Wireless Communication Systems HW2
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We have the Doppler frequency shift formula

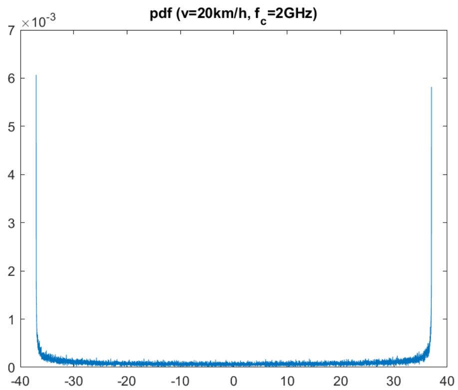
$$f_{D,n}(t) = f_m \cos\theta_n(t)$$

where $f_m = \frac{v}{\lambda_c}$ and λ_c is the wavelength
Furthermore, $\lambda_c = \frac{c}{f_c} \Rightarrow f_m = \frac{vf_c}{c}$ where c is the speed of light

For the subproblems (a) and (b) of problem 1, I sample $F_{D,n}(t) = f_m \cos\Theta$ 100000 times and divide $[-fm, fm]$ into 10000 subintervals and then count how many samples lie in each subinterval so that simulate the pdf of $F_{D,n}(t)$
(Instead of $[-fm, fm]$, I use the minimum and maximum of sample to build the interval)

For the subproblem (c), I assume V and Θ are independent so it just need sampling V and Θ respectively and componentwise multipling the samples of V by those of Θ to get new samples. Then, do the same thing as (a) and (b)

(a) v=20km/h, f_c=2GHz



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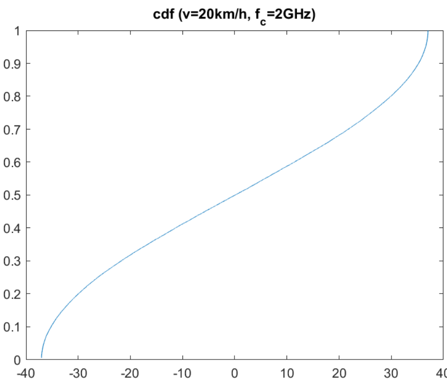

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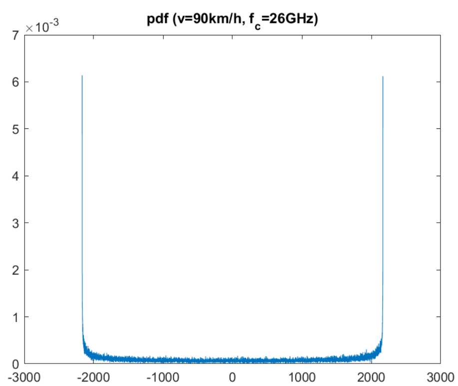
Figure 1: v=20km/h, f_c=2GHz

(b) v=90km/h, f_c=26GHz



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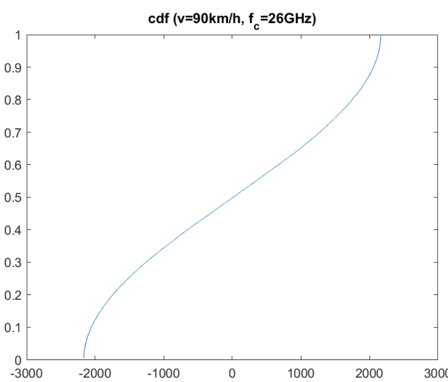

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Figure 2: v=90km/h, f_c=26GHz

(c) $V \sim U(20, 90)$km/h, f_c=2GHz

Here, I ssume V and Θ are independent

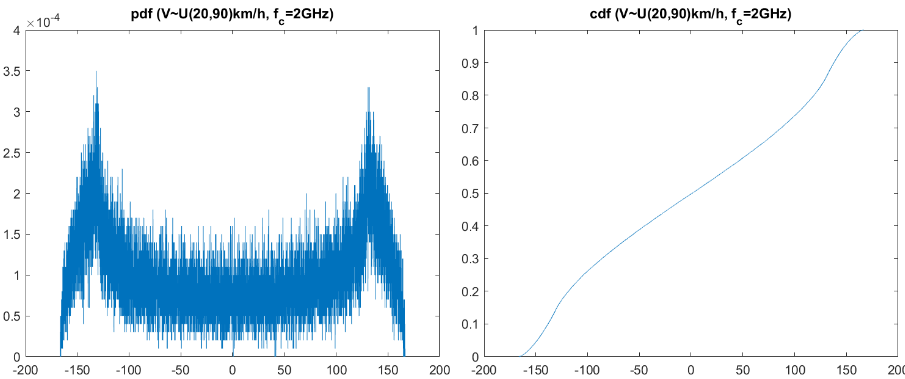

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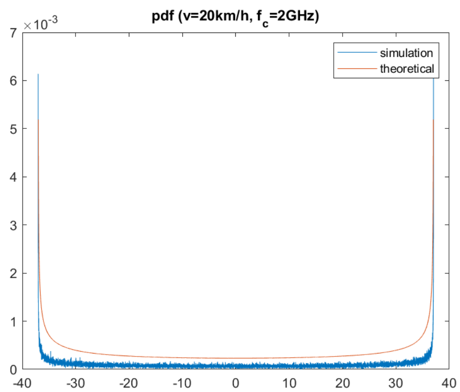
Figure 3: $v \sim U(20, 90)$km/h, f_c=2GHz

(d) Let $\Theta \sim U(-\pi, \pi)$, $X \sim U(0, \pi)$,then, $\cos\Theta = \cos X$ since cos is even

Thus, consider $Y = f_m \cos X$

$$
\begin{aligned}
F_Y(y) &= P(Y \leq y) \\
&= P(f_m \cos X \leq y) \\
&= P\left(X \geq \cos^{-1}\left(\frac{y}{f_m}\right)\right) \\
&= P\left(X \leq \cos^{-1}\left(\frac{-y}{f_m}\right)\right) \\
&= \frac{\cos^{-1}\left(\frac{-y}{f_m}\right)}{\pi} \qquad\qquad \text{for } y \in [-f_m, f_m] \\
\Rightarrow f_Y(y) &= \frac{\partial}{\partial y} F_Y(y) \\
&= \frac{1}{\pi f_m \sqrt{1 - \left(\frac{y}{f_m}\right)^2}}
\end{aligned}
$$

Now, take v=20km/h, f_c=2GHz to compare the corresponding $F_Y(y)$ and $f_Y(y)$ with simulation of (a)



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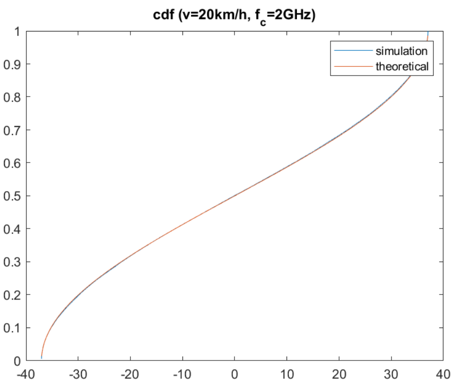

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Figure 4: compare theoretical result with simulation of (a)

Note that, since $f_Y(y) \to \infty$ as $y \to \pm 1$ in order to plot $f_Y(y)$ and the simulation pdf of (a) in one screen, I discard the points $(y, f_Y(y))$ that are near ± 1 at first axis

Furthermore, the theoretical cdf and the simulative cdf are perfectly consistent but it is not the case for pdf. And it may be due to the fact that $f_Y(y) \to \infty$ as $y \to \pm 1$ but simulation pdf can't approach ∞ no matter how near are y and ± 1