sqrt{(1 + a' t^2)[1 + (a' + b') t^2]} \,. \tag{36}$$

In these variables, the following is clearly indicated

$$\text{nabs} = v2,No \sqrt{\frac{Yud}{3}} \int dt \{ \frac{a'}{1 + a' t^2}$$

$$+ 2(a, + b, + (1 + 11)/t2)(1 - -e\,,,112)[17(1 + a, t2)\,\{a, + (b,/2) + (1 + 11)/t2$$

$$+ e(a' + (1 + 17)/t2) + [(b'/2) - e(a' + (1 + 17)/t2)]\, e\text{-}17112\}]\text{-}1\}. \tag{37}$$

In the limiting case $a' = b' = e = 0$ we get the same result as in GI. (20), if one sets $a = 0$ there. This is not exactly the case, since GI. (27) is only approximately valid because of the heating of the *Fermi* approximation.
is. It follows from GI. (20) for $a = 0$:

$$n_{abs} = 8 N_0 v_r^2 \sqrt{\gamma_U \pi d}/3 \,, \tag{38}$$

on the other hand from GI. (37) for $a' = b' = e = 0$:

$$\text{nabs} = 4 NovV \sqrt{Yu\ 7r\ d/V3}\,.$$

Thus, the *Fermi* method yields a value that is about 10% too high. To the
The accuracy of the *Fermi* method can be judged well from this result. In order to
obtain the correct result in the limiting case $a' = b' = e = 0$, reduce
we increase the second member of the right-hand side of (37) by the amount in question
and obtain the approximate final formula:

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$$nabs = 2 Nov, \int_{-\infty}^{+\infty} \frac{dt}{1 + a' t^2} \left(a' \right.$$

$$+ 4(a' + b' + (1 + 17)It2)(1 - dIIID/3[17\{a' + (b'12) + (1 + 17/t2)$$

$$+ e(a' + (1 + 17)/t2) + [(b'12) - e(a' + (1 + 17)/t2)]e\text{-}11112\}l\text{-}1\,.\} \tag{39}$$

The calculation of *w is* then done as in GI. (20) to (22). We begin the discussion of the physical meaning of formulae (20) and (39) with the

Case 1: $b = b' = 0$. The equation for η_{abs} contains here two members. The first gives the absorption, which is caused by the fact that/3 neutrons in the plate <are slowed down by a collision with, say, an O atom to such an extent that they are subsequently trapped in a U atom. This term therefore agrees with the expression for absorption in homogeneous solutions: [Section] 11.2a, GI. (24). The second term in (20) or (39) represents the absorption of neutrons entering the plate from region 2 with energy close to the resonance energy. This element decreases with increasing values of *a',* since the scattering of neutrons in the plate allows a part of the neutrons to leave the plate again; thus the increase of *a'* naturally increases the "albedo", i.e. the absorption is reduced, despite the greater distance that some neutrons have to travel in the plate. In the case of very thin plates, the latter effect outweighs the former, i.e. the absorption decreases with increasing scattering, but in the case of thicker plates, the first element outweighs the former, and the absorption increases with increasing scattering.

In the case of scattering without deceleration (case 2: $q = 0$, b 0), only the second effect is naturally gema/3, the number of absorbed neutrons decreases decreases with increasing scattering. However, it would be incorrect to conclude from this result that the presence of scattering substances in the plate is favorable for practical use in the uranium machine. This is because this scattering generally has a negative effect on the absorption of *thermal* neutrons.
I, as it is for the resonance neutrons; what can be achieved by a 17
The variation of the first term in the second term should be compensated by the corresponding changes in the thermal region. The variation of the second member should therefore compensate itself against the corresponding changes in the thermal region, and it is important to keep the first member in the same order.
(20) and (39) as small as possible. It is therefore expedient, instead of U308 always use metallic uranium in the plates.

The numerical evaluation of the formulas (20) and (39) shows that/3 for U3Os plates of thickness up to about 3 cm the influence/3 of the braking and scattering in the plate is small. For such plates, therefore, the formulas of I, Gin. (15) to (17) and (60) to (62) remain valid, if the expression - $A.Ilg$ J <lurch { i Yi [1 +((1 - $aJI\ aJ$ lg(1 - ai)]}-1. For this reason, one can also conclude that for the cylinders of the *Anderson-Fermi-Szilard* experiment, whose diameter was *5* cm, the formulas of Part I can be used, taking into account that the cylinders of the *Anderson-Fermi-Szilard* experiment had a diameter of *5* cm.
riicksichtigung of the mentioned change in good approximation can be counted. Here, too, the scattering and braking in the uranium layers should play only a minor role. For this most important case, in which scattering and braking in the uranium layer do not play a significant role, the formula for w is written down again explicitly. From the above considerations in Part I, in agreement with [Section] 11.2b, GI. (38), it follows that/3 is given by the number of absorbed neutrons <lby

$$\eta_{abs.} = L\ Nor\ v;\ \underline{VYvr}_{7r} f\ df\ VX - \tag{40}$$

Here the integral is to extend fiber the surface of all uranium tiicke;
Vx means the mean value of the square root of the values obtained from a rectilinear

8 neutron passing through uranium. In this averaging it is to be taken into account that the incident neutrons are distributed according to the cosine law. (Sieve Part I): For the flat plate $VX = 4$ *v1dl3.* For an infinitely long cylinder of diameter dist for a point on the cylindrical shell $x = 0.95$ *Vet.* The number of absorbed neutrons mull now be by the number of emitted ones. Let the total volume of all U-beads (or U3Os-beads) be V_1, the area 2 available for neutrons be V_2. Then, finally, the probability of capture of neutrons

$$w = \sum_r \sqrt{\gamma_{U,r}\pi} \int df \sqrt{x}$$

$$- \{Vi\, L\, YP> [1+((1+ aj}la;) \lg(1-a;)] + ViL\, Y\2> [1+((1-a;)/a;) \lg(1-a;)]\} - 1- \quad (41)$$

This formula will be used later for the discussion of the *Anderson-Fermi-Szilard* ex periment.

If the scattering and braking m the uranium layers are not neglected The results for the flat plates are the only ones available for the time being: Eqs. (22) and (39). In the case $b = 0$, i.e. no scattering by heavy atoms, the result for small values of *a,* i.e. of *d La yp>,* can be calculated according to powers of *ya can* be developed. One then obtains a formula that is clearer than Eq. (20):

$$\text{nabs.} = no,v;f\, VYu.dn\{1-3\, Vna112/4+a-Vna312/4+a2/10- + \ldots\} . \quad (42)$$

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11.3 Evaluation of the results of Joliot, Halban, Kowarski and Perrin. Comparison with other experiments

Some time after the completion of Part I, a work by *Joliot, Ha/ban, Kowarski,* and *Perrin came* to the attention of the author, the results of which can be compared with the conclusions and assumptions in Part I. The work of *Joliot, Ha/ban, Kowarski, and Perrin is a very interesting* one.

Firstly, this paper gives the probability *w* of resonance capture at different concentrations of uranium in water. According to [Section] 1.2a, *w* must increase with the square root of the uranium content. This is confirmed by the measurements:

N_H/N_U	= 140	65	30	
w	= **0,11 ±0,02**	0,14±0,02	0,02±0,02	(1)
$w\sqrt{N_H/N_U}$ =	**1,3 ± 0,2**	1,13 ±0,16	1,10±0,1	

As an average we can set

$$w\sqrt{N_H/N_U} = 1{,}15 \ . \quad (2)$$

The comparison with formula (22) of part I shows that the undefined

Gro/3e has the value 2.13 ± 0.2, which is at the upper limit of the values discussed in I (1/25 κ 5 2). This means that/3 the lowest resonance point at 25 eV contributes only a small part to the total absorption. For GI. (25) in [Section] 11.2a gives the contribution of the lowest resonance point (let us now see $a_8 = 14.8 \cdot 10\text{-}24$ and $a_0 = 3 \cdot 10\text{-}24$ [cm2] set):

$$w, = \frac{r, Va,}{E,} \cdot \frac{VaH+½ao}{aH+½\text{-}0.114ao} = 0{,}106 \cdot VNtt \qquad (3)$$

Thus, using the values $r, = 0.12$ eV, $a, = 2700 \cdot 10\text{-}24$ cm2 , the lowest resonance line contributes only about 100Jo to the total resonance absorption. This somewhat implausible result indicates that/3 the values for $r,$ and $a,$ at the lowest resonance line still are not correctly determined. One can 20 but conversely use the *Halban-Joliot* work to determine the magnitude of the reso nance capture. From (3) and (2) it follows that

$$\frac{r,}{Va,} = \frac{1024\text{-}27}{E,} \pm 3 \text{ [cm]} \qquad (4)$$

All the following calculations will be carried out with this numerical value. *Ha/ban, Joliot* etc. also investigate mixtures of U_{308} with small amounts of HzO in concentration ratios of 1:3, 1:2 and 1:1. In the calculation of $w,$ a certain uncertainty arises here for the highest U concentrations, since the magnitude $/J;$ for oxygen: $/J; = 0.12$ can perhaps no longer be regarded as large/3 against the line width. For the lowest resonance line is $/J;$ however, still large/3 against the line width; we therefore first assume this for all lines and set $e; = 1$; the oxygen atoms then already contribute significantly to the increase of $w.$ One obtains <by substituting in [Sect.] 11.2a, GI. (28) by using (4):

NH/Nu=	3	2	1	0
w =	0,50	0,585	0,73	0,99

(5)

One could now use these data to calculate the number of fission neutrons X (cf. [Sect.] 1.1) emitted per captured thermal neutron in uranium from the experimental results on neutron absorption in the substances mentioned. It turns out, however, that at high concentrations this number depends so sensitively on w that it is more expedient to assume as good a value of X as possible and to calculate w from the experimental results.

to determine the results of the experiments. The experimental results have already been discussed in detail by *Ha/ban, Joliot,* etc., so it is sufficient to give the result here: For $X = 2.0$ one obtains

	3H	2H	1H	OH
$w_{exp.}$ =	0.50	0,585	0,66	(-1)
$w_{theor.}$ =	0.50	0,585	0,73	0,99

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The upper agreement at 3 H is <l made by the number $X = 2.0$. At 1 H (i.e. 1/2 H20 per U) the upper agreement is not as good, which is probably
is due to the fact that the line widths here are no longer all small compared to the size $P;$ for oxygen. (For the deepest resonance line, if the specifications I',=0,12eV, a=2700- 10-24[cm2] are correct, the line width in the variable *u has* the value 0,05, while $P; = 0{,}12$. It is quite possible that the widths
of other resonance lines are larger than 0.1 and this has led to a reduction van *w*). So we consider

$$X=2{,}0 \qquad (6)$$

as the most probable value according to the measurements van *Ha/ban* etc. In Part I, X <lby GI. (1) was related to w and κ (Eq. (19)). If one sets, as is the case with GI. (2) of this section, $\kappa = 2.13$, it follows from Part I, GI. (1) and (19): $X = 1{,}86$ in sufficient agreement with
(6). However, the fundamentals of the van win Part I calculation still require improvement before the final comparison of the *Fermi* and *Joliot Halban* measurements can be made.

In Part I, the density of U308 used by *Anderson, Fermi* and *Szilard in* their experiment was wrongly given as 8, whereas in reality it was only 3.3. Moreover, for the calculation of the exact formula ([Section] III.2b, GI. (41)) (braking and scattering in the uranium can probably be ignored here). From [Section] 11.3, GI. (4) we find that at a density of U308 of 3,3

$$\frac{I:}{r} = 0.0578 \text{ n2}$$

22 Furthermore, in U308: $Yo = 0.059$ n4 in water YH = 0.87 and $Yo = 0.10$. The characteristic factor $1 + (1- a)/a \lg(1 - a)$ has the value 0.114 for O. The total surface area of all cylinders in *Fermi's* experiment was n (5 - 60 + 2 - 6.25) - 52 = n - 16250 cm2, their total volume $V1 = n$ - 6.25 - 60 - 52 = n - 19500 cm3.
Finally, according to [Section] 11.2b, GI. (41):

$$W = \frac{n - 3535}{n - 131 + 0.88 \cdot V2}$$

A certain uncertainty now arises in the calculation of $V2$. The neutron density is certainly too low towards the edge of the vessel. The volume "available" to the neutrons must be considered to be at least that volume which lies between the $U_3 0_8$ cylinders and directly at their edge.
According to the drawing in the paper, this

amounts to n(322 - 70-19500) = n - 24200

cm3 -

However, it could also have a radius that is about 5 cm larger, and you would then have to cut it to about

n(322 - 70-19500) = n - 52100 cm3

are used. The corresponding values for w are 0.165 and 0.077. One will probably have to conclude that w lies between the two values. If the neutron density is much greater inside the apparatus than outside, as can be inferred from the experiments of *Ha/ban* etc., then w is probably close to the upper limit. If one now uses the GI. (1) of Part I: $x = 3.2/(1 - w) - 2$, then one obtains from the *Anderson-Fermi-Szilard* experiment:

1,84.::::X;.:;1,47.

In this case, X should be close to the upper limit. The *Fermi* value for X is therefore 10 - 200/o lower than the *half-range* value.

Furthermore, recently a paper by *Bradt* (Helv. Physica Acta **XII,** 553, 23 1939) has been published, which also intends a measurement of X. Since in the calculation of the experimental results the capture of a neutron in U ! 8 was completely neglected, the values of X given by *Bradt* cannot be used. The experimental results must therefore be re-evaluated: *Bradt* uses a neutron source in a large water container, which is surrounded once by a U3O8, another time by an equally scattering PbO layer. He measures the number N of thermal neutrons in the water and finds that this total number is 5.6±0.70Jo higher in the experiment with $U_3 O_8$ than in that with PbO:

$$\frac{\Delta N}{N} = 0{,}056$$

The neutron density in the vicinity of the U_3 Os layer was 5.63 - 10-5 N [a], and it is assumed that the density inside the layer had this same value everywhere. The number of U atoms in the layer was 4.24 - 1024- The number of fission neutrons produced in uranium per sec is thus (the capture cross section for U at thermal velocity is 3.4 - 10-24cm2)

$$n = 5.63 \cdot 10^{-5} \cdot N \cdot 4.44 \cdot 10^{24} \cdot X \cdot 3.4 \cdot 10^{-24}$$
$$= \mathbf{8.5.}\ 10^{-4} \cdot v_{1h.} N \cdot X .$$

The number of neutrons absorbed per sec in water is

$$N \cdot v_{ih} \cdot uH \cdot NH = \Delta N \cdot v_{1h} \cdot uH \cdot 6.7 \cdot 10^{22} \cdot$$

If the capture of neutrons in resonance levels of U is completely abated, and if one further assumes that PbO does not absorb at all, then by difference formation one obtains

$$8{,}5 \cdot 10^{-4} \cdot v_{th.} N(X-1) = \Delta N \cdot v_{th.} u_{H.} 6.7 \cdot 10^{22},$$

$$X - 1 = \frac{\Delta N}{N} uw 7{,}9 - 10^{25}.$$

Before inserting the experimental values of $\Delta N/N$, one can add a

Apply correction ftir the fission processes caused by fast neutrons in U. The corrected value of *LJN/N* is:

LJNIN= 0.051 ±0.007 .

The value of *X* now depends mainly on $a_{,H}$. In part I $a_{,H}$ = 0,3 - 10-24 cm2 was assumed. This value is somewhat lower than that measured by *Fermi* and *Amaldi* (0.31 - 10-24cm2). Recently, however
Frisch, Ha/ban and *Koch* found an even considerably lower value: 0.22 - 10-24 cm2 (after conversion with the value of v_1 h used here). If one considers - which is probably somewhat arbitrary - the values 0.3 and 0.22 as upper and lower limits, respectively, one finds:

1.89 :$ *X* :$ 2,21 .

By taking into account the resonance absorption, which has no great influence in this experiment, these values would be approximately increased; the consideration of a possible absorption of PbO would lower the values. Since nothing is known about the absorption of PbO, no great weight can be attached to the determination of *X* by *Bradt.*

Finally, *Zinn* and *Szilard* have undertaken a determination of *X*, which they themselves, however, describe as premature and imprecise. They find

X-1,4

If one summarizes all previous experimental results, one will find about

X= 1,9±0,2

to be applied. The probable error ± 0.2 of this result is unfortunately still so large that it is of decisive importance for the possibility of the uranium machine.

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11.4 Improvement of previous results based on new experimental data

Provided that

$$X=1,9 \quad \text{and} \quad \underline{Er..Va.\cdot 1024lE.=:=2.7cm} \qquad (1)$$

the most important oats of the uranium machine are to be recalculated. First, a solution of uranium in which the *isotope* 235 *is enriched* by a factor of *f is* to be studied in water. If the effective cross sections for capture and fission in ordinary uranium are 1.4 - 10-24cm2 and 2 - 10-24cm2 respectively, then the uranium machine is to be recalculated.
they are in enriched uranium 1,4 - 1-0 24 and *2f* - 1-0 24- For each
fission process3, on average (1.9 - 3.4)/2 = 3.23 neutrons are produced. Denoted

again the ratio of the atomic numbers of U and H is given by *a(a = Nu/NH), then* by analogy with [Part] I, Eq. (22), taking into account [Part] 11.3, Eq. (2):

$$v = -vNH \cdot 10\text{-}24(0{,}3+(1{,}4+2j)a\text{-}2f \cdot 3{,}23a(1\text{-}1{,}15\ Va)) = -vNw\ 10\text{-}24(0{,}3\text{-}(4{,}46/\text{-}1{,}4)a+7{,}43/\text{-}a312) \quad (2)$$

The maximum value of v is

$$a_{max} = \frac{(23\,4{,}46/\text{-}\ 1{,}4)^2}{7{,}43/} \quad (3)$$

and amounts to

$$v_{max} = -v N_H \cdot 10^{-24}(0{,}3 - 4/27 \cdot (4{,}46f - 1{,}4)^3/(7{,}43f)^2) \quad (4)$$

The result is therefore
for

f	1	1,5	2	2,5	3
amax=	0,0775	0,100	0,114	0,123	0,129
Vmax =	- 0.223	- 0,124	-0,014	+0,098	+0,212

Neutron multiplication would therefore only occur if the isotope 235 were enriched by a factor of 2.5 or more. The fact that this result is much more uncertain than in Part I is due to the unexpectedly high value of κ, which follows from *Halban's* calculations, and also to the error noted by *Bothe*.
ler in the calculations for Table 1 in Part I. If one uses the *Frisch-Halban-Koch* value *a,=* 0.22 - 10-24 in (2) for the capture of neutrons in hydrogen, an enrichment by a factor of $f = 2$ would already suffice for the neutron increase. If instead of the solution an arrangement with individual layers is used, an even smaller value off is probably sufficient.

Secondly, a solution of uranium in heavy water is to be investigated. Equation (27) in [Section] 11.2a gives here $w = 3.83$ *Va- (a= Nu!No).* The cross section *a,* for the trapping in heavy water (per D-atom
we want to leave undefined for the time being. Then

$$v_{D_2O} = -v \cdot N_D \cdot 10^{-24}(\sigma_r \cdot 10^{24} - 0{,}0069) \ .$$

For pure heavy water, according to the assumptions of Tei! I *a,= 0* .0045
• 10-24 cm2; for this v_{020} would become positive, and neutron multiplication would occur. If, however, the heavy water contains 1OJo of ordinary water, then, according to the assumptions of Part I, v_{020} would be negative; the solution would then no longer be suitable for the production of the uranium machine. However, the conditions will certainly be considerably more favourable if individual uranium plates in D_2 0 are used instead of the solution. If the experimental values assumed here are correct, heavy water will in any case be suitable for the manufacture of the uranium machine.

For a mixture of uranium with helium under high pressure, according to (27) in [Section] 11.2a: $w = 5.76$, if the cross-section of action for the elastic scattering of neutrons of 25 eV by helium nuclei is assumed to be 3 - 10-24cm2.

n becomes. Since neutrons cannot be trapped in helium, I so

VttE = -*v* - Ntte - 10-24(-3.06a+37.2a312)

The maximum value is

$$\nu_{He} = +\nu \cdot N_{He} \cdot 0{,}0031 \cdot 10^{-24}$$

Thus neutron multiplication occurs. The corresponding diffusion ring is

/He= 31.6/dtte(cm) ,

where dtte represents the density of helium under high pressure. This length is still quite large at the technically representable densities, so that even at a pressure of 1000 atm vessels of several meters diameter were necessary for the production of the uranium machine.

Finally, we consider a mixture of U and pure Kahle. Here $w = 16{,}1$ \la, thus

Ve= - *v* -*Ne*- 10-24 *(a,*- 1024 - 3.06 *a*+ 104 *a312)* ,

and the maximum value

vc'.!ax = - *v* -*Ne*- 10-24 *(a,*- 1024- 0,00034)

If *a, as* assumed in Part I, is about 0.003 - l-024 cm2, then pure Kahle is not sufficient for the production of the uranium machine. Here too, however, the conditions can be more favourable if uranium plates are used.

The arrangements in which uranium layers alternate with layers of other substances shall not be further treated in this report, since *v. Weiz* has carried out detailed calculations on this subject and will probably communicate *them in the* near future.