Exercises Set 2

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September 14, 2023

Abstract

Only the questions with a * are compulsory (but do all of them!).

1 Systems of Linear Equations

Reduced Row Echelon Form Find the Reduced Row Echelon Form of the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 6 & 2 & 3 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

Gaussian Elimination (*) Solve the following linear system using Gaussian Elimination:

$$2x + y - z = 4$$
$$3x + 2y + z = 5$$
$$x - y + 3z = 7$$

Start by writing the augmented matrix for the system and perform the necessary row operations to find the solution.

2 Vector Spaces

Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

Show that $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ are linearly independent.

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Exercise 2: Let $\mathbb{P}_2$ be the vector space of polynomials of degree at most 2. Consider the following polynomials:

$$p_1(x) = 1, \quad p_2(x) = 2x, \quad p_3(x) = 3x^2 - 1$$

(a) Determine whether the polynomials $p_1(x)$, $p_2(x)$, and $p_3(x)$ form a spanning set for $\mathbb{P}_2$.

(b) If they do, express an arbitrary polynomial $q(x) \in \mathbb{P}_2$ as a linear combination of $p_1(x)$, $p_2(x)$, and $p_3(x)$.