

Derivatives

Question 1. *Calculate the derivative of the following functions:*

- $f_0(x) = 3x^2$
- $f_1(x) = 5x^2 - 18$
- $f_2(x) = 5x^2 - 18x + 39$
- $f_3(x) = \sin(x)$
- $f_4(x) = \sin(x) * x^2$
- $f_5(x) = \frac{5x^3 - 2x + 1}{2x - 7}$
- $f_6(x) = ax^2 + bx + c$

Question 2. *Calculate the second order derivative of the same functions:*

- $f_0(x) = 3x^2$
- $f_1(x) = 5x^2 - 18$
- $f_2(x) = 5x^2 - 18x + 39$
- $f_3(x) = \sin(x)$
- $f_4(x) = \sin(x) * x^2$
- $f_5(x) = \frac{5x^3 - 2x + 1}{2x - 7}$
- $f_6(x) = ax^2 + bx + c$

Question 3. *Find the anti-derivative of the following functions:*

- $g_0(x) = 3x^2$
- $g_1(x) = 5x^2 - 18$
- $g_2(x) = 5x^2 - 18x + 39$
- $g_3(x) = \sin(x)$
- $g_4(x) = ax^2 + bx + c$

Question 4. *Calculate the following partial derivatives:*

- $h_1(x, y) = 3x^2 + y^2$ w.r.t. x ($\frac{\partial h_1}{\partial x}$)
- $h_1(x, y) = 3x^2 + y^2$ w.r.t. y ($\frac{\partial h_1}{\partial y}$)

- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ w.r.t. x ($\frac{\partial h_2}{\partial x}$)
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ w.r.t. y ($\frac{\partial h_2}{\partial y}$)
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ w.r.t. z ($\frac{\partial h_2}{\partial z}$)

Question 5. Calculate the following second / third order partial derivatives:

- $h_1(x, y) = 3x^2 + y^2$ w.r.t. x then y ($\frac{\partial^2 h_1}{\partial x \partial y}$)
- $h_1(x, y) = 3x^2 + y^2$ w.r.t. y then x ($\frac{\partial^2 h_1}{\partial y \partial x}$)
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ w.r.t. x and x ($\frac{\partial^2 h_2}{\partial x^2}$)
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ w.r.t. y and x ($\frac{\partial^2 h_2}{\partial y \partial x}$)
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ w.r.t. z then x and y ($\frac{\partial^3 h_2}{\partial x \partial y \partial z}$)

Question 6. Calculate 5 steps of gradient descent with learning rate of $\lambda = 0.8$, starting from $x_0 = -0.25$ for the function $f(x) = x^2 - x + 3$.

Conjecture what is the exact minimum of f ; how far is x_1 from it? and x_4 ?

What happens if the learning rate is $\lambda = 1$? and $\lambda = 2$? and $\lambda = 0.1$? and $\lambda = 0.01$? (do only 3 steps)