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# 無線通訊系統 (Wireless Communications Systems)

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Prof. Tsai

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## Chapter 3 Physical Layer Technologies for Wireless Communication Systems

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# Modulation Schemes

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## Modulation

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- Message information can be transmitted in the **amplitude**, **frequency** or **phase** of the carrier
- For mobile radio applications:
  - We shall use bandwidth and power resources most efficiently
- **Bandwidth efficiency:** measured in bits/sec/Hz
  - The modulation must have compact power density spectrum
- **Good BER performance:**
  - Good BER must be maintained in mobile radio environments
  - The modulation must be able to prevail over fading, ISI, Doppler spread, adjacent and co-channel interference, and thermal noise
- **Envelope properties:**
  - Portable transmitters normally use **nonlinear power amplifiers**
  - Modulation with a **relatively constant envelope** shall be used

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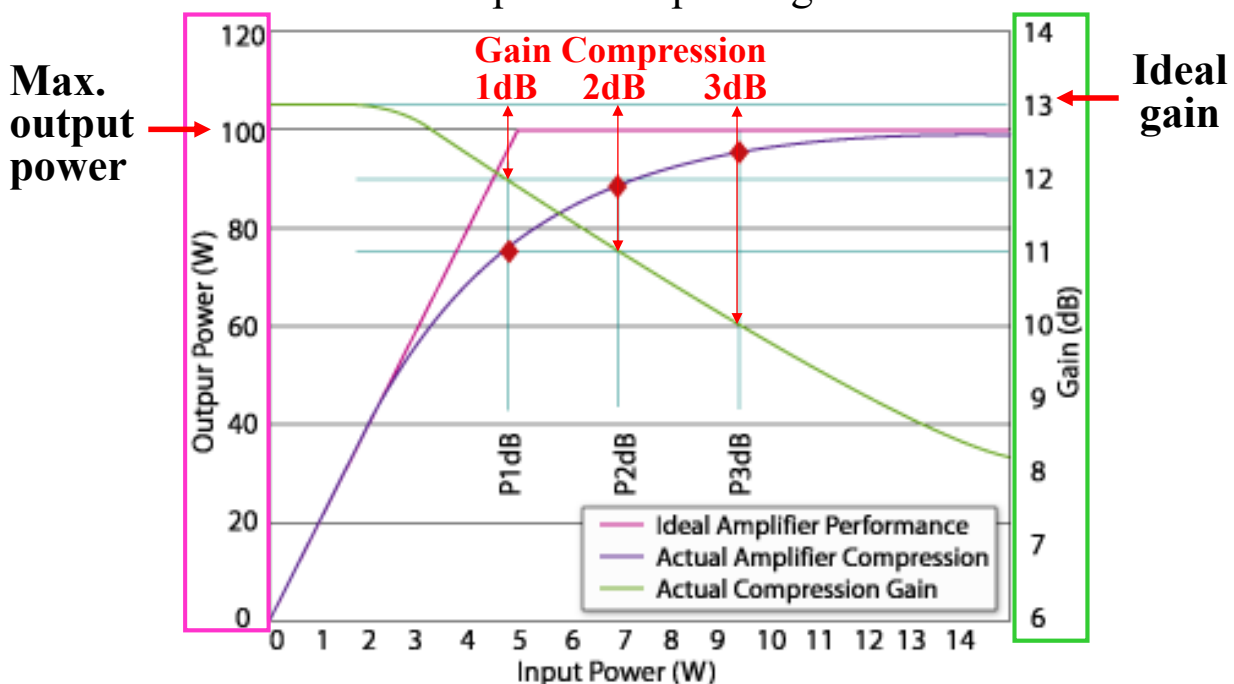
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# Power Amplifier

- A RF **power amplifier** (PA) can amplify a small input RF signal of a certain shape to a larger one with the same shape at the output.
- The **gain** is defined as the ratio of output power to input power.
- Since the maximum output power is limited, a PA will definitely be **saturated** when input power gradually increases.
- The saturation of a PA is defined as the situation that the output signal is **not proportional** to the input signal.
  - **Distortion** occurs in the output signal
- It is highly desirable to operate a PA in the **linear region**
  - The input signal is faithfully re-produced at the output

## Power Amplifier (Cont.)

- Theoretical and actual power-amplifier gain



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## Power Amplifier (Cont.)

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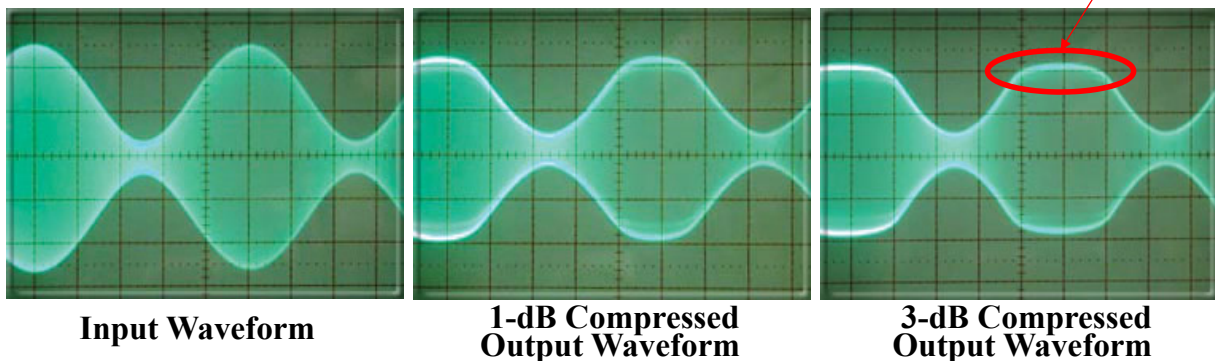
- The amount of **usable power** depends on the applications
  - It depends on how much **distortion** can be tolerated
- A commonly used figure-of-merit is the **1-dB compression point**. This is the output power level at which the gain is compressed by 1 dB.
  - However, the decreased gain affects only those parts of the signal with amplitudes **close to or greater than** the 1-dB compression point.

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## Power Amplifier (Cont.)

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- For 1-dB compressed output waveform, the maximum input power is equal to the “**P1dB**” input power
  - **Smaller average output power with less distortion**
- For 3-dB compressed output waveform, the maximum input power is equal to the “**P3dB**” input power
  - **Larger average output power with more distortion**



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## Power Amplifier (Cont.)

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- We may select the required amplifier power by **backing off** an acceptable amount from P1dB to prevent signal distortion.
- For example, if an application requires a maximum output power of **50 W**, the system designer may select a PA with a **P1dB of 100 W or higher** to ensure that the PA is within its **linear region** of operation.
  - It depends upon the **linearity requirement** of the system.
- For the same tolerable distortion, a signal with a high peak-to-average ratio will **degrade** the **average output signal power**, as well as the efficiency of the PA.
  - Or equivalently, it increases the **linearity** and **dynamic range** requirements of the PA

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## Modulation for Different Systems

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- First generation systems:
  - **FM modulation** (AMPS, TACS, NMT)
- Second generation systems:
  - **$\pi/4$ -DQPSK** (IS-136, PDC, PHS)
  - **GMSK** (GSM, DCS1800, DECT)
- Third generation systems:
  - CDMA – **QPSK/Offset QPSK** (cdma2000, WCDMA)
- Fourth generation systems:
  - OFDM – **BPSK, QPSK, 16-QAM, 64-QAM** (WiMAX, LTE)
- Fifth generation systems:
  - OFDM – ..., **256-QAM, 512-QAM, 1024-QAM**

# Phase Shift Keying (PSK)

- The  $M$ -PSK complex envelope can be expressed as

$$\tilde{s}(t) = A \sum_n b(t - nT, \mathbf{x}_n)$$

$$b(t, \mathbf{x}_n) = h_a(t) \exp(j\theta_n)$$

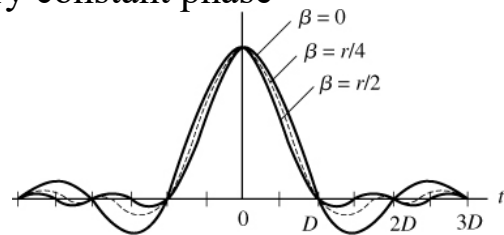
- $\theta_n = \frac{2\pi}{M} x_n + \theta_0$  and  $\theta_0$  is an arbitrary constant phase

- $x_n = n, n \in \{0, 1, \dots, M-1\}$

- $M$  is the alphabet size

- $h_a(t)$  is the amplitude shaping pulse

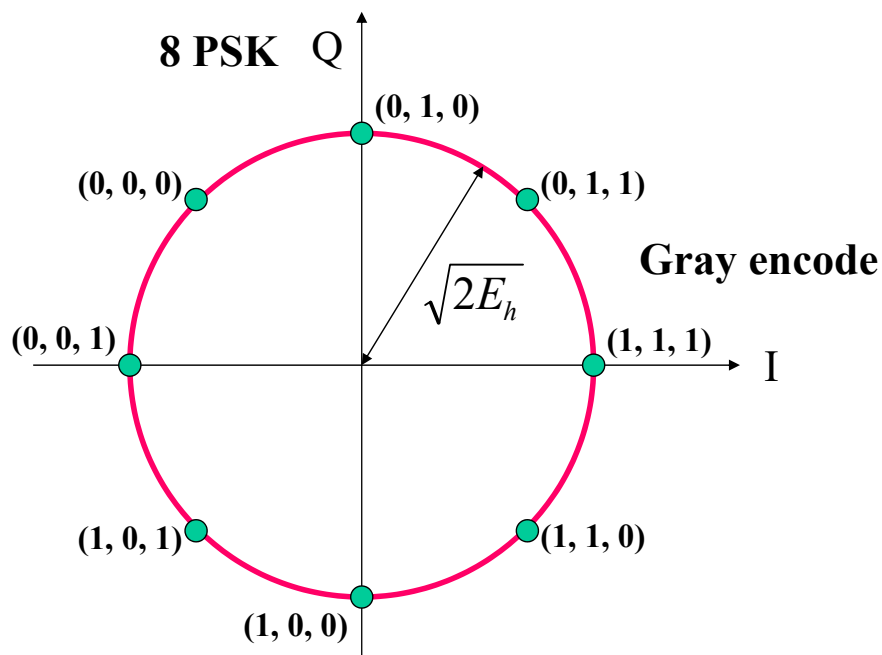
- Normally chosen to be a raised cosine function



- The complex envelope is

$$\tilde{s}_n(t) = Ah_a(t)e^{j\theta_n}, \quad n = 0, 1, \dots, M-1$$

# Phase Shift Keying (PSK) (Cont.)



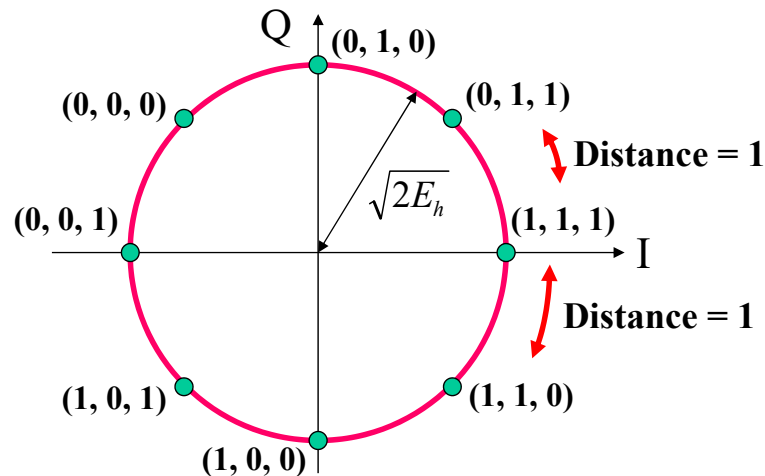
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## Question

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- **Question:**

- Why **Gray Encode** is generally used in modulation?
- Bit Error Rate  $\cong$  Symbol Error Rate / 3



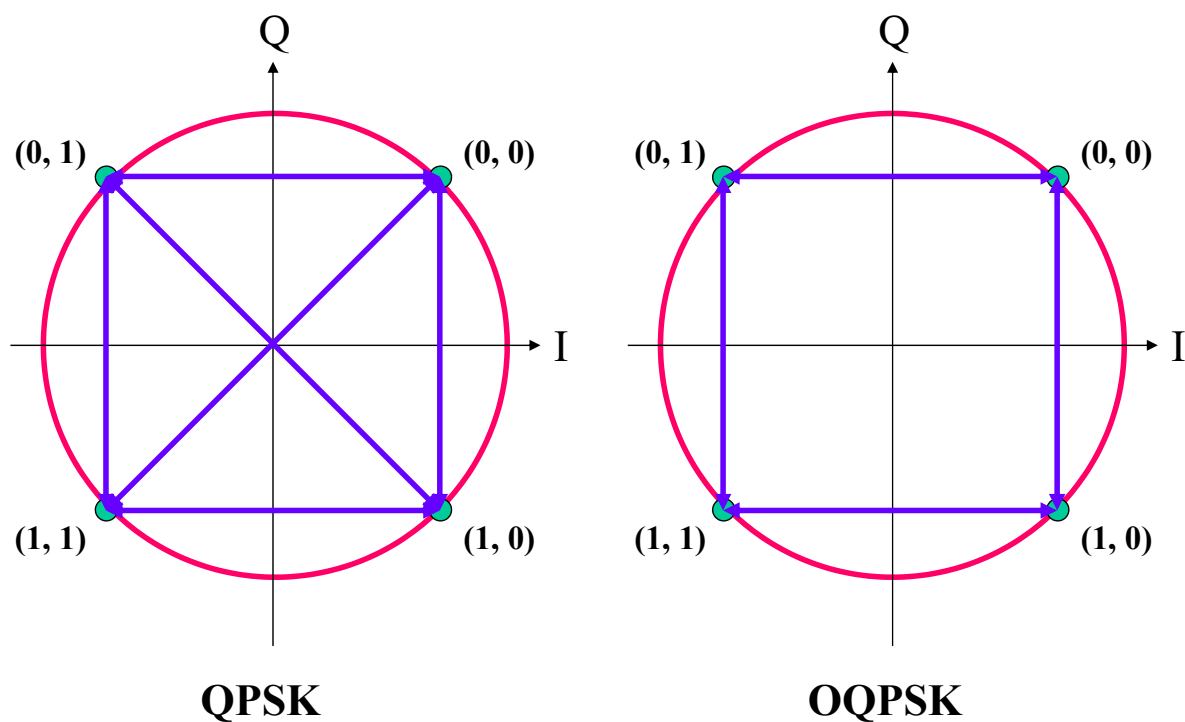
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## Offset QPSK (OQPSK)

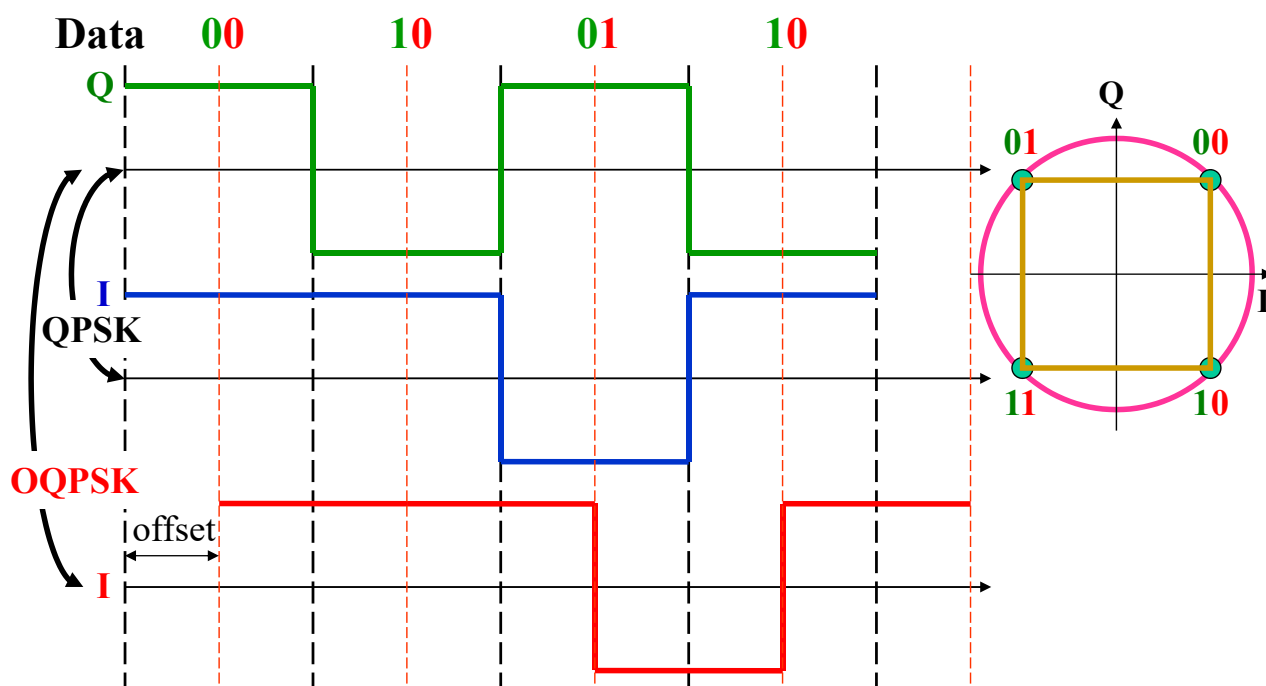
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- The QPSK signal can have either  $\pm 90^\circ$  or  $\pm 180^\circ$  phase shift from one baud interval to the next
- The OQPSK signal can change by only  $\pm 90^\circ$
- The phase trajectory does not pass through **the origin**
  - Reduce the peak-to-average ratio of the complex envelope
  - Make the OQPSK signal less sensitive to amplifier nonlinearity than the QPSK signal

## Offset QPSK (OQPSK) (Cont.)



## Offset QPSK (OQPSK) (Cont.)





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## $\pi/4$ -Differential QPSK ( $\pi/4$ -DQPSK)

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- QPSK: **4** carrier phases (**Coherent detection is essential**)
- $\pi/4$ -DQPSK: **8** carrier phases (**Non-coherent detection**)
- For quaternary source sequence  $\{x_n\}$ ,  $x_n \in \{\pm 1, \pm 3\}$

$$\Delta\theta_n = \theta_n - \theta_{n-1} = \begin{cases} -3\pi/4, & x_n = -3 \\ -\pi/4, & x_n = -1 \\ +\pi/4, & x_n = +1 \\ +3\pi/4, & x_n = +3 \end{cases}$$

- The phase differences must be  $\pm \pi/4$  and  $\pm 3\pi/4$

$$\Delta\theta_n = x_n \frac{\pi}{4}$$

- The complex envelope is expressed as

$$\tilde{s}(t) = A \sum_n b(t - nT, \mathbf{x}_n)$$

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## $\pi/4$ -Differential QPSK ( $\pi/4$ -DQPSK) (Cont.)

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- where

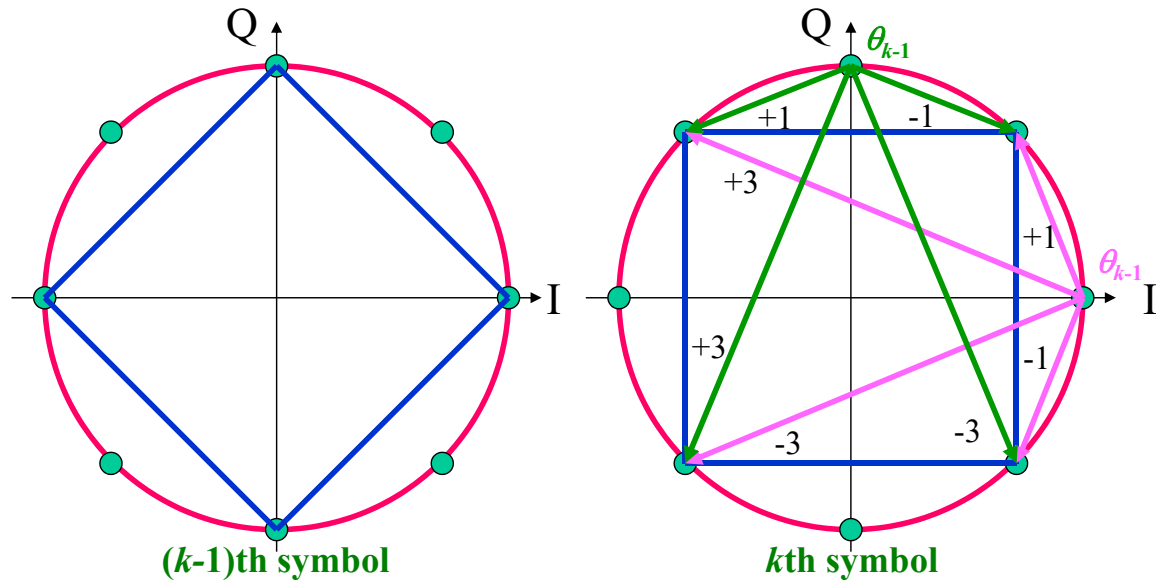
$$\begin{aligned} b(t, \mathbf{x}_n) &= h_a(t) \exp \left\{ j \left( \theta_{n-1} + x_n \frac{\pi}{4} + \theta_0 \right) \right\} \\ &= h_a(t) \exp \left\{ j \frac{\pi}{4} \left( \sum_{k=-\infty}^{n-1} x_k + x_n \right) + j \theta_0 \right\} \end{aligned}$$

- Change the phase for every symbol: the **symbol synchronization** is much easier than QPSK
- **Non-coherent detection** can be applied in receivers
- There are two signal sets:

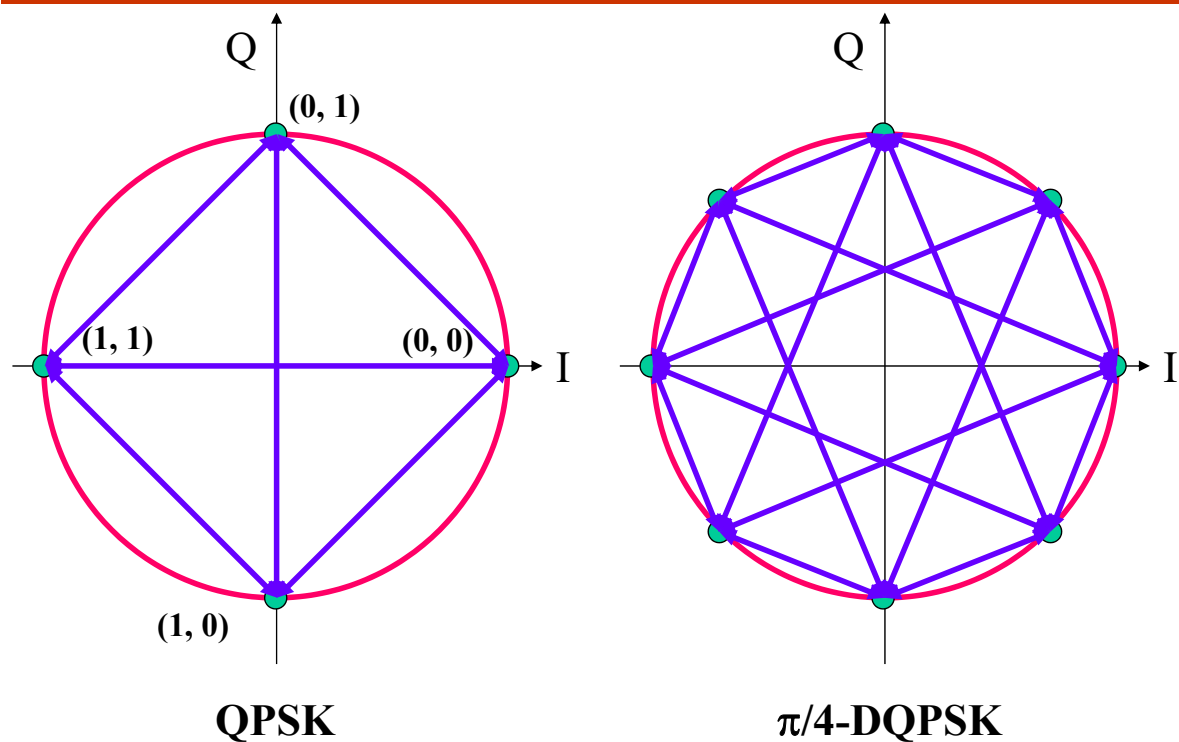
$$\begin{aligned} &\{0, \pi/2, \pi, 3\pi/2\} \\ &\{\pi/4, 3\pi/4, 5\pi/4, 7\pi/4\} \end{aligned}$$

## $\pi/4$ -Differential QPSK ( $\pi/4$ -DQPSK) (Cont.)

- The phase trajectory does not pass through **the origin**
  - Reduce the linearity and dynamic range requirements of the power amplifier



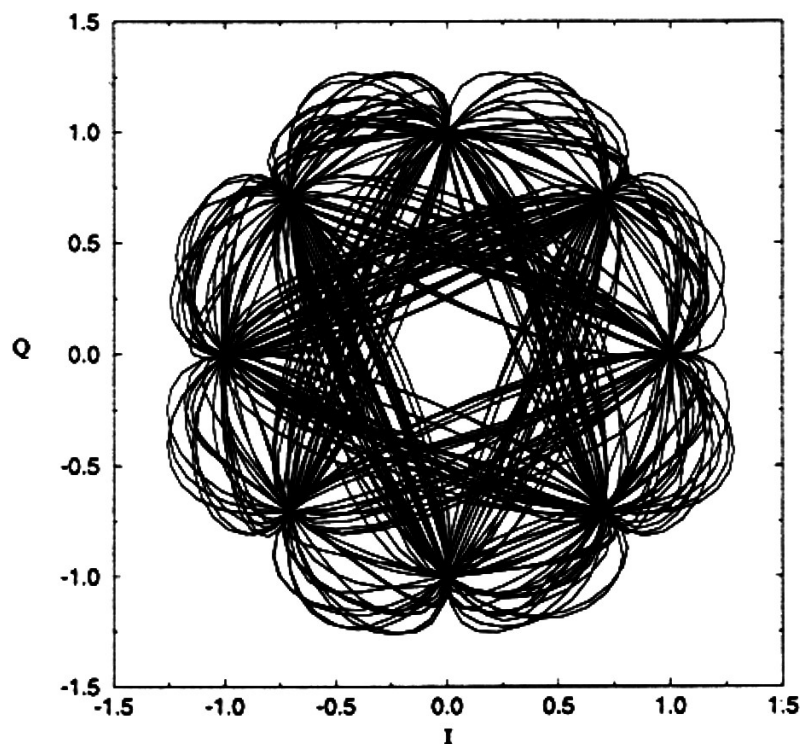
## $\pi/4$ -Differential QPSK ( $\pi/4$ -DQPSK) (Cont.)



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## $\pi/4$ -Differential QPSK ( $\pi/4$ -DQPSK) (Cont.)

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## Continuous Phase Modulation (CPM)

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- Continuous Phase Modulation: the carrier phase varies in a **continuous** manner
- CPM is attractive because it has **constant envelope** and **excellent spectral characteristics**
- The complex envelope is

$$\tilde{s}(t) = A \exp \{ j(\phi(t) + \theta_0) \}$$

$$\phi(t) = 2\pi \int_0^t \sum_{k=0}^{\infty} h_k x_k h_f(\tau - kT) d\tau$$

- $\phi(t)$ : the **excess phase**: the phase difference to the zero phase
- $\{x_k\}$ : the data symbol sequence
- $\{h_k\}$ : the sequence of modulation indices
- $h_f(t)$ : the frequency shaping function

# Continuous Phase Modulation (CPM) (Cont.)

- $h_f(t)$  is zero for  $t < 0$  and  $t > LT$ 
  - Full response CPM:  $L = 1$
  - Partial response CPM:  $L > 1$

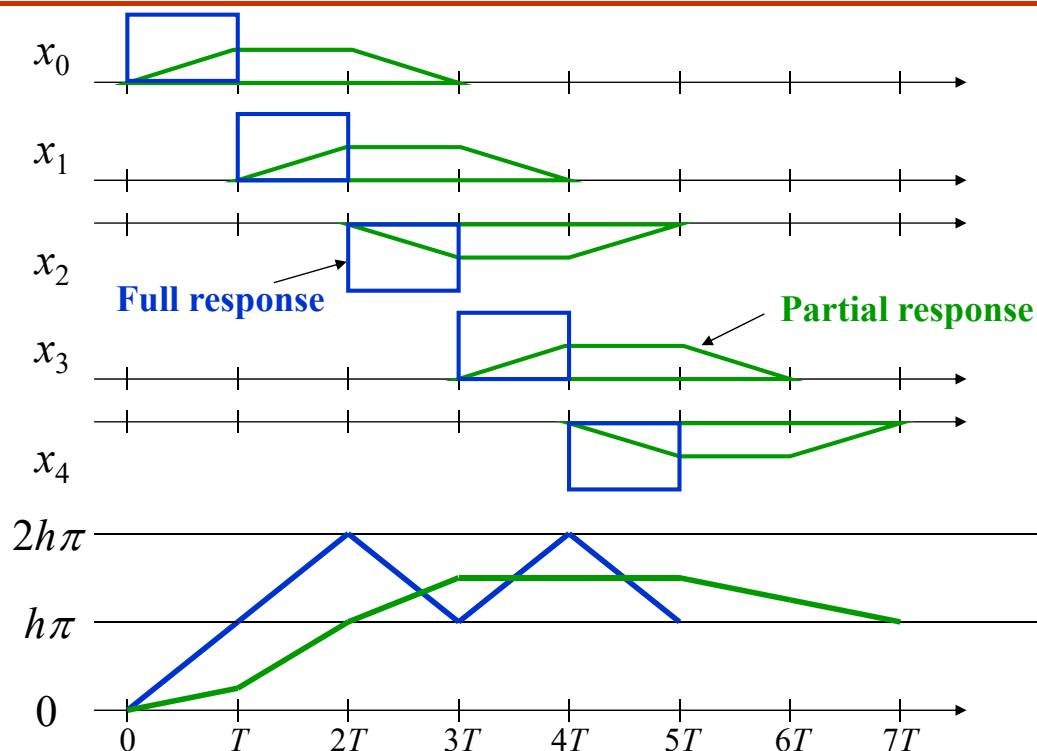
- The phase shaping function:

$$\beta(t) = \begin{cases} 0, & t < 0 \\ \int_0^t h_f(\tau) d\tau, & 0 \leq t \leq LT \\ 1/2, & t \geq LT \end{cases}$$

- For a full response CPM and  $h_k = h$ , the excess phase is

$$\begin{aligned} \phi(t) &= 2\pi h \left[ \underbrace{\int_0^{nT} \sum_{k=0}^{n-1} x_k h_f(\tau - kT) d\tau}_{\text{Response of the previous symbols}} + \underbrace{\int_{nT}^t x_n h_f(\tau - nT) d\tau}_{\text{Response of the current symbol}} \right] \\ &= \pi h \sum_{k=0}^{n-1} x_k + 2\pi h x_n \beta(t - nT) \end{aligned}$$

# Continuous Phase Modulation (CPM) (Cont.)



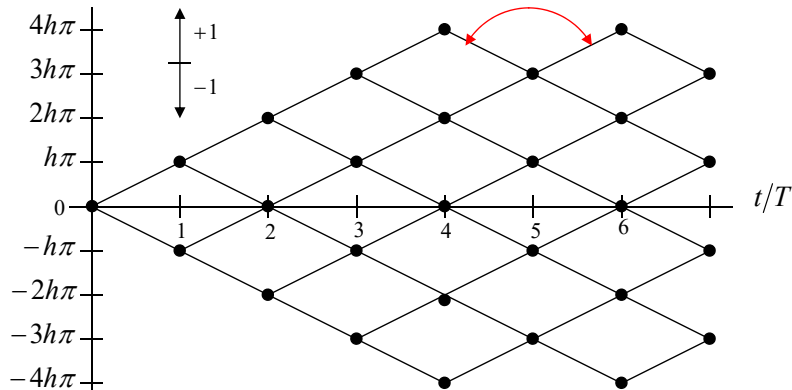
# Continuous Phase Frequency Shift Keying(Cont.)

- Continuous phase frequency shift keying (CPFSK):
  - A special type of full response CPM
  - By using the **rectangular** shaping function  $h_f(t) = u_T(t)$

$$\beta(t) = \begin{cases} 0, & t < 0 \\ t/2T, & 0 \leq t \leq T \\ 1/2, & t \geq T \end{cases}$$

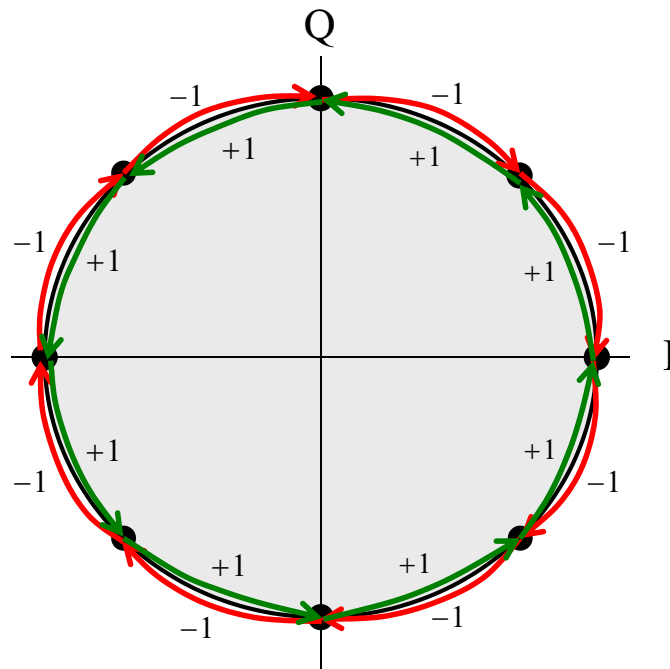
Two slopes  
 $\Rightarrow$  Two frequencies  
 Representing '+1' & '-1'

- The phase tree:



# Continuous Phase Frequency Shift Keying(Cont.)

- Binary CPM signal with  $h = 1/4$



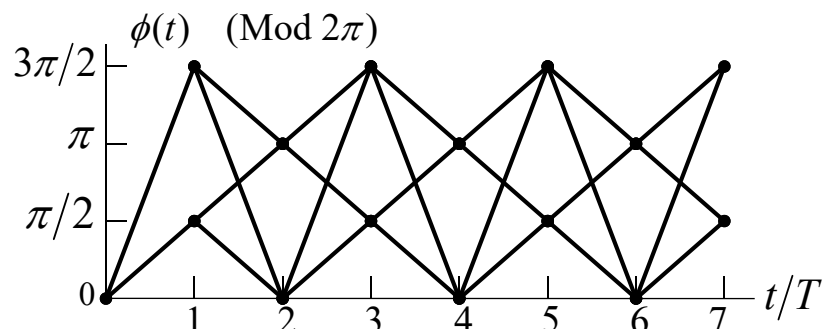
## Minimum Shift Keying (MSK) (Cont.)

- Minimum Shift Keying is a special case of binary CPFSK
  - With modulation index  $h = 1/2$

- Carrier phase:** Zero phase

$$\begin{aligned}\phi_c(t) &= 2\pi f_c t + \frac{\pi}{2} \sum_{k=0}^{n-1} x_k + \frac{\pi}{2} x_n \frac{t - nT}{T} \\ &= \left( 2\pi f_c + \frac{\pi x_n}{2T} \right) t + \frac{\pi}{2} \sum_{k=0}^{n-1} x_k - \frac{\pi n}{2} x_n\end{aligned}$$

- Excess phase:



## Minimum Shift Keying (MSK) (Cont.)

- The MSK band-pass waveform is

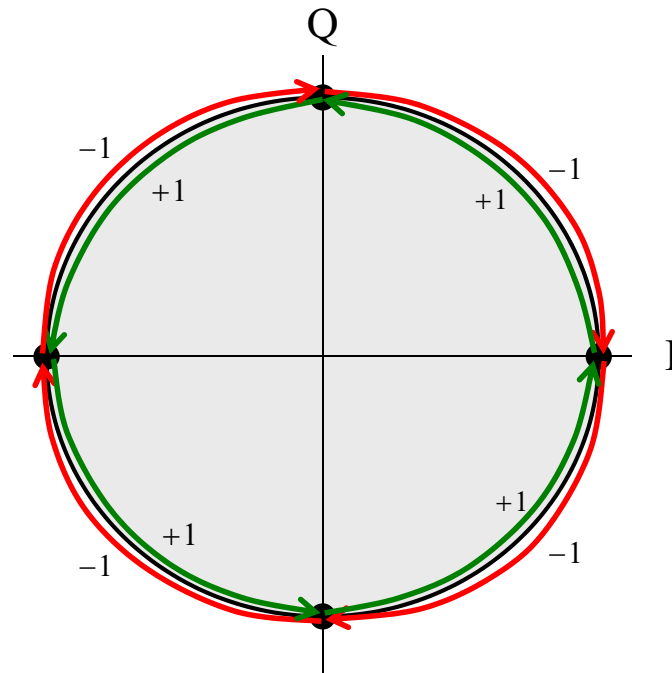
$$s(t) = A \cos \left( 2\pi \left( f_c + \frac{x_n}{4T} \right) t + \frac{\pi}{2} \sum_{k=0}^{n-1} x_k - \frac{\pi n}{2} x_n \right), \quad nT \leq t \leq (n+1)T$$

- Note that an MSK signal has one of two possible frequencies:

$$f_L = f_c - \frac{1}{4T}, \quad \text{and} \quad f_U = f_c + \frac{1}{4T}$$

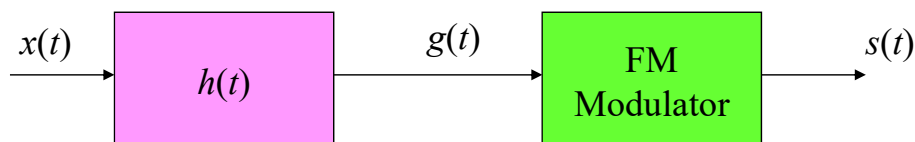
- The difference  $\Delta f = f_U - f_L = \frac{1}{2T}$  is the minimum frequency separation to ensure **orthogonality** between two sinusoid of duration  $T$  with coherent demodulation

## Minimum Shift Keying (MSK) (Cont.)



## Gaussian Minimum Shift Keying (GMSK)

- **Gaussian Minimum Shift Keying** is achieved by **low-pass filtering** the modulating signal prior to modulation
  - The filtering removes the higher frequency components
  - ⇒ It results in a more **compact spectrum**
  - ⇒ It introduces the **inter-symbol interference (ISI)**



- The transfer function of the pre-modulation filter (low-pass) is

$$H(f) = \exp\left\{-\left(\frac{f}{B}\right)^2 \frac{\ln 2}{2}\right\}$$

- where  $B$  is the 3 dB bandwidth of the filter

## Gaussian Minimum Shift Keying (Cont.)

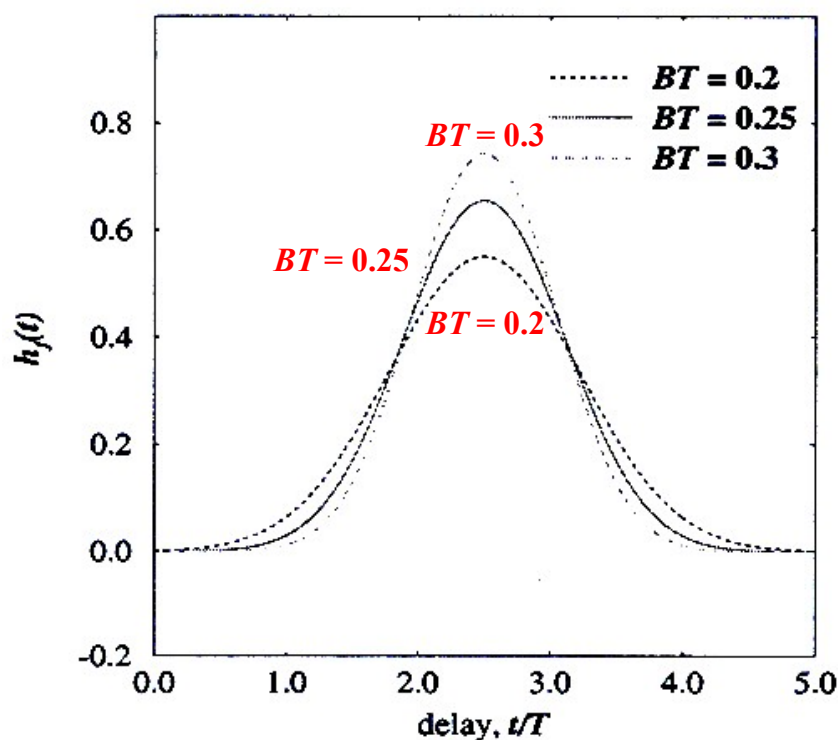
- The GMSK frequency shaping pulse filter:
  - The normalized filter bandwidth is  $BT$
  - The shaping pulse duration  $> T \Rightarrow$  ISI
  - $BT$  decreases  $\Rightarrow$  more compact in power spectral density  $\Rightarrow$  ISI increases  $\Rightarrow$  the bit error rate performance is degraded
- In GSM systems:

$$x_n \in \{-1, +1\}$$

$$h = 1/2$$

$$BT = 0.3$$

## Gaussian Minimum Shift Keying (Cont.)





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# Question

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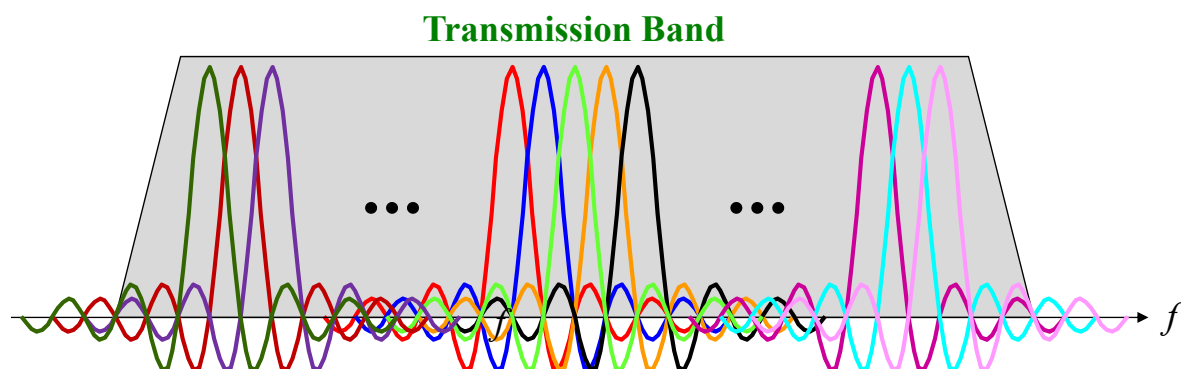
- **Question:**
  - Why **amplitude modulation** is generally not used in mobile communications?
- There is no amplitude reference in the received signal
  - Fast fading
  - Channel gain estimation is not easy
- **Question:**
  - Why **phase/frequency modulation** is generally used in mobile communications? (the phase may change randomly)
- Although there is no phase reference in the received signal, the non-coherent detection can still be applied
  - $\pi/4$ -DQPSK or MSK

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# OFDM

# Orthogonal Frequency Division Multiplexing

- **Orthogonal frequency division multiplexing (OFDM)** is a promising technique because of its
  - High bandwidth efficiency and
  - Resistance to multipath fading
- OFDM transmission can be regarded as a kind of **multi-carrier transmission**.

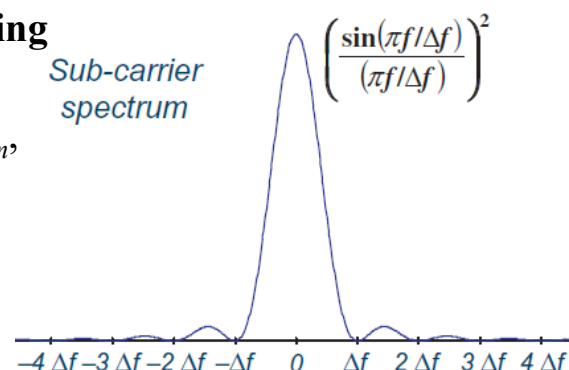


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## OFDM (Cont.)

- However, the following characteristics distinguish OFDM from a straightforward multi-carrier extension:
  - The use of a typically **very large number** of relatively narrowband subcarriers (e.g., several hundred subcarriers)
  - Simple **rectangular pulse shaping** is used
- ⇒ A sinc-square-shaped per-subcarrier spectrum
- **Tight frequency-domain packing** of the subcarriers
- ⇒ A subcarrier spacing  $\Delta f = 1/T_{sym}$ ,  
 $T_{sym}$  is the per-subcarrier modulation-symbol duration



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## OFDM (Cont.)

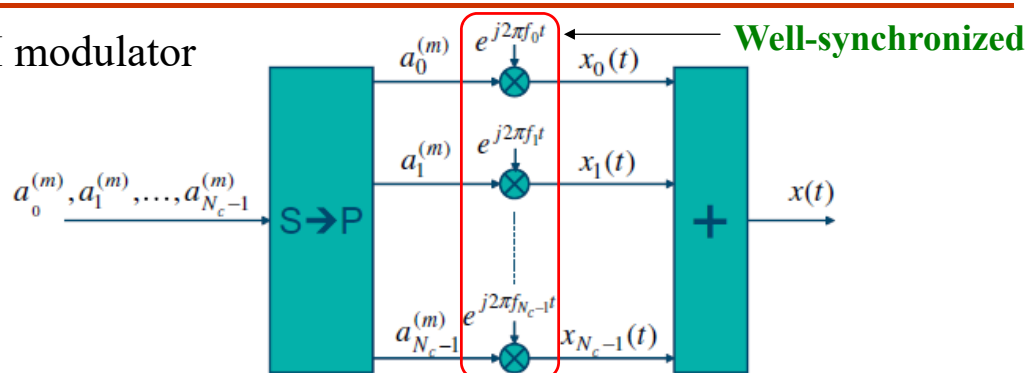
- **OFDM** can be regarded as a **frequency-division multiplexing (FDM)** scheme.
- A large number of closely-spaced **orthogonal subcarriers** are used to carry data.
- In complex baseband notation, a basic OFDM signal  $x(t)$  during the time interval  $mT_{\text{sym}} \leq t \leq (m+1)T_{\text{sym}}$  is expressed as:

$$x(t) = \sum_{k=0}^{N_c-1} x_k(t) = \sum_{k=0}^{N_c-1} a_k^{(m)} e^{j2\pi k \Delta f t}$$

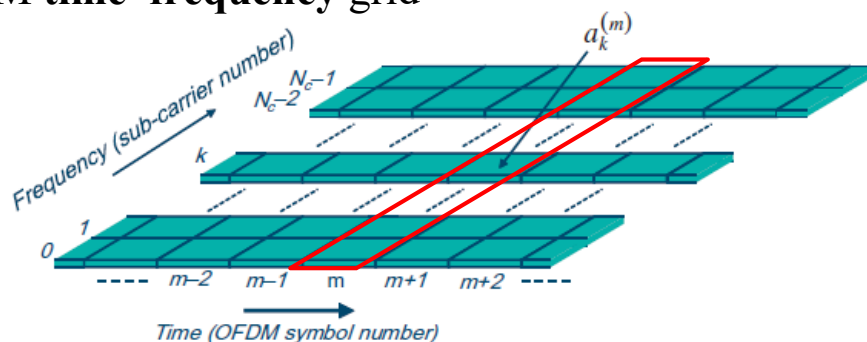
- $x_k(t)$  is the  $k$ -th modulated subcarrier with  $f_k = k\Delta f$
- $a_k^{(m)}$  is the modulation symbol applied to the  $k$ -th subcarrier during the  $m$ -th OFDM symbol interval
- $N_c$  is number of the modulated subcarriers

## OFDM Modulation

- OFDM modulator

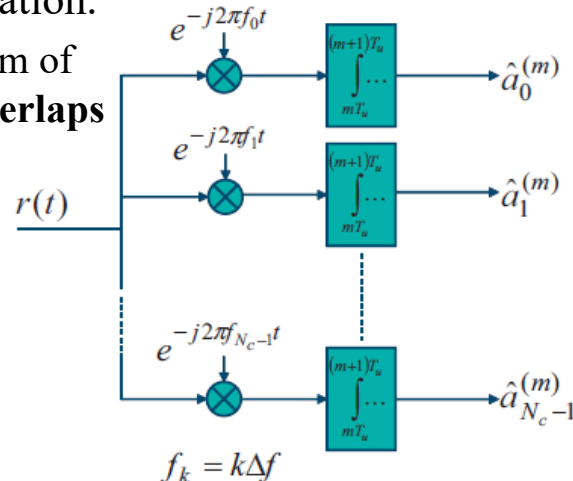


- OFDM time–frequency grid



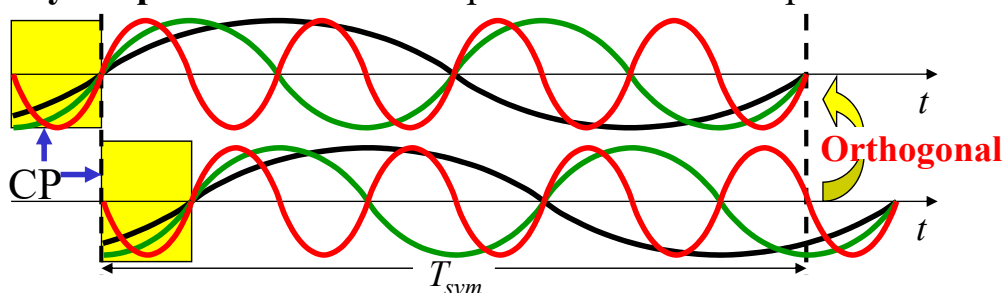
# OFDM Demodulation

- The basic principle of OFDM demodulation consists of a **bank of correlators**, each of which corresponds to a subcarrier.
- Because of the orthogonality between subcarriers, in the ideal case, two OFDM subcarriers **do not** cause any interference to each other after demodulation.
  - Despite that the spectrum of neighbor subcarriers **overlaps**
- OFDM demodulator



# OFDM Orthogonality

- The **subcarrier orthogonality** is due to
  - The specific **frequency-domain structure** of each subcarrier
  - The specific choice of a **subcarrier spacing**  $\Delta f$  equal to the **per-subcarrier modulation-symbol rate**  $1/T_{sym}$
- Any **corruption** of the frequency-domain structure may lead to destruction in orthogonality  $\Rightarrow$  causing **mutual interference**
  - Time dispersion, **Doppler spread**, ...
  - Cyclic prefix** is inserted to prevent ISI in multipath channels



# OFDM Implementation

- OFDM allows for low-complexity implementation by means of computationally efficient **Fast Fourier Transform (FFT)** processing.
  - The **sampling rate**  $f_s$  is a multiple of the subcarrier spacing  $\Delta f$ 

$$f_s = 1/T_s = N \times \Delta f$$
  - The FFT size  $N$  should exceed  $N_c$  with a sufficient margin,  $N > N_c$
- The time-discrete OFDM signal can be expressed as
  - The sequence  $x_n$  is the Inverse Discrete Fourier Transform (IDFT) of the symbols  $a_0, a_1, \dots, a_{N_c-1}$ , extended with zeros to length  $N$

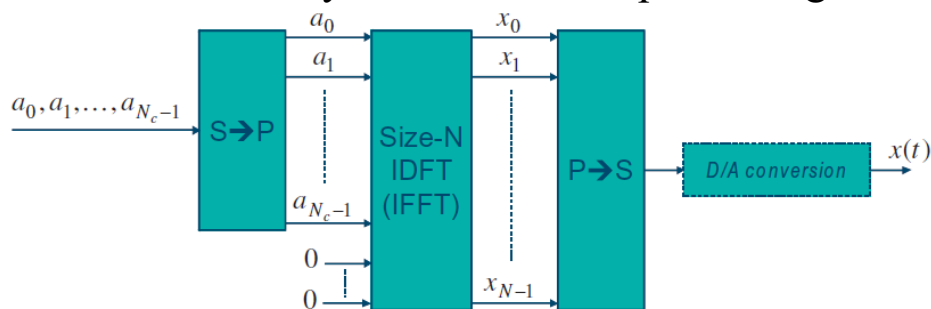
$$x_n = x(nT_s) = \sum_{k=0}^{N_c-1} a_k e^{j2\pi k \Delta f n T_s}$$

$$= \sum_{k=0}^{N_c-1} a_k e^{j2\pi kn/N} = \sum_{k=0}^{N-1} a'_k e^{j2\pi kn/N}$$

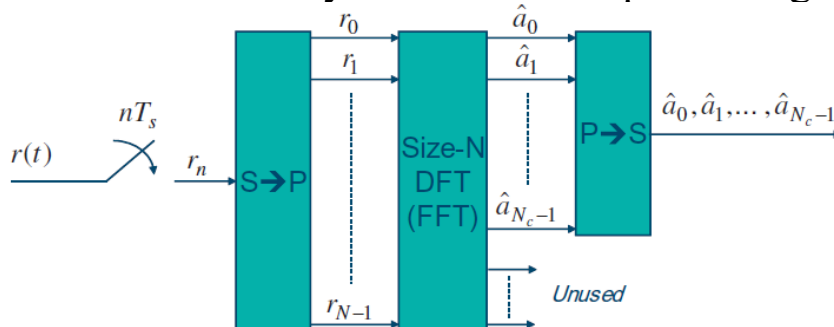
$$a'_k = \begin{cases} a_k & 0 \leq k < N_c \\ 0 & N_c \leq k < N \end{cases}$$

# OFDM Implementation

- OFDM **modulation** by means of **IFFT** processing



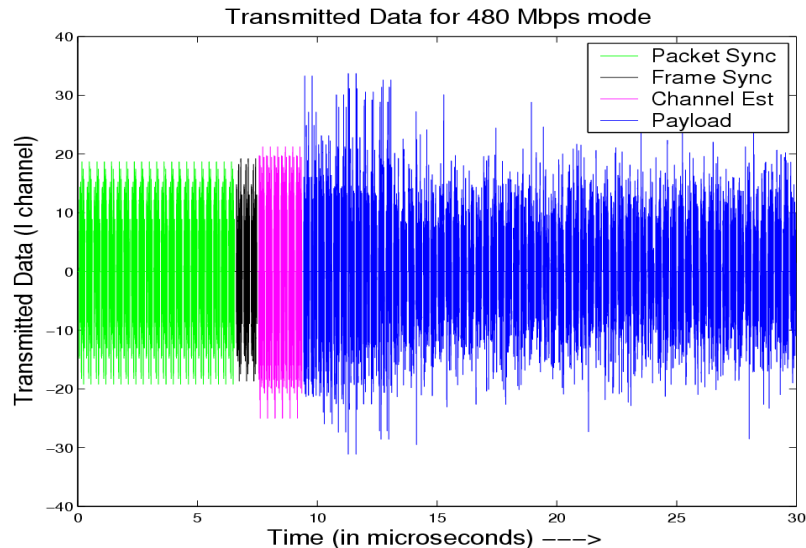
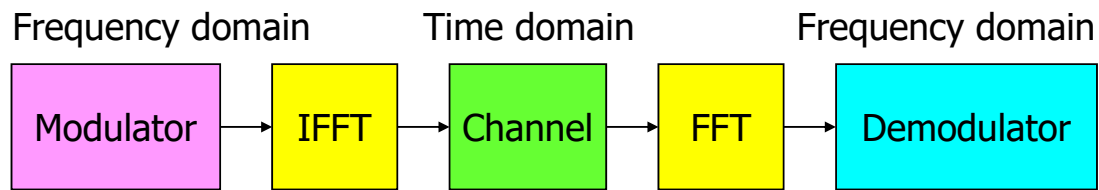
- OFDM **demodulation** by means of **FFT** processing



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# OFDM Transmission

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## OFDM Transmission (Cont.)

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- The data are divided into several **parallel data streams** or channels, one for each subcarrier.
- Each subcarrier applies a **conventional modulation scheme** (PSK, QAM, ...) at a low symbol rate.
- OFDM may be regarded as using many **slowly-modulated narrowband signals** rather than one rapidly-modulated wideband signal.
  - The channel equalization is simplified
- OFDM requires very accurate **frequency synchronization** between the receiver and the transmitter
  - Including the **carrier** frequency and **sampling** frequency
  - Otherwise **inter-carrier interference (ICI)** is introduced

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## Question

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- **Question:**
  - Why **QAM modulation** has been adopted in OFDM systems for mobile communications?
- The amplitude reference can be obtained via channel estimation
  - Some **pilot subcarriers** are inserted in OFDM systems
  - Channel gain (including phase) estimation for all subcarriers can be easily achieved
- **Question:**
  - Why **low peak-to-average ratio (PAR) modulation** is not considered in OFDM systems?
- The PAR problem in time-domain is mainly due to the signal sum of multiple subcarriers. The PAR of a subcarrier plays almost no important role.

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## Digital Signaling on Flat Fading Channels

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## Average BER – BPSK/QPSK

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- For BPSK, the probability of bit error can be expressed as

$$P_b(\gamma_b) = Q(\sqrt{2\gamma_b})$$

$$\gamma_b = \alpha^2 E_b / N_0$$

- The pdf of energy-to-noise ratio for **flat Rayleigh fading** channels is

$$p_{\gamma_b}(x) = (1/\bar{\gamma}_b) e^{-x/\bar{\gamma}_b}$$

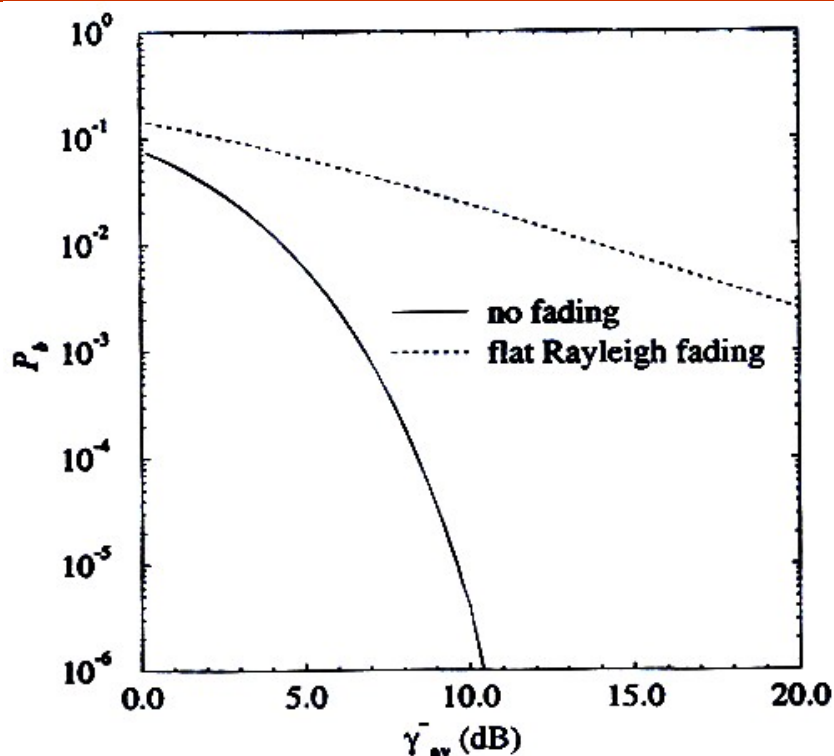
- The average probability of bit error becomes

$$\begin{aligned} P_b &= \int_0^\infty P_b(x) p_{\gamma_b}(x) dx \\ &= \int_0^\infty Q(\sqrt{2x}) p_{\gamma_b}(x) dx \\ &= \frac{1}{2} \left[ 1 - \sqrt{\frac{\bar{\gamma}_b}{1 + \bar{\gamma}_b}} \right] \approx \frac{1}{4\bar{\gamma}_b} \quad \text{for } \bar{\gamma}_b \gg 1 \end{aligned}$$

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## Average BER – BPSK/QPSK (Cont.)

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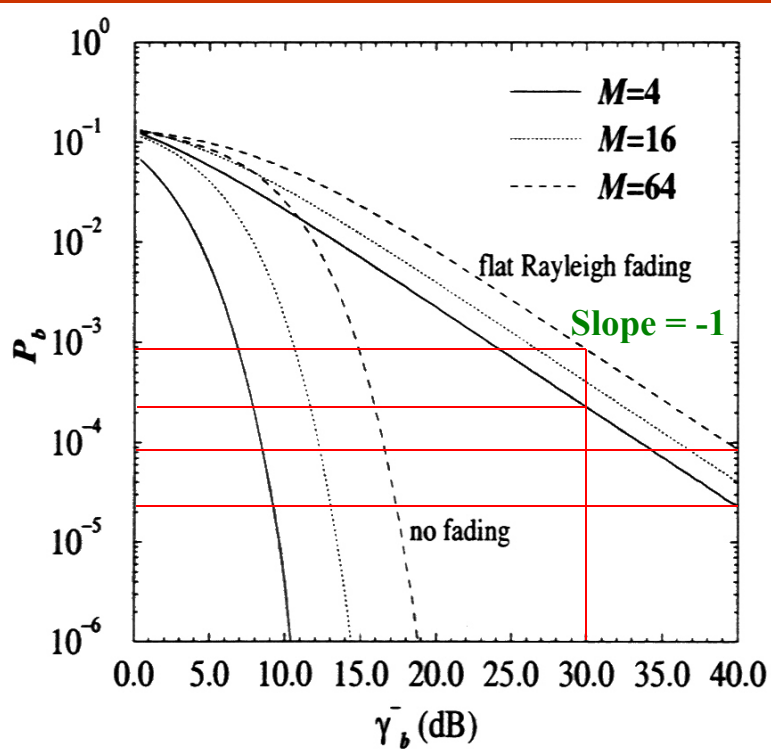




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## Average BER – M-QAM

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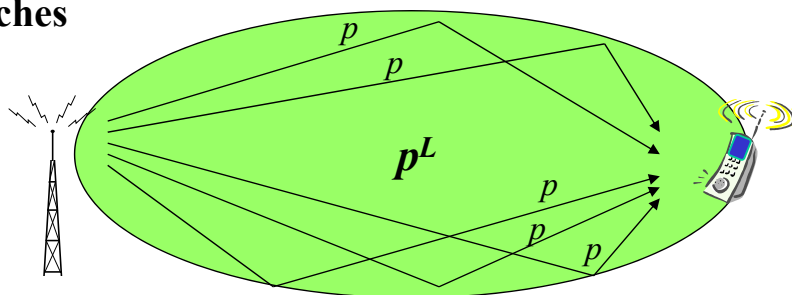
## Diversity Techniques

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# Diversity Techniques

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- **Diversity** is a very effective solution for combating fading:
  - It can provide the receiver with multiple faded replicas of the same information signal
  - Let  $p$  denote the probability that the instantaneous signal-to-noise ratio is **below** some critical threshold **on a particular diversity branch**
  - $p^L$  is the probability that all the instantaneous signal-to-noise ratios are **below** the same critical threshold **on all  $L$  diversity branches**



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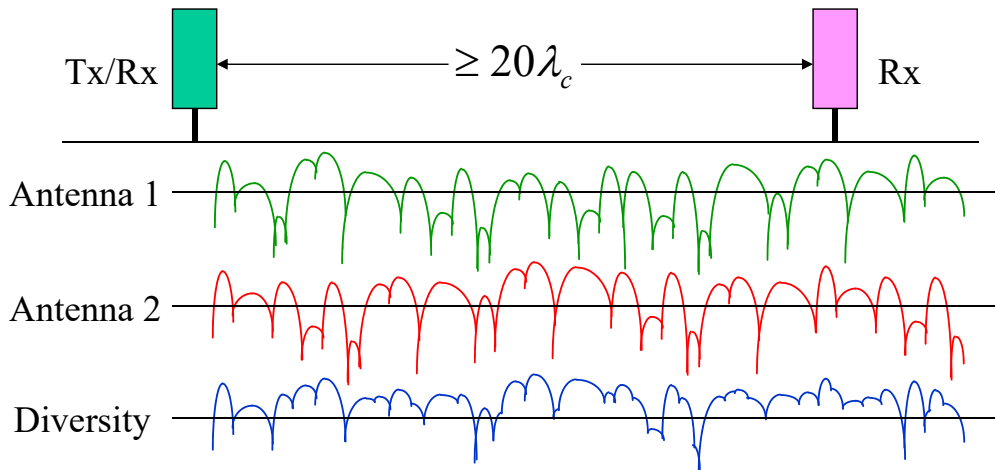
## Diversity Techniques (Cont.)

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- Diversity techniques:
  - Space
  - Angle
  - Polarization
  - Frequency
  - Multipath
  - Time

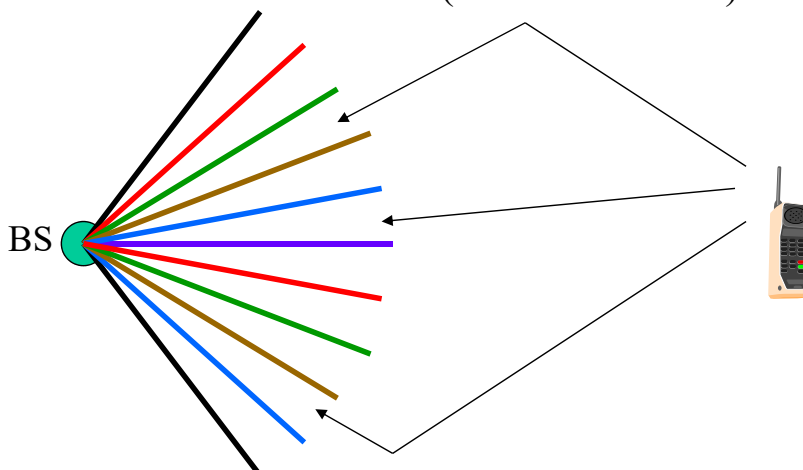
# Space Diversity

- Space diversity (generally applied at BSs):
  - Achieved by using **multiple receive antennas**
  - The **spatial separation** between the antennas is chosen so that the diversity branches experience **uncorrelated fading**



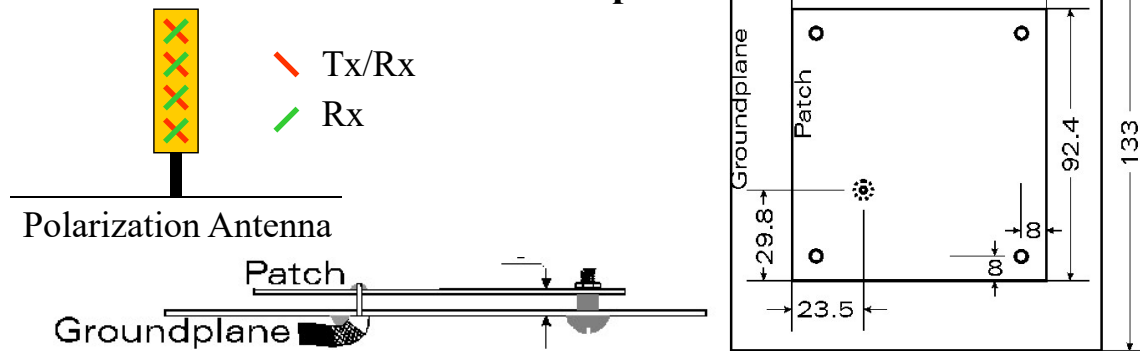
# Angle Diversity

- Angle diversity:
  - A number of **directional antennas** are required
  - Each antenna selects plane waves arriving from a narrow range of angles
  - All branches are uncorrelated (WSSUS channel)



# Polarization Diversity

- Polarization diversity:
  - The scattering environment tends to **depolarize** a signal
  - Receive antennas having different polarizations can be used to obtain diversity
  - At subscriber units: combination of **monopole** and **patch** antennas
  - At base stations: **slant 45° monopole**

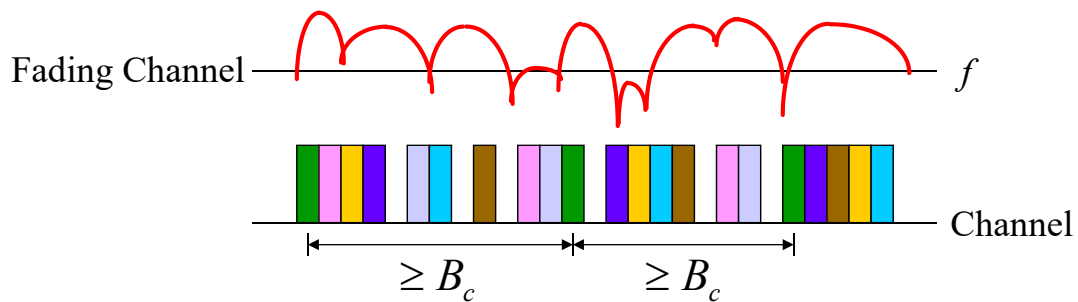


## Question

- **Question:**
  - Is **Polarization Diversity** suitable for any propagation environments?
- No. If the environment does not depolarize the desired signal, the performance of **Polarization Diversity** is bad
  - Only vertical polarization exists
- **Question:**
  - Should **Space Diversity** or **Polarization Diversity** be applied in urban area (or open area)? Why?
- In urban area, **Polarization Diversity** should be applied
  - Reduce the cost of the antenna system
- In open area, **Space Diversity** should be applied
  - To get an acceptable performance

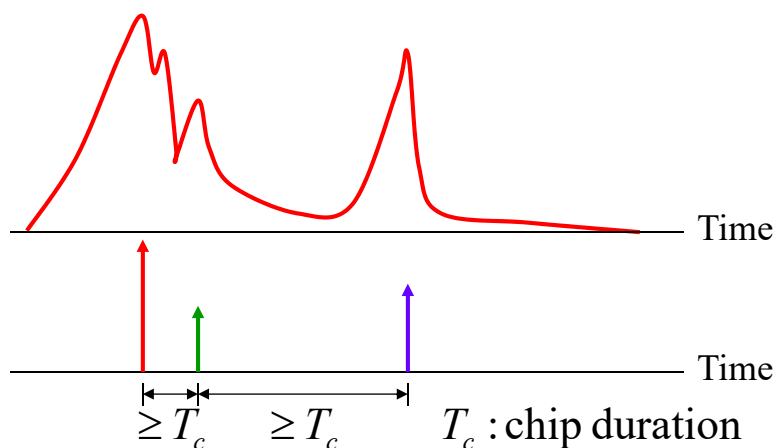
# Frequency Diversity

- Frequency diversity:
  - Using multiple frequency channels
  - Each channel is separated by at least the **coherence bandwidth** of the channel
  - **Frequency hopping spread spectrum (FHSS)** systems provide frequency diversity through the fast frequency hopping
    - ⇒ Each symbol is transmitted on multiple hops (carriers)



# Multipath Diversity

- Multipath diversity:
  - Resolving multipath components at different delays
  - Time resolution must be high enough ⇒ **Wideband signals**
  - Using **direct sequence spread spectrum (DSSS)** signaling along with a **RAKE receiver** can achieve multipath diversity

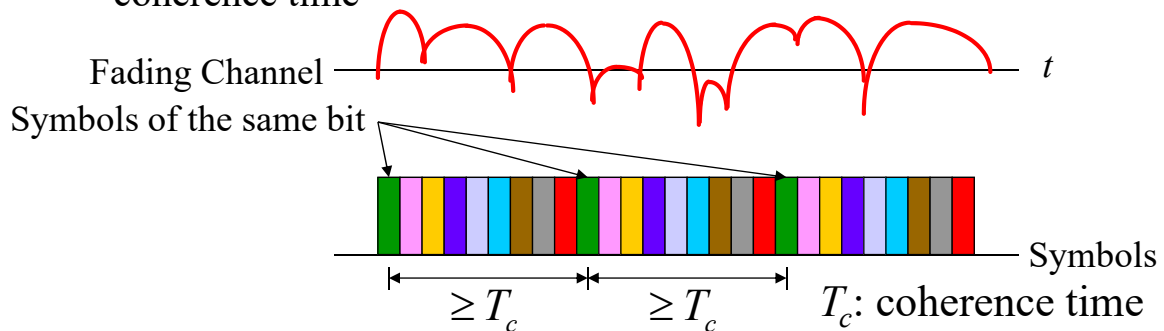


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# Time Diversity

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- Time diversity:
  - Using multiple time slots that are separated by at least the **coherence time** of the channel
  - **Error correction coding** with **interleaving** is an efficient method that provides time diversity
  - The coherence time of the channel depends on the **velocity** of the MS, and slow moving MSs have the channel with a large coherence time



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## Forward Error Correction Coding and Interleaving

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# Forward Error Correction Coding

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- **Forward error correction (FEC)** coding (or known as channel coding) is a technique used for controlling errors in data transmission
  - Over **noisy** and/or **unreliable (fading)** communication channels
- By introducing **redundancy** into an encoded message, the receiver can detect a limited number of errors that may occur anywhere in the message.
  - The **bandwidth efficiency** (throughput) is reduced, depending on the **code rate** ( $0 < r \leq 1$ )
  - For a large (small) code rate, the bandwidth efficiency is high (low), but the error-correction capability is low (high)
- FEC is also applied to **mass storage devices** to enable recovery of corrupted data and to improve the device reliability.

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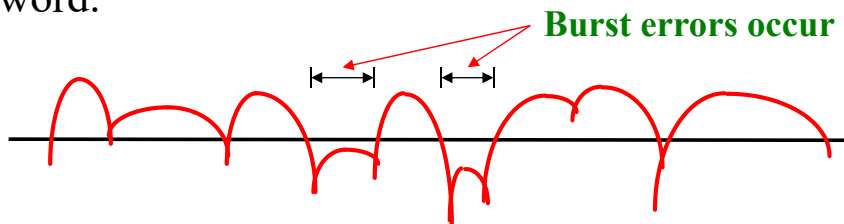
## Forward Error Correction Coding (Cont.)

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- There are two major categories of FEC codes:
  - **Block codes:** work on a fixed, predetermined number of bits or symbols (a fixed block size), such as Hamming codes, Reed-Solomon codes, BCH codes, LDPC codes, ...
    - The error-correction capability depends on both the **code rate** and the **block size**.
  - **Convolutional codes:** work on a bit or symbol stream of **arbitrary length**
    - The most often used decoding approach is soft decoding with the **Viterbi algorithm**.
    - The error-correction capability depends on both the **code rate** and the **constraint length** (the memory size in the encoder).
- FEC codes generally expects errors to be **uniformly** distributed.

# Interleaving

- **Interleaving** is frequently used in digital communications and storage systems to improve the performance of FEC codes.
- Many communication channels suffer **burst errors**, not random (independent) errors.
  - For an **AWGN channel**, the error pattern is random errors
  - For a **fading channel**, the error pattern is burst errors
- If the number of errors within a code word exceeds the error-correction capability, the decoder fails to recover the original code word.



## Interleaving (Cont.)

- Interleaving mitigates the impact of burst errors by shuffling **contiguous coded symbols** across time, frequency, or other domains
  - To make the burst errors become **uniformly distributed errors**
- Considering a sequence of coded symbols with a block size of 8 symbols, each row of symbols corresponds to a codeword
  - Without interleaving, the transmission order is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, ...

Without interleaving

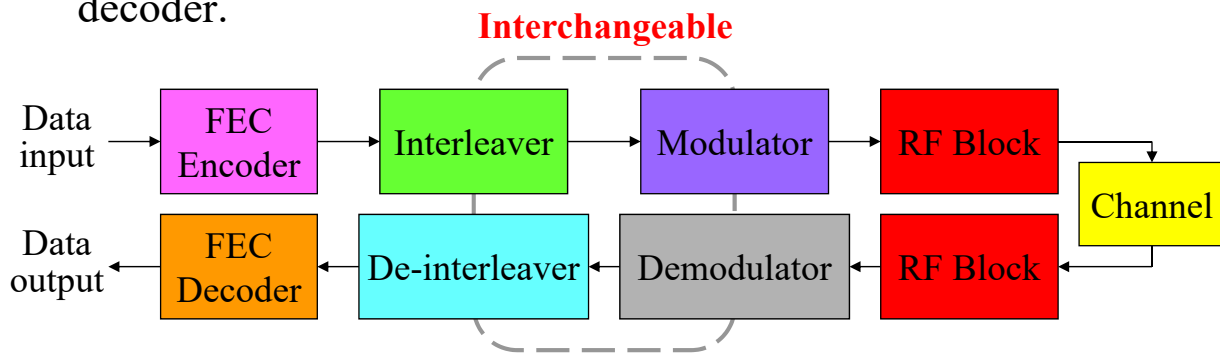
1	2	3	4	5	6	7	8
9	10	11	12	13	14	15	16
17	18	19	20	21	22	23	24
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
449	450	451	452	453	454	455	456

With interleaving



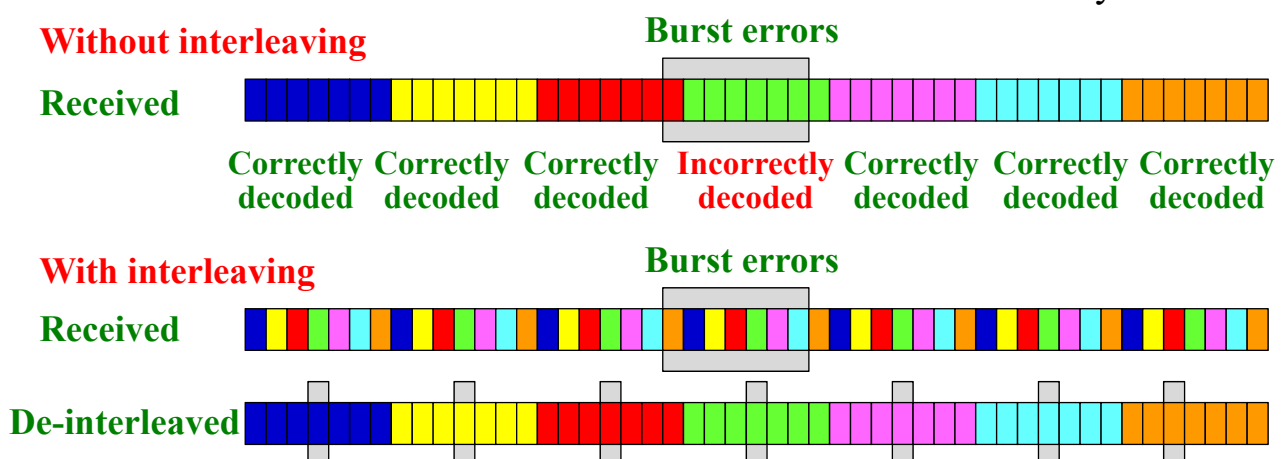
## Interleaving (Cont.)

- With interleaving, the transmission order becomes 1, 9, 17, ..., 449, 2, 10, 18, ...
  - The adjacent symbols belong to different codewords.
  - The symbols of the same codeword are interleaved across time.
- At the transmitter, the **interleaver** is set after the FEC encoder.
- At the receiver, the **de-interleaver** is set in front of the FEC decoder.



## Interleaving (Cont.)

- When passing through the propagation channel, burst errors still occur in the received bit/symbol stream
  - Without interleaving, the receiver fails to decode some blocks
  - With interleaving, the burst errors becomes almost **uniformly distributed errors** and all blocks can be decoded correctly.

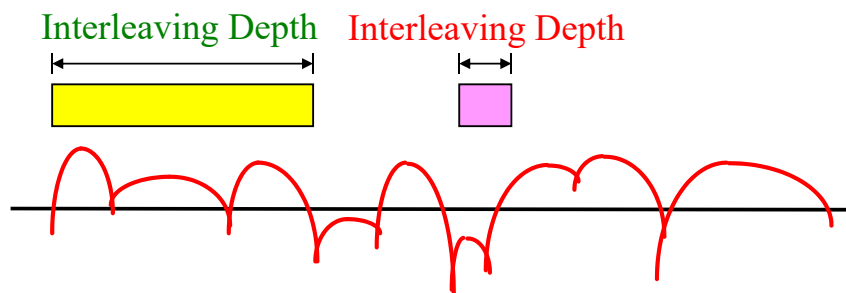


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# Interleaving and Average Fade Duration

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- The design of the **interleaving depth** relies on the **average envelope fade duration**.
- If the interleaving depth is **smaller** than the average envelope fade duration, all the symbols of a codeword may experience **the same fade duration**  $\Rightarrow$  burst errors occur in a codeword
- The interleaving depth must be **larger** than the average envelope fade duration.



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## Diversity Combining

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# Diversity Combining

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- Diversity combining refers to the method by which the signals from the diversity branches are combined
  - **Predetection** combining: diversity combining takes place **before** detection (takes place at RF)
  - **Postdetection** combining: diversity combining takes place **after** detection (takes place at baseband)
- For ideal coherent detection:
  - There is no difference in performance between predetection and postdetection combining

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## Diversity Combining (Cont.)

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- Diversity combining methods:
  - **Selective Combining (SC)**
  - **Maximal Ratio Combining (MRC)**
  - **Equal Gain Combining (EGC)**
  - **Switched Combining (SW)**
- The received complex envelopes of the diversity branches:

$$\tilde{r}_k(t) = g_k \tilde{s}(t) + \tilde{n}_k(t), \quad k = 1, \dots, L$$

- $g_k = \alpha_k e^{j\phi_k}$  is the complex fading gain associated with the  $k^{\text{th}}$  branch
  - $\tilde{n}_k(t)$  is assumed to be independent from branch to branch
- The received signal vectors are

$$\tilde{\mathbf{r}}_k = g_k \tilde{\mathbf{s}} + \tilde{\mathbf{n}}_k, \quad k = 1, \dots, L$$

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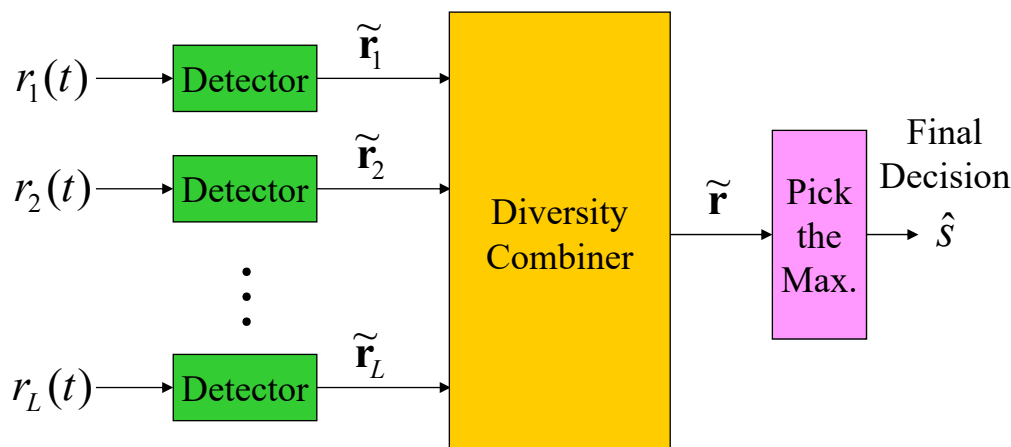
## Diversity Combining (Cont.)

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- where  $\tilde{\mathbf{r}}_k = [\tilde{r}_{k_1}, \tilde{r}_{k_2}, \dots, \tilde{r}_{k_M}]$  is the received signal vector corresponding to the  $M$  possible symbols

$$\tilde{r}_{k_m} = g_k \tilde{s}_m + \tilde{n}_{k_m}, \quad m = 1, \dots, M$$

### Postdetection combining



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## Impact of Fading Gain Correlation

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- The fading gains of the various diversity branches typically have **some degree of correlation**, which depends on
  - The type of diversity being used
  - The propagation environment
- For analytical purposes, the diversity branches are generally **assumed to be uncorrelated**
  - Branch correlation reduces the achievable diversity gain
  - The uncorrelated branch assumption gives **optimistic results**

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# Impact of Fade Distribution

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- The **fade distribution** will affect the diversity gain
  - The relative advantage of diversity is **greater for Rayleigh fading** than Ricean fading
  - Ricean factor  $K$  increases  $\Rightarrow$  less difference between the instantaneous received SNR on the various diversity branches
  - The performance will always **be better with Ricean than Rayleigh fading**

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## Selective Combining

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## Predetection Selective Combining (SC)

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- The branch with the highest signal-to-noise ratio is selected at any instant
- With Rayleigh fading, the instantaneous received bit energy-to-noise ratio on  $k$ -th diversity branch has the exponential pdf

$$p_{\gamma_k}(x) = \frac{1}{\bar{\gamma}_c} e^{-x/\bar{\gamma}_c}$$

- $\bar{\gamma}_c$  is the average received bit energy-to-noise ratio for each diversity branch
- With SC, the branch with the **largest** bit energy-to-noise ratio is always selected

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## Predetection Selective Combining (SC) (Cont.)

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- The effective instantaneous bit energy-to-noise ratio is

$$\gamma_s^S = \max\{\gamma_1, \gamma_2, \dots, \gamma_L\}$$

- $L$  is the number of diversity branches
- If the diversity branches are independently faded, the cumulative distribution function (cdf) of  $\gamma_s^S$  is

$$F_{\gamma_k}(x) = \int_0^x \frac{1}{\bar{\gamma}_c} \exp\left(-\frac{y}{\bar{\gamma}_c}\right) dy = -\exp\left(-\frac{y}{\bar{\gamma}_c}\right) \Big|_0^x = 1 - e^{-x/\bar{\gamma}_c}$$

$$F_{\gamma_s^S}(x) = P_r[\gamma_1 \leq x, \gamma_2 \leq x, \dots, \gamma_L \leq x] = \left[1 - e^{-x/\bar{\gamma}_c}\right]^L$$

- Differentiating the cdf gives the pdf

$$p_{\gamma_s^S}(x) = \frac{L}{\bar{\gamma}_c} \left[1 - e^{-x/\bar{\gamma}_c}\right]^{L-1} e^{-x/\bar{\gamma}_c}$$

## Predetection Selective Combining (SC) (Cont.)

- The average bit energy-to-noise ratio with SC is

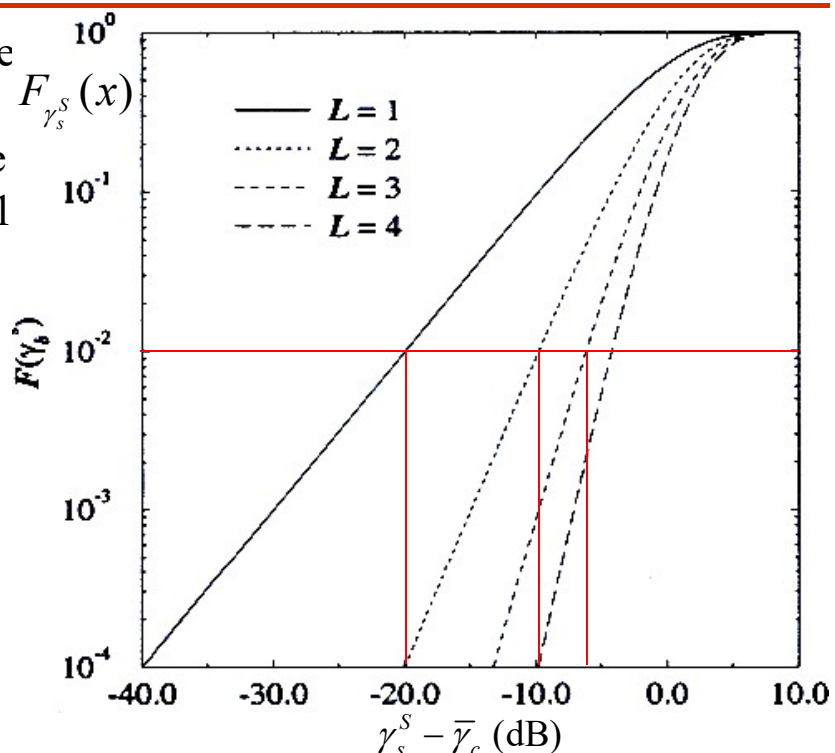
$$\begin{aligned}\bar{\gamma}_s^S &= \int_0^\infty x p_{\gamma_s^S}(x) dx \\ &= \int_0^\infty \frac{Lx}{\bar{\gamma}_c} \left[1 - e^{-x/\bar{\gamma}_c}\right]^{L-1} e^{-x/\bar{\gamma}_c} dx \\ &= \bar{\gamma}_c \sum_{k=1}^L \frac{1}{k}\end{aligned}$$

- The largest diversity gain is obtained by using **2-branch diversity**:
  - $L = 1 \rightarrow 2 \Rightarrow 1.5/1 \Rightarrow 1.761 \text{ dB}$
  - $L = 2 \rightarrow 3 \Rightarrow 1.83/1.5 \Rightarrow 0.8715 \text{ dB}$

## Predetection Selective Combining (SC) (Cont.)

- Assume the desired SNR is  $\gamma$  and the acceptable outage probability is 0.01

- For  $L = 1$   
 $\gamma - \bar{\gamma}_c = -20 \text{ (dB)}$   
 $\bar{\gamma}_c = \gamma + 20 \text{ (dB)}$
- For  $L = 2$   
 $\gamma - \bar{\gamma}_c = -10 \text{ (dB)}$   
 $\bar{\gamma}_c = \gamma + 10 \text{ (dB)}$
- For  $L = 3$   
 $\gamma - \bar{\gamma}_c = -6 \text{ (dB)}$   
 $\bar{\gamma}_c = \gamma + 6 \text{ (dB)}$



## Predetection Selective Combining (SC) (Cont.)

- The DPSK bit error probability with differential detection is:

$$P_b(\gamma_s) = \frac{1}{2} e^{-\gamma_s}$$

- With SC, the bit error probability for DPSK is:

$$\begin{aligned}
 P_b &= \int_0^\infty P_b(x) p_{\gamma_s}(x) dx \\
 &= \int_0^\infty \frac{1}{2} \exp(-x)^{\frac{L}{\bar{\gamma}_c}} [1 - \exp(-\frac{x}{\bar{\gamma}_c})]^{L-1} \exp(-\frac{x}{\bar{\gamma}_c}) dx \\
 &= \int_0^\infty \frac{L}{2\bar{\gamma}_c} \exp[-(1 + \frac{1}{\bar{\gamma}_c})x] [1 - \exp(-\frac{x}{\bar{\gamma}_c})]^{L-1} dx \\
 &= \frac{L}{2\bar{\gamma}_c} \sum_{n=0}^{L-1} C_n^{L-1} (-1)^n \int_0^\infty \exp[-(1 + \frac{1}{\bar{\gamma}_c} + \frac{n}{\bar{\gamma}_c})x] dx \\
 &= \frac{L}{2\bar{\gamma}_c} \sum_{n=0}^{L-1} C_n^{L-1} (-1)^n \frac{\bar{\gamma}_c}{\bar{\gamma}_c + n + 1} = \frac{L}{2} \sum_{n=0}^{L-1} \frac{C_n^{L-1} (-1)^n}{\bar{\gamma}_c + n + 1}
 \end{aligned}$$

Binomial expansion

$$(1-x)^{L-1} = \sum_{n=0}^{L-1} C_n^{L-1} (-1)^n x^n$$

## Predetection Selective Combining (SC) (Cont.)

- Diversity offers a very large improvement in the BER performance
- The bit error probability is now proportional to  $\bar{\gamma}_c^{-L}$ , not proportional to  $\bar{\gamma}_c^{-1}$
- The largest diversity gain is achieved with 2-branch diversity



## Predetection Selective Combining (SC) (Cont.)

- BER of DPSK

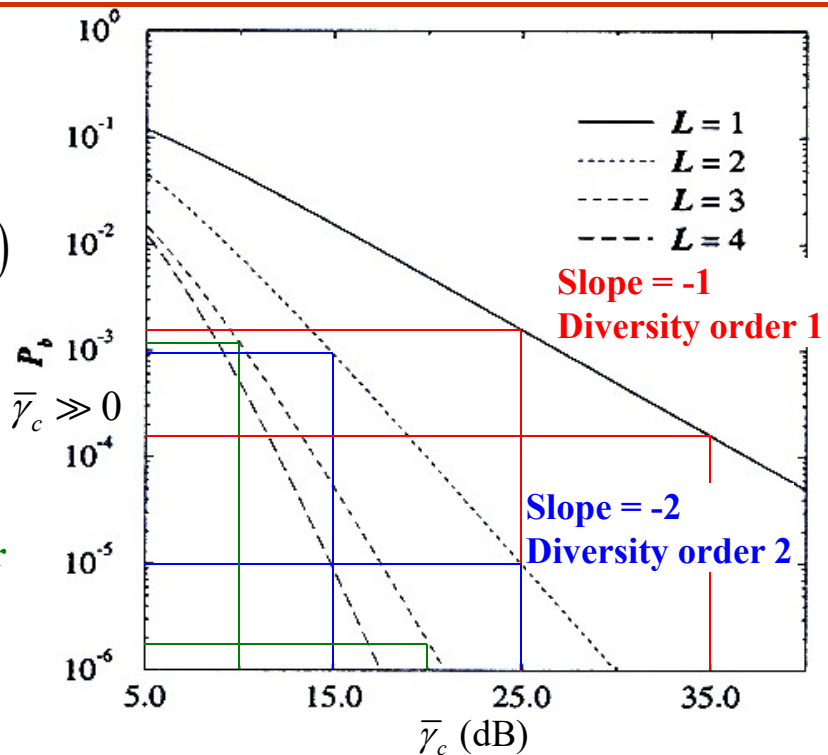
$$\text{Assume } P_b = \beta \bar{\gamma}_c^{-L}$$

$$\log_{10}(P_b) = \log_{10}(\beta \bar{\gamma}_c^{-L})$$

$$= \log_{10} \beta - L \log_{10}(\bar{\gamma}_c)$$

$$\text{Slop: } \frac{\log_{10}(P_b)}{\log_{10}(\bar{\gamma}_c)} \approx -L,$$

**Diversity order**



## Maximal Ratio Combining

# Maximal Ratio Combining (MRC)

- For Maximal Ratio Combining (MRC), the diversity branches must be **weighted** by their respective **complex fading gains** and then combined
- MRC results in a **maximum likelihood (ML)** receiver giving the **best possible performance** among the diversity combining techniques
- The ML receiver must have **complete knowledge** of the channel gain vector  $\{\mathbf{g}\}$
- Postdetection combining:
  - The weighting and combining is performed after integration
- Predetection combining:
  - The weighting and combining is performed before integration

## Maximal Ratio Combining (MRC) (Cont.)

- The received signal vector:

$$\tilde{\mathbf{r}} \triangleq (\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_L); \quad \tilde{r}_k(t) \triangleq g_k \tilde{s}(t) + \tilde{n}_k(t)$$

- The ML receiver chooses the symbol  $\tilde{s}_m$  that maximizes the metric

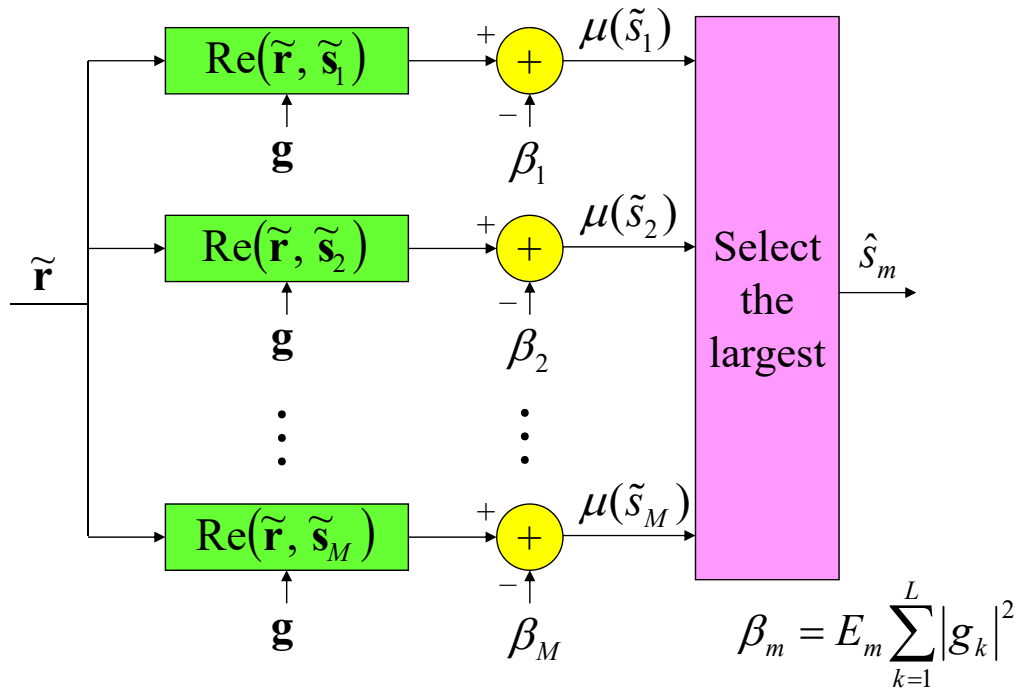
$$\mu_1(\tilde{s}_m) = -\sum_{k=1}^L |\tilde{r}_k - g_k \tilde{s}_m|^2 = -\left( \sum_{k=1}^L |\tilde{r}_k|^2 - 2 \operatorname{Re}\{\tilde{r}_k, g_k \tilde{s}_m\} + |g_k \tilde{s}_m|^2 \right)$$

$$\begin{aligned} \mu_2(\tilde{s}_m) &= \sum_{k=1}^L \operatorname{Re}\left\{ \int_0^T \tilde{r}_k(t) \times g_k^* \tilde{s}_m^*(t) dt \right\} - \sum_{k=1}^L |g_k|^2 E_m \\ &= \sum_{k=1}^L \operatorname{Re}\left\{ g_k^* \int_0^T \tilde{r}_k(t) \tilde{s}_m^*(t) dt \right\} - E_m \sum_{k=1}^L |g_k|^2 \end{aligned}$$

$2\mu_2(\tilde{s}_m)$   
 May be different for different  $S_m$

**The inner product:**  $(u, v) = \int u(t) \times v^*(t) dt$        $\|\tilde{s}_m\|^2 = 2E_m$

# Metric Computer for MRC



## Maximal Ratio Combining (MRC) (Cont.)

- If all symbols have equal energy, the metric becomes

$$\mu_3(\tilde{s}_m) = \sum_{k=1}^L \operatorname{Re} \left\{ g_k^* \int_0^T \tilde{r}_k(t) \tilde{s}_m^*(t) dt \right\}$$

Equal energy symbols  
Postdetection combining

- An alternative form of the ML receiver can also be obtained by rewriting the metric

$$\mu_4(\tilde{s}_m) = \int_0^T \operatorname{Re} \left\{ \sum_{k=1}^L g_k^* \tilde{r}_k(t) \times \tilde{s}_m^*(t) \right\} dt - E_m \sum_{k=1}^L |g_k|^2$$

Predetection combining

- Equivalently, the signal used for demodulation is

$$\tilde{r}(t) = \sum_{k=1}^L g_k^* \tilde{r}_k(t) = \sum_{k=1}^L g_k^* g_k \tilde{s}(t) + \sum_{k=1}^L g_k^* \tilde{n}_k(t)$$

$g_k = \alpha_k e^{j\phi_k}; \quad g_k^* g_k = \alpha_k^2$

**Weighting factor**  
**Channel gain**

## Maximal Ratio Combining (MRC) (Cont.)

- The channel-gain envelope of the composite signal is

$$\alpha_c = \sum_{k=1}^L \alpha_k^2$$

- Assuming that all branches have the same noise power, the sum of the branch **noise powers** is

$$\sigma_{\tilde{n},tot}^2 = N_0 \sum_{k=1}^L \alpha_k^2 \quad \leftarrow \text{Weighted by the channel gains}$$

- The symbol energy-to-noise ratio with MRC is

$$\gamma_s^{MR} = \frac{\alpha_c^2 E_{av}}{\sigma_{\tilde{n},tot}^2} = \sum_{k=1}^L \left( \alpha_k^2 \sum_{j=1}^L \alpha_j^2 E_{av} \right) / \left( N_0 \sum_{j=1}^L \alpha_j^2 \right) = \sum_{k=1}^L \frac{\alpha_k^2 E_{av}}{N_0} = \sum_{k=1}^L \gamma_k$$

— where  $E_{av}$  is the average received symbol energy and  
**Average received symbol energy**

$$\gamma_k = \alpha_k^2 E_{av} / N_0$$

## Maximal Ratio Combining (MRC) (Cont.)

- $\gamma_s^{MR}$  is the sum of the bit energy-to-noise ratios of all diversity branches
- If all diversity branches provide the same average power and the branches are uncorrelated, then  $\gamma_s^{MR}$  has a chi-square distribution with  $2L$  degrees of freedom

$$p_{\gamma_s^{MR}}(x) = \frac{1}{(L-1)! (\bar{\gamma}_c)^L} x^{L-1} e^{-x/\bar{\gamma}_c}$$

— where

$$\bar{\gamma}_c = E[\gamma_k], \quad k = 1, \dots, L$$

- The cdf of  $\gamma_s^{MR}$ :

$$F_{\gamma_s^{MR}}(x) = 1 - e^{-x/\bar{\gamma}_c} \sum_{k=0}^{L-1} \frac{1}{k!} \left( \frac{x}{\bar{\gamma}_c} \right)^k$$

## Maximal Ratio Combining (MRC) (Cont.)

- The average symbol energy-to-noise ratio with MRC is

$$\bar{\gamma}_s^{MR} = E[\gamma_s^{MR}] = E\left[\sum_{k=1}^L \gamma_k\right] = \sum_{k=1}^L E[\gamma_k] = \sum_{k=1}^L \bar{\gamma}_k = \sum_{k=1}^L \bar{\gamma}_c = L\bar{\gamma}_c$$

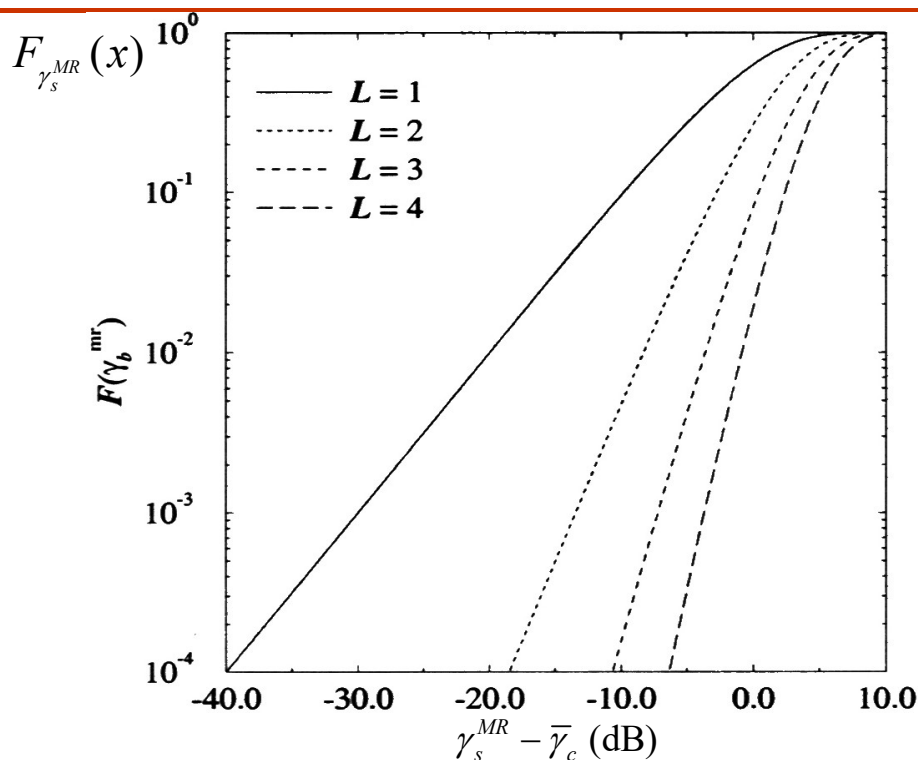
- For SC,  $L = 2$ :  $F_{\gamma_s^S}(x) = 10^{-4}$  at  $\gamma_s^S - \bar{\gamma}_c = -20$  dB
- For MRC,  $L = 2$ :  $F_{\gamma_s^{MR}}(x) = 10^{-4}$  at  $\gamma_s^{MR} - \bar{\gamma}_c = -18$  dB
  - MRC is 2 dB more effective than SC
- The bit error probability of BPSK is

$$P_b = \int_0^\infty P_b(x) p_{\gamma_s^{MR}}(x) dx = \int_0^\infty Q(\sqrt{2x}) \frac{1}{(L-1)!(\bar{\gamma}_c)^L} x^{L-1} e^{-x/\bar{\gamma}_c} dx$$

$$= \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} C_k^{L-1+k} \left(\frac{1+\mu}{2}\right)^k$$

$$\mu = \sqrt{\frac{\bar{\gamma}_c}{1+\bar{\gamma}_c}}$$

## Maximal Ratio Combining (MRC) (Cont.)

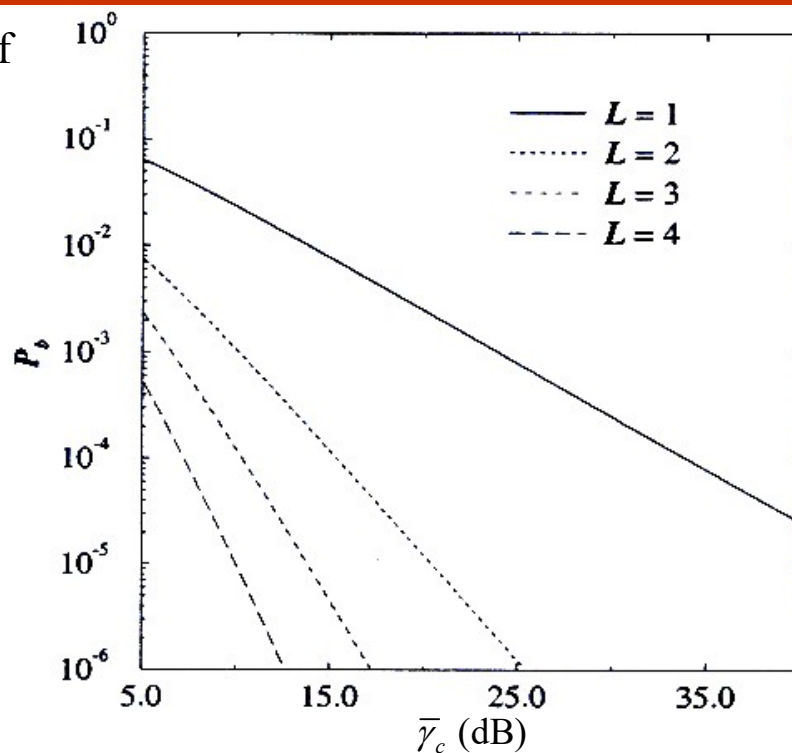


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## Maximal Ratio Combining (MRC) (Cont.)

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- BER of BPSK



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## Coherent Equal Gain Combining

# Coherent Equal Gain Combining (EGC)

- The **coherent** Equal Gain Combining (EGC) is similar to MRC, but the diversity branches are **not weighted**
- EGC is useful for the modulation techniques having **equal energy symbols** (no channel information  $\mathbf{g}$  is required), e.g. M-PSK
- For signals with unequal energy, the channel vector  $\mathbf{g}$  is required and MRC (**best possible performance**) should be used
- The ML receiver chooses the symbol  $\tilde{s}_m$  that maximizes the metric

$$\mu(\tilde{s}_m) = \sum_{k=1}^L \operatorname{Re} \left\{ \tilde{r}_k e^{-j\phi_k}, \tilde{s}_m \right\} = \sum_{k=1}^L \operatorname{Re} \left\{ \int_0^T \underbrace{e^{-j\phi_k}}_{\text{Phase information is still required}} \tilde{r}_k(t) \tilde{s}_m^*(t) dt \right\}$$

# Coherent Equal Gain Combining (EGC) (Cont.)

$$\mu(\tilde{s}_m) = \operatorname{Re} \left\{ \sum_{k=1}^L e^{-j\phi_k} \tilde{r}_k, \tilde{s}_m \right\} = \int_0^T \operatorname{Re} \left\{ \left( \sum_{k=1}^L e^{-j\phi_k} \tilde{r}_k(t) \right) \tilde{s}_m^*(t) \right\} dt$$

- For pre-detection combining, the signal after the diversity combiner is
- $$\tilde{\mathbf{r}}(t) = \sum_{k=1}^L e^{-j\phi_k} \tilde{r}_k(t) = \sum_{k=1}^L e^{-j\phi_k} g_k \tilde{s}(t) + \sum_{k=1}^L e^{-j\phi_k} \tilde{n}_k(t)$$
- The vector  $\tilde{\mathbf{r}}$  is then applied to the metric computer for MRC with  $\beta_m = 0, m = 1, \dots, L$
  - The channel-gain envelope of the composite signal is

$$\alpha_E = \sum_{k=1}^L \alpha_k$$

**The symbol energy  $E_m$  is excluded**

- Assuming that all branches have the same noise power, the sum of the branch **noise powers** is  $LN_0$

## Coherent Equal Gain Combining (EGC) (Cont.)

- The symbol energy-to-noise ratio with EGC is

$$\gamma_s^{EG} = \frac{\alpha_E^2 E_{av}}{LN_0}$$

– where  $E_{av}$  is the average symbol energy

- The cdf and pdf of  $\gamma_s^{EG}$  do not exist in closed form for  $L > 2$
- For  $L = 2$  and  $\bar{\gamma}_1 = \bar{\gamma}_2 = \bar{\gamma}_c$ , the cdf and pdf are

$$F_{\gamma_s^{EG}}(x) = 1 - e^{-2x/\bar{\gamma}_c} - \sqrt{\pi \frac{x}{\bar{\gamma}_c}} e^{-x/\bar{\gamma}_c} \left( 1 - 2Q\left(\sqrt{2 \frac{x}{\bar{\gamma}_c}}\right) \right)$$

$$p_{\gamma_s^{EG}}(x) = \frac{1}{\bar{\gamma}_c} e^{-2x/\bar{\gamma}_c} + \sqrt{\pi} e^{-x/\bar{\gamma}_c} \left( \frac{1}{2\sqrt{x\bar{\gamma}_c}} - \frac{1}{\bar{\gamma}_c} \sqrt{\frac{x}{\bar{\gamma}_c}} \right) \times \left( 1 + 2Q\left(\sqrt{2 \frac{x}{\bar{\gamma}_c}}\right) \right)$$

## Coherent Equal Gain Combining (EGC) (Cont.)

- The average symbol energy-to-noise ratio with EGC is

$$\bar{\gamma}_s^{EG} = \frac{E_{av}}{LN_0} E[\alpha_E^2] = \frac{E_{av}}{LN_0} E\left[\left(\sum_{k=1}^L \alpha_k\right)^2\right] = \frac{E_{av}}{LN_0} \sum_{k=1}^L \sum_{\ell=1}^L E[\alpha_k \alpha_\ell]$$

- For an **uncorrelated** Rayleigh fading channel:

$$E[\alpha_k^2] = 2\sigma^2 \text{ and } E[\alpha_k \alpha_\ell] = E[\alpha_k] E[\alpha_\ell] = \left(\sqrt{\pi/2}\sigma\right)^2 = \pi\sigma^2/2$$

$$\begin{aligned} \bar{\gamma}_s^{EG} &= \frac{E_{av}}{LN_0} \left( 2L\sigma^2 + L(L-1) \frac{\pi\sigma^2}{2} \right) \\ &= \frac{2\sigma^2 E_{av}}{N_0} \left( 1 + (L-1) \frac{\pi}{4} \right) = \bar{\gamma}_c \left( 1 + (L-1) \frac{\pi}{4} \right) \leq L\bar{\gamma}_c \end{aligned}$$

- The bit error probability of coherent BPSK (2-branch):

$$P_b = \int_0^\infty P_b(x) p_{\gamma_s^{EG}}(x) dx = \frac{1}{2} \left( 1 - \sqrt{1 - \mu^2} \right), \quad \mu = \frac{1}{1 + \bar{\gamma}_c}$$



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# Switched Combining

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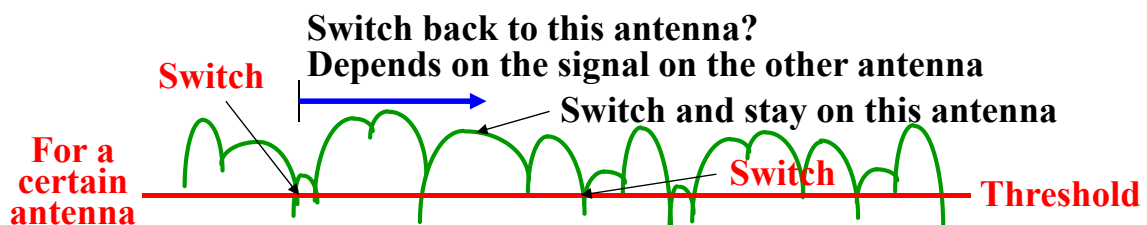
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## Predetection Switched Combining (SW)

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- A switched combiner **scans** through the diversity branches until it finds one that has a signal-to-noise ratio **exceeding a specified threshold**
- This diversity branch is used until the signal-to-noise ratio again **drops below the threshold**
- Two-branch **Switch and Stay Combining (SSC)**:
  - The receiver switches to, and stays with, the alternate branch when the bit energy-to-noise ratio drops below the threshold
  - It does this regardless of whether or not the bit energy-to-noise ratio of the alternate branch is **above or below** the threshold



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## Predetection Switched Combining (SW) (Cont.)

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- Assume that the symbol energy-to-noise ratios associated with the two branches are  $\gamma_1$  and  $\gamma_2$  and the switching threshold is  $T$
- The pdf of the received bit energy-to-noise ratio is

$$p_{\gamma_k}(x) = \frac{1}{\bar{\gamma}_c} e^{-x/\bar{\gamma}_c}$$

- Define

$$q = \Pr[\gamma_i < T] = 1 - e^{-T/\bar{\gamma}_c}$$

$$p = \Pr[\gamma_i \leq S] = 1 - e^{-S/\bar{\gamma}_c}$$

**$T$  is the threshold  
 $S$  is an arbitrary value**

- The symbol energy-to-noise ratio at the output of the switched combiner is  $\gamma_s^{SW}$

$$\Pr[\gamma_s^{SW} \leq S] = \Pr[\gamma_s^{SW} \leq S | \gamma_s^{SW} = \gamma_1] \cup \Pr[\gamma_s^{SW} \leq S | \gamma_s^{SW} = \gamma_2]$$

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## Predetection Switched Combining (SW) (Cont.)

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- $\gamma_1$  is statistically identical to  $\gamma_2 \Rightarrow$  Assuming that branch 1 is currently in use, we have the cdf of  $\gamma_s^{SW}$  expressed as

$$\Pr[\gamma_s^{SW} \leq S] = \begin{cases} \Pr[\{\gamma_1 < T\} \cap \{\gamma_2 \leq S\}], & S < T \\ \Pr[\{T \leq \gamma_1 \leq S\} \cup \{\gamma_1 < T \cap \gamma_2 \leq S\}], & S \geq T \end{cases}$$

- Since  $\gamma_1$  and  $\gamma_2$  are independent, we have

$$\Pr[\{\gamma_1 < T\} \cap \{\gamma_2 \leq S\}] = qp$$

$$\Pr[\{T \leq \gamma_1 \leq S\} \cup \{\gamma_1 < T \cap \gamma_2 \leq S\}] = (p - q) + qp$$

$$\Pr[\gamma_s^{SW} \leq S] = \begin{cases} qp, & S < T \\ p - q + qp, & S \geq T \end{cases}$$

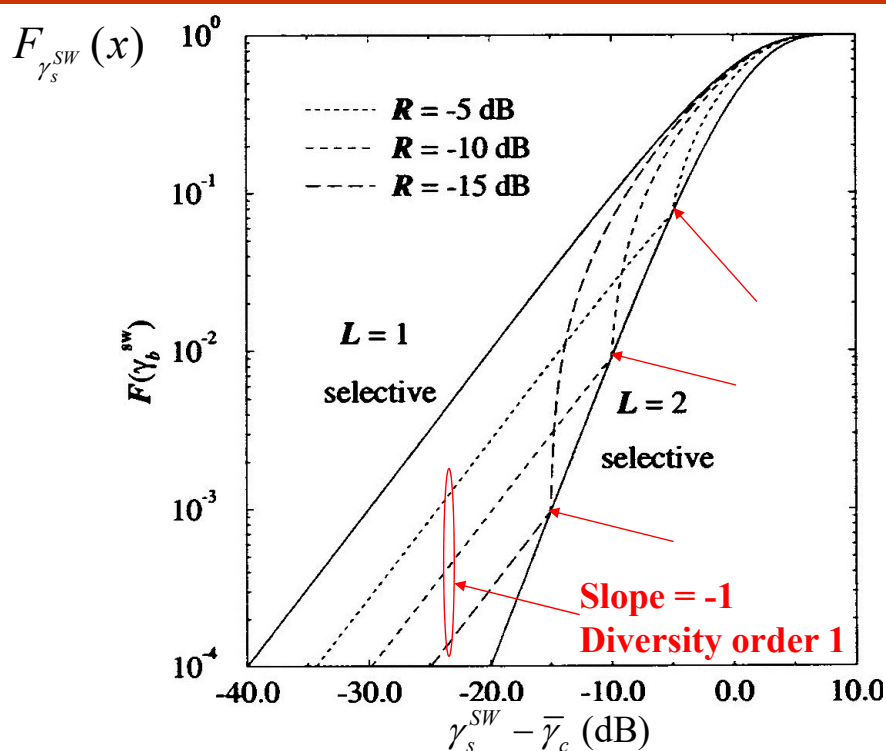
## Predetection Switched Combining (SW) (Cont.)

- The normalized threshold is defined as

$$R = 10 \log_{10}(T/\bar{\gamma}_c) \text{ (dB)}$$

- SSC always performs **worse than SC**
  - Except at the **switching threshold**, where the performance is the same as SC
- The threshold level  $T$  should be chosen as  $\gamma_{th}$ 
  - The minimum acceptable instantaneous symbol energy-to-noise ratio
- The optimum threshold,  $T = R\bar{\gamma}_c$ , depends on  $\bar{\gamma}_c$ 
  - Since  $\bar{\gamma}_c$  varies due to path loss and shadowing, the normalized threshold  $R$  must be adaptive

## Predetection Switched Combining (SW) (Cont.)



## Predetection Switched Combining (SW) (Cont.)

- The pdf of  $\gamma_s^{SW}$  is

$$p_{\gamma_s^{SW}}(x) = \begin{cases} q \frac{1}{\bar{\gamma}_c} e^{-x/\bar{\gamma}_c}, & x < T \\ (1+q) \frac{1}{\bar{\gamma}_c} e^{-x/\bar{\gamma}_c}, & x \geq T \end{cases}$$

- If binary DPSK is used, the probability of error is

$$P_b = \int_0^\infty P_b(x) p_{\gamma_s^{SW}}(x) dx$$

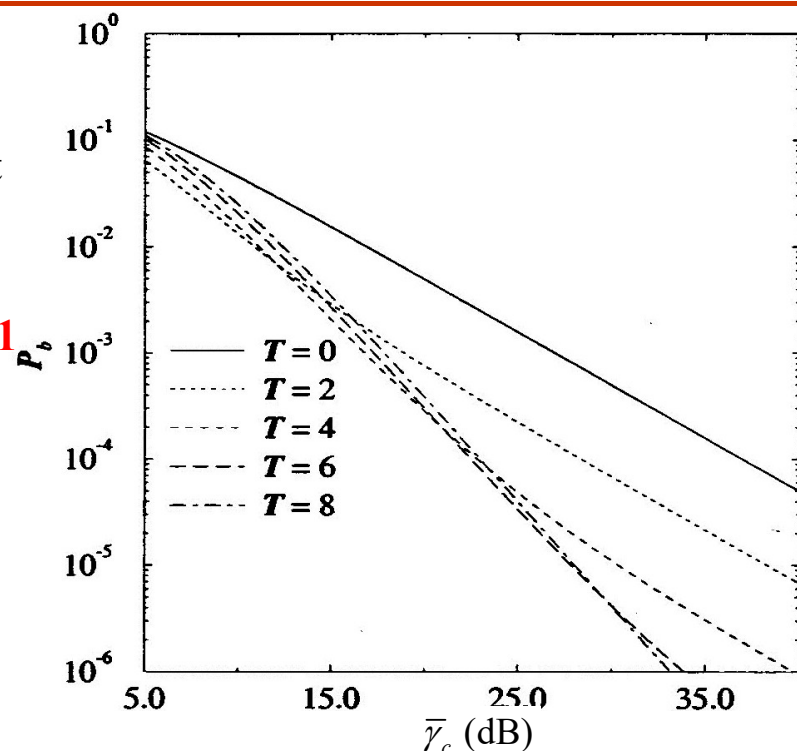
$$\text{Diversity order} = 1 = \frac{1}{2(1+\bar{\gamma}_c)} \underbrace{(q + (1-q)e^{-T})}_{\text{Almost a constant for } \bar{\gamma}_c \gg 0}$$

- The performance with  $T = 0$  (no switching): no diversity at all
- The performance changes little for  $T > 6$

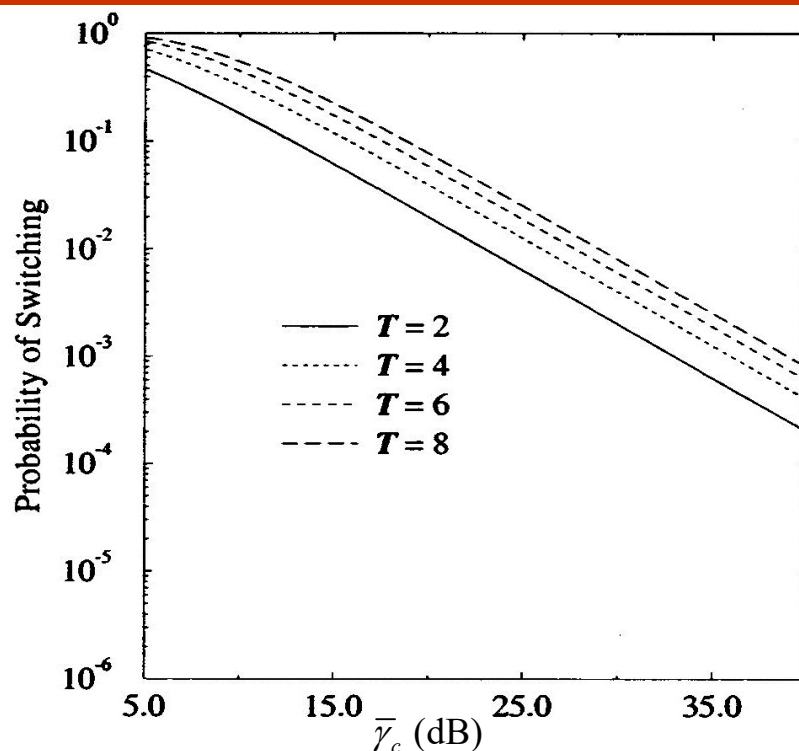
## Predetection Switched Combining (SW) (Cont.)

- BER of DPSK
- When  $\bar{\gamma}_c \gg 0$ , the receiver almost always stays at the same antenna

$\Rightarrow$  **Diversity order = 1**



## Predetection Switched Combining (SW) (Cont.)



## Question

- In theory, the performance comparison for different diversity schemes is  $MRC > EGC > SC > SW$
- **Question:**
  - Is the performance of MRC surely better than that of EGC in practical situations?
- Not necessary. The performance of MRC depends on the accuracy of channel gain estimation.
  - Only when channel gain estimation is good enough
- **Question:**
  - If the signal is received by two or more BSs, can diversity combining still be applied? Which schemes can be applied?
- When MS is in the soft-handoff state. Only the selective combining scheme can be applied (Combine multiple decoded frames - Macroscopic Diversity).

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# Transmit Diversity

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## Transmit Diversity (TD)

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- Transmit diversity uses **multiple transmit antennas** to provide the receiver with multiple uncorrelated replicas of the same signal
  - The complexity of having multiple antennas may be shared among many receivers
  - Generally used in **forward link** (base-to-mobile)
- The receiver with only **one** antenna can still benefit from a diversity gain

# Space-Time Transmit Diversity

- Assume that two transmit antennas and one receiver antenna are used
- In any given baud period, two data symbols are transmitted simultaneously from the two transmit antennas
  - The symbols transmitted from Antenna 1 and 2 are  $\tilde{\mathbf{s}}_{(1)}$  and  $\tilde{\mathbf{s}}_{(2)}$
  - During the next baud period, the symbols transmitted from Antenna 1 and 2 are  $-\tilde{\mathbf{s}}_{(2)}^*$  and  $\tilde{\mathbf{s}}_{(1)}^*$
- The channel gains for the two antennas are  $g_1(t)$  and  $g_2(t)$
- Assume that the channel stays **constant** over **two** baud intervals

$$g_k(t) = g_k(t + T) = g_k = \alpha_k e^{j\phi_k}$$

- where  $T$  is the baud period

## Space-Time Transmit Diversity – 2×1

- The received complex signal vectors are:

$$\tilde{\mathbf{r}}_{(1)} = g_1 \tilde{\mathbf{s}}_{(1)} + g_2 \tilde{\mathbf{s}}_{(2)} + \tilde{\mathbf{n}}_{(1)}$$

$$\tilde{\mathbf{r}}_{(2)} = -g_1 \tilde{\mathbf{s}}_{(2)}^* + g_2 \tilde{\mathbf{s}}_{(1)}^* + \tilde{\mathbf{n}}_{(2)}$$

- where  $\tilde{\mathbf{r}}_{(1)}$  and  $\tilde{\mathbf{r}}_{(2)}$  represent the received vectors at  $t$  and  $t+T$

- The diversity combiner constructs the following two signal vectors:

$$\tilde{\mathbf{v}}_{(1)} = g_1^* \tilde{\mathbf{r}}_{(1)} + g_2 \tilde{\mathbf{r}}_{(2)}^*$$

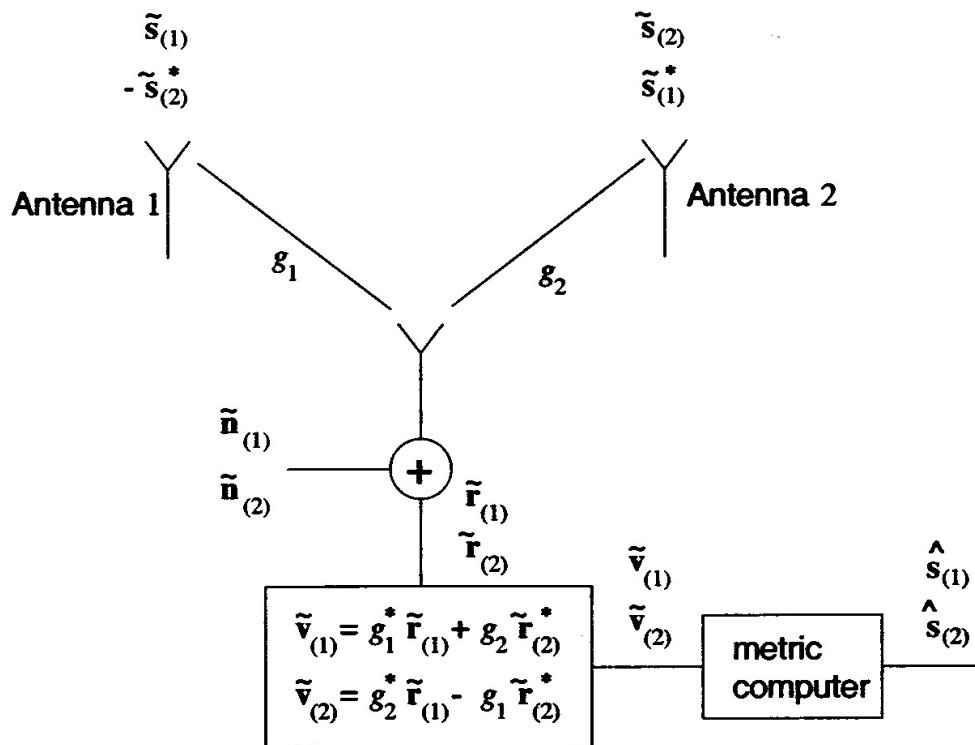
$$\tilde{\mathbf{v}}_{(2)} = g_2^* \tilde{\mathbf{r}}_{(1)} - g_1 \tilde{\mathbf{r}}_{(2)}^*$$

$$\tilde{\mathbf{v}}_{(1)} = (\alpha_1^2 + \alpha_2^2) \tilde{\mathbf{s}}_{(1)} + g_1^* \tilde{\mathbf{n}}_{(1)} + g_2 \tilde{\mathbf{n}}_{(2)}^*$$

$$\tilde{\mathbf{v}}_{(2)} = (\alpha_1^2 + \alpha_2^2) \tilde{\mathbf{s}}_{(2)} - g_1 \tilde{\mathbf{n}}_{(2)}^* + g_2^* \tilde{\mathbf{n}}_{(1)}$$

$$g_1^* g_1 = (\alpha_1 e^{-j\phi_1})(\alpha_1 e^{j\phi_1}) = \alpha_1^2$$

## Space-Time Transmit Diversity – 2×1 (Cont.)



## Space-Time Transmit Diversity – 2×1 (Cont.)

- Applying the metric computer for MRC, we can make the decisions by maximizing the metric

$$\mu(\tilde{s}_{(1),m}) = \text{Re}(\tilde{v}_{(1)}, \tilde{s}_{(1),m}) - E_m(|g_1|^2 + |g_2|^2)$$

$$\mu(\tilde{s}_{(2),m}) = \text{Re}(\tilde{v}_{(2)}, \tilde{s}_{(2),m}) - E_m(|g_1|^2 + |g_2|^2)$$

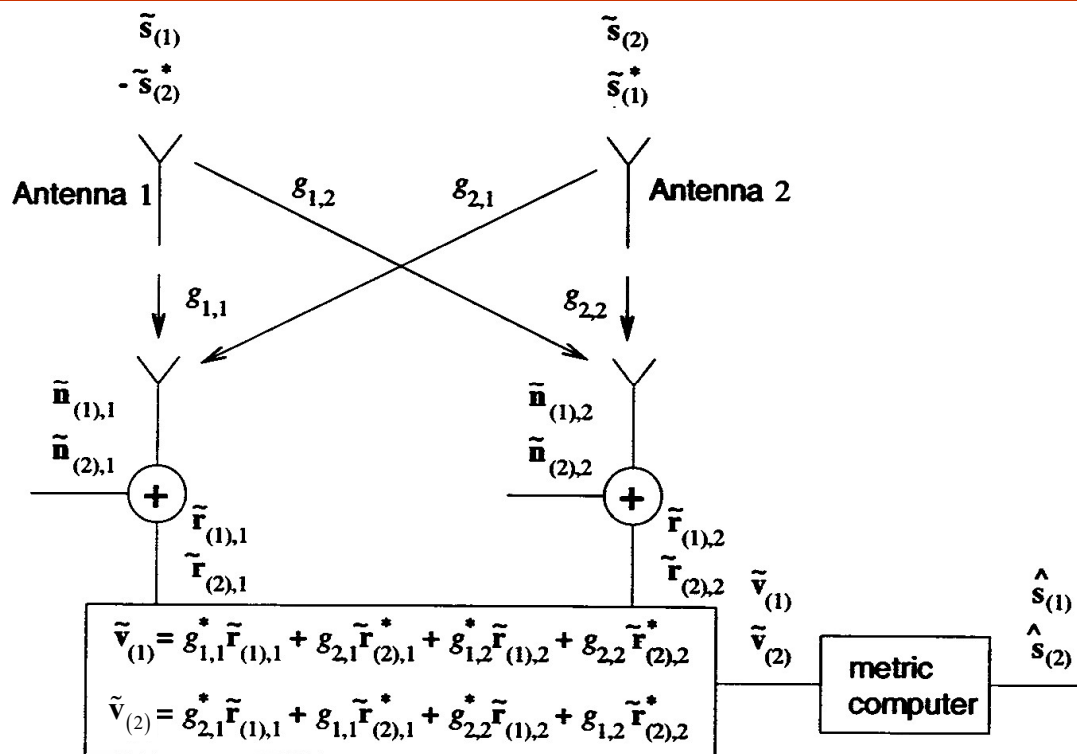
- Compared with the output of the MRC with  $L = 2$

$$\begin{aligned} \tilde{\mathbf{r}} &= g_1^* \tilde{\mathbf{r}}_1 + g_2^* \tilde{\mathbf{r}}_2 \\ &= (\alpha_1^2 + \alpha_2^2) \tilde{\mathbf{s}}_m + g_1^* \tilde{\mathbf{n}}_1 + g_2^* \tilde{\mathbf{n}}_2 \end{aligned}$$

- The combined signals are the same, except for the **phase rotations of the noise vectors**



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## Signal Space Diversity (SSD)

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- The SSD technology was proposed to exploit **diversity gain** for the transmission through fading channels.
- At the **transmitter**, the input information bits are modulated to form a set of complex symbols, such as M-PSK or QAM.
- Subsequently, the modulated signals are **phase rotated** by a predetermined angle and followed by an ideal **component interleaving** process.
  - Interleave the **I-part** and **Q-part** components of multiple symbols **to form a set of new symbols**, in each of which **the two components come from two original symbols**.
- Then the new (**interleaved and combined**) symbols are transmitted through the propagation channels.

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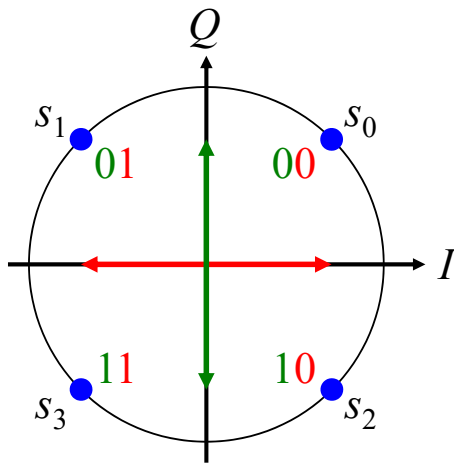
## Signal Space Diversity (SSD) (Cont.)

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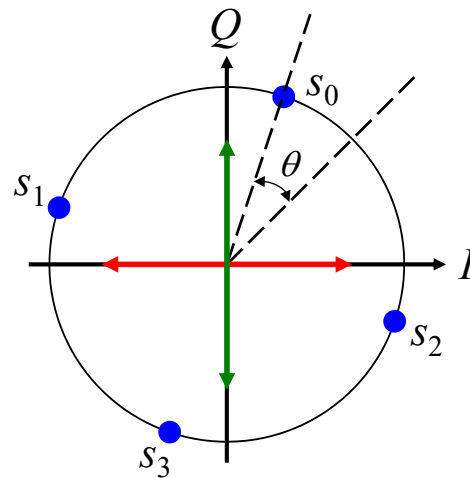
- At the **receiver**, the signals received on all channels are **component de-interleaved** to reconstruct the symbols corresponding to the original symbols.
- Multiple original symbols are involved in using the common channels
  - All related symbols are jointly detected based on the **maximum likelihood (ML)** criterion.
- If  $L$  original symbols are involved in SSD, an diversity order  $L$  can be achieved.
- No additional power or bandwidth resources are required.

## Signal Space Diversity (SSD) (Cont.)

- Without phase rotation, **I-part/Q-part** depends only on one bit
- With phase rotation, **I-part/Q-part** depends on the two bits



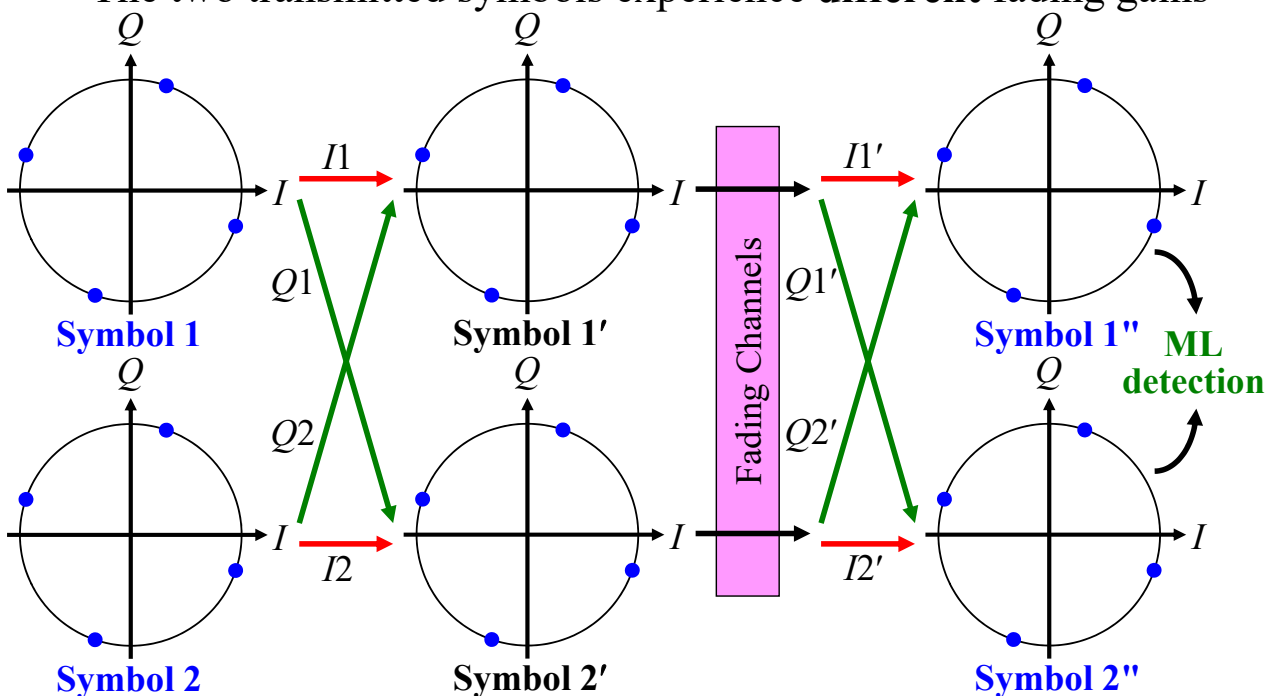
↔ I-part: representing the bit 0 or 1  
 ↔ Q-part: representing the bit 0 or 1



↔ I-part: representing the two bits  
 ↔ Q-part: representing the two bits

## Signal Space Diversity (SSD) (Cont.)

- The two transmitted symbols experience **different** fading gains



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# Macroscopic Diversity

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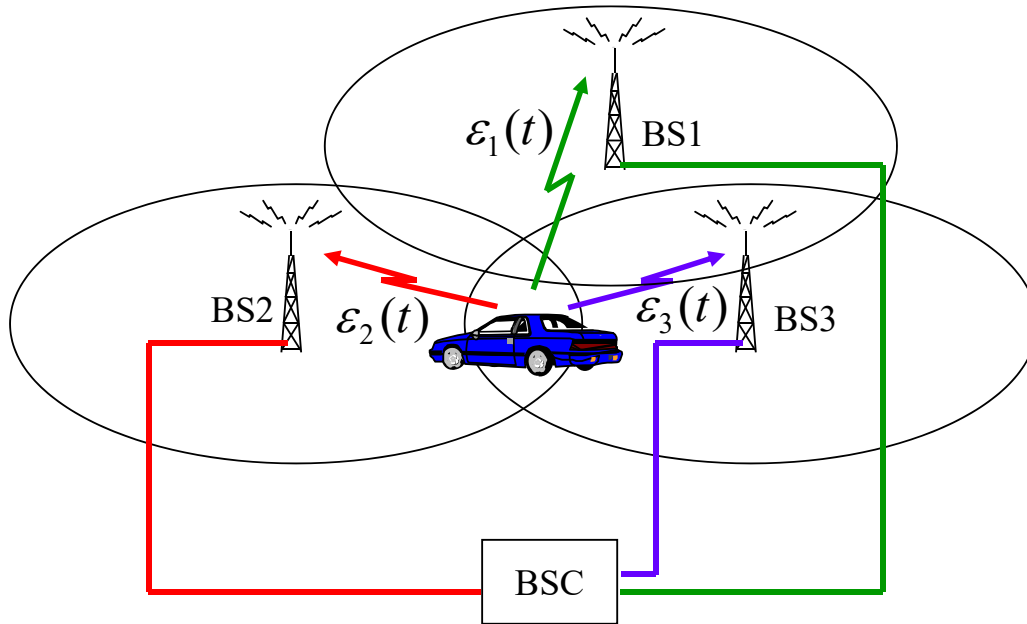
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## Macroscopic Diversity

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- **Microscopic diversity** applies some diversity techniques to mitigate the effects of envelope fading (fast fading)
- **Macroscopic diversity** is a diversity technique that is used to combat the effects of **shadow fading**
  - **Up-link** signals from a mobile station are received by **two or more** geographically separated base stations
  - The received signals are diversity combined
  - The diversity advantage is obtained if the signals experience some degree of **uncorrelated shadowing**
- Assume that there are  $M$  diversity branches (BSs)
- The average received bit energy-to-noise ratio per diversity branch is  $\bar{\gamma}_c$  (in dB), including path loss and shadowing

## Macroscopic Diversity (Cont.)



## Macroscopic Diversity (Cont.)

- We have  $p(\bar{\gamma}_{c(\text{dB})}) = \frac{1}{\sqrt{2\pi}\sigma_{\bar{\gamma}_c}} \exp\left\{-\left(\bar{\gamma}_{c(\text{dB})} - \mu_{\bar{\gamma}_c}\right)^2 / 2\sigma_{\bar{\gamma}_c}^2\right\}$ 
  - where  $\mu_{\bar{\gamma}_c} = E[\bar{\gamma}_{c(\text{dB})}]$
- The most effective and simplest method for macroscopic diversity is **selective combining**
  - The BS that provides the largest average received bit energy-to-noise ratio is selected as the serving BS
- Let  $\bar{\gamma}_{th(\text{dB})}$  be a specified threshold level, and  $\bar{\gamma}_{ck(\text{dB})}$  be the average received bit energy-to-noise ratio from BS  $k$

$$\begin{aligned} \Pr(\bar{\gamma}_{ck(\text{dB})} \leq \bar{\gamma}_{th(\text{dB})}) &= \int_{-\infty}^{\bar{\gamma}_{th(\text{dB})}} \frac{1}{\sqrt{2\pi}\sigma_{\bar{\gamma}_c}} \exp\left\{-\left(\bar{\gamma}_{ck(\text{dB})} - \mu_{\bar{\gamma}_{ck}}\right)^2 / 2\sigma_{\bar{\gamma}_c}^2\right\} d\bar{\gamma}_{ck} \\ &= Q\left((\mu_{\bar{\gamma}_{ck}} - \bar{\gamma}_{th(\text{dB})}) / \sigma_{\bar{\gamma}_c}\right) \end{aligned}$$

## Macroscopic Diversity (Cont.)

- Let  $\mathbf{d} = (d_0, d_1, \dots, d_{M-1})$  be the set of distances between the MS and the  $M$  different BSs
- The outage probability of thermal noise:

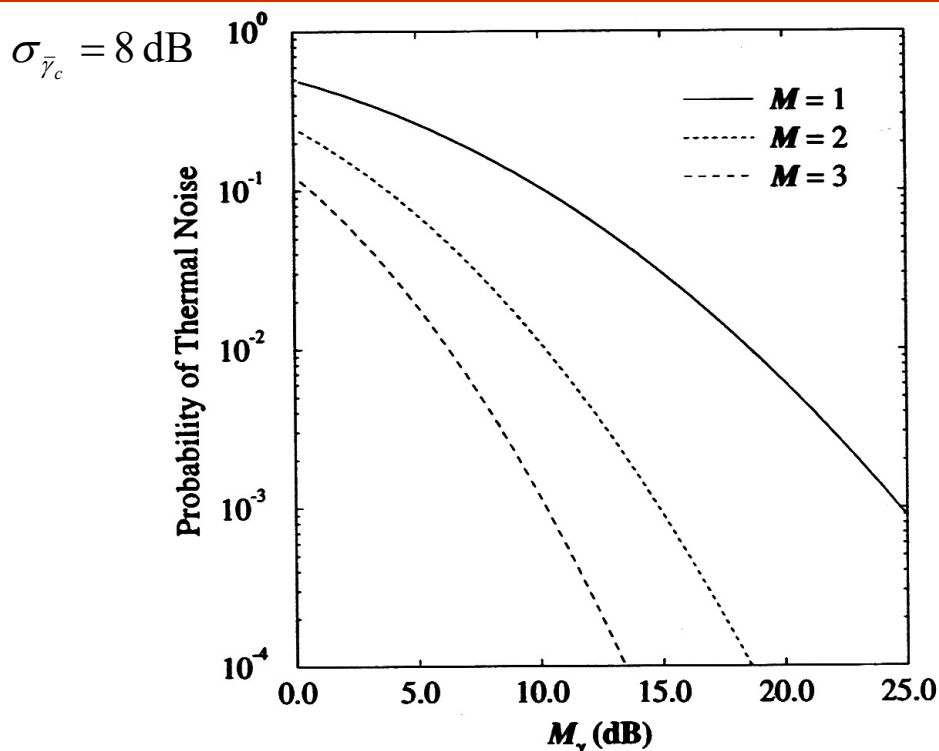
$$O(\mathbf{d}) = \prod_{k=0}^{M-1} \Pr(\bar{\gamma}_{ck(\text{dB})} \leq \bar{\gamma}_{th(\text{dB})}) = \prod_{k=0}^{M-1} Q\left(\frac{\mu_{\bar{\gamma}_{ck}} - \bar{\gamma}_{th(\text{dB})}}{\sigma_{\bar{\gamma}_c}}\right)$$

- Assume that  $\mu_{\bar{\gamma}_{ck}}$  are all equal to  $\mu_{\bar{\gamma}_c}$ , i.e.,  $d_0 = d_1 = \dots = d_{M-1}$ 
  - The outage probability becomes

$$O = \left[ Q\left(\frac{\mu_{\bar{\gamma}_c} - \bar{\gamma}_{th(\text{dB})}}{\sigma_{\bar{\gamma}_c}}\right) \right]^M$$

- The thermal noise margin:  $M_\gamma = \mu_{\bar{\gamma}_c} - \bar{\gamma}_{th(\text{dB})}$

## Macroscopic Diversity (Cont.)



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# Precoding

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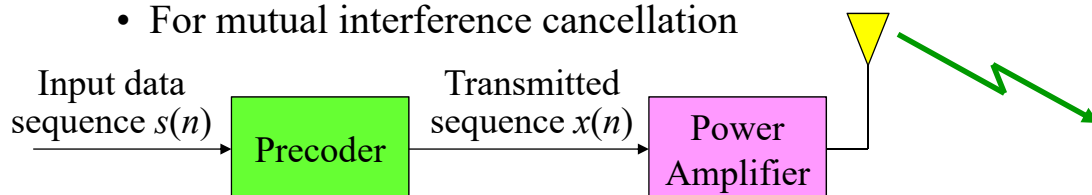
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## Precoding

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- **Precoding** is a modification of a sequence of signals (symbols) **before transmission** to meet a specific goal/requirement.
- There are various purposes for using precoding in wireless transmissions
  - To decrease the envelope variations of the transmitted signals
  - To mitigate the inter-symbol interference induced by channels
  - To achieve radio resource mapping for different signals
    - For diversity reception
    - For multiple access
    - For mutual interference cancellation



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# Precoding – Transmit Diversity

- Space-time transmit diversity is an example of using precoding
- The input data signals are transformed to new modified signals for transmission
  - To achieve diversity reception at the receiver
- For 2×1 transmit diversity, the transmitted signals are

**Original data signals**

**Transmitted signals**

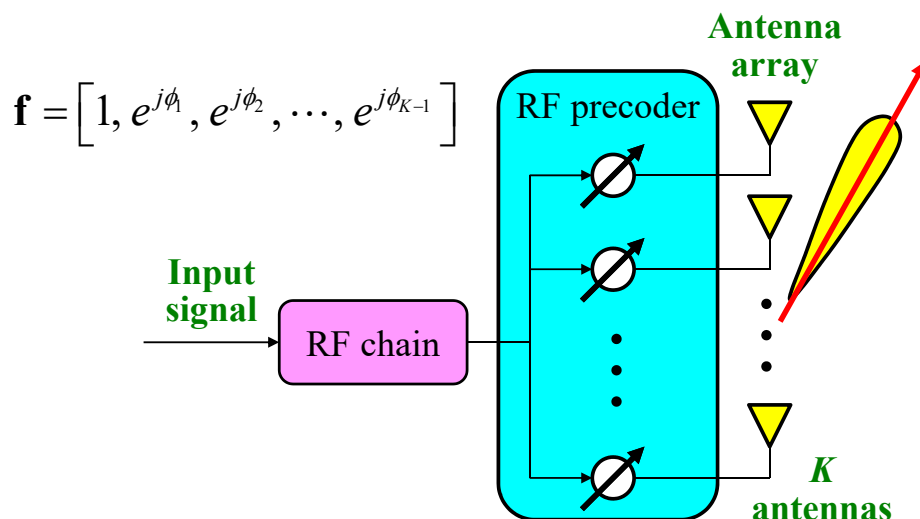
$$\begin{bmatrix} \tilde{s}_{(1)} \\ \tilde{s}_{(2)} \end{bmatrix} \xrightarrow{\text{Precoding}} \begin{bmatrix} \tilde{x}_{(1),1} & \tilde{x}_{(1),2} \\ \tilde{x}_{(2),1} & \tilde{x}_{(2),2} \end{bmatrix} = \begin{bmatrix} \tilde{s}_{(1)} & \tilde{s}_{(2)} \\ -\tilde{s}_{(2)}^* & \tilde{s}_{(1)}^* \end{bmatrix}$$

Antenna index

**Alamouti's code**

# Precoding – Beamforming

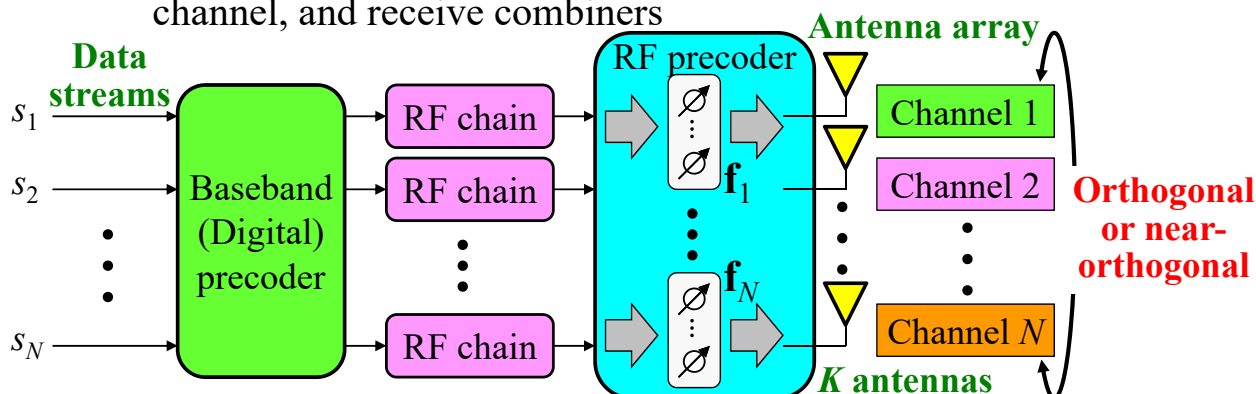
- The same signal is emitted from each of the transmit antennas with appropriately chosen **weighting (phase and/or gain)**
  - Such that the signal power is **maximized** in a desired direction
  - For RF precoding, a **constant amplitude** constraint is required





## Precoding – Spatial Multiplexing

- **Multiple data streams** are emitted from the transmit antennas with **different** and appropriately chosen weightings
  - Such that the transmitter can **simultaneously** transmit multiple data streams to the receiver/receivers using **the same resources**
  - The goal is to **maximize** the total system throughput
  - **Multiple spatial channels** are formed through the precoders, channel, and receive combiners



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## Precoding – DFT-Spread OFDM

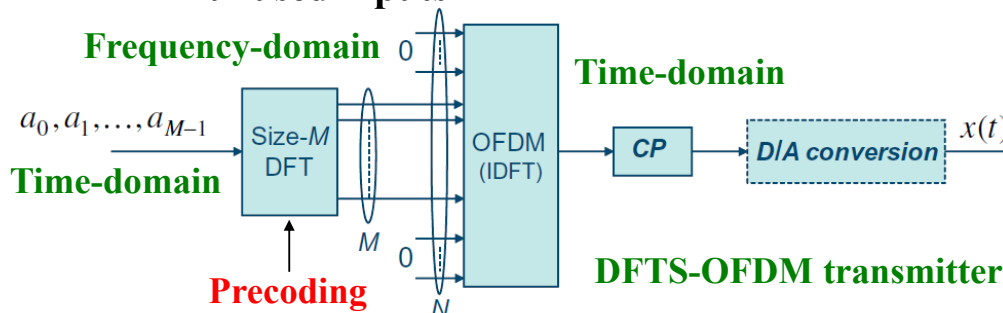
- **Flexible bandwidth assignment:** For OFDM-based **uplink** transmission, **dynamically allocating** different number of subcarriers to different terminals depending on their instantaneous channel conditions
- Conventional OFDM transmission results in a signal with **high PAPR** (peak-to-average power ratio)
- To achieve the goal of **low-PAPR** at a user equipment, “**single-carrier**” transmission is used
  - The **DFT-spread OFDM** (DFTS-OFDM) transmission scheme
  - Small variations in the instantaneous signal power (low PAPR)
  - Possibility for low-complexity high-quality **equalization** in the **frequency domain** (Property of OFDM-based transmission)
  - Possibility for FDMA with **flexible bandwidth** assignment

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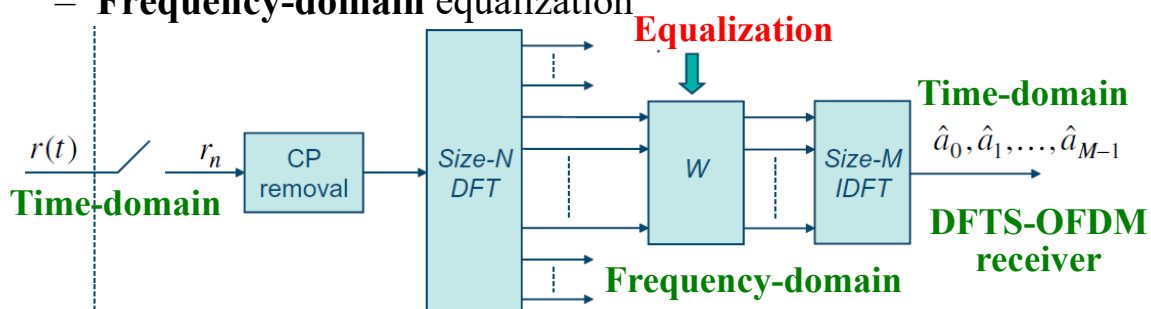
## Precoding – DFT-Spread OFDM (Cont.)

- The DFTS-OFDM transmission can be regarded as normal OFDM transmission with a **DFT-based precoding**
  - A block of  $M$  modulation symbols (BPSK, QPSK, 16QAM, ...) is first applied to a size- $M$  DFT
  - The output is then applied to consecutive inputs (**subcarriers**) of an OFDM modulator, depending on the **bandwidth assignment**
  - The OFDM modulator is a size- $N$  inverse DFT (IDFT) with  $N > M$  and the **unused inputs** of the IDFT are set to zero



## Precoding – DFT-Spread OFDM (Cont.)

- The demodulation of a DFTS-OFDM signal is similar to the demodulation of an OFDM signal
  - Using size- $N$  DFT processing with removal of the frequency samples **not corresponding** to the desired signal
  - Using size- $M$  inverse DFT processing to obtain data estimate
- An equalizer is needed to compensate for the radio-channel **frequency selectivity**
  - Frequency-domain** equalization



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# Multi-Antenna Techniques

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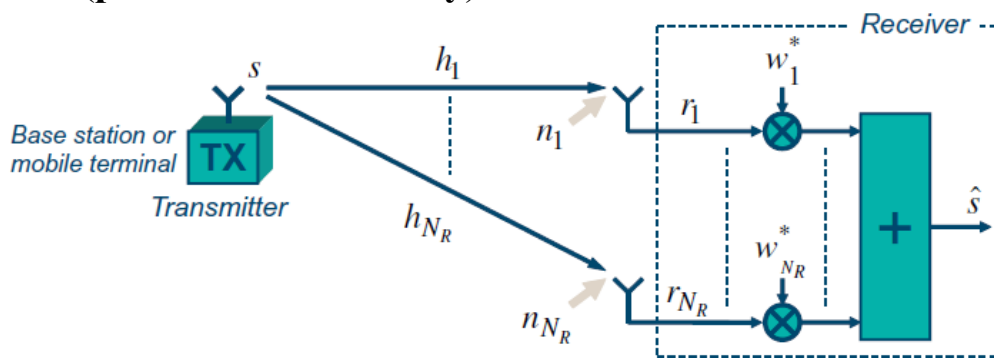
## Antenna Distance

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- An important characteristic of multi-antenna configuration is the **distance** between the different antenna elements
  - The **mutual correlation** between the **radio-channel fading** experienced by the signals at the different antennas
  - A **large** inter-antenna distance  $\Rightarrow$  a **low** mutual fading correlation
    - Almost independent fast fading
  - A **small** inter-antenna distance  $\Rightarrow$  a **high** mutual fading correlation
    - Very similar fast fading
- Whether **high or low correlation** is desirable depends on what is to be achieved with the multi-antenna configuration
  - Diversity, beamforming, or spatial multiplexing

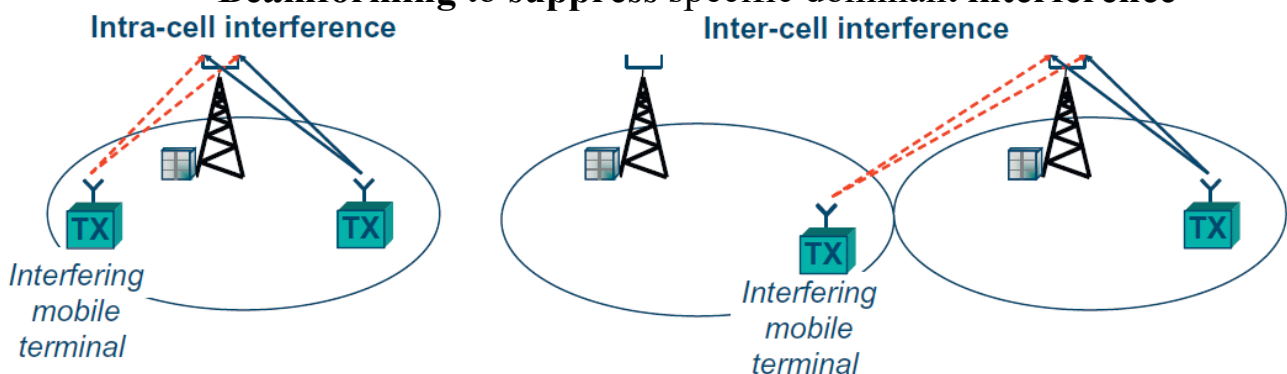
## Benefits of Multi-Antenna: Diversity

- Using multiple antennas at the transmitter **and/or** the receiver to provide additional **diversity** against channel fading
  - Low mutual correlation** between different antennas, implying
    - The need for a sufficiently **large inter-antenna distance** (**spatial diversity**) or
    - The use of **different antenna polarization** directions (**polarization diversity**)



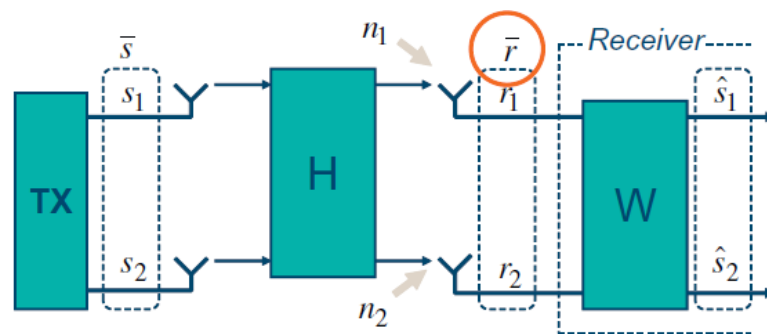
## Benefits of Multi-Antenna: Beamforming

- Using multiple antennas at the transmitter **and/or** the receiver to “**shape**” the overall **antenna beam pattern**
  - Very high mutual correlation** or **low mutual correlation** between different antennas
    - Beamforming** to **maximize** the overall antenna gain in the direction of the target receiver/transmitter
    - Beamforming** to **suppress** specific dominant **interference**



# Benefits of Multi-Antenna: Spatial Multiplexing

- **Simultaneously** using multiple antennas at the transmitter and the receiver to support **multiple parallel channels** over the radio interface with the same radio resources
  - **Spatial multiplexing** to achieve very high bandwidth efficiency
  - Also referred to as **MIMO (Multi-Input Multi-Output)** antenna processing
- The case of  $2 \times 2$  antenna configuration



## Benefits of Multi-Antenna: SM (Cont.)

- The received signals can be expressed as:

$$\mathbf{r} = \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} h_{1,1} & h_{1,2} \\ h_{2,1} & h_{2,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

- Assuming no noise and that the channel matrix  $\mathbf{H}$  is **invertible**, both signals  $s_1$  and  $s_2$  can be **perfectly** recovered at the receiver

$$\hat{\mathbf{s}} = \begin{bmatrix} \hat{s}_1 \\ \hat{s}_2 \end{bmatrix} = \mathbf{W}\mathbf{r} = \mathbf{H}^{-1}\mathbf{r} = \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \underline{\mathbf{H}^{-1}\mathbf{n}} \leftarrow \text{Noise enhancement}$$

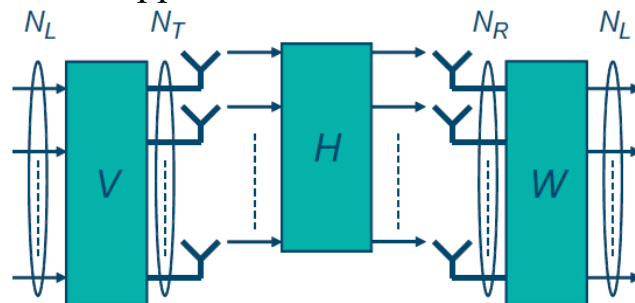
- The properties of  $\mathbf{H}$  will determine the resultant noise level
  - The closer the channel matrix is to being a **singular matrix**, the larger the **increase** in the noise level

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## Benefits of Multi-Antenna: SM (Cont.)

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- Consider a multiple-antenna configuration consisting of  $N_T$  transmit antennas and  $N_R$  receive antennas.
- The number of parallel signals that can be spatially multiplexed is **upper limited** by  $N_L = \min\{N_T, N_R\}$ 
  - No more than  $N_T$  different signals can be transmitted from  $N_T$  transmit antennas
  - With  $N_R$  receive antennas, a maximum of  $N_R - 1$  interfering signals can be suppressed



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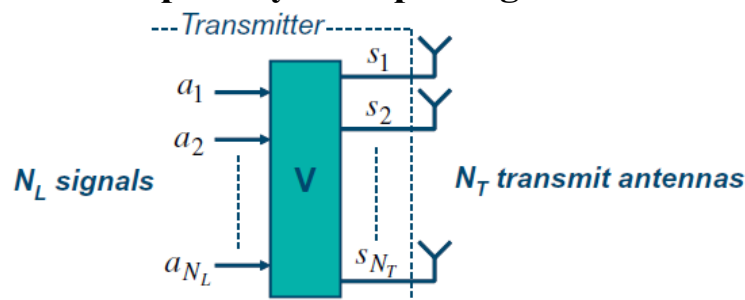
## Benefits of Multi-Antenna: SM (Cont.)

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- However, in many cases, the number of spatially multiplexed signals will be **less than**  $N_L$
- In the case of **very bad** channel conditions (low signal-to-noise ratio, SNR) there is **no gain** of spatial multiplexing
  - The channel capacity is a linear function of the SNR
  - The multiple antennas should be used for **beamforming** to improve the SNR, rather than for spatial multiplexing
- The spatial-multiplexing order should be determined based on the properties of the size  $N_R \times N_T$  channel matrix
  - Any **excess antennas** should be used to provide **beamforming**
  - Such combined beamforming and spatial multiplexing can be achieved by means of **precoder-based spatial multiplexing**

# Precoder-Based Spatial Multiplexing

- Linear precoding for spatial multiplexing is achieved by using a size  $N_T \times N_L$  **precoding matrix** at the transmitter side
- In the general case with  $N_L \leq N_T$ ,  $N_L$  signals are spatially multiplexed and transmitted using  $N_T$  transmit antennas
  - If  $N_L = N_T$ , the precoding can be used to “**orthogonalize**” the parallel transmissions, allowing for improved signal isolation
  - If  $N_L < N_T$ , the precoding provides the mapping including the combination of **spatially multiplexing** and **beamforming**



## Precoder-Based Spatial Multiplexing (Cont.)

- In the LTE system, the codebook-based precoding allows for a maximum of **four antenna ports** (output signals) and, as a consequence, a maximum of **four layers** (input signals)
- Precoder matrices for two antenna ports and one and two layers
  - One layer

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{W}\mathbf{x}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} +1 \\ +1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} +1 \\ -1 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} +1 \\ +j \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} +1 \\ -j \end{bmatrix}$$

- Two layers

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{W}\mathbf{x} = \mathbf{W} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\frac{1}{\sqrt{2}} \begin{bmatrix} +1 & 0 \\ 0 & +1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} +1 & +1 \\ +1 & -1 \end{bmatrix}$$

$$\frac{1}{2} \begin{bmatrix} +1 & +1 \\ +j & -j \end{bmatrix}$$

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# Equalization

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## Adaptive Equalizer

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- An **adaptive equalizer** is an adjustable filters at the receiver
  - Used to mitigate the combined effect of ISI and noise
- Two broad categories of equalizers:
  - **Symbol-by-symbol** equalizers
    - Include a decision device to make symbol-by-symbol decisions on the received symbol sequence
  - **Sequence estimators**
    - Make decisions on a **sequence of received symbols**
- Sequence estimators are generally more complex than symbol-by-symbol equalizers, but can potentially offer better performance



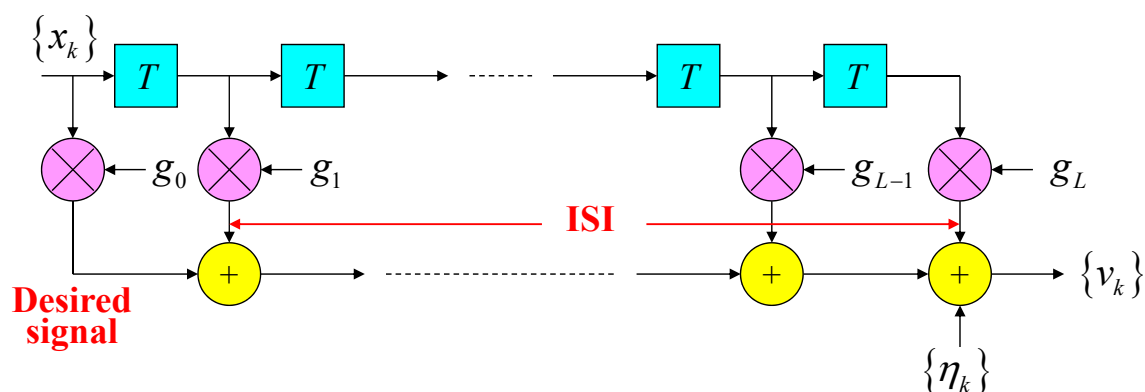
# Modeling of ISI Channels

- The multipath channel can be represented by a **discrete-time transversal filter** with coefficients:

$$\mathbf{g} = [g_0, g_1, \dots, g_{L-1}, g_L]^T$$

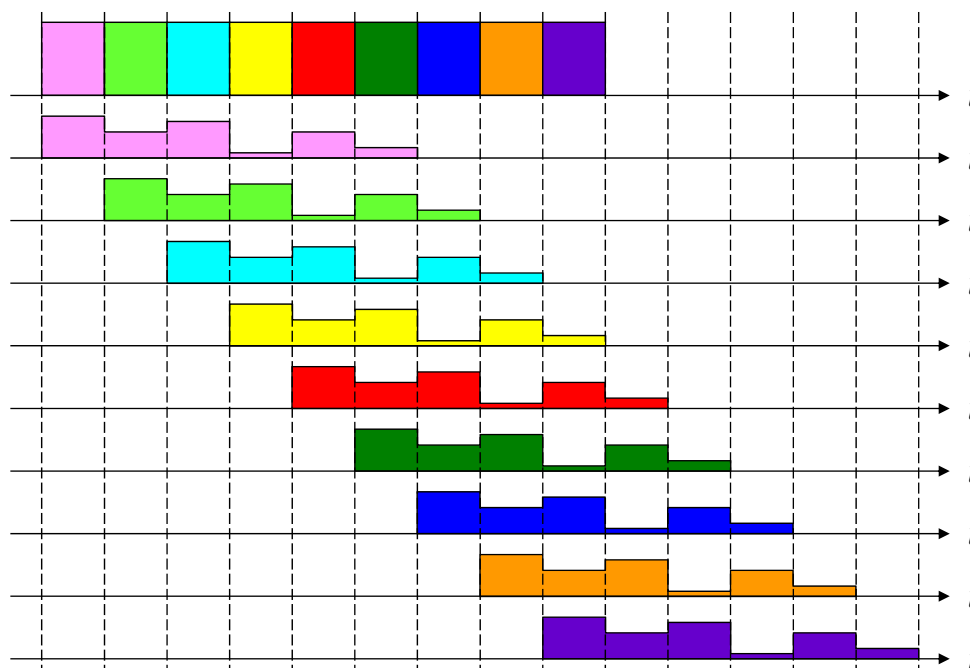
- If the channel gain is time-variant, the coefficients become

$$\mathbf{g}(k) = [g_0(k), g_1(k), \dots, g_{L-1}(k), g_L(k)]^T$$



## Modeling of ISI Channels (Cont.)

- Each input symbol triggers a channel impulse response in time

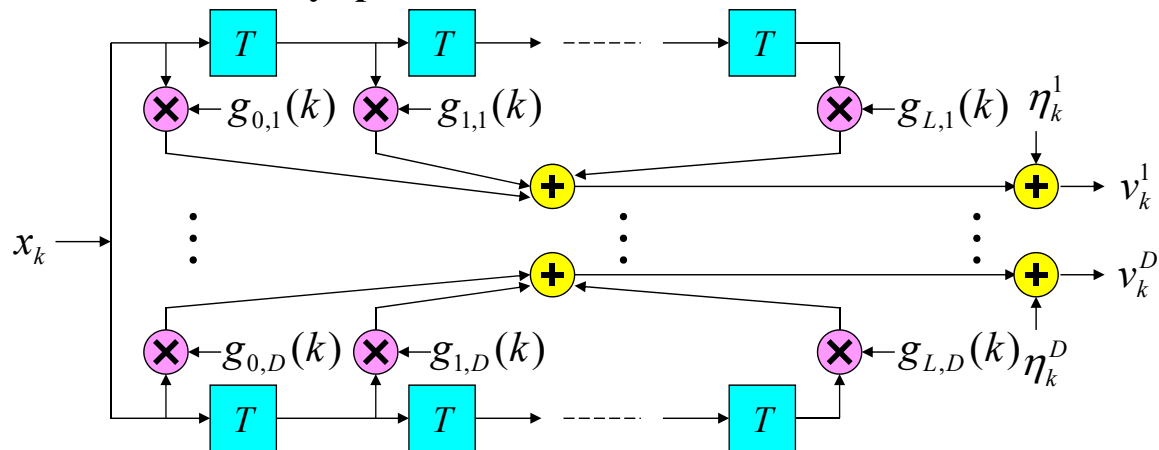


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## Modeling of ISI Channels (Cont.)

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- The overall channel can be modeled by a collection of  $D$  transversal filters that are  $T$ -spaced and have  $L+1$  taps
  - $D$  is the number of **diversity branches**
  - $L$  is the **delay spread**

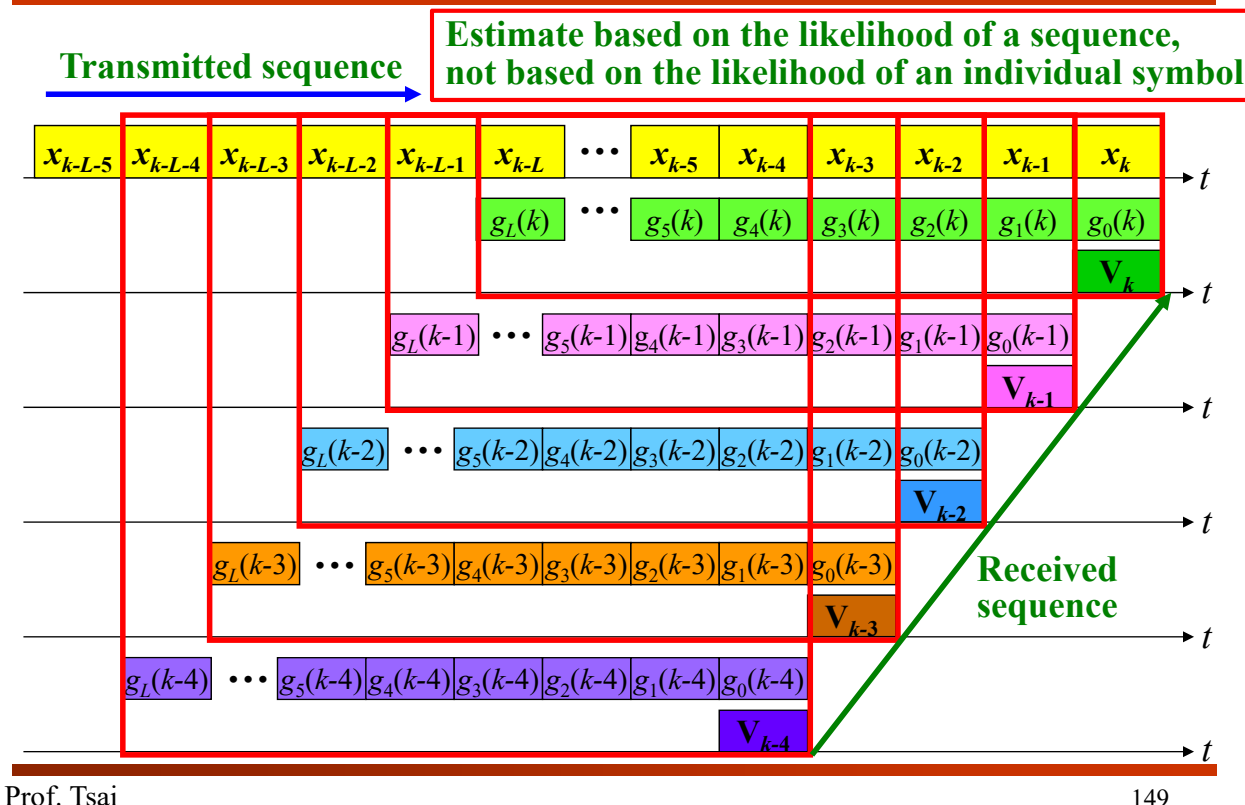


- Use **maximum likelihood sequence estimation (MLSE)**

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## Maximum Likelihood Sequence Estimation

# Maximum Likelihood Sequence Estimation



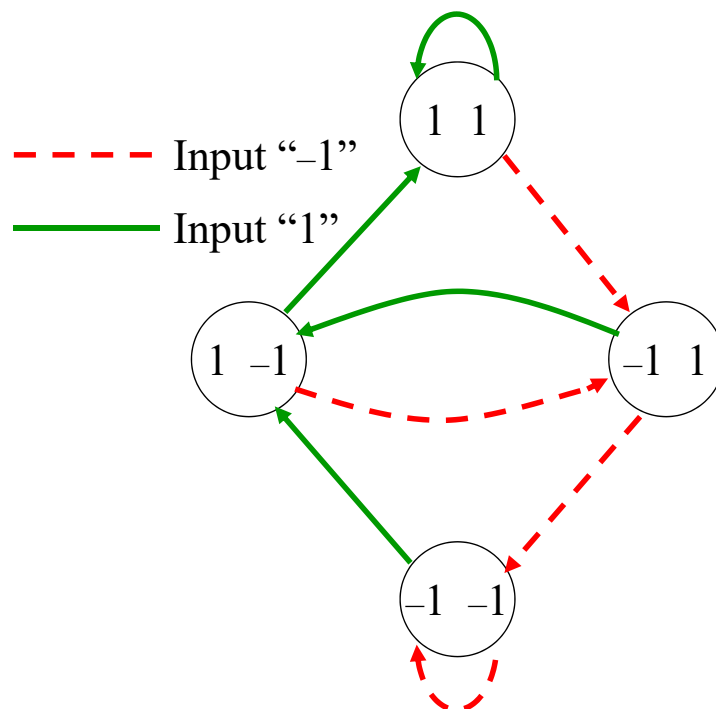
## MLSE (Cont.)

- The channel has a finite number of states
  - If the size of the signal constellation is  $2^n$  ( $M$ -ary modulation)
  - There are a total of  $N_s = 2^{nL}$  states ( $L$  shift registers)
- The state at epoch  $k$  is:

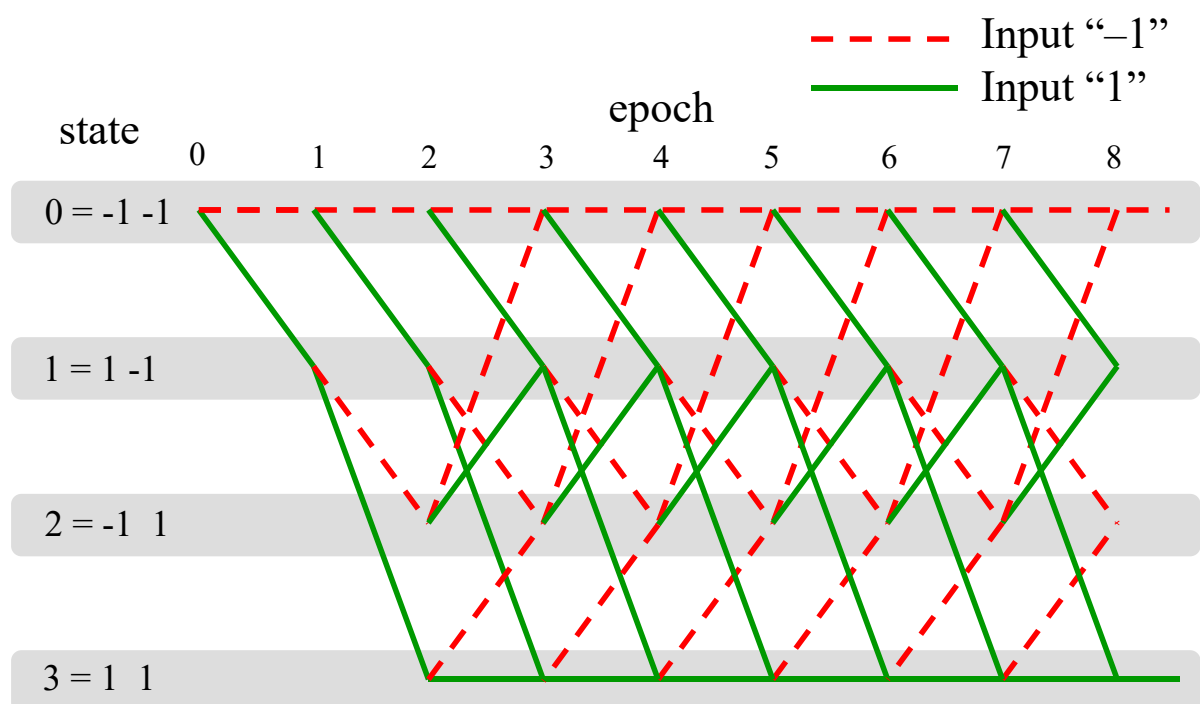
$$S_k = (x_{k-1}, x_{k-2}, \dots, x_{k-L})$$

- For example: the binary sequence  $\mathbf{x}$ ,  $x_n \in \{-1, +1\}$ , is transmitted over a three-tap static ISI channel with channel vector  $\mathbf{g} = (1, 1, 1)$
- There are four states ( $3\text{-tap} \Rightarrow 2\text{ shift registers} \Rightarrow 2^2 = 4\text{ states}$ )
- The system can be described by the **state diagram**
- The system state diagram can be used to construct the **trellis diagram**
  - The initial zero state is assumed to be  $S_0^{(0)} = (-1, -1)$

## State Diagram for Three-Tap ISI Channel



## Trellis Diagram for Three-Tap ISI Channel



## MLSE (Cont.)

- Let the vector of signals received on all diversity branches at epoch  $n$  be

$$\mathbf{V}_n = (v_{n,1}, v_{n,2}, \dots, v_{n,D})$$

- After receiving the sequence  $\{\mathbf{V}_n\}_{n=1}^k$ , the ML receiver decides in favor of the sequence  $\{x_n\}_{n=1}^k$  that maximizes the **likelihood function** or **log-likelihood function**

$$p(\mathbf{V}_k, \dots, \mathbf{V}_1 | x_k, \dots, x_1), \quad \log p(\mathbf{V}_k, \dots, \mathbf{V}_1 | x_k, \dots, x_1)$$

- Possible input data sequences:

$$\begin{aligned} \mathbf{x}_1 &= (x_{k,1}, x_{k-1,1}, \dots, x_{2,1}, x_{1,1}) \Rightarrow p_1 = p(\mathbf{V}_k, \dots, \mathbf{V}_1 | \mathbf{x}_1) \\ \mathbf{x}_2 &= (x_{k,2}, x_{k-1,2}, \dots, x_{2,2}, x_{1,2}) \Rightarrow p_2 = p(\mathbf{V}_k, \dots, \mathbf{V}_1 | \mathbf{x}_2) \\ &\vdots \\ \mathbf{x}_{M^k} &= (x_{k,M^k}, x_{k-1,M^k}, \dots, x_{2,M^k}, x_{1,M^k}) \Rightarrow p_{M^k} = p(\mathbf{V}_k, \dots, \mathbf{V}_1 | \mathbf{x}_{M^k}) \end{aligned}$$

## MLSE (Cont.)

- Find the maximum value of  $p_j$  (or  $\log p_j$ )

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x}_j, 1 \leq j \leq M^k} \{p(\mathbf{V}_k, \dots, \mathbf{V}_1 | \mathbf{x}_j)\}$$

- Since  $\mathbf{V}_n$  depends only on the  $L$  most recent transmitted symbols, the log-likelihood function at epoch  $k$  is denoted as

$$\begin{aligned} &\log p(\mathbf{V}_k, \dots, \mathbf{V}_1 | x_k, \dots, x_1) \\ &= \log p(\mathbf{V}_k | x_k, \dots, x_{k-L}) + \log p(\mathbf{V}_{k-1}, \dots, \mathbf{V}_1 | x_{k-1}, \dots, x_1) \end{aligned}$$

– where  $x_{k-L} = 0$  for  $k - L \leq 0$

- Assume that the second term of the right-hand side has been calculated previously at epoch  $k - 1$ 
  - Only the first term (**branch metric**) has to be computed for each incoming signal vector  $\mathbf{V}_k$  at epoch  $k$

## MLSE (Cont.)

$$\begin{aligned}
 & \oplus \max [\log p(\mathbf{V}_1 | x_1)] \\
 & \oplus \max [\log p(\mathbf{V}_2, \mathbf{V}_1 | x_2, x_1)] \\
 & \quad \vdots \\
 & \oplus \max [\log p(\mathbf{V}_{L+1}, \dots, \mathbf{V}_1 | x_{L+1}, \dots, x_1)] \\
 & \oplus \max [\log p(\mathbf{V}_{L+2}, \dots, \mathbf{V}_1 | x_{L+2}, \dots, x_1)] \\
 & \quad = \max \left[ \underbrace{\log p(\mathbf{V}_{L+2} | x_{L+2}, \dots, x_2)}_{\text{New item to be calculated}} + \underbrace{\log p(\mathbf{V}_{L+1}, \dots, \mathbf{V}_1 | x_{L+1}, \dots, x_1)}_{\text{Has been calculated}} \right] \\
 & \quad \quad \quad \vdots \\
 & \oplus \max [\log p(\mathbf{V}_k, \dots, \mathbf{V}_1 | x_k, \dots, x_1)] \\
 & \quad = \max \left[ \underbrace{\log p(\mathbf{V}_k | x_k, \dots, x_{k-L})}_{\text{New item to be calculated}} + \underbrace{\log p(\mathbf{V}_{k-1}, \dots, \mathbf{V}_1 | x_{k-1}, \dots, x_1)}_{\text{Has been calculated}} \right]
 \end{aligned}$$

## MLSE (Cont.)

- The conditional pdf of the received signal vector:

$$p(\mathbf{V}_k | x_k, \dots, x_{k-L}) = \frac{1}{(\pi N_0)^D} \exp \left\{ -\frac{1}{N_0} \sum_{d=1}^D \left| v_{k,d} - \sum_{i=0}^L g_{i,d} x_{k-i} \right|^2 \right\}$$

- For  $\log p(\mathbf{V}_k | x_k, \dots, x_{k-L})$ , the **branch metric** is

$$\mu_k = -\sum_{d=1}^D \left| v_{k,d} - \hat{v}_{k,d}(x_k, \dots, x_{k-i}) \right|^2 = -\sum_{d=1}^D \left| v_{k,d} - \sum_{i=0}^L g_{i,d} x_{k-i} \right|^2$$

- The **Viterbi algorithm** can be used to implement the ML receiver by searching through the  $N_s$ -state trellis for most likely transmitted sequence  $\mathbf{x}$
- This search process is called maximum likelihood sequence estimation (**MLSE**)

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# MLSE and Viterbi Algorithm

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## MLSE and Viterbi Algorithm

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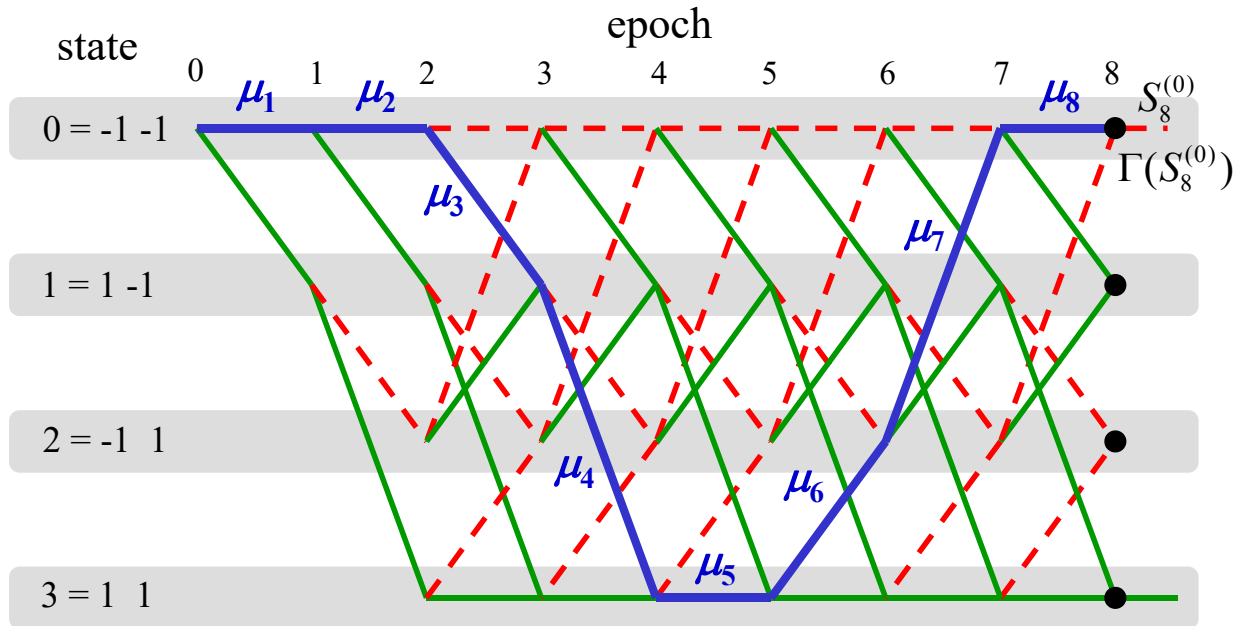
- At epoch  $k$ , assume that the algorithm has stored  $N_s$  **surviving sequences**  $\tilde{\mathbf{x}}(S_k^{(i)})$  (paths through the trellis)
  - with associated **path metrics**  $\Gamma(S_k^{(i)})$  (distances from the received sequence)

$$\Gamma(S_k^{(i)}) = \sum_{\{k\}} \mu_k$$

- $\{\mu_k\}$  is the set of **branch metrics** along the surviving path  $\tilde{\mathbf{x}}(S_k^{(i)})$

# Example of Three-Tap ISI Channel

— : a surviving path



## MLSE and Viterbi Algorithm (Cont.)

- After the vector  $\mathbf{V}_k$  has been received, the Viterbi algorithm executes the following steps for each state  $S_{k+1}^{(j)}, j = 0, \dots, N_s - 1$ 
  - Compute the set of path metrics for all possible paths through the trellis that terminate in state  $S_{k+1}^{(j)}$

$$\Gamma(S_k^{(i)} \rightarrow S_{k+1}^{(j)}) = \Gamma(S_k^{(i)}) + \underline{\mu(S_k^{(i)} \rightarrow S_{k+1}^{(j)})}$$

- Find  $\Gamma(S_{k+1}^{(j)}) = \max_i \Gamma(S_k^{(i)} \rightarrow S_{k+1}^{(j)})$ , where the maximization is over all possible paths through the trellis that terminate in  $S_{k+1}^{(j)}$
- Store  $\Gamma(S_{k+1}^{(j)})$  and its associated **surviving sequence**  $\tilde{\mathbf{x}}(S_{k+1}^{(j)})$ , and **drop all other paths**

- $\mu(S_k^{(i)} \rightarrow S_{k+1}^{(j)})$  is the branch metric associated with the transition  $S_k^{(i)} \rightarrow S_{k+1}^{(j)}$  and is computed as

$$\mu(S_k^{(i)} \rightarrow S_{k+1}^{(j)}) = - \sum_{d=1}^D \left| v_{k,d} - g_{0,d} x_k(S_k^{(i)} \rightarrow S_{k+1}^{(j)}) - \sum_{m=1}^L g_{m,d} \underline{x_{k-m}(S_k^{(i)})} \right|^2$$

**Symbols in the surviving sequence**

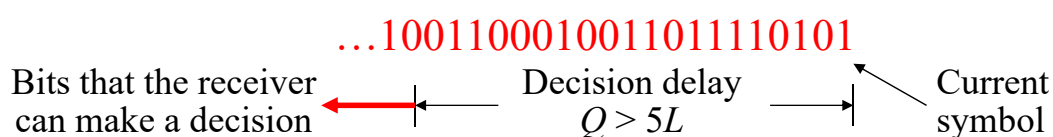


## MLSE and Viterbi Algorithm (Cont.)

- $x_k(S_k^{(i)} \rightarrow S_{k+1}^{(j)})$  is a symbol that is uniquely determined by the transition  $S_k^{(i)} \rightarrow S_{k+1}^{(j)}$ , and the  $L$  most recent symbols  $\{x_{k-m}(S_k^{(i)})\}_{m=1}^L$  are uniquely specified by the previous states  $S_k^{(i)}$
- When two paths merged into a state, **why only the path with a larger path metric is stored** (as the **surviving path**)?
- Consider the two sequences s1="1001100" and s2="0101000" merged at State 0, with the path metric  $M1 > M2$ 
  - Assume that the subsequent sequence with the maximum path metric is "...101101"
  - Then, the metrics "1001100101101" > "0101000101101" > "0101000XXXXXX"  $\Rightarrow$  So, the sequence s2 can be discarded
- After all states have been processed, the time index  $k$  is **incremented** and the whole algorithm repeats

## MLSE and Viterbi Algorithm (Cont.)

- **Theoretically**, the ML receiver waits until the **entire sequence**  $\{\mathbf{V}_n\}_{n=1}^{\infty}$  has been received before making a decision
- **In practice**, a decision about  $x_{k-Q}$  is usually made when  $\mathbf{V}_k$  is received and processed
  - If  $Q > 5L$ , the performance degradation caused by the resulting **path metric truncation** is negligible
- The **decision delay** shall be  $Q > 5L$ 
  - Consider a sequence currently with the maximum path metric



# Cumulative Path Metrics

- Suppose that the data sequence  $\mathbf{x} = (-1, 1, 1, -1, 1, 1, -1, -1, \dots)$  is transmitted

- The noiseless received sequence is  $\mathbf{v} = (v_0, v_1, v_2, v_3, v_4, \dots)$

$$\begin{aligned} v_n &= g_0 x_n + g_1 x_{n-1} + g_2 x_{n-2} \\ &= x_n + x_{n-1} + x_{n-2} \end{aligned}$$

$$\begin{aligned} \mathbf{g} &= (1, 1, 1) \\ S_0^{(0)} &= (-1, -1) \end{aligned}$$

- Then the noiseless received sequence is

$$\mathbf{v} = (-3, -1, 1, 1, 1, 1, -1, \dots)$$

– where the previous two symbols are assumed to be  $(-1, -1)$

- Assume that the noisy received sequence is

$$\mathbf{v} = (-3.2, -1.1, 0.9, 0.1, 1.2, 1.5, 0.7, -1.3, \dots)$$

- The path metrics are equal to the square Euclidean distance between surviving sequence  $\tilde{\mathbf{x}}(S_k^{(i)})$  and received sequence  $\mathbf{v}$

## Cumulative Path Metrics (Cont.)

$$\Gamma(S_0^{(0)}) = 0; \Gamma(S_0^{(1)}) = 0; \Gamma(S_0^{(2)}) = 0; \Gamma(S_0^{(3)}) = 0;$$

$$S_k^{(0)} \rightarrow S_{k+1}^{(0)} \Rightarrow v_{k+1} = -3; S_k^{(0)} \rightarrow S_{k+1}^{(1)} \Rightarrow v_{k+1} = -1;$$

$$S_k^{(1)} \rightarrow S_{k+1}^{(2)} \Rightarrow v_{k+1} = -1; S_k^{(1)} \rightarrow S_{k+1}^{(3)} \Rightarrow v_{k+1} = +1;$$

$$S_k^{(2)} \rightarrow S_{k+1}^{(0)} \Rightarrow v_{k+1} = -1; S_k^{(2)} \rightarrow S_{k+1}^{(1)} \Rightarrow v_{k+1} = +1;$$

$$S_k^{(3)} \rightarrow S_{k+1}^{(2)} \Rightarrow v_{k+1} = +1; S_k^{(3)} \rightarrow S_{k+1}^{(3)} \Rightarrow v_{k+1} = +3;$$

- Epoch 1

$$\mu(S_0^{(0)} \rightarrow S_1^{(0)}) = -|-3.2 - (-3)|^2 = -0.04; \Rightarrow \Gamma(S_0^{(0)} \rightarrow S_1^{(0)}) = -0.04$$

$$\mu(S_0^{(0)} \rightarrow S_1^{(1)}) = -|-3.2 - (-1)|^2 = -4.84; \Rightarrow \Gamma(S_0^{(0)} \rightarrow S_1^{(1)}) = -4.84$$

- Epoch 2

$$\mu(S_1^{(0)} \rightarrow S_2^{(0)}) = -|-1.1 - (-3)|^2 = -3.61; \Gamma(S_1^{(0)} \rightarrow S_2^{(0)}) = -0.04 - 3.61 = -3.65$$

$$\mu(S_1^{(0)} \rightarrow S_2^{(1)}) = -|-1.1 - (-1)|^2 = -0.01; \Gamma(S_1^{(0)} \rightarrow S_2^{(1)}) = -0.04 - 0.01 = -0.05$$

$$\mu(S_1^{(1)} \rightarrow S_2^{(2)}) = -|-1.1 - (-1)|^2 = -0.01; \Gamma(S_1^{(1)} \rightarrow S_2^{(2)}) = -4.84 - 0.01 = -4.85$$

$$\mu(S_1^{(1)} \rightarrow S_2^{(3)}) = -|-1.1 - (+1)|^2 = -4.41; \Gamma(S_1^{(1)} \rightarrow S_2^{(3)}) = -4.84 - 4.41 = -9.25$$

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## Cumulative Path Metrics (Cont.)

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- Epoch 3

$$\begin{aligned}\mu(S_2^{(0)} \rightarrow S_3^{(0)}) &= -|0.9 - (-3)|^2 = -15.21; \Gamma(S_2^{(0)} \rightarrow S_3^{(0)}) = -3.65 - 15.21 = -18.86 \otimes \\ \mu(S_2^{(2)} \rightarrow S_3^{(0)}) &= -|0.9 - (-1)|^2 = -3.61; \Gamma(S_2^{(2)} \rightarrow S_3^{(0)}) = -4.85 - 3.61 = -8.46 \\ \mu(S_2^{(0)} \rightarrow S_3^{(1)}) &= -|0.9 - (-1)|^2 = -3.61; \Gamma(S_2^{(0)} \rightarrow S_3^{(1)}) = -3.65 - 3.61 = -7.26 \otimes \\ \mu(S_2^{(2)} \rightarrow S_3^{(1)}) &= -|0.9 - (+1)|^2 = -0.01; \Gamma(S_2^{(2)} \rightarrow S_3^{(1)}) = -4.85 - 0.01 = -4.86 \\ \mu(S_2^{(1)} \rightarrow S_3^{(2)}) &= -|0.9 - (-1)|^2 = -3.61; \Gamma(S_2^{(1)} \rightarrow S_3^{(2)}) = -0.05 - 3.61 = -3.66 \\ \mu(S_2^{(3)} \rightarrow S_3^{(2)}) &= -|0.9 - (+1)|^2 = -0.01; \Gamma(S_2^{(3)} \rightarrow S_3^{(2)}) = -9.25 - 0.01 = -9.26 \otimes \\ \mu(S_2^{(1)} \rightarrow S_3^{(3)}) &= -|0.9 - (+1)|^2 = -0.01; \Gamma(S_2^{(1)} \rightarrow S_3^{(3)}) = -0.05 - 0.01 = -0.06 \\ \mu(S_2^{(3)} \rightarrow S_3^{(3)}) &= -|0.9 - (+3)|^2 = -4.41; \Gamma(S_2^{(3)} \rightarrow S_3^{(3)}) = -9.25 - 4.41 = -13.66 \otimes\end{aligned}$$

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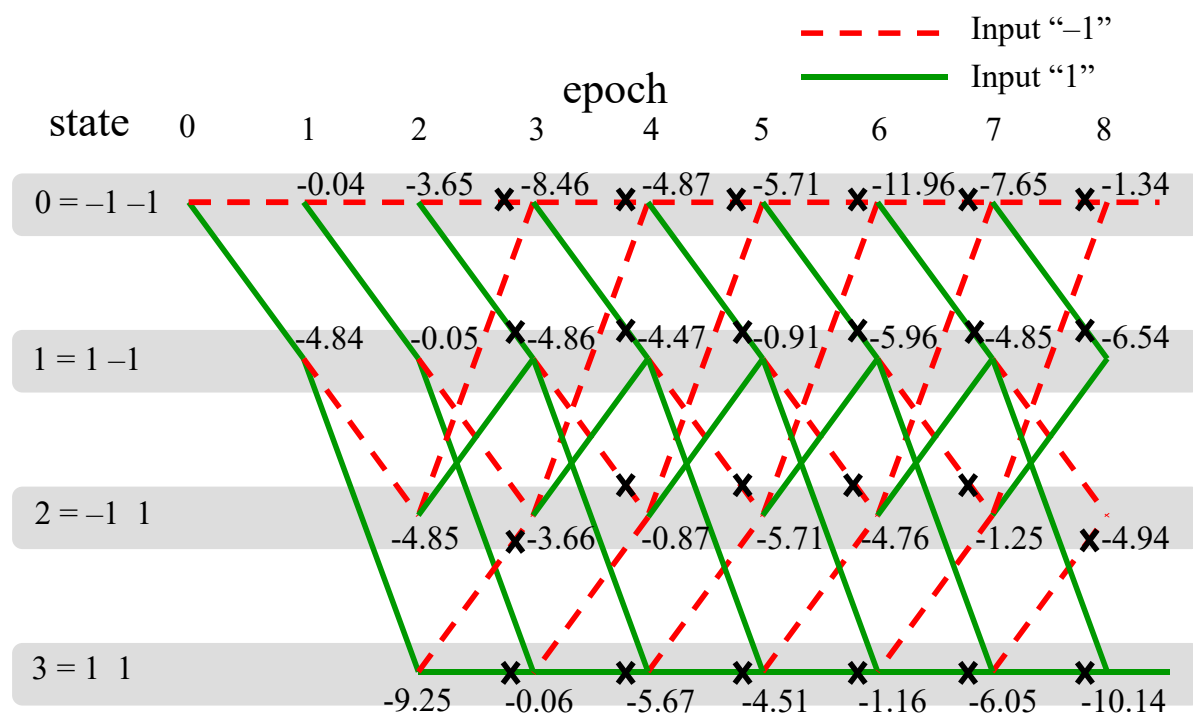
## Cumulative Path Metrics (Cont.)

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- Epoch 4

$$\begin{aligned}\mu(S_3^{(0)} \rightarrow S_4^{(0)}) &= -|0.1 - (-3)|^2 = -9.61; \Gamma(S_3^{(0)} \rightarrow S_4^{(0)}) = -8.46 - 9.61 = -18.07 \otimes \\ \mu(S_3^{(2)} \rightarrow S_4^{(0)}) &= -|0.1 - (-1)|^2 = -1.21; \Gamma(S_3^{(2)} \rightarrow S_4^{(0)}) = -3.66 - 1.21 = -4.87 \\ \mu(S_3^{(0)} \rightarrow S_4^{(1)}) &= -|0.1 - (-1)|^2 = -1.21; \Gamma(S_3^{(0)} \rightarrow S_4^{(1)}) = -8.46 - 1.21 = -9.67 \otimes \\ \mu(S_3^{(2)} \rightarrow S_4^{(1)}) &= -|0.1 - (+1)|^2 = -0.81; \Gamma(S_3^{(2)} \rightarrow S_4^{(1)}) = -3.66 - 0.81 = -4.47 \\ \mu(S_3^{(1)} \rightarrow S_4^{(2)}) &= -|0.1 - (-1)|^2 = -1.21; \Gamma(S_3^{(1)} \rightarrow S_4^{(2)}) = -4.86 - 1.21 = -6.07 \otimes \\ \mu(S_3^{(3)} \rightarrow S_4^{(2)}) &= -|0.1 - (+1)|^2 = -0.81; \Gamma(S_3^{(3)} \rightarrow S_4^{(2)}) = -0.06 - 0.81 = -0.87 \\ \mu(S_3^{(1)} \rightarrow S_4^{(3)}) &= -|0.1 - (+1)|^2 = -0.81; \Gamma(S_3^{(1)} \rightarrow S_4^{(3)}) = -4.86 - 0.81 = -5.67 \\ \mu(S_3^{(3)} \rightarrow S_4^{(3)}) &= -|0.1 - (+3)|^2 = -8.41; \Gamma(S_3^{(3)} \rightarrow S_4^{(3)}) = -0.06 - 8.41 = -8.47 \otimes\end{aligned}$$

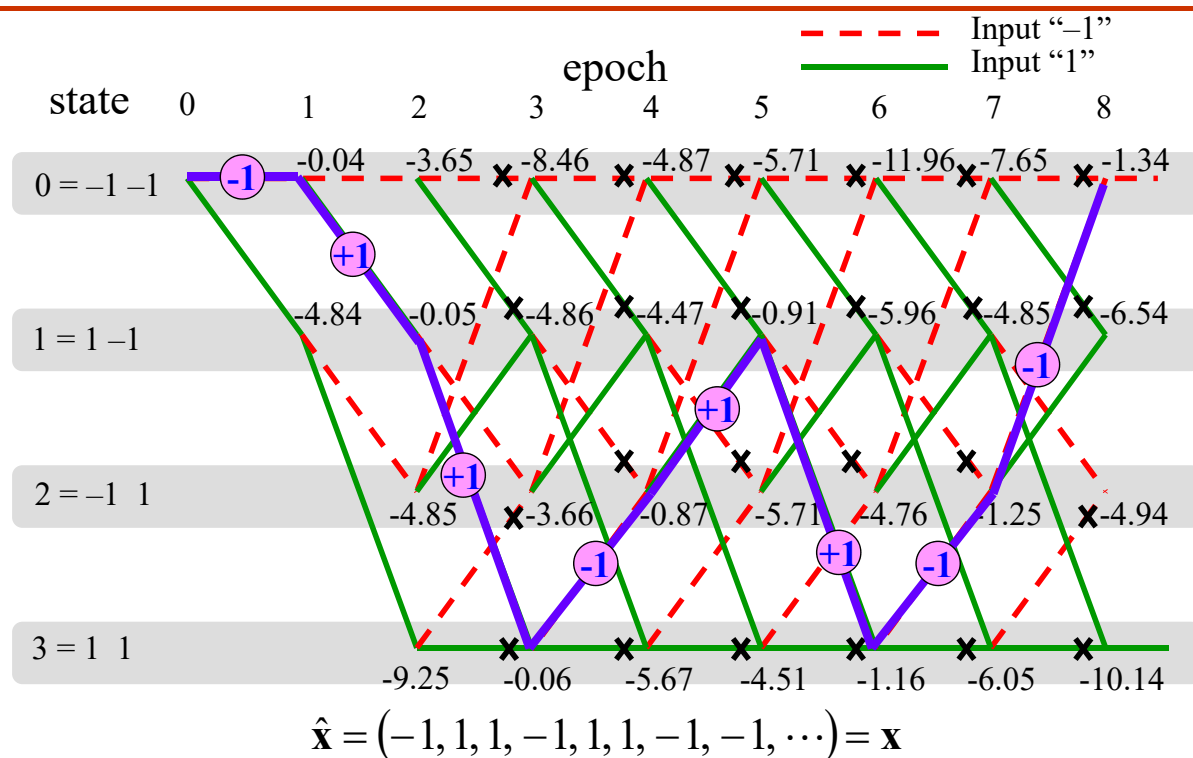
## Cumulative Path Metrics (Cont.)



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## Trace Back



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## Question

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- **Question:**
  - Why will channel equalization fail for high mobility users?
- For a high mobility user, the propagation environment changes rapidly
  - The channel vector  $\mathbf{g}$  is not constant. The channel equalization algorithm (such as MLSE) cannot perform well.

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## Training Sequence

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# Training Sequence

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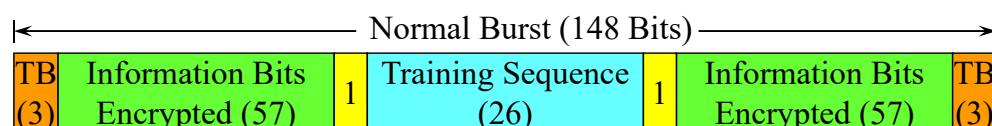
- The role of training sequence is to train the adaptive equalizer in **training mode**
- The training sequence is a **fixed-pattern (known)** sequence, for the adaptive equalizer to estimate the channel vector **g**
- For **TDMA** systems, the training sequence should be transmitted **in each slot**
- **Long** training sequences:
  - Good estimation of the channel
  - Wasting the spectrum resource
- **Short** training sequences:
  - Bad estimation of the channel  $\Rightarrow$  degrade the performance

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## Training Sequence (Cont.)

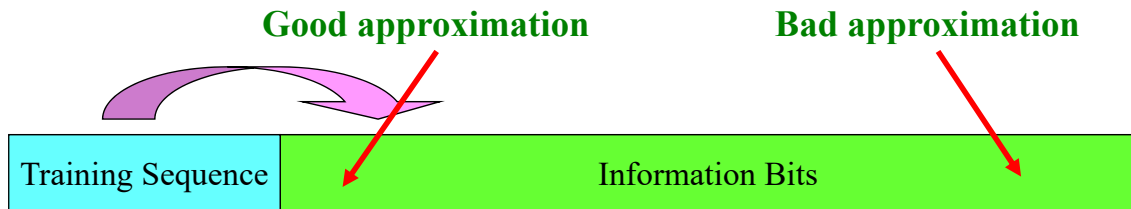
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- The position of the training sequence could be in:
  - The **head** of a frame
  - The **middle** of a frame
  - The **tail** of a frame
- For GSM systems, the training sequence is located in the **middle** of a burst
  - The length of the training sequence is 26 bits
  - There are a total of 8 training sequence codes (TSC)
- GSM systems use the maximum likelihood sequence estimation (MLSE) with Viterbi algorithm
- It reflects the maximum delay of **15  $\mu$ s** (4 bits)

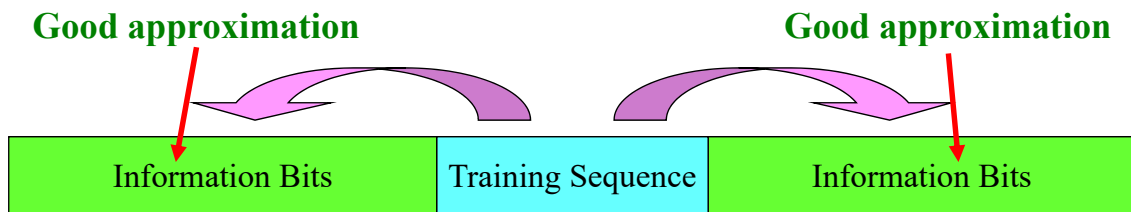


# Question

- **Question:**
  - Where should we put the training sequence in a frame?
- In the head (or tail) of a frame

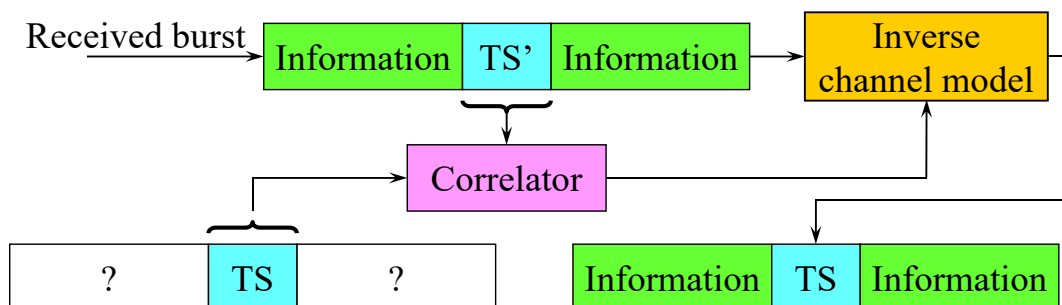


- In the middle of a frame



## Training Sequence (Cont.)

TSC 0 (0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1)  
 TSC 1 (0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1)  
 TSC 2 (0, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 1, 1, 1, 0)  
 TSC 3 (0, 1, 0, 0, 0, 1, 1, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 0, 1, 1, 1, 1, 0)  
 TSC 4 (0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1)  
 TSC 5 (0, 1, 0, 0, 1, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 1, 0, 1, 0)  
 TSC 6 (1, 0, 1, 0, 0, 1, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 1, 1, 1)  
 TSC 7 (1, 1, 1, 0, 1, 1, 1, 1, 0, 0, 0, 1, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 1, 1, 0, 0)



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# Link Adaptation

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## Link Adaptation

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- **Link adaptation** is a technique used to improve the transmission efficiency in wireless communications
  - Adapt the **modulation, coding** and other parameters to the transmission conditions (e.g., the propagation loss, the interference, the sensitivity of a receiver, the available transmit power margin, etc.).
- Link adaptation systems generally require some **channel state information (CSI)** at the transmitter.
  - For **TDD** systems, the transmitter can approximate the CSI of the **Tx-to-Rx link** as the CSI estimate of the **Rx-to-Tx link**
    - Under the reciprocal theorem and slow varying assumption
  - For **FDD** systems, the receiver needs to feed back the CSI estimate to the transmitter
    - It may only feed back the decoding results (Success/Failure) or the **received signal strength (RSS)**

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## Link Adaptation (Cont.)

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- The most popular link adaptation scheme adapts only **modulation** and **coding**
- According to the **channel quality**, the transmitter jointly choose the modulation type and code rate for transmission
  - If the transmission condition is **favorable**, the transmitter uses high-order modulation and high-rate coding
    - To improve the throughput
  - If the transmission condition is **unfavorable**, the transmitter may downgrade the modulation order and/or the code rate
    - To ensure successful reception
- If high-order modulation and high-rate coding are used in an **unfavorable** transmission condition
  - The packet error rate will be very high and the throughput is still degraded

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## Link Adaptation – GPRS

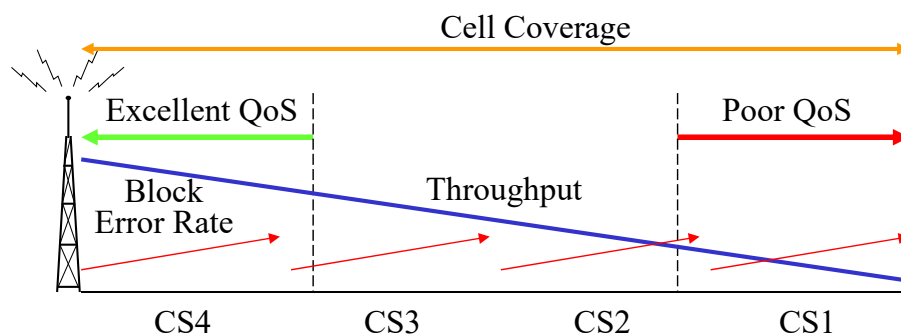
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- For example, **GPRS (General Packet Radio Service)** in GSM systems uses a rate adaptation algorithm that adapts the **coding scheme (CS)** according to the quality of the radio channel
  - Modulation: GMSK (1 bit/symbol);
  - Code rate: 1.0, 3/4, 2/3, 1/2;
- The code rate adaptation is achieved by **bit puncturing**
  - The **same encoder** (code rate 1/2) is used for different code rates

Scheme	Code Rate	Punctured bits	Data rate (kbps)
CS-1	1/2	0	9.05
CS-2	2/3	132	13.4
CS-3	3/4	220	15.6
CS-4	1	-	21.4

## Link Adaptation – GPRS (Cont.)

- When a receiver is close to the BS (with an **excellent channel quality**), high-rate transmission (e.g., CS4) is used
  - If low-rate transmission is used, throughput will be degraded
- When a receiver is at the cell border (with a **poor channel quality**), low-rate transmission (e.g., CS1) is used
  - If high-rate transmission (e.g., CS4) is used, throughput is also degraded because of a **very high packet error probability**



## Link Adaptation – EDGE

- For example, **EDGE (Enhanced Data rates for GSM Evolution)** uses a rate adaptation algorithm that adapts the **modulation and coding scheme (MCS)** according to the quality of the radio channel
  - Modulation: GMSK (1 bit/symbol); 8PSK (3 bits/symbol);
  - Code rate: 1.0, 0.8, 0.66, 0.53 (GMSK); 1.0, 0.92, 0.76, 0.49, 0.37 (8PSK);

Scheme	Code rate	Modulation	Data rate (kb/s)	Scheme	Code rate	Modulation	Data rate (kb/s)
MCS-4	1.0	GMSK	17.6	MCS-9	1.0	8PSK	59.2
MCS-3	0.80		14.8/13.6	MCS-8	0.92		54.4
MCS-2	0.66		11.2	MCS-7	0.76		44.8
MCS-1	0.53		8.8	MCS-6	0.49		29.6/27.2
				MCS-5	0.37		

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## Link Adaptation – HSDPA

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- **HSDPA (High-Speed Downlink Packet Access)** in 3G CDMA systems uses a rate adaptation algorithm that adapts the modulation and coding scheme, known as the **AMC (Adaptive Modulation and Coding)** scheme, according to the quality of the radio channel
  - Modulation: QPSK (2 bits/symbol); 8-PSK (3 bits/symbol); 16-QAM (4 bits/symbol); 64-QAM (6 bits/symbol);
  - Code rate: 3/4, 1/2, 1/4 (QPSK); 3/4 (8-PSK); 3/4, 1/2 (16-QAM); 3/4 (64-QAM);
- Traditionally, CDMA systems have used **fast power control** as the preferred method for link adaptation.
  - Control the transmission power, not control the transmission rate
- The AMC scheme has offered an alternative link adaptation method that promises to raise the **overall system capacity**.

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## Link Adaptation – HSDPA (Cont.)

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- With AMC, the power of the transmitted signal is held **constant** over a **frame interval**
  - The modulation and coding format is changed to match the current received signal quality or channel conditions.

Scheme	Modulation	Code Rate	Data rate (Mbps)
MCS1	QPSK	1/4	1.2
MCS2	QPSK	1/2	2.4
MCS3	QPSK	3/4	3.6
MCS4	8-PSK	3/4	5.4
MCS5	16-QAM	1/2	4.8
MCS6	16-QAM	3/4	7.2
MCS7	64-QAM	3/4	10.8

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# Random Access Techniques

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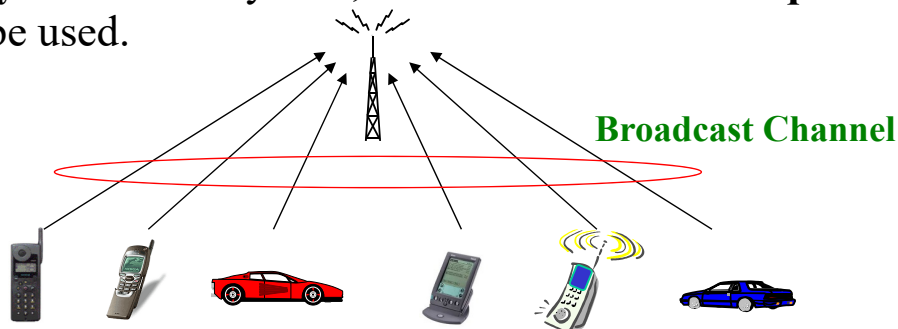
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## Random Access

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- In a **broadcast channel** (such as a wireless channel), one of the key issue is to determine who gets the right of using the channel when there is **competition** for it.
- In a **synchronous** system (with a **controller**), the **controlled-access** (multiple access) **techniques**, such as TDMA or CDMA, can be applied to prevent/reduce **signal collision** or **mutual interference**.
- In an **asynchronous** system, **random access techniques** should be used.



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# Random Access Schemes

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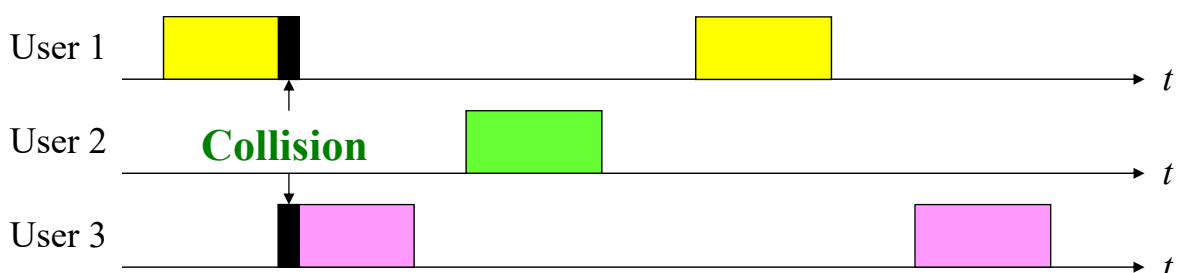
- Random access techniques are completely **decentralized**
- Each user determines its transmission depends solely on a **local random mechanism** and the **local observation**
- It is possible that two or more users decide to transmit their signals in an **overlapping** time interval
  - It is called as a **collision**
- Several random access schemes have been proposed for wireless communications
  - **Pure ALOHA**
  - **Slotted ALOHA**
  - **CSMA/CD**
  - **MACA**

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## Pure ALOHA

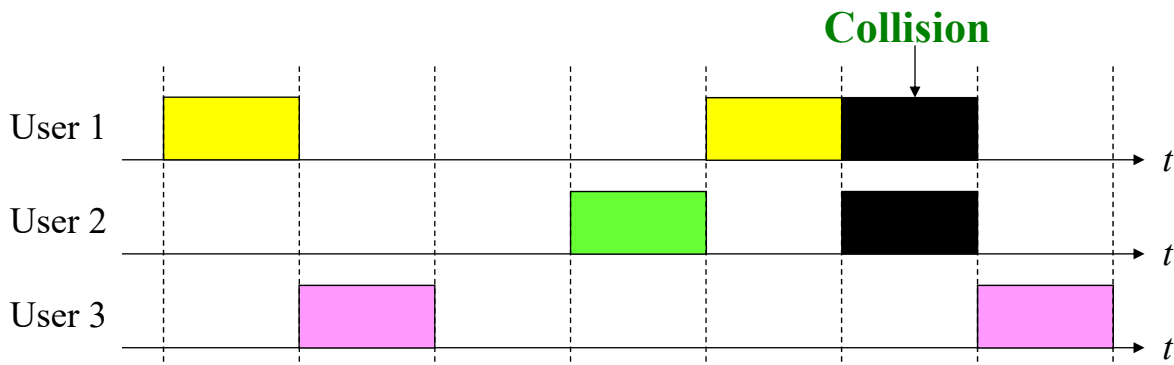
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- Pure ALOHA is the **simplest** random access scheme
- Each user transmits the data packet **immediately** whenever it has a packet for transmission
- There is a feedback mechanism to acknowledge the **success/failure** of the transmitted packet
  - Such as the **positive acknowledgement with timeout**
- When a collision occurs, the user will **re-transmit** the packet after a **random backoff time**



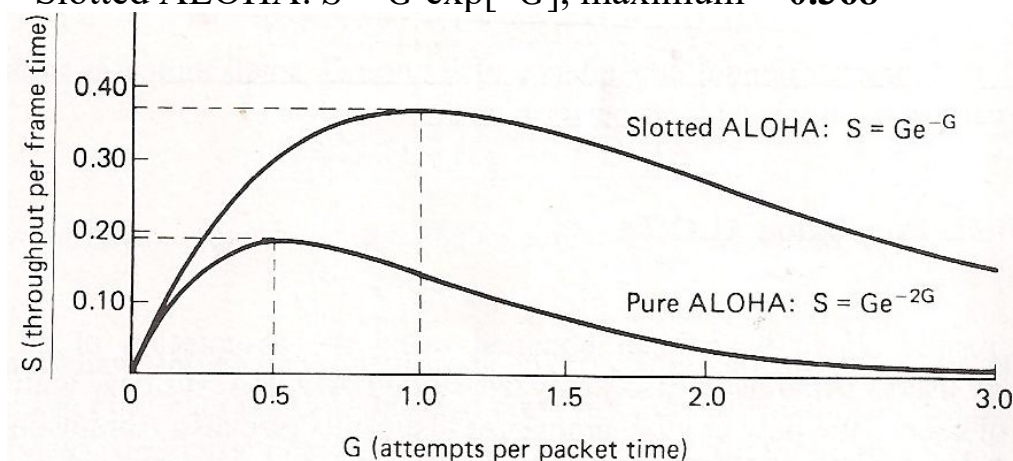
# Slotted ALOHA

- In slotted ALOHA, the time scale is slotted into **multiple time slots** with the duration equal to the packet length
- Users can transmit data packets at the **beginning** of each time slot only
- The other processes are the same as the pure ALOHA scheme



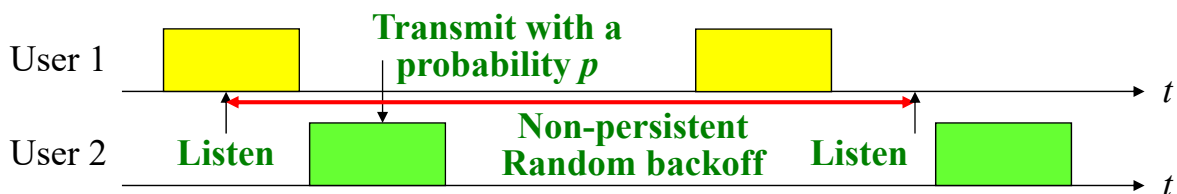
## ALOHA – Average Throughput

- Denote  $G$  as the average **traffic load** (in attempts per packet time) of the channel
- The average throughput (in packets per packet time) is
  - Pure ALOHA:  $S = G \exp[-2G]$ , maximum = **0.184**
  - Slotted ALOHA:  $S = G \exp[-G]$ , maximum = **0.368**



# CSMA

- In **Carrier Sense Multiple Access (CSMA)** scheme, each node listens to a carrier and act accordingly.
- When a node has a data packet to send, it listens to the channel to see if there is anyone else under transmission
  - **$p$ -persistent**: If there is any, it continually senses and waits for the channel to go idle and then transmits **with a probability  $0 \leq p \leq 1$** , and defers (a random backoff) with probability  $q = 1 - p$
  - **Non-persistent**: If there is any, it defers a random backoff
- When a collision occurs, the user **re-transmits** the packet after a **random backoff time**

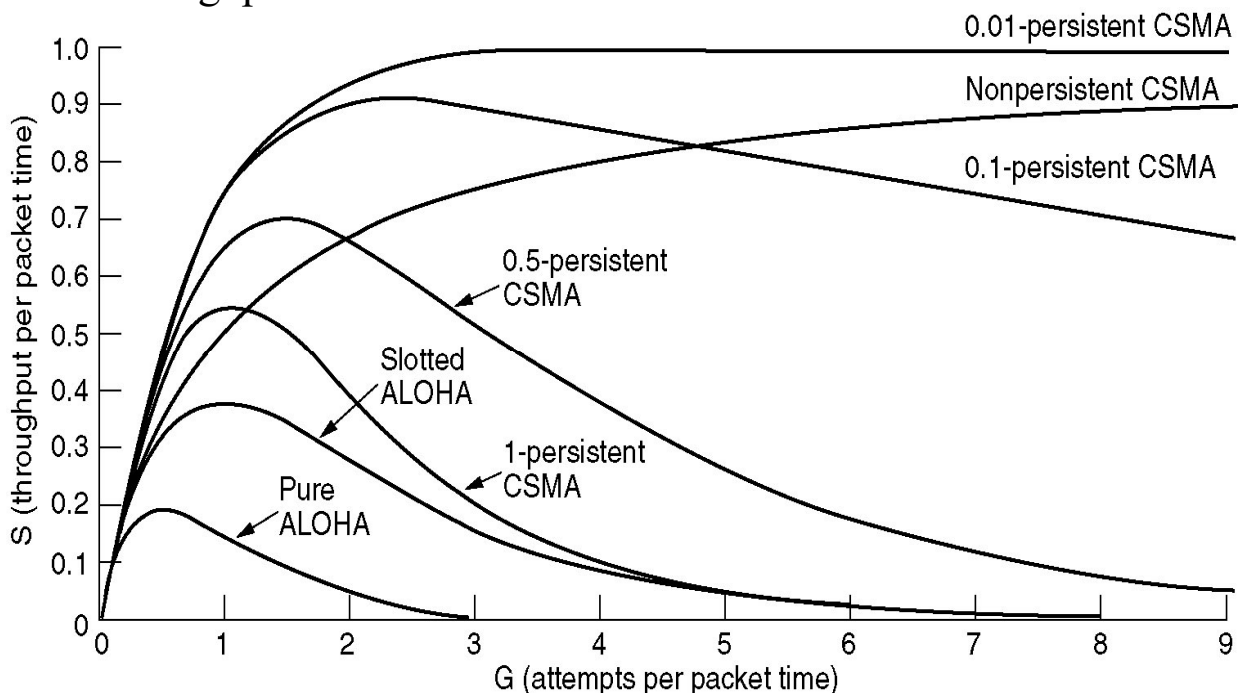


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## CSMA – Average Throughput

- Throughput versus Traffic load

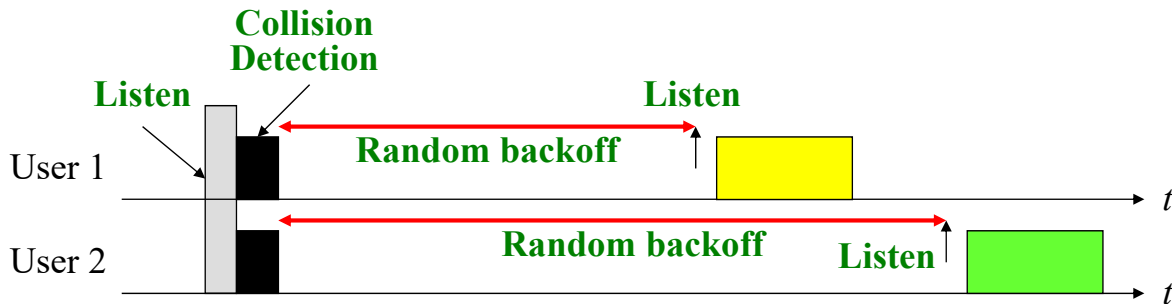


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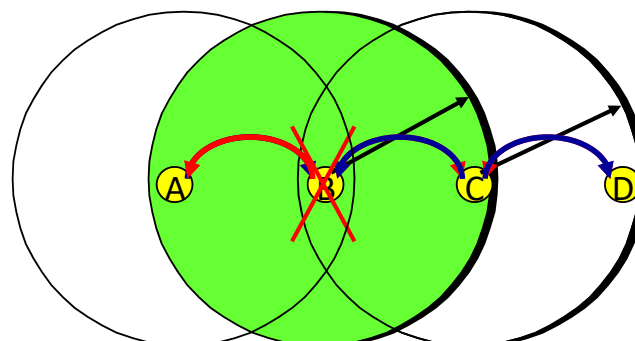
# CSMA/CD

- **Carrier Sense Multiple Access with Collision Detection (CSMA/CD)**: based on CSMA, collision detection is added for performance improvement
- The stations **abort** their transmission as soon as they **detect a collision**



## Problems in Mobile Ad Hoc Networks

- In **mobile ad hoc networks**, random access is more complicated
- **Hidden nodes problem**:  $A \rightarrow B$  and  $C \rightarrow B$ : collide at B
  - Neither A nor C is aware of this collision
- **Exposed node problem**: Suppose that B is sending to A
  - Since C can hear B's transmission, it would be a mistake for C to conclude that it cannot transmit to D
  - C's transmission will not interfere with A's receiving





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# Multiple Access with Collision Avoidance

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- 802.11 addresses these problems with an algorithm called **Multiple Access with Collision Avoidance (MACA)**
  - Before the sender actually transmits any data, the sender and receiver exchange **control frames** with each other
- The sender transmits a **Request to Send (RTS)** frame to the receiver
  - Includes a field that indicates **how long** the sender wants to hold the medium
- The receiver replies with a **Clear to Send (CTS)** frame
  - Echoes the **length field** back to the sender
- Any node that sees the CTS frame knows that it is close to the receiver and **cannot transmit** for the period of time

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## MACA (Cont.)

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- There are two more details for MACA
- The receiver sends an **ACK** to the sender after successfully receiving a frame
  - All nodes must **wait for this ACK** before trying to transmit
- Should two or more nodes detect an idle link and try to transmit an RTS frame at the same time
  - The RTS frames will collide with each other
  - The senders realize the collision has happened when they do not receive the CTS frame after a period of time
  - They wait a random amount of time before trying again
    - **Exponential backoff algorithm** is generally used

## MACA (Cont.)

- RTS/CTS/data/ACK

NAV: network allocation vector

