Wireless Communication Systems HW2

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1. Consider that a MS with a velocity v receives an unmodulated carrier with a frequency f_c*. The incidence angle* $\theta(t)$ *of the incoming wave is assumed to be uniformly distributed between* $-\pi$ *and* π*.*

(a) If $v = 20km/hr$ *and* $f_c = 2\ GHz$*, find the distribution function(cdf) and the probability density function(pdf) of the observed Doppler shift via simulation.*

1.1.1 Answer–(a)

模擬假設的 sample points N 越大，計算得 PDF 跟 CDF 曲線就越平滑，武斷取 N 為 100 萬個，用 rand()隨機產生 100 萬個平均分布於正負π的 incident angle $\theta(t)$，並利用 *histcounts()*跟 *cdfplot()*得到 PDF 跟 CDF，其他參數依題意代入條件去模擬。

$$f_{D,n}(t) = f_m \cos\theta_n(t),\ f_m = \frac{v}{\lambda_c} = \frac{v}{v_c/f_c}$$

1.1.2 Code–(a)

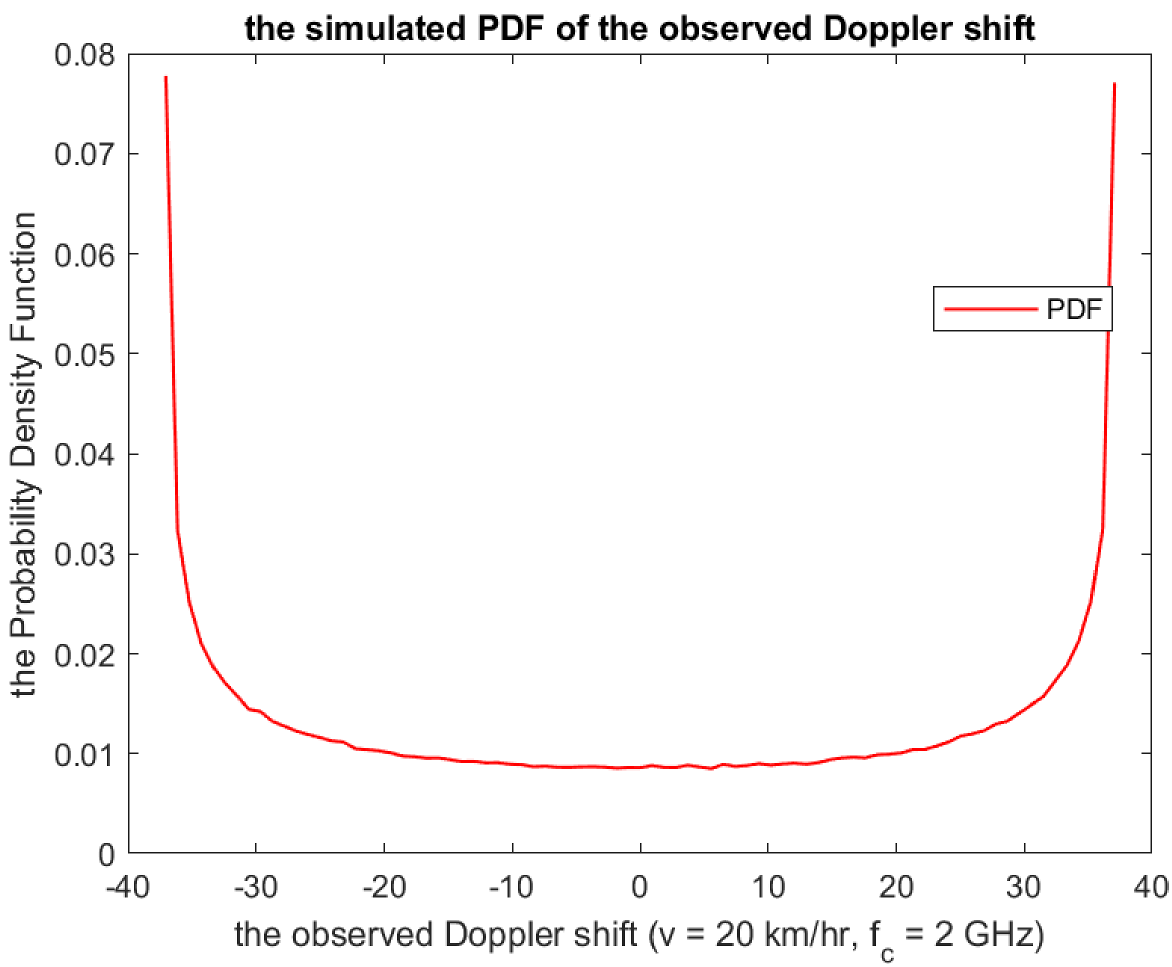
```
%
% Wireless Communication Systems HW2, 通訊所一年級 110064533 陳劭珩
%
% 1.Consider that a MS with a velocity v receives an unmodulated carrier
%   with a frequency f_c. The incidence angle θ(t) of the incoming wave is
%   assumed to be uniformly distributed between -π and π.
%  (a) If v = 20 km/hr and f_c = 2 GHz, find the distribution function(cdf)
%      and the probability density function(pdf) of the observed Doppler
%      shift via simulation.
%
clear;
clc;
%
N = 1e6;              % number of samples N, say, 1 million samples
%
% from $ doc rand, we can generate N random numbers in the interval (a, b)
```

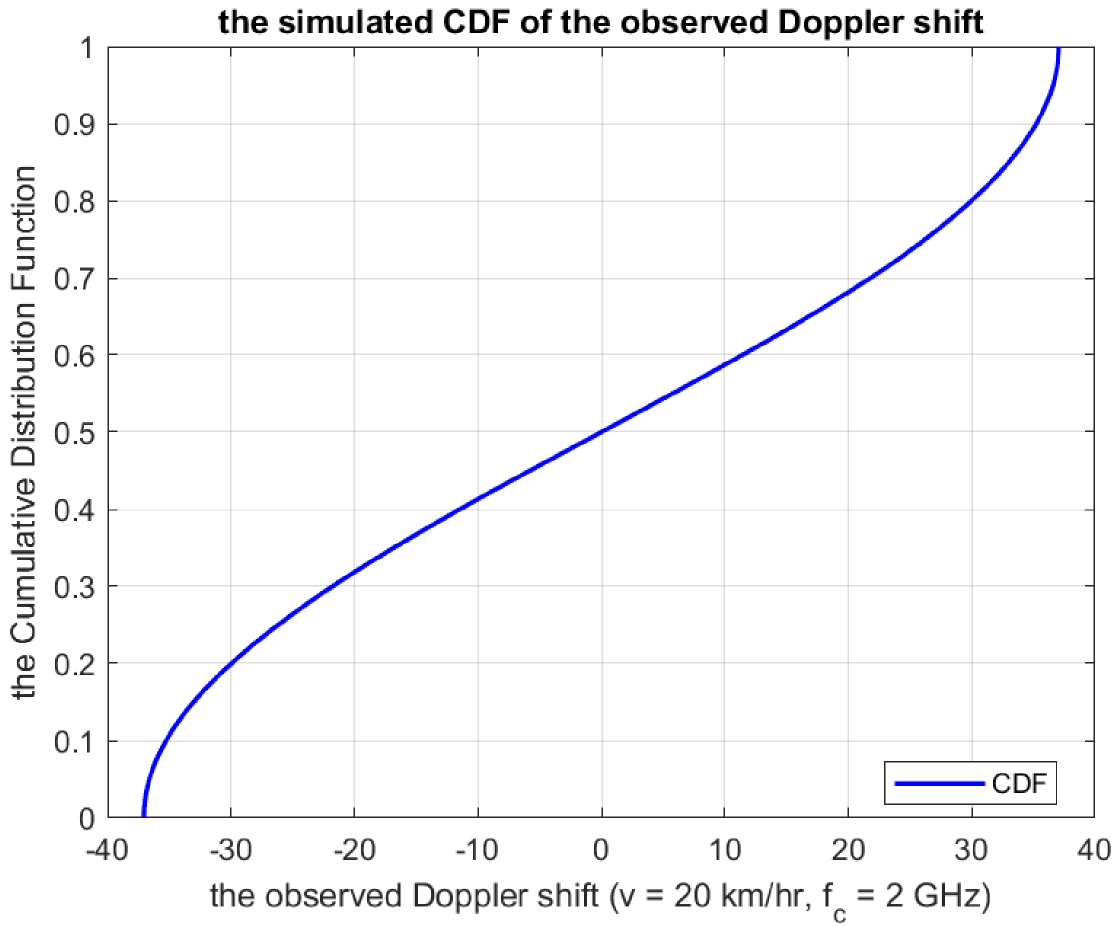
```
% in array r = a + (b-a).*rand(1, N), in our case incidence angle θ(t) are
% uniformly distributed between -π and π
theta = -pi + (pi + pi).*rand(1, N); % incidence angle θ(t)
v = 20*1000/(60*60);  % MS velocity v = 20 km/hr = 20*1000/3600 m/sec
%
fc = 2*1e9;           % unmodulated carrier frequency f_c = 2 GHz
vc = 299792458;       % speed of light v_c = 3*10^8 m/sec
fm = v / (vc/fc);     % f_m = v / λ_c = v / (v_c/f_c)
%
fDn = fm*cos(theta);  % doppler frequency shift f_Dn = f_m*cos(θ(t))
%
% [N, edges] = histcounts(X, edges, 'Normalization','pdf', ...
%                         'BinLimits', [BMIN, BMAX]);
% X - Data to distribute among bins, specified as a vector, or n-D array.
% edges - Number of bins, specified as a positive integer. If not specified
%         then histcountsautomatically calculates how many bins to use
%         based on the values in X.
% 'Normalization' - Type of normalization, 'count'(default) | 'probability'
%                   | 'countdensity' | 'pdf' | 'cumcount' | 'cdf'
% 'BinLimits' - Specified as a two-element vector, [bmin, bmax]
%
[fDn_num, edges] = histcounts(fDn, 'normalization', 'pdf', ...
                              'BinLimits', [-fm, fm]);

%
% linspace(x1, x2, n) - Generates n points, and spacing between the points
%                       is (x2 - x1) / (n - 1). Create a vector of numbers
%                       with n evenly spaced points between x1 and x2.
%
x = linspace(edges(1), edges(end), length(edges) - 1);  % x-coordinate
figure(1); % figure1 plot PDF
plot(x, fDn_num, 'Color', [1, 0, 0], 'LineWidth', 1, 'LineStyle', '-');
legend('PDF', 'Location', 'best');
xlabel('the observed Doppler shift (v = 20 km/hr, f_c = 2 GHz)');
ylabel('the Probability Density Function');
title('the simulated PDF of the observed Doppler shift');
% save the plot as 'HW2_a_pdf_110064533.png'
% print('HW2_a_pdf_110064533.png', '-dpng');
%
%
```

```
figure(2); % figure2 plot CDF
CDF = cdfplot(fDn);
set(CDF, 'LineWidth', 1.5, 'LineStyle', '-', 'Color', 'b');
legend('CDF', 'Location', 'best');
xlabel('the observed Doppler shift (v = 20 km/hr, f_c = 2 GHz)');
ylabel('the Cumulative Distribution Function');
title('the simulated CDF of the observed Doppler shift');
% save the plot as 'HW2_a_cdf_110064533.png'
% print('HW2_a_cdf_110064533.png', '-dpng');
%
%
```

1.1.3 Plot–(a)





(b) If $v = 90km/hr$ *and* $f_c = 26\ GHz$*, find the distribution function(cdf) and the probability density function(pdf) of the observed Doppler shift via simulation.*

1.2.1 Answer–(b)

模擬跟(a)小題差不多，一樣假設的 sample points N 越大，計算得 PDF 跟 CDF 曲線就越平滑，一樣武斷取 N 為 100 萬個，用 rand()隨機產生 100 萬個平均分布於正負π的 incident angle $\theta(t)$，並利用 *histcounts()*跟 *cdfplot()*得到 PDF 跟 CDF，其他參數一樣依題意代入條件去模擬，基本上只要改v跟f_c即可。

$$f_{D,n}(t) = f_m \cos\theta_n(t),\ f_m = \frac{v}{\lambda_c} = \frac{v}{v_c/f_c}$$

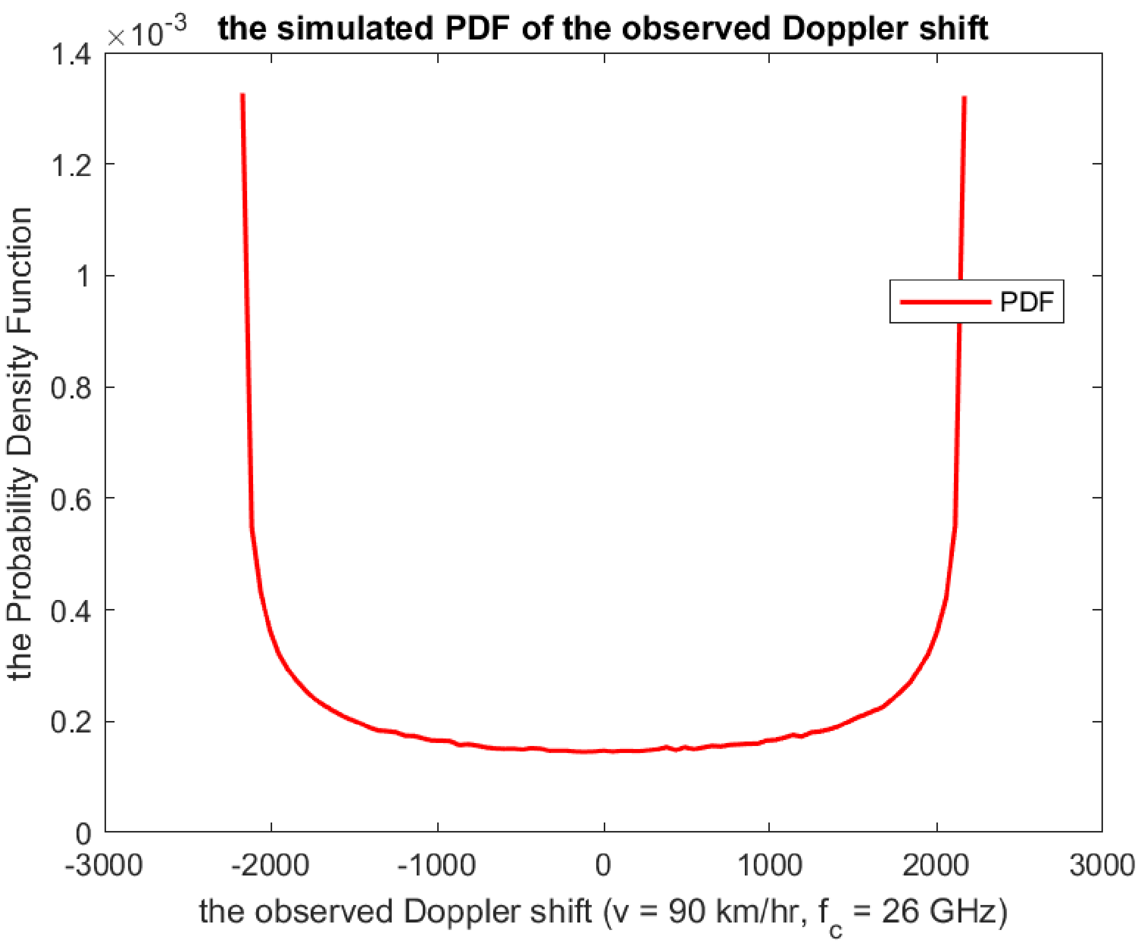
1.2.2 Code–(b)

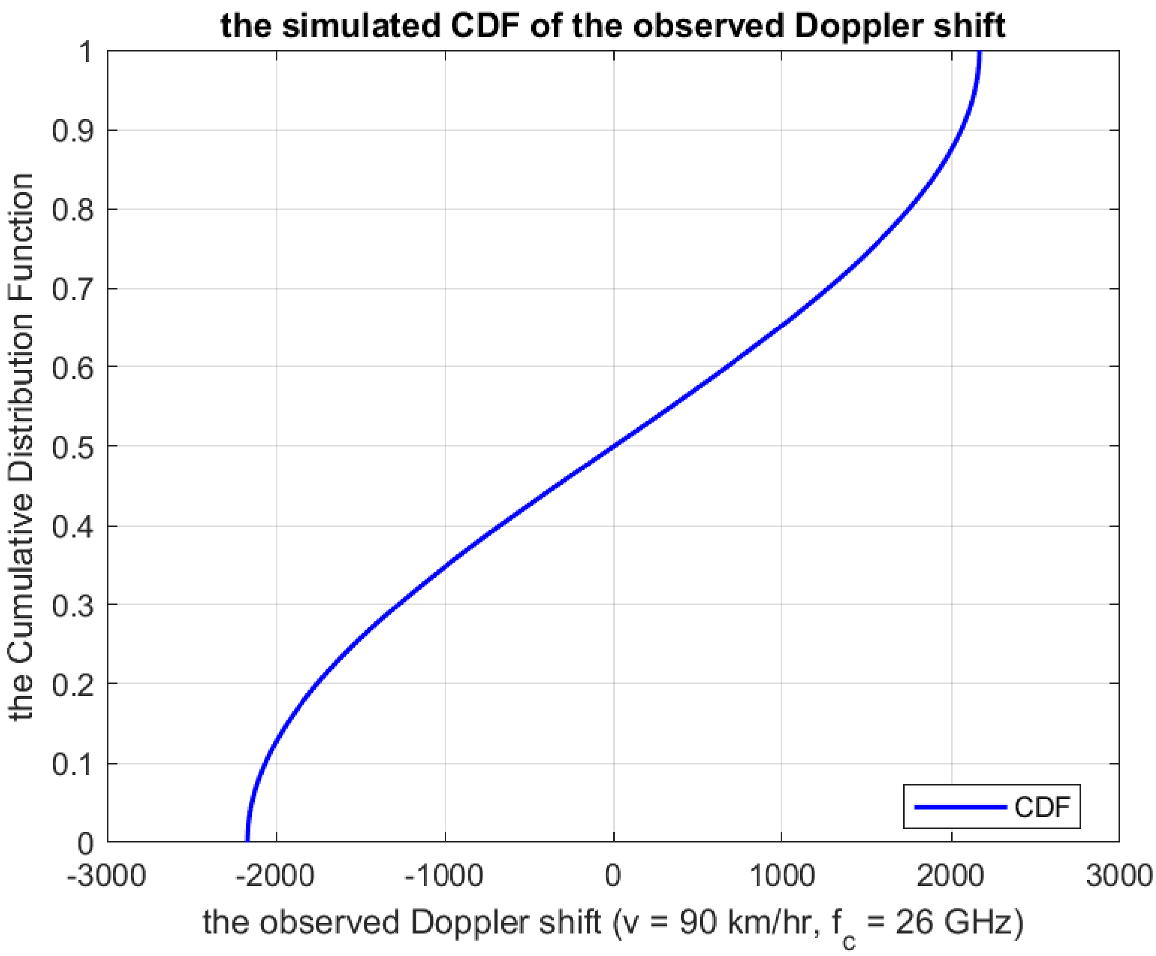
```
%
% Wireless Communication Systems HW2, 通訊所一年級 110064533 陳劭珩
%
% 1.Consider that a MS with a velocity v receives an unmodulated carrier
```

```
%   with a frequency f_c. The incidence angle θ(t) of the incoming wave is
%   assumed to be uniformly distributed between -π and π.
%  (b) If v = 90 km/hr and f_c = 26 GHz, find the cdf and the pdf of the
%      observed Doppler shift via simulation.
%
clear;
clc;
%
N = 1e6;                % number of samples N, say, 1 million samples
%
% from $ doc rand, we can generate N random numbers in the interval (a, b)
% in array r = a + (b-a).*rand(1, N), in our case incidence angle θ(t) are
% uniformly distributed between -π and π
theta = -pi + (pi + pi).*rand(1, N); % incidence angle θ(t)
v = 90*1000/(60*60);   % MS velocity v = 90 km/hr = 20*1000/3600 m/sec
%
fc = 26*1e9;            % unmodulated carrier frequency f_c = 26 GHz
vc = 299792458;         % speed of light v_c = 3*10^8 m/sec
fm = v / (vc/fc);       % f_m = v / λ_c = v / (v_c/f_c)
%
fDn = fm*cos(theta);   % doppler frequency shift f_Dn = f_m*cos(θ(t))
%
% [N, edges] = histcounts(X, edges, 'Normalization','pdf', ...
%                          'BinLimits', [BMIN, BMAX]);
% X - Data to distribute among bins, specified as a vector, or n-D array.
% edges - Number of bins, specified as a positive integer. If not specified
%         then histcountsautomatically calculates how many bins to use
%         based on the values in X.
% 'Normalization' - Type of normalization, 'count'(default) | 'probability'
%                   | 'countdensity' | 'pdf' | 'cumcount' | 'cdf'
% 'BinLimits' - Specified as a two-element vector, [bmin, bmax]
%
[fDn_num, edges] = histcounts(fDn, 'normalization', 'pdf', ...
                              'BinLimits', [-fm, fm]);
%
% linspace(x1, x2, n) - Generates n points, and spacing between the points
%                       is (x2 - x1) / (n - 1). Create a vector of numbers
%                       with n evenly spaced points between x1 and x2.
%
x = linspace(edges(1), edges(end), length(edges) - 1);  % x-coordinate
figure(1); % figure1 plot PDF
plot(x, fDn_num, 'Color', [1, 0, 0], 'LineWidth', 1.5, 'LineStyle', '-');
```

```
legend('PDF', 'Location', 'best');
xlabel('the observed Doppler shift (v = 90 km/hr, f_c = 26 GHz)');
ylabel('the Probability Density Function');
title('the simulated PDF of the observed Doppler shift');
% save the plot as 'HW2_b_pdf_110064533.png'
% print('HW2_b_pdf_110064533.png', '-dpng');
%
%
figure(2); % figure2 plot CDF
CDF = cdfplot(fDn);
set(CDF, 'LineWidth', 1.5, 'LineStyle', '-', 'Color', 'b');
legend('CDF', 'Location', 'best');
xlabel('the observed Doppler shift (v = 90 km/hr, f_c = 26 GHz)');
ylabel('the Cumulative Distribution Function');
title('the simulated CDF of the observed Doppler shift');
% save the plot as 'HW2_b_cdf_110064533.png'
% print('HW2_b_cdf_110064533.png', '-dpng');
```

1.2.3 Plot–(b)





(c) If $f_c = 2\ GHz$ *and* v *is uniformly distributed between* $20km/hr$ *and* $20km/hr$, *find the cdf and the pdf of the observed Doppler shift via simulation.*

1.3.1 Answer–(c)

模擬跟(a)(b)小題差不多，一樣假設的 sample points N 越大，計算得 PDF 跟 CDF 曲線就越平滑，一樣武斷取 N 為 100 萬個，用 rand()隨機產生 100 萬個平均分布於正負π的 incident angle $\theta(t)$，並利用 *histcounts()*跟 *cdfplot()*得到 PDF 跟 CDF，剩餘參數一樣依題意代入條件，其中 MS 的速度v改為用 rand()產生 100 萬個平均分布於$20 \sim 90km/hr$的隨機值去模擬。

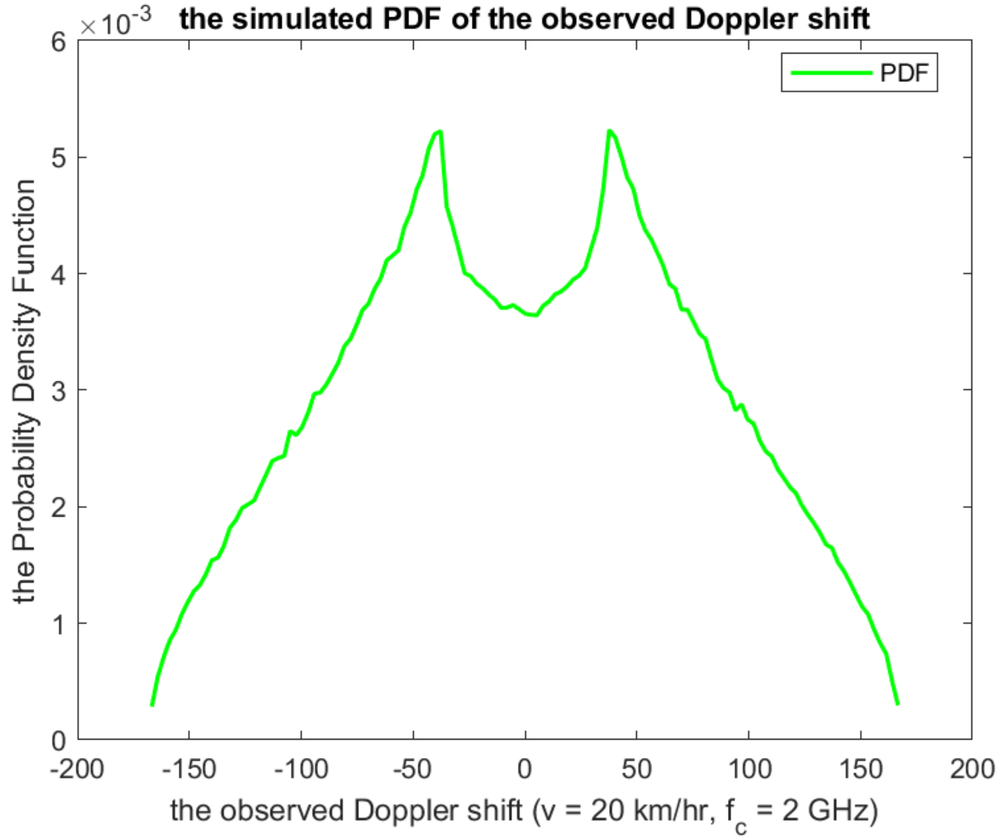
$$f_{D,n}(t) = f_m \cos\theta_n(t),\ \ f_m = \frac{v}{\lambda_c} = \frac{v}{v_c/f_c}$$

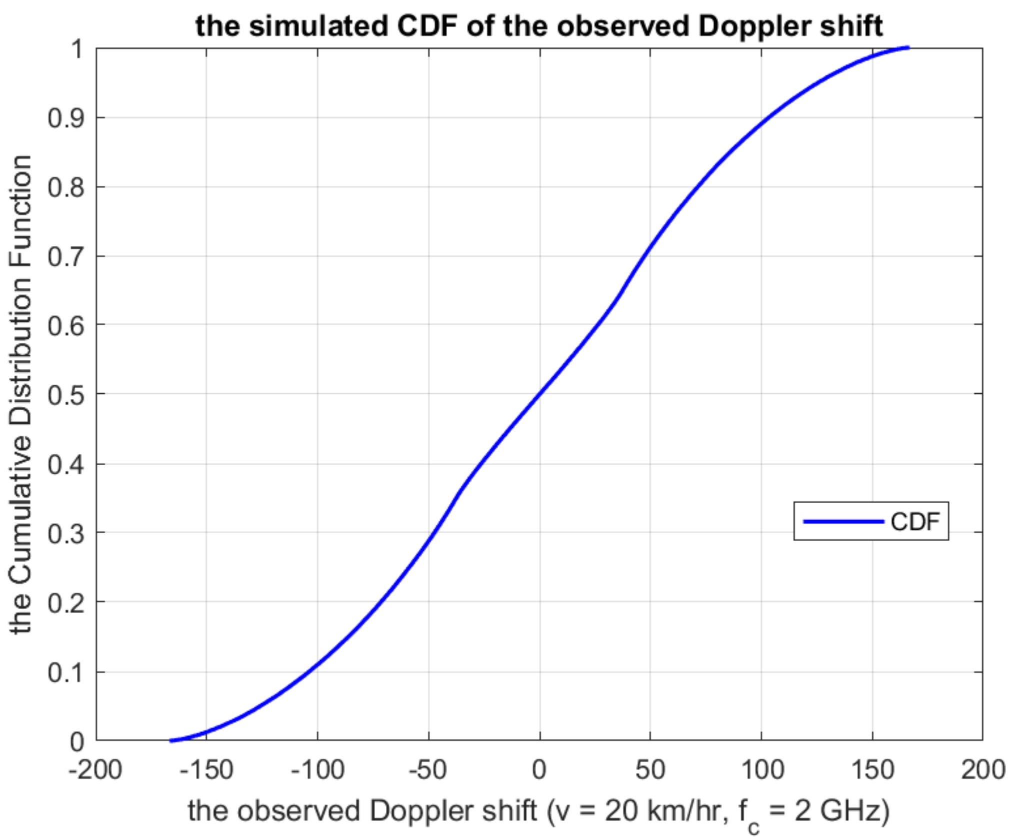
1.3.2 Code–(c)

```
%
% Wireless Communication Systems HW2, 通訊所一年級 110064533 陳劭珩
%
% 1.Consider that a MS with a velocity v receives an unmodulated carrier
%   with a frequency f_c. The incidence angle θ(t) of the incoming wave is
%   assumed to be uniformly distributed between -π and π.
%  (c) If f_c = 2 GHz and v is uniformly distributed between 20 km/hr and
%      90 km/hr, find the cdf and the pdf of the observed Doppler shift
%      via simulation.
%
clear;
clc;
%
%
N = 1e6;              % number of samples N, say, 1 million samples
%
% from $ doc rand, we can generate N random numbers in the interval (a, b)
% in array r = a + (b-a).*rand(1, N), in our case incidence angle θ(t) are
% uniformly distributed between -π and π
theta = -pi + (pi + pi).*rand(1, N);          % incidence angle θ(t)
v = (20 + (90-20).*rand(1, N))*1000/(60*60); % MS velocity v = 20~90 km/hr
%
fc = 2*1e9;           % unmodulated carrier frequency f_c = 2 GHz
vc = 299792458;       % speed of light v_c = 3*10^8 m/sec
fm = v / (vc/fc);     % f_m = v / λ_c = v / (v_c/f_c)
%
% because fm is vector, to perform elementwise multiplication, use ".*"
fDn = fm.*cos(theta); % doppler frequency shift f_Dn = f_m*cos(θ(t))
%
% [N, edges] = histcounts(X, edges, 'Normalization','pdf', ...
%                          'BinLimits', [BMIN, BMAX]);
% X - Data to distribute among bins, specified as a vector, or n-D array.
% edges - Number of bins, specified as a positive integer. If not specified
%         then histcountsautomatically calculates how many bins to use
%         based on the values in X.
% 'Normalization' - Type of normalization, 'count'(default) | 'probability'
%                   | 'countdensity' | 'pdf' | 'cumcount' | 'cdf'
% 'BinLimits' - Specified as a two-element vector, [bmin, bmax]
%
```

```
BMAX = 90*1000/(60*60) / (vc/fc); % v_max / λ_c = v_max / (v_c/f_c) = f_max
BMIN = (-1)*BMAX;                 % v_min / λ_c = v_min / (v_c/f_c) = f_min
[fDn_num, edges] = histcounts(fDn, 'normalization', 'pdf', ...
                              'BinLimits', [BMIN, BMAX]);
%
% linspace(x1, x2, n) - Generates n points, and spacing between the points
%                       is (x2 - x1) / (n - 1). Create a vector of numbers
%                       with n evenly spaced points between x1 and x2.
%
x = linspace(edges(1), edges(end), length(edges) - 1);  % x-coordinate
figure(1); % figure1 plot PDF
plot(x, fDn_num, 'Color', [0, 1, 0], 'LineWidth', 1.5, 'LineStyle', '-');
legend('PDF', 'Location', 'best');
xlabel('the observed Doppler shift (v = 20 km/hr, f_c = 2 GHz)');
ylabel('the Probability Density Function');
title('the simulated PDF of the observed Doppler shift');
% save the plot as 'HW2_c_pdf_110064533.png'
% print('HW2_c_pdf_110064533.png', '-dpng');
%
%
figure(2); % figure2 plot CDF
CDF = cdfplot(fDn);
set(CDF, 'LineWidth', 1.5, 'LineStyle', '-', 'Color', 'b');
legend('CDF', 'Location', 'best');
xlabel('the observed Doppler shift (v = 20 km/hr, f_c = 2 GHz)');
ylabel('the Cumulative Distribution Function');
title('the simulated CDF of the observed Doppler shift');
% save the plot as 'HW2_c_cdf_110064533.png'
% print('HW2_c_cdf_110064533.png', '-dpng');
%
%
```

1.3.3 Plot–(c)


the simulated PDF of the observed Doppler shift
×10^-3
6
5
4
3
2
1
0
-200
-150
-100
-50
0
50
100
150
200
PDF
the Probability Density Function
the observed Doppler shift (v = 20 km/hr, f_c = 2 GHz)


the simulated CDF of the observed Doppler shift
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-200
-150
-100
-50
0
50
100
150
200
CDF
the Cumulative Distribution Function
the observed Doppler shift (v = 20 km/hr, f_c = 2 GHz)

(d) Derive the cdf and the pdf of the observed Doppler shift for fixed v and f_c. Compare the simulation results with the theoretical results.

1.4.1 Answer–(d)

(d) derive the cdf and pdf of the observed Doppler shift for fixed v and f_c.

$$f_{D,n}(t) = f_m \cos\theta_n(t) \rightarrow \frac{f_{D,n}(t)}{f_m} = \cos\theta_n(t), \quad \theta_n(t) \sim U[-\pi, \pi]$$

$\frac{f_{D,n}(t)}{f_m} \Downarrow Y$, $\theta_n(t) \Downarrow W$

Let $Y = \cos W \rightarrow P(W \le \pi) = \frac{\omega}{\pi}$

cdf $F_Y(y) = P(Y \le y)$

$= P(\cos W \le y)$

$= P(W \ge \cos^{-1}(y))$

$= P(W \le \cos^{-1}(-y))$

$= \cos^{-1}(-y)/\pi$ #

pdf $f_Y(y) = \frac{d}{dy} F_Y(y)$

$= \frac{1}{\pi} \frac{d}{dy} \cos^{-1}(-y)$

$\cos^{-1} y = ?$

令 $y = \cos^{-1} x \rightarrow \cos y = x$

$$\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}} = \frac{1}{\frac{d}{dy}\cos y} = \frac{1}{-\sin y}$$

$\sin^2 y = 1 - \cos^2 y$

$\sin y = \pm\sqrt{1-\cos^2 y}$ (取正)

$$\frac{dy}{dx} = \frac{1}{-\sin y} = \frac{-1}{\sqrt{1-\cos^2 y}}$$

$$\Rightarrow \frac{d}{dx}\cos^{-1} x = \frac{-1}{\sqrt{1-x^2}}$$

$$\boxed{\frac{d}{dy}\cos^{-1} y = \frac{-1}{\sqrt{1-y^2}}}, \quad -1 \le y \le 1$$

$\Rightarrow \frac{1}{\pi} \cdot \frac{d}{dy} \cos^{-1}(-y)$

$= \frac{1}{\pi} \cdot \frac{1}{\sqrt{1-y^2}}$, $y = \frac{f_{D,n}(t)}{f_m} = \cos\theta_n(t)$

$\therefore \frac{f_{D,n}(t)}{f_m}$ 的 theoretical PDF

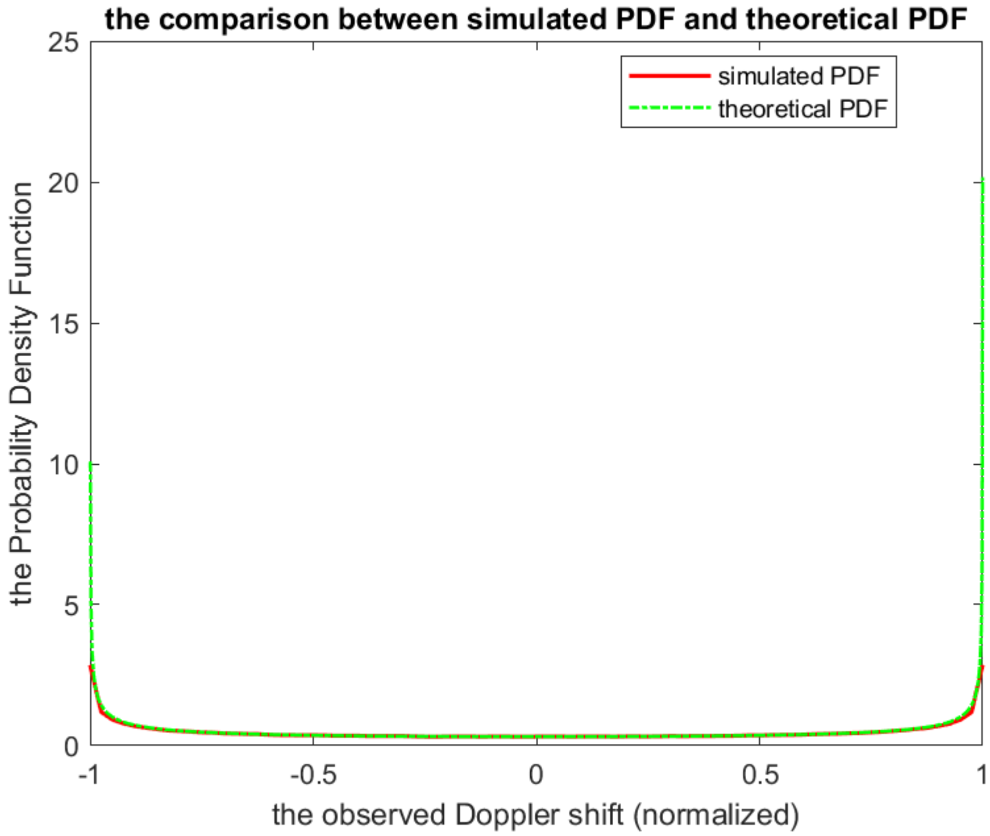
$$= \frac{1}{\pi} \cdot \frac{1}{\sqrt{1 - \left(\frac{f_{D,n}(t)}{f_m}\right)^2}}$$ #

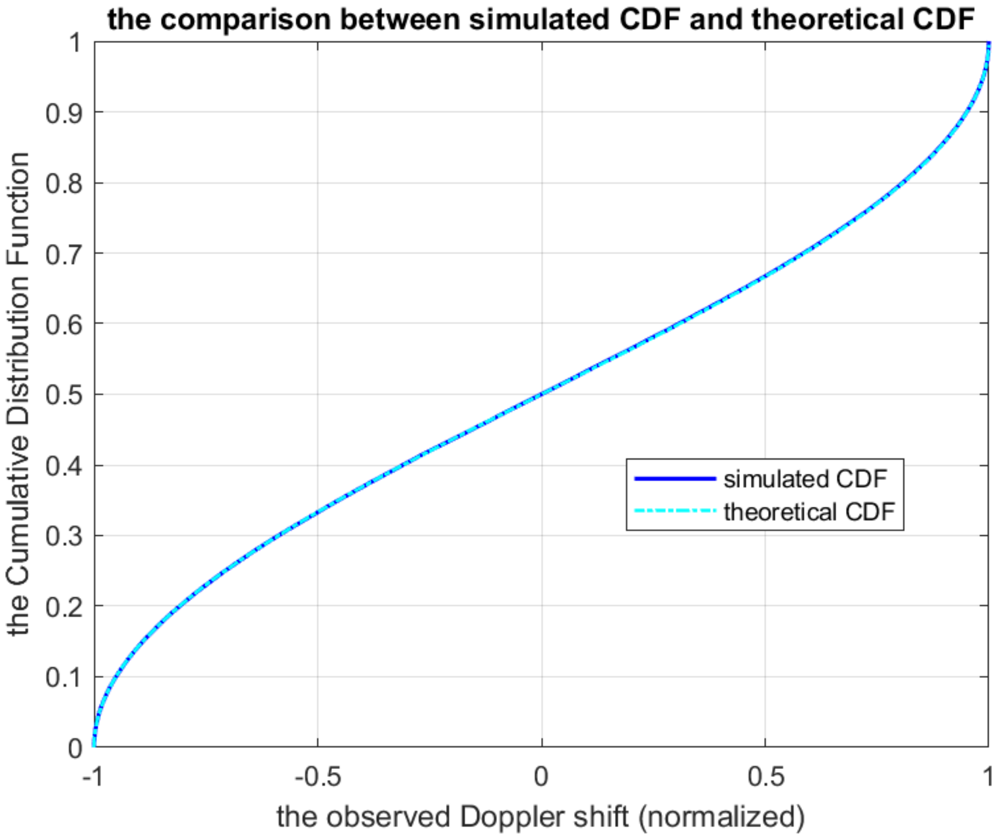
1.4.2 Code–(d)

```
%
% Wireless Communication Systems HW2, 通訊所一年級 110064533 陳劭珩
%
% 1.Consider that a MS with a velocity v receives an unmodulated carrier
%   with a frequency f_c. The incidence angle θ(t) of the incoming wave is
%   assumed to be uniformly distributed between -π and π.
%  (d) Derive the cdf and the pdf of the observed Doppler shift for fixed v
%      and fc. Compare the simulation results with the theoretical results.
%
clear;
clc;
%
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
%
% simulated result, settings are the same as before.
%
N = 1e6; % number of samples N, say, 1 million samples
%
theta_simulated = -pi + (pi + pi).*rand(1, N); % incidence angle θ(t)
fDn_simulated = cos(theta_simulated); % normalized fDn = fDn/fm = cosθ(t)
%
[fDn_num, edges] = histcounts(fDn_simulated, 'normalization', 'pdf', ...
                              'BinLimits', [-1, 1]);
%
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
%
% theoretical result, settings are base on the derived formula
%
theta_theoretical = linspace((-1)*pi, pi, 200);
fDn_theoretical = cos(theta_theoretical);
pdf_theoretical = 1 ./ (pi * sqrt(1 - (fDn_theoretical).^2));
cdf_theoretical = acos((-1)*fDn_theoretical) / pi;
% cdf_theoretical = 1 - (acos(fDn_theoretical) / pi);
%
% % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % % %
%
% plot the comparison results
%
```

```
%
figure(1); % figure1 plot the comparison of PDFs
%
% first, the simulated PDF, in red solid line
x = linspace(edges(1), edges(end), length(edges) - 1);  % x-coordinate
plot(x, fDn_num, 'Color', 'r', 'LineWidth', 1.5, 'LineStyle', '-');
hold on;
%
% then, the theoretical PDF, in green, dash-dot line
plot(cos(theta_theoretical), pdf_theoretical, 'g-.', 'LineWidth', 1.2);
%
xlabel('the observed Doppler shift (normalized)');
ylabel('the Probability Density Function');
title('the comparison between simulated PDF and theoretical PDF');
legend('simulated PDF', 'theoretical PDF', 'Location', 'best');
hold off;
% save the plot as 'HW2_d_pdf_110064533.png'
% print('HW2_d_pdf_110064533.png', '-dpng');
%
%
figure(2); % figure2 plot the comparison of CDFs
%
% first, the simulated CDF, in blue solid line
simulated_CDF = cdfplot(fDn_simulated);
set(simulated_CDF, 'LineWidth', 1.5, 'LineStyle', '-', 'Color', 'b');
hold on;
%
% then, the theoretical CDF, in cyan, dash-dot line
plot(cos(theta_theoretical), cdf_theoretical, 'c-.', 'LineWidth', 1.2);
%
xlabel('the observed Doppler shift (normalized)');
ylabel('the Cumulative Distribution Function');
title('the comparison between simulated CDF and theoretical CDF');
legend('simulated CDF', 'theoretical CDF', 'Location', 'best');
hold off;
% save the plot as 'HW2_d_cdf_110064533.png'
% print('HW2_d_cdf_110064533.png', '-dpng');
%
```

1.4.3 Plot–(d)


the comparison between simulated PDF and theoretical PDF
simulated PDF
theoretical PDF
the Probability Density Function
the observed Doppler shift (normalized)
25
20
15
10
5
0
-1
-0.5
0
0.5
1


the comparison between simulated CDF and theoretical CDF
simulated CDF
theoretical CDF
the Cumulative Distribution Function
the observed Doppler shift (normalized)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-1
-0.5
0
0.5
1