Exercises Set 5

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September 25, 2023

Abstract

Only the questions with a * are compulsory (but do all of them!).

1 Change of Basis

Let $B = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be a basis for $\mathbb{R}^3$ and let P be the matrix whose columns are the coordinates of $\mathbf{v}_1$, $\mathbf{v}_2$, and $\mathbf{v}_3$ with respect to the standard basis. That is,

$$P = \begin{bmatrix} \mathbf{v}_1 & \mathbf{v}_2 & \mathbf{v}_3 \end{bmatrix}$$

Suppose we have another basis $B' = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ for $\mathbb{R}^3$ and let Q be the matrix whose columns are the coordinates of $\mathbf{u}_1$, $\mathbf{u}_2$, and $\mathbf{u}_3$ with respect to the standard basis. That is,

$$Q = \begin{bmatrix} \mathbf{u}_1 & \mathbf{u}_2 & \mathbf{u}_3 \end{bmatrix}$$

We want to find the change of basis matrix C that allows us to express the coordinates of vectors in B in terms of the coordinates in B'. This can be done using the formula:

$$C = Q^{-1}P$$

To find C:

1. Find the inverse of matrix Q, denoted as Q^{-1}.
2. Multiply Q^{-1} by P to obtain the change of basis matrix C.

Now, let's calculate C and use it to express the vector $\mathbf{v}$ in terms of the basis B', where $\mathbf{v}$ is a vector in $\mathbb{R}^3$ with coordinates $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ with respect to basis B.

$$\mathbf{v} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

To express $\mathbf{v}$ in terms of the basis B', we can use the change of basis matrix C as follows:

$$\mathbf{v}' = C\mathbf{v}$$

Find $\mathbf{v}'$.