The possibility of technical energy production from uranium fission

From **W. Heisenberg** [1]

Summary: The considerations compiled here, which are intended to provide a more precise answer to the question raised in *Fliigge's* article, are based on the following assumptions: It is assumed that the conclusions drawn by *Bohr* and *Wheeler* (Phys. Rev. **56,** 426, 1939) are essentially correct, that in particular U i5 is the isotope responsible for fission by thermal neutrons. Furthermore, it is assumed that the figures given in the *Anderson-Fermi-Szi/ard* paper (Phys. Rev. **56,** 284, 1939) are correct. Under these conditions it is investigated whether it is possible to produce mixtures of substances which, when bombarded with neutrons, emit more neutrons than they absorb, and in what way machines for the production of energy which work with such mixtures act.

1) The Anderson-Fermi-Szilard Experiment

Anderson, Fermi and *Szilard* have found that the number of neutrons absorbed in a large water vessel increases by 10% when a certain quantity of uranium is introduced into the water. It was found that the uranium as a whole absorbs about half of the neutrons emitted by the source as thermal neutrons in this arrangement, so that if *n* neutrons per sec are emitted from the source situated in the centre of the water container, then without uranium *n are* also absorbed in the water; but with uranium, according to the measurement, 1.1 - *n are* absorbed in the water and 0.5 - *n in the uranium,* so that 0.6 - *n* neutrons per sec must be emitted again by the uranium. This means that from one thermal neutron absorbed by uranium (after taking into account fission, capture, resonance capture of the fission neutrons, etc.) about 1.2 thermal neutrons are produced.

2 In order to calculate from this the number of neutrons that are initially produced per absorbed thermal neutron, it is necessary to determine how probable it is that a fast neutron is captured in the U i8 , for example by resonance processes, *before* it reaches the thermal energies. (The trapping in water can be completely neglected here.) We call this probability *w.* Then one can improve the just discussed calculation as follows: the number per sec of neutrons emitted by the source is *n,* of which *n* (1- w) reach the thermal region; in water 1.1 - *n* thermal neutrons are absorbed, in uranium 0.5 - *n.* In the course of this process, the neutrons are split into *n* (1- w). Thereby are produced by fission

[1] Undated, but according to KWI-List dated 6 December 1939. (Editor)

n. *X* neutrons, of which 0.5 - *n* (1- w) - *X* enter the thermal region. One thus obtains the equation:

$$(n+0.5-X-n)(1-w)=1.1-n+0.5-n, \text{ from which}$$
$$1 + 0{,}5 - X = 1{,}6/(1-w), \quad X = 3.2/(1-w)-2 . \tag{1}$$

The size *w* is determined in the following section.

2) The capture of neutrons in the U^{8} at resonance sites

Uranium atoms can be mixed in two ways with other substances serving for neutron scattering: uranium can either be dissolved in some compound in another substance *(Ha/ban, Joliot, Kowarski;* Na ture **143,** 680, 1939), or uranium is concentrated in large amounts and surrounded by the other substance *(Anderson, Fermi* and *Szilard)*.

a) In the former case, the probability *w* that a neutron is captured in a resonance site during its deceleration depends on the concentration ratio. We assume that there are *Nu* uranium atoms and *N* atoms of the deceleration determining substance in the ccm. Then the probability of trapping at a collision is

$$\frac{Nuau}{Na+Nuau} \tag{2}$$

where *a is* the elastic effective cross-section of the scattering substance, *au is* the capture effective cross-section van U. 3
cross section van U. Furthermore, according to *Breit* and *Wigner,* in the vicinity of the resonance line *Ek*

$$\text{ow- - }_{ac} \quad \frac{(\Gamma')2}{T_2 k} \qquad (E-Ek)^2 + (-\tfrac{1}{2}-I'k)^2 \tag{3}$$

The average number of collisions between the energy *E1* and *E2* is

$$\int_{E1}^{E_2} \frac{dE}{E} \qquad \frac{1}{(-\lg o)'} \tag{4}$$

if **O** is the fraction to which, on average, the energy of a neutron is reduced in an elastic collision. *(o* = 1/2 for the collision with protons.) Generally, **O** = 1-[2M1M2/(M1+M2)2], where M1 or *M2* is the mass of the colliding particle. Finally, it follows for w if

$$\frac{\sigma N}{\sigma_k N_U} = \eta_k \tag{5}$$

is set,

$$w = \frac{1}{-\lg \delta} \int \frac{dE}{E} \sum_k \frac{1}{\eta_k [(E-E_k)^2/(\frac{1}{2}\Gamma_k)^2 + 1] + 1}$$

$$= -\frac{1}{\lg \delta} \sum_k \frac{\pi \Gamma_k}{2E_k^2} [\eta_k (1+\eta_k)]^{-1/2} . \qquad (6)$$

According to *Bohr* and *Wheeler*, the deepest resonance point in uranium is the following

$$\Gamma_1 = 0{,}12 \text{ eV} \; ; \quad E_1 = 25 \text{ eV} \; ; \quad \sigma_1 = 2700 \cdot 10^{-24} \text{ cm}^2 \qquad (7)$$

So the contribution herrtihrende of this becomes *w:*

$$w_1 = \frac{0{,}0075}{-\lg c, \sqrt{l\,(1+17i)}} \qquad (8)$$

In most practical cases, *1711,* so 0.0075

4 $$w_1 = \frac{}{-\lg c,} \qquad (9)$$

From the fact that the vibrational resonance points contribute about 2.5 times as much to the thermal cross-section as those below, one can assume that *w can* also be about 3 to 4 times as large as *w1 - the* roof is dependent on this quite uncertain estimate. We therefore set

$$w = \frac{0{,}026}{-\lg c, \sqrt{1h}} K, \qquad (10)$$

where *K is* an unknown numerical factor that is likely to be between 1/2 and 2.

b) If uranium or a uranium compound fills large regions of space unmixed, and if these regions of space are surrounded by the substance intended for deceleration, then only neutrons can be captured which have received an energy in the immediate vicinity of the uranium surface in the order of the resonance energy. (We assume here that in the uranium itself no noticeable slowing down of the neutrons can occur in the energy region in question). This is because the neutrons which, after reaching the surface of the uranium, collide again with another atom, migrate out of the resonance region again due to the stol3. Let *l* be the mean free path length of the neutrons in the decelerating substance (in the energy region in question). Then the probability that a neutron at a distance *x from* the upper surface of the uranium will reach it (its velocity soil with the flat normal include the angle *r'J* (cos *r'J* = + O) by

$$P(x) = e^{-x/A(} \tag{11}$$

is given. The average probability that a neutron at distance x reaches the surface, is therefore

$$\underline{P(x)} = \frac{1}{2}\int_0 P(x)\, d(= \frac{1}{2}\int_0 d(e^{-xi)(} . \tag{12}$$

If the mean neutron density is denoted by *(2,* then for every 5 surface units

$$\int \varrho\, \overline{\beta(x)}\, dx = \frac{\varrho}{2}\int_0^\infty dx \int_0^1 d\zeta\, e^{-x/\lambda\zeta} = \varrho\frac{\lambda}{4} \tag{13}$$

Their directional distribution follows the cosine law *(dw* = cos rJ *dQ)* if the neutron density in the strip of width *Al* 4 does not vary appreciably. We can calculate as if all neutrons in a layer of width A./4 around the primordial corpuscle reach it, while all other neutrons suffer a collision with another atom before.

We now consider a neutron hitting the surface. Let x be the average distance it had to travel when crossing the uranium block, *and let E be* its energy. *The* probability that it is absorbed in the uranium is then given by

$$1\text{-}e\text{-}xauNu, \qquad \left(Au = \frac{1}{auNu}\right),$$

given. This probability depends very much on the energy near the resonance line.

Let's form

$$\Delta E = \int dE\,(1 - e^{-x\sigma_U N_U}) , \tag{14}$$

we get the "equivalence width" of the absorption line. We can then calculate as if all neutrons whose energy is within .1£ are absorbed in the uranium block, while the ions pass freely. For the absorption line *Ek*

$$\Delta E = \int dE\,(1 - \exp\{-x\sigma_k N_U (1/2\Gamma_k)^2/[(E-E_k)^2 - (1/2\Gamma_k)^2]\})$$

$$= \frac{1}{2}\Gamma_k \int_{-\infty}^{+\infty} d\xi\,(1 - e^{-(a/1+\xi^2)}) ,$$

$$\left(\xi = \frac{E-E_k}{\frac{1}{2}\Gamma_k}\right) ,$$

where *xakNu* = a is set.

6Fi .ir $a \blacktriangleright 1$, which is true in all practical cases, will be

$$L1E = \frac{J}{2} rk \int_{-oo}^{} d<;(1 - e\text{-}a/^{2}) = rk4 \Big|_{Or/} <1 - e^{-a''^{2}})$$

$$= Tk \int_{o}^{00} dry 2ae\text{-}0^{2} = TkVna = rkVnxakNu. \qquad (15)$$

In the experiment of *Anderson, Fermi* and *Szilard,* uranium oxide ($U308$) is bent in cylindrical rods of 5 cm diameter. One can therefore set $x = 5$ cm. Therefore $a = 230$ and

$$L1El = 0.12 \sqrt{n} \cdot 5 \cdot 2700 \cdot 1.7 \cdot 10\text{-}2 = 3.2\text{eV} \qquad (16)$$

If we denote the total number of neutrons in this energy region by *ne,* those of them which are located in the surface layer of thickness *Al4* around the uranium block by *neo*, the contribution of the deepest resonance site becomes w:

$$\frac{L1El}{Egg(\text{-}\lg o)} \frac{neo}{n_e} = \frac{0{,}13}{-\lg \delta} \frac{neo}{n_e} \qquad (17)$$

For *w* itself one obtains as estimation 0,45

$$w = \frac{neo}{\text{-}\lg o \; ne} K, \qquad (18)$$

where *K* should be between 1/2 and 2 as in (10). In the experiment of *Ander son, Fermi* and *Szilard,* the ratio *ne0I ne,* as can be seen from the geometry of the arrangement, is approximately between 1/20 and 1/6; thus one obtains

$$w'''' 0.008 K. \qquad (19)$$

The decisive factor for all further discussions is the

$$X = \frac{22}{1\text{-}w}^{2},$$

71 which indicates the average number of neutrons released during the fission process/3 per thermal neutron captured in ordinary uranium, is therefore probably between 1.34 and 1.81:

$$1{,}34 \leq X \leq 1{,}81 . \qquad (20)$$

A more precise indication of *X* cannot be obtained from the above-mentioned experiment.

If one compares the two methods for slowing down the neutrons, one sees that/3 in the case of the *uranium/dsungen* the number of neutrons trapped away in resonance sites increases with the square root of the uranium concentration an-

increases, i.e., for a given amount of decelerating substance, it increases with the square root of the amount of uranium. In an arrangement where larger uranium tiicks are surrounded by the scattering substance, the number of neutrons trapped away increases with the surface area and the square root of the thickness of the uranium layer, i.e. with the 5/6th power of the amount of uranium for a given shape.

The second method has the further disadvantage compared with the first that under certain circumstances not the whole quantity of uranium can be used for the absorption of the thermal neutrons, since possibly only a few thermal neutrons reach the interior of the uranium block. On the other hand, in the arrangement of *Anderson, Fermi* and *Szilard,* the magnitude *w is* only about 2/3 of that which would be expected if the same amount of uranium were dissolved in water. If *Anderson, Fermi* and *Szilard* had arranged the uranium oxide in a compact cylindrical ring of the radii 16 cm, and 24 cm, they had reduced *w with* it again on the half. By clever geometrical arrangement, therefore, *w can be* considerably reduced by the second method, to which *Harteck* has drawn attention.

3) Solutions of uranium in water and heavy water

a) We first consider a solution of uranium in water containing Nu uranium atoms and NH H atoms in cc; the effect of the 0- 8 atoms and possibly other atoms in the uranium compound shall be neglected. For the relative time change of the neutron number *n, one* obtains, if the cross-section of action for trapping a thermal neutron in the H nucleus 0.3 - 10-24cm2, in the uranium nucleus 3.4 - 10-24cm2, the velocity ther of mixed neutrons *v* and the ratio Nu/NH= *a* is set:

$$v=-\frac{1}{n}\frac{dn}{dt}=-vNH'\,10^{-24}\,(0.3+3.4a[1-X(1-w)]). \qquad (21)$$

The cross section for elastic collisions with the proton in the area of the resonance points is 13 - 10-24cm2, so according to (5)

$$T/=t\,\frac{13}{2700a}$$

and according to (10) (a= 1/2)

$$w = 0.54\sqrt{a_K}, \quad \text{thus}$$

$$v = -\,vNH'\,10\text{-}24(0.3 + 3.4a[1-X(1-0.54\,J;'a\,K)]) \qquad (22)$$

If we substitute *X* from the *Fermi* experiment, we get

$$v= -vNH'\,10\text{-}24\,fo,3\text{-}3,4a0,\frac{2+0,24K}{1\text{-}0,08K}+1,835a312K\,\frac{1\text{-}2+0,16K)}{1\text{-}0,08K}- \qquad (23)$$

The maximum of the bracket as a function of *a* is at *am,* which is given by

$$1+1.2K = t/a:(4.86+0.65K)K \tag{24}$$

is given. It follows:

$$v_m = -vNwl0\text{-}24(03\text{-}0227 \quad \frac{(1+1,2K)^3}{(1 - 0.08\ K)(4.86 + 0.65\ K)^2\ K2}) \tag{25}$$

so the numerical values

K	0,5	1,0	1,5	2,0
$\frac{v_m}{v N_H \cdot 10^{-24}}$	-0,15	-0,21	-0,22	-0,23

9 In a uranium-water mixture, therefore, *neutron reduction* always occurs; the uranium-water mixture is *not* suitable for energy generation from the fission process.

b) The safest method for the realization of energy production consists in the enrichment of the isotope U§f responsible for the fission.
increase by the factor *f* simply leads to an increase of *X* by the factor *f.* Replacing *X* by *f X* in (22) and performing the same calculations as above, we obtain fi.ir

Table 1

f	*K*	*vmlv -NH-* 10-24
	0,5	-0,15
	1	-0,21
	1,5	-0,22
	2	-0,23
1,1	0,5	+0,043
	1	-0,14
	1,5	-0,18
	2	-0,19
1,2	0,5	+0,31
	1	-0,044
	1,5	-0,11
	2	-0,15
1,3	0,5	+0,62
	1	+0,37
	1,5	-0,065
	2	-0,11

It can be seen from the table that an enrichment of 30% would probably be sufficient to enable energy production with the help of the uranium-water mixture.

(v > 0). With an enrichment of 500/o one could be practically sure of success.

c) Since a considerable enrichment of the u ? in large quantities is probably connected with large costs, one must look for still other metho ds of the realization of the energy production. Here, first of all, the slowing down of neutrons by other substances than water presents itself. These other substances do not have to perform the slowing down much worse than **H20** and may only absorb much less. The following table gives a summary of the cross sections for trapping *(ar)* and for elastic scattering in the case of thermal neutrons (a1h), in the case of neutrons
of 25 V (a25) land for fast neutrons (asch) in the elements H, D, C, 0, 10 and U.
The values are partly derived from measurements, partly from theoretical estimations. Well determined values are underlined [bold]. Errors of 500/o or more are quite possible in the case of the tibratory values. The effect cross sections are given in the unit 1-0 ^{24} cm2.

Table 2

Substance		H	D	C	0	u
Capture cross section	*a,*	**0,3**	0,003	0,003	0,003	**3,4**
Scatter-cuts	{ a ,$_{h}$	**40**	7	4	3,3	10
	$_{a25}$	**13**	3	3,5	3	
	Gsch	2	2	2	2	6

The following table shows the mean free path lengths for elastic dispersion Aih, A_{25} , Asch and for capture Ar resulting from these cross sections, furthermore the "diffusion length" / = *VD! -* = *V11Hr* Aih *(D* = Ath is the diffusion constant) for H_2 0, D_2 0 (mixed with 10/o H_2 0), C (spec. weight 1.25) and U308 (spec. weight 8). Furthermore, the distance /' is estimated which an initially fast neutron (-3 MeV) has travelled on average in each direction until it arrives at thermal velocities. Since an exact theory of simultaneous diffusion and deceleration is not available, one has to rely on a very rough estimate of /'. (Length in [cm], Ar= 1/(Naeinr))

Table 3

Substance	H20	D20+ 1% H20	C	U308
A,	**5,0**	2000	5200	**17**
Ath	**0,3**	1,7	4	3,1
A2s	**1**	3,2	4,5	
Asch	5	5	8	5
l	**2,1**	34	83	4,2
/'	10	13	38	

11 The correction of these tables by more accurate experimental data would be an important prerequisite for a more reliable discussion of the possibilities of energy production.

The numbers in the table show that/3 primarily heavy water is considered for the slowing down of the neutrons. If again the ratio of the U-atoms to the D-atoms is denoted by *a,* then

$$W = 1.32\, VaK \tag{26}$$

and

$$v = -vNo - 10\text{-}24 fo.0075\text{-}3.4a \left(\frac{0.2+0.24K}{1\text{-}0{,}08K} + 4.48a312K \frac{1\text{-}2+0.16K}{1\text{-}0{,}08K} \right). \tag{27}$$

Fi.ir the gi.inst values of *a* and *v you* get:

Table 4

K	= 0,5		1,5	2
<Xin	= 0,106	0,Q38	0,0235	0,016
vm Iv N0 - 10-24 = 0.050		0,020	0,013	0,011

Fi.ihrt one again a characteristic length / by the equation /2 = *Div* where $D = v_{\text{Arh is}}$ the diffusion constant, then fi.ir

Table 5

K = 0,5		1,5	2
I = 13	21	26	28 cm

Table 4 shows that a mixture of heavy water and uranium would already be suitable for neutron multiplication, i.e. for energy production. This is of course only valid under the condition that the numbers given in table 2 are correct. Also, the rather large lengths / of Table 5 show that for this production of the uranium machine large quantities of heavy water are necessary, which again leads to considerable costs.

12 Perhaps the conditions in heavy water are even somewhat more favourable than those shown in Table 4, since in heavy water the process possible with fast neutrons still produces new neutrons.

nes are formed. The corresponding process/3 n p + n has at a proton energy of 5.1 MeV has an effective cross-section2 of about 1.4 - 10-26 cm2. The minimum energy at which it can occur is 3.25 **MeV.** Since the cross section, as a theoretical calculation shows, is approximately proportional to the square of the Oberschu/3energie

[2] W. H. Barkas and M. G. White: Phys. Rev. 56,288, 1939

it can already be about 2 - 10-25cm at 10MeV. So if the energy of the neutrons released during fission is on average very grail (10 MeV), an additional increase of *X* and thus of the neutron increase by about 100Jo can occur. If, however, as *Zinn* and *Szilard3* claim, most of the fission neutrons have less than 3.5 MeV, then the effect mentioned does not play a noticeable role.

d) Finally, the question of whether fission <by fast neutrons at U 8 and Th 0 can be used to produce the ketene reactions will be briefly discussed at this point. The cross sections for fission by 2.5 MeV neutrons are, according to *Ladenburg, Kanner, Barsha/1* and *Voorhis* (Phys. Rev. **56,168,** 1939), *5* - 10-25cm2and 1 - 10-25cm2and decrease rapidly with decreasing energy. The effective cross section for scattering, which will correspond approximately to the nuclear cross section, is therefore about 10 and 50 times gr6-13er. Since scattering is generally allowed to be inelastic and is therefore associated with energy loss, a neutron will usually have lost its energy.
I before it has the opportunity to cause the cleavage procel3. From a 13
So there can hardly be any question of a chain reaction here.

4) The diffusion equation and its implications

a) Simultaneously with the change of the total neutron number a spreading by diffusion takes place. This diffusion can be characterized
<lby the diffusion coefficient *D,* which is related to the mean free path length *I* and the velocity *v* by the formula

$$D = vA.!3 = \frac{v}{3Na} \qquad (28)$$

Here *a* is the cross section for elastic scattering. If there are several atomic species with the numbers *N;* per ccm, the following applies

$$1/D = 1/v \sum_i 3N;a; . \qquad (29)$$

A difficulty in the application of this equation lies in the fact that the *a;* depend on the velocity and that3 neutrons of quite different velocities occur in the uranium-water mixture. However, for the U - D2O mixtures of low U concentration discussed in [Section] 1b, it can be assumed that the neutrons spend by far the longest time in the thermal region. For these mixtures, one can therefore simply insert the values of *a* and *l in* the thermal region in (28).

For the calculation of energy production, it is also important to note that the diffusion constant depends on the temperature *T* . In the first approximation, *I* can be regarded as independent of the temperature, then *D* increases with Vran.

3 W. H. Zinn and L. Szilard: Phys. Rev. **56,** 619, 1939

For the neutron density ϱ in the substance under consideration, the following equation is valid

$$\frac{\partial \varrho}{\partial t} = D\,\Delta\varrho + \nu\varrho \quad (30)$$

14 We now consider a stationary spherically symmetric density distribution I in a sphere of matter of radius *R.* To determine the solution we need boundary conditions at the positioner= *R.* If we assume that the neutrons diffuse outside the sphere of matter into the air, it follows that/3 at the boundary $r = R$ (except for large quantities of the order of the free path length) the density must vanish. We therefore first set as boundary condition

$$Q = 0 \text{ ftir} \qquad r = R\,. \quad (31)$$

For the mixtures considered in [Sect.] 3b and c, v is positive and we set (cf. Table 5)

$$Div = {}^{12}\text{-} \quad (32)$$

/ is the characteristic length of the mixture, which corresponds to the diffusion length according to *Amaldi* and *Fermi.* For the mixture considered in equation (27), under the assumption $\kappa = 1$

$$No/Nu = 26, \qquad I = 21 \text{ cm} \quad (33)$$

The stationary solution *(oQlot* = 0) is now:

$$Q = (air)\ \text{sirr}\ (\tfrac{R-r}{\cdot}) \quad . \quad (34)$$

As long as $R < nI$, there is a positive density everywhere and a singularity at $r = 0$. This means that/3 such a density distribution can only be maintained by a neutron source in the center. Let us assume, for example, that/3 there is a neutron source of radius r_0 <i1 *I in* the center which delivers n neutrons per sec. Then mu/3 hold:

$$n = -D4\,nd\,o/or\,[(air)\ \sin((R - r)I/)],= {}_{0,,,}\,4\,na\,D\,\sin(RII)$$

$$\{2 = \frac{n}{4nrD}\,\frac{\sin((R-r)//)}{\sin(RI/)} \quad (35)$$

The number of neutrons escaping per sec at $r = R$ is then

$$n' = -D4nR28Qlor = n\ \frac{R}{/\sin(RI/)}$$

$$n'ln = \frac{(RI/)}{\sin(RI/)}$$

by n17t01

Syn c and corr ecti ons

(36)

So there is an increase of the neutron number, which for small values 15 of R (R 4l; /) is approximately <lurch

$$n'ln = 1 + 1/6(R//)^2 \tag{37}$$

is given. If **B.** approaches the value $n\,I$, then n' tends towards infinity. If $R > nI$, there is no stationary neutron distribution at all, where the density is positive everywhere, so there is first no neutron distribution at all, because of $oelot > 0$. according to (30) an unlimited multiplication of neutrons takes place. However, with such an unlimited neutron multiplication, since the neutron multiplication originates from the U-fission, a considerable temperature increase goes hand in hand. With it the diffusion coefficient increases and with it the Lange /. Thus, when the temperature is high enough, $R < nI$ again, so thatJ3 the unlimited N;utron propagation stops again. So there is always automatically a temperature, which is to be calculated from the equation $R = nI$. Since / asVT vari ized, you can

$$T = 300° \left(\frac{!!}{7r\,/300°} \right)^4 \tag{38}$$

set. This temperature remains independent of any energy dissipated to the outside. From this energy depends only the number of decaying U-atoms. From these considerations follows also, thatJ3 for a machine, where $R > n$ /300°, the neutron source in the center plays practically no more role and can therefore be omitted.

Eq. (38) ceases to hold when the temperature becomes so high thatJ3 the coefficients v are no longer temperature-independent. For the usual mixture of U§ 8 and U§i5 this occurs already at some eV neutron energy, since from this energy on the capture in U§i8 considerably outweighs the fission of U§i5. Thus, in mixtures with ordinary uranium, the temperature will not be able to rise above the temperature of the good explosives known so far. By enrichment of U§i5, however, the temperature would further increase. If it is possible to enrich U§i5 to such an extent thatJ3 temperatures can be achieved which correspond to a neutron energy of about 300 eV, which according to (38) include a radius of R '°" 10 nI, then the temperature of U§i5 would rise even without further enrichment. increase of the radius, the temperature would rise in one fell swoop to - 1012 degrees, i. e. the entire radiation energy of all available uranium atoms would be reduced to are released once. Because at 300 eV the effective cross-section of U§i5 for fission has decreased to about 5 - 10-24cm2 and for geometrical reasons it will not decrease further. Above 300 eV neutron energy therefore / does not increase further with temperature. This explosive transformation of the uranium atoms can only occur in almost pure U§i5, because already at low additions
The neutrons in the resonance point of U§ 8 are trapped away by the oscillations of U§ 8.

b) In the solutions of (30) discussed so far, it was assumed thatJ3 the neutrons escape from the sphere into air. Obviously the conditions for the neutron production become even more favorable, if the neutrons escape into a material.

in which they diffuse slowly and are absorbed very little, that is, a substance with a high "albedo". We first assume as an example that fi.ir the up3er material *D* becomes as large as in the interior, but that the absorption coefficient on3en is very small (so on3en soil $V_a <$ 0 but
$|val| V_i |$; *DI* - V_a= *1*). Then the solution

$$inside: (! = (alr)sin((r+b)ll) \; ; \qquad auj]en: (! = (a'/r)e^{-'11-} \tag{39}$$

e and *Dae/at* mi.issen at the boundary surface *r* =Rhave the same value on both sides. From this follows:

$$-1/R + (1/1)\,\mathrm{cotg}((R + b)/1) = -1/R-1/la \; , \tag{40}$$

17 that is, when $l a \blacktriangleright R$ is:

$$\mathrm{cotg}((R+b)/1) = 0 \; , \qquad b = (n/2)1-R \; . \tag{41}$$

Inside is then

$$(! = (air)\cos((R - r)/ 1) \tag{42}$$

Fi.ir the source in the center one gets

$$n = 4naDcos(R/l) \; , \tag{43}$$

fi.ir the neutrons leaving at $r = R$

$$n' = 4naD \tag{44}$$

Sync and corrections by n17t01

$$\frac{n'}{n} \quad \frac{1}{cos(Rll)} \tag{45}$$

The neutron multiplication thus starts faster than in the case treated fri.her, and already at R = (n/2)*1 the* spontaneous neutron generation begins. Such an effect can already be achieved <by water as a auf3er substance. We consider, to show this, the uranium-D2O mixture of [Sec.] 3 b, fi.ir which is set *1* = 21 cm, A.th= 1.7 cm. Fi.ir H2O is *1* = 2.1 cm, A.th= 0.3 cm (according to *Amaldi* and *Fermi).* Instead of (40) we get the equation

$$-1/R+(1/l)\mathrm{cotg}((R+b)/l) = DH_2\,olDo_2\,o\,(\cdot|\cdot|) \quad = -1 \quad (\cdot|\cdot|) \quad , \qquad R \quad la \quad 5{,}7 \quad R \quad la$$

(46)

$$cotg((R+b)/1) = -1{,}75+0{,}825(//R). \tag{47}$$

Spontaneous neutron generation starts at the radius ***R*** where from (47)

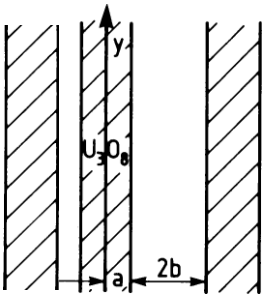
the value $b = 0$ follows; this is the case for $R = 2.51$. - Already for $R = 0.5\ l$, thus $R = 10$ cm, a neutron increase of about 130Jo could be expected.

On the whole, it can be calculated that in the case of the D2O-uranium mixture, a sphere of about 60 cm radius surrounded by water (containing about 1000 l of D2O and 1200 kg of uranium) already provides spontaneous energy generation; its static temperature would be about 800° Celsius.

Much more favorable, of course, would be uranium, in which the isotope 235 is enriched. For enrichment by 300Jo (for $\kappa = 1$) according to (22) and table 1 l - 2 cm. Thus, small spheres of 8 cm radius already gave spontaneous energy production. 18 However, the size is already limited downwards by the average range for fast neutrons and it is therefore questionable whether spheres with a radius of less than 20 cm were useful.

5) Arrangements in which the uranium is separated from the neutron decelerating substance

It has been pointed out by *Harteck* that <by suitable geome trical arrangement of the uranium tiicks in the decelerating substance a more favorable result can be obtained than in the solution. In order to be able to s u r v e y the very numerous possibilities offered by this, we shall first dispense with the absolute size of the apparatus, i. e. we shall first assume the apparatus to be arbitrarily large and ask how the strongest neutron multiplication can be obtained per unit volume. Under this assumption, the most favorable arrangement probably consists of a series of uranium (or U3O8) plates of equal thickness separated <by plates of decelerating substance (Fig. 1). Let the plates be infinite in the y- and z-directions.



[Fig. 1]

extended. In the x-direction the thickness of the uranium plates is 2a, that of the intermediate spaces 2b. Since the arrangement is periodically repeated in the x-direction and symmetrical in the $+x$- and -x-directions, the solution of the dif fusion equation *must* also *be* periodic and symmetrical. If we denote the magnitudes related to the uranium compound with the index U, it follows from

$$D\Delta\varrho + \nu\varrho = 0\ , \qquad \frac{\nu}{D} = \frac{1}{l^2}\ , \tag{48}$$

that the density distribution in the middle uranium layer is determined by

$$e = eocosh- \frac{x}{lu} \quad (49)$$

must be given. In the case of intermediate regions, which are filled with a neutron-slowing substance, it is to be noted that in | them the uranium (or any other neutron source). We call *n the* thermal neutrons produced per sec and unit volume and assume that *n* has the same value everywhere, which will then be fulfilled in a very good approximation if the neutrons originate from uranium. In the scattering substance, the diffusion equation then holds (v < 0!)

$$\frac{ae}{8t} = DLle+ ve+ n \quad (50)$$

The stationary L5solution, which also satisfies the symmetry conditions, is in the first interspace to the right of $x = 0$:

$$e = --\frac{n}{|v|} + e1\ cash\frac{x-a-b}{1} \quad (51)$$

We denote the Gr58s related to the scattering substance by the subscript s:

$$\frac{Ds}{Vs} \quad 1_s^2 \quad \{! = --\frac{n}{|V|} \quad \{\ ca\ sh\frac{x-a-b}{ls} \quad (52)$$

At the boundary {! and *D 8el8x* must be continuous:

$$eocosh\frac{a}{lu} = -\frac{n}{|Vs|}+e1\ cosh\frac{b}{ls} \quad (53)$$

$$(D_U/l_U)\,\varrho_0 \sinh\frac{a}{l_U} = -\frac{D_s}{l_s}\,\varrho_1 \sinh\frac{b}{l_s}\ , \quad \text{also} \quad (54)$$

$$\varrho_1 = -\frac{D_U\, l_s}{D_s\, l_U}\,\frac{\sinh(a/l_U)}{\sinh(b/l_s)}\,\varrho_0\,; \qquad -\frac{n}{|Vs|} = eocosha\frac{}{lu} -+\frac{D-u-sl}{Ds\ lu}sm\cdot h\frac{a}{lu}-coth\frac{b}{ls} \quad (55)$$

The number of neutrons absorbed per cm2 in uranium between 0 and *a* is

$$aJelvuldx=eolvullus\cdot mh\left(\frac{-a}{lu}\right)_o. \quad (56)$$

For each absorbed neutron *X* fission neutrons are produced, of which *X* (1- w) enter the thermal region. If {!o is given, then in a region of the base area 1 cm2 and of the height *a*+ *b in* the whole

$$\{lo\,|\,Vo\,|\,lvs\text{-}mha\text{-}(1\text{-}w)X\text{-}\{loI\,VsI\,b(\cos a\ h\text{-}+Du\text{--}lss\cdot mha\text{-}cothb\text{-})$$

$$/u \qquad Iv \quad Dslv \qquad Iv \qquad ls \tag{57}$$

Neutrons. We thus obtain the mean relative neutron multiplication v if we divide this expression by

$$^{a+Jb}\{ldx = \{lolvs\cdot mha\text{-}+\{lob(\cos ha\ \text{-}+D\text{-}u\text{-}lss\ mh\ a\text{-}acothb\text{-})$$

$$o \qquad Iv \qquad Iv \quad Dslv \qquad Iv \qquad ls$$

$$-\{lo\,I\,\frac{Duis}{Dslv}\,.\,\frac{h}{Iv}\,\frac{a}{}\tag{58}$$

divide. If one still uses the relations $Vs = -Dsl\,t;;$ $vu = -Dul\,IG$ and sets20

on

e *al Iv= <;', blls = 17,* then one obtains

$$v=lvsl\ \frac{(1\text{-}w)X\text{-}17[coth17+(DslvlDvls)coth<;']}{Vslvv+1]\,[coth17+(DslvlDvls)coth<;)\text{-}1} \tag{59}$$

Here w still depends on <;' and *17.* To calculate this dependence, we must first calculate the mean value of the aquivalence width of the resonance line for a given slab thickness. The incident neutrons are distributed in the direction according to the cosi nus law; the aquivalence width varies with the square root of the distance traversed. Thus we obtain as the mean aquivalence width of the resonance *Ek* according to (15)

$$-\quad LIE = rk\ \frac{\int_0^1 d<;'\,<;\,\overline{V2al<;'}}{\int_0^1 d<;'\,<;'} = \frac{4}{3}I'k\ \overline{V2rraakNv} \tag{60}$$

For the ratio *ne0lne* (cf. GI. (17)) applies

$$neo\ Ine = -A\,\frac{5}{4b} = {}^{A1s251}(4ls1J)\ . \tag{61}$$

Finally, fi.ir w (fi.ir U 30 $_{8)}$

$$w = .\pm._{3\ E1(\text{-}lgJ)}\ \frac{3.5Ii}{}\quad \frac{V2naa1Nv\ A\ 5}{4ls1J}K = \frac{0.195}{\text{-}lgJ}\ \frac{A\ 5}{ls\quad 1J}\ K \tag{62}$$

Substituting the numerical values fi.ir HiO as the scattering substance from Table 3, we obtain fi.ir V at the most gi.inst values fi.ir <;' and *17* the following results, which, to facilitate comparison with Table 1, we also relate to the number per cm3 *Ntt of* the H atoms in water:

Table 6

K =	0,5	1,5	2

vlvNH - 10-24 = -0,026	-0,047	-0,056	-0,054

Thus, neutron reduction always occurs again, but the reduction is considerably less than in the case of the most favorable solution of uranium in water. Nevertheless, the result of Eq. (25) is confirmed, that the production of energy from uranium fission is impossible if water is used to slow down the neutrons.

21 A positive value of V results only if other substances are used to slow down the neutrons. For heavy water you get,
substituting the data of Table 3 in (59) and (62), fi.ir $\kappa = 1$

$$V = \nu N_0 \cdot 10\text{-}24 \cdot 0.045 \tag{63}$$

as the most gi.inst value obtained fi.ir <; = 0.5, r, = 0, 17. This value is, like the considerably gi.instiger than the corresponding value in Table 4. The corresponding characteristic length / becomes, neglecting the difference of diffusion parallel to the plates and perpendicular to the plates, about 17 cm, compared with 21 cm for the uranium D2O solution. The most suitable plate thicknesses are about 4 cm for the $U_3 O_8$ plate and 11.5 cm for the interstices filled with D2O.

When calculating the absolute size of the apparatus in connection with the
[It should also be noted that certain quantities of neutrons are lost because some of the fission neutrons from the uranium layers at the edge leave the apparatus before they have reached thermal velocities. If this effect is taken into account, it can be estimated that energy production could already occur with an apparatus thickness of (1.2 cm)³. By an even more appropriate geometrical arrangement of the uranium plates, the volume could perhaps be reduced by a factor of 2 or 3.

Pure coal is also suitable for slowing down the neutrons. If one substitutes into (59) the values of Table 3 of C, one finds the most gi.inst value of $v(K = 1)$ approximately at (= 1/2, r, = 1/5

$$\bar{\nu} = 2,3\,|\nu_s|\ . \tag{64}$$

The characteristic length becomes l= 55 cm. The greatest thickness of the uranium plates becomes about 4 cm, their greatest distance 33 cm. Accordingly, the absolute size of the apparatus must be increased considerably. One estimates after (38), so that at a size of (3 m)³ energy production would occur. One needs to this apparatus therefore about 30 t pure coal and 25 t

22 Uranium oxide. Schliel3lich I could reduce the absolute Grol3e thereby substantially. D2O and C are used in the following way: The U3Os plates are covered on both sides with a layer of D2O about 5 cm thick. This had the effect of reducing w to about the third part, since the neutrons coming from the coal must first mi.iss the D2O layer before they are captured in the uranium. The intermediate spaces to be filled with coal could accordingly be reduced to about the third part, the uranium layers somewhat reduced in size, so that the whole apparatus could be reduced to about (1.2 m)³. It will then contain about 2 to 3 t $U_3\ 0_8$, 600 l heavy water and 1 t coal. These figures can also be reduced by gi.in-

geometric arrangement can be reduced by a factor of 2.

When using Koble, the following should also be noted: If the fission neutrons often have high energy, they can trigger the reaction (n, *a)* (and possibly (n, p)) in C. The necessary energies are 5.4 MeV (and 13 MeV). The energies required for this are 5.4 MeV (and 13 MeV). This would require an additional absorption, which could make the energy production impossible. Therefore, if the fission neutrons have a high energy on average, one *must* surround the U_{308} with layers Dz() which are not too thin. For the same reason, other light elements, such as Li, Be and B, are not suitable for neutron deceleration, since nuclear reactions (n, *a)* can be triggered in them by rather slow neutrons, which absorb the neutrons.

6) The mode of operation of the uranium machine

As was pointed out in section 4a, a uranium machine of the type discussed here would have to maintain itself automatically at a higher temperature <due to the size of the apparatus>; it would supply as much energy <due to fission processes> at any one time as was removed from it externally. This goes on until the original amount of uranium is considerably reduced, or until the contamination <by the fission products has increased the absorption so much that the Temperature I is dropping. The contamination will occur sooner than the consumption. 23 of uranium. This limit can be calculated: We assume approximately an average effective cross section for input of the fission products to be 50 - 10-24 cm2. If we now consider a uranium machine consisting of 1 cbm Dz()+ U308, the absorption becomes comparable with the neutron generation, when there is one atom of the fission products for every 1000 D atoms in the solution; thus, when 0.7 - 1026 atoms of the fission products are formed on the whole. Up to this time

$$0.35 \cdot 10^{26} \cdot 190 \cdot 10^{6}\,\text{eV} = 0.35 \cdot 10^{26} \cdot 190 \cdot 10^{6} \cdot 1.6 \cdot 10^{-12}\,\text{erg}$$
$$= 1.05 \cdot 10^{22}\,\text{erg} = 2.5 \cdot 10^{14}\,\text{cal}$$

since about 190 MeV of energy are released in each fission process. The temperature of the machine would therefore only begin to fall when energy of the order 10^{13} cal had been removed from it; 10^{13} cal is about one tenth of the annual output of a large power station.

With the energy production an extraordinarily intensive neutron radiation and y-radiation would go hand in hand. During each fission process about 7 - 1012 cal are released. For each calorie removed from the machine, therefore, 0.5 - 10^{12} neutrons and about the same number of y-quanta are emitted. At a power of only 10 kW, 10^{15} neutrons and y-quanta would be emitted per sec; this corresponds to the y-radiation intensity of 10 kg Ra or the neutron intensity of a Ra-Be source containing 10^{5} kg Ra. The radiation power would thus be about 10^{5} times the power of a large cyclotron. Even if a considerable part of this radiation intensity is absorbed inside the machine, the operation of the machine would obviously be quite extensive.

make extensive radiation protection measures necessary. This applies in particular when the machine is " switched on", i.e. when the individual parts are filmed together. At the moment when the temperature rises to the steady-state value, e.g. 100 °C, in this example ^{108} cal are used to heat the machine.

24 needs, i.e. 5 - ^{1019} l neutrons and y quanta are released. This corresponds to the total y-radiation of 1 kg Ra in the course of one day.

Summary

The fission processes on uranium discovered by *Hahn* and *Straj]mann* can, according to the data available so far, also be used for large-scale energy production. The safest method of producing a machine suitable for this purpose consists in the enrichment of the isotope U§f The further the enrichment is carried out, the smaller the machine can be built. The enrichment of U§ ^{5} is the only method by which the volume of the machine can be made small to 1 cbm. It is also the only method of producing explosives which exceed by several powers of ten the explosive power of the most powerful explosives yet produced. However, normal uranium can also be used to produce energy without enriching U ^{5} by combining uranium with another substance that slows down the neutrons of uranium without absorbing them. Water is not suitable for this purpose. On the other hand, according to the data available so far, heavy water and quite pure Kahle fulfil this purpose. Small impurities can always make energy production impossible. According to the results available so far, the most favourable arrangement of the machine consists of U_3 O_8 layers of about 4 cm thickness, which are covered on both sides with D2O layers of about 5 cm thickness. The space between the various triple layers of this type should be about 10 - 20 cm and filled with pure brine; the whole machine is also appropriately surrounded with pure brine. The minimum area of the layers is about 1 $^{m2-}$ A machine of this type would be kept at a constant temperature by the splitting processes, the height of which depends on the size of the appa rature. The absolute temperature increases with the fourth power of the diameter of the apparatus. The machine would at the same time be an extraordinarily intense source of neutrons and y-radiation. - To check these results, a verification of Tables 2 and 3 would be of great importance.