



**Table 2.1** Zero-order hold sampling of a continuous-time system,  $G(s)$ . The table gives the zero-order-hold equivalent of the continuous-time system,  $G(s)$ , preceded by a zero-order hold. The sampled system is described by its pulse-transfer operator. The pulse-transfer operator is given in terms of the coefficients of

$$H(q) = \frac{b_1 q^{n-1} + b_2 q^{n-2} + \dots + b_n}{q^n + a_1 q^{n-1} + \dots + a_n}$$

$G(s)$	$H(q)$ or the coefficients in $H(q)$
$\frac{1}{s}$	$\frac{h}{q-1}$
$\frac{1}{s^2}$	$\frac{h^2(q+1)}{2(q-1)^2}$
$\frac{1}{s^m}$	$\frac{q-1}{q} \lim_{a \rightarrow 0} \frac{(-1)^m}{m!} \frac{\partial^m}{\partial a^m} \left( \frac{q}{q - e^{-ah}} \right)$
$e^{-sh}$	$q^{-1}$
$\frac{a}{s+a}$	$\frac{1 - \exp(-ah)}{q - \exp(-ah)}$
$\frac{a}{s(s+a)}$	$b_1 = \frac{1}{a} (ah - 1 + e^{-ah})$ $b_2 = \frac{1}{a} (1 - e^{-ah} - ahe^{-ah})$ $a_1 = -(1 + e^{-ah})$ $a_2 = e^{-ah}$
$\frac{a^2}{(s+a)^2}$	$b_1 = 1 - e^{-ah}(1 + ah)$ $b_2 = e^{-ah}(e^{-ah} + ah - 1)$ $a_1 = -2e^{-ah}$ $a_2 = e^{-2ah}$
$\frac{s}{(s+a)^2}$	$\frac{(q-1)he^{-ah}}{(q - e^{-ah})^2}$
$\frac{ab}{(s+a)(s+b)}$	$b_1 = \frac{b(1 - e^{-ah}) - a(1 - e^{-bh})}{b - a}$ $b_2 = \frac{a(1 - e^{-bh})e^{-ah} - b(1 - e^{-ah})e^{-bh}}{b - a}$ $a_1 = -(e^{-ah} + e^{-bh})$ $a_2 = e^{-(a+b)h}$
$a \neq b$	

Table 2.1 continued

$G(s)$	$H(q)$ or the coefficients in $H(q)$
$\frac{(s+c)}{(s+a)(s+b)}$ $a \neq b$	$b_1 = \frac{e^{-bh} - e^{-ah} + (1 - e^{-bh})c/b - (1 - e^{-ah})c/a}{a-b}$ $b_2 = \frac{c}{ab} e^{-(a+b)h} + \frac{b-c}{b(a-b)} e^{-ah} + \frac{c-a}{a(a-b)} e^{-bh}$ $a_1 = -e^{-ah} - e^{-bh} \quad a_2 = e^{-(a+b)h}$
$\frac{\omega_0^2}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$	$b_1 = 1 - \alpha \left( \beta + \frac{\zeta\omega_0}{\omega} \gamma \right) \quad \omega = \omega_0 \sqrt{1 - \zeta^2} \quad \zeta < 1$ $b_2 = \alpha^2 + \alpha \left( \frac{\zeta\omega_0}{\omega} \gamma - \beta \right) \quad \alpha = e^{-\zeta\omega_0 h}$ $a_1 = -2\alpha\beta \quad \beta = \cos(\omega h)$ $a_2 = \alpha^2 \quad \gamma = \sin(\omega h)$
$\frac{s}{s^2 + 2\zeta\omega_0 s + \omega_0^2}$	$b_1 = \frac{1}{\omega} e^{-\zeta\omega_0 h} \sin(\omega h) \quad b_2 = -b_1$ $a_1 = -2e^{-\zeta\omega_0 h} \cos(\omega h) \quad a_2 = e^{-2\zeta\omega_0 h}$ $\omega = \omega_0 \sqrt{1 - \zeta^2}$
$\frac{a^2}{s^2 + a^2}$	$b_1 = 1 - \cos ah \quad b_2 = 1 - \cos ah$ $a_1 = -2 \cos ah \quad a_2 = 1$
$\frac{s}{s^2 + a^2}$	$b_1 = \frac{1}{a} \sin ah \quad b_2 = -\frac{1}{a} \sin ah$ $a_1 = -2 \cos ah \quad a_2 = 1$
$\frac{a}{s^2(s+a)}$	$b_1 = \frac{1-\alpha}{a^2} + h \left( \frac{h}{2} - \frac{1}{a} \right) \quad \alpha = e^{-ah}$ $b_2 = (1-\alpha) \left( \frac{h^2}{2} - \frac{2}{a^2} \right) + \frac{h}{a} (1+\alpha)$ $b_3 = - \left[ \frac{1}{a^2} (\alpha - 1) + \alpha h \left( \frac{h}{2} + \frac{1}{a} \right) \right]$ $a_1 = -(\alpha + 2) \quad a_2 = 2\alpha + 1 \quad a_3 = -\alpha$