

# Derivatives

**Question 1.** *Calculate the derivative of the following functions:*

- $f_0(x) = 3x^2$
- $f_1(x) = 5x^2 - 18$
- $f_2(x) = 5x^2 - 18x + 39$
- $f_3(x) = \sin(x)$
- $f_4(x) = \sin(x) * x^2$
- $f_5(x) = \frac{5x^3 - 2x + 1}{2x - 7}$
- $f_6(x) = ax^2 + bx + c$

**Question 2.** *Calculate the second order derivative of the same functions:*

- $f_0(x) = 3x^2$
- $f_1(x) = 5x^2 - 18$
- $f_2(x) = 5x^2 - 18x + 39$
- $f_3(x) = \sin(x)$
- $f_4(x) = \sin(x) * x^2$
- $f_5(x) = \frac{5x^3 - 2x + 1}{2x - 7}$
- $f_6(x) = ax^2 + bx + c$

**Question 3.** *Find the anti-derivative of the following functions:*

- $g_0(x) = 3x^2$
- $g_1(x) = 5x^2 - 18$
- $g_2(x) = 5x^2 - 18x + 39$
- $g_3(x) = \sin(x)$
- $g_4(x) = ax^2 + bx + c$

**Question 4.** *Calculate the following partial derivatives:*

- $h_1(x, y) = 3x^2 + y^2$  w.r.t.  $x$  ( $\frac{\partial h_1}{\partial x}$ )
- $h_1(x, y) = 3x^2 + y^2$  w.r.t.  $y$  ( $\frac{\partial h_1}{\partial y}$ )

- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$  w.r.t.  $x$  ( $\frac{\partial h_2}{\partial x}$ )
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$  w.r.t.  $y$  ( $\frac{\partial h_2}{\partial y}$ )
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$  w.r.t.  $z$  ( $\frac{\partial h_2}{\partial z}$ )

**Question 5.** Calculate the following second / third order partial derivatives:

- $h_1(x, y) = 3x^2 + y^2$  w.r.t.  $x$  then  $y$  ( $\frac{\partial^2 h_1}{\partial x \partial y}$ )
- $h_1(x, y) = 3x^2 + y^2$  w.r.t.  $y$  then  $x$  ( $\frac{\partial^2 h_1}{\partial y \partial x}$ )
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$  w.r.t.  $x$  and  $x$  ( $\frac{\partial^2 h_2}{\partial x^2}$ )
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$  w.r.t.  $y$  and  $x$  ( $\frac{\partial^2 h_2}{\partial y \partial x}$ )
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$  w.r.t.  $z$  then  $x$  and  $y$  ( $\frac{\partial^3 h_2}{\partial x \partial y \partial z}$ )

As a reminder, one step of gradient descent is done using the formula

$$x_{n+1} = x_n - \lambda f'(x_n)$$

where  $x_n$  is the current point,  $\lambda$  is the 'learning rate', and  $f'$  (sometimes written  $\frac{df}{dx}$ ) is the derivative of  $f$ ,  $f$  being the function to minimize.

**Question 6.** Calculate 5 steps of gradient descent with learning rate of  $\lambda = 0.8$ , starting from  $x_0 = -0.25$  for the function  $f(x) = x^2 - x + 3$ .

Conjecture what is the exact minimum of  $f$ ; how far is  $x_1$  from it? and  $x_4$ ?

What happens if the learning rate is  $\lambda = 1$ ? and  $\lambda = 2$ ? and  $\lambda = 0.1$ ? and  $\lambda = 0.01$ ? (do only 3 steps)