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CSCI 5622 - Machine Learning: Learnability (HW 4) Discussion

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1) argue about an ordering of hypothesis classes in terms of complexity: hyperplanes through the origin, arbitrary hyperplanes, and axis-aligned rectangles (you can use your experiments as a guide, but simply reporting those numbers is not sufficient; you must make a mathematical argument)
Solution:
2) prove that your frequency correctly classifies any training set (up to floating point precision on the computer).
Solution: From the additional reading, svmtutorial.pdf: If we choose some number l, and find l points that can be shattered, we choose the points to be:

$$x_i = 10^{-i}, i = 1, ..., l$$

Then we specify any labels:

$$y_1, y_2, ..., y_l, y_i \in \{-1, 1\}$$

Then $f(\alpha)$ gives this labeling if we choose α to be:

$$\alpha = \pi(1 + \sum_{i=1}^{l}(\frac{1 - y_i)10^i}{2})$$

Thus the VC dimension of this is infinite.
3) Suppose we are classifying real numbers, not integers. The classifier returns positive (1) if the point is greater than the sin function and negative (0) otherwise.

$$h_\omega(x : x \in \mathbb{R}) \equiv \begin{cases} 1 & \text{if } \sin(\omega x) \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

Give an example of four points that cannot be shattered by this classifier. How does this relate to the VC dimension?
Solution: Four equa-distant points cannot be shattered by this classifier. For example $x_i = 1, 2, 3, 4$, $y_i = 0, 0, 1, 0$.
Despite having a VC dimension that is infinite, there still exist some points that cannot be shattered by $\sin(\omega x)$. This doesn't say anything about the upper bound because the upper bound says that *any* set of points can't be shattered by the classifier.