Derivatives

Question 1. *Calculate the derivative of the following functions:*

- $f_0(x) = 3x^2$
- $f_1(x) = 5x^2 - 18$
- $f_2(x) = 5x^2 - 18x + 39$
- $f_3(x) = \sin(x)$
- $f_4(x) = \sin(x) * x^2$
- $f_5(x) = \frac{5x^3 - 2x + 1}{2x - 7}$
- $f_6(x) = ax^2 + bx + c$

Question 2. *Calculate the second order derivative of the same functions:*

- $f_0(x) = 3x^2$
- $f_1(x) = 5x^2 - 18$
- $f_2(x) = 5x^2 - 18x + 39$
- $f_3(x) = \sin(x)$
- $f_4(x) = \sin(x) * x^2$
- $f_5(x) = \frac{5x^3 - 2x + 1}{2x - 7}$
- $f_6(x) = ax^2 + bx + c$

Question 3. *Find the anti-derivative of the following functions:*

- $g_0(x) = 3x^2$
- $g_1(x) = 5x^2 - 18$
- $g_2(x) = 5x^2 - 18x + 39$
- $g_3(x) = \sin(x)$
- $g_4(x) = ax^2 + bx + c$

Question 4. *Calculate the following partial derivatives:*

- $h_1(x, y) = 3x^2 + y^2$ *w.r.t.* x $(\frac{\partial h_1}{\partial x})$
- $h_1(x, y) = 3x^2 + y^2$ *w.r.t.* y $(\frac{\partial h_1}{\partial x})$
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ *w.r.t.* x $(\frac{\partial h_2}{\partial x})$
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ *w.r.t.* y $(\frac{\partial h_2}{\partial y})$

- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ *w.r.t.* z ($\frac{\partial h_2}{\partial z}$)

Question 5. *Calculate the following second / third order partial derivatives:*

- $h_1(x, y) = 3x^2 + y^2$ *w.r.t.* x *then* y ($\frac{\partial^2 h_1}{\partial x \partial y}$)
- $h_1(x, y) = 3x^2 + y^2$ *w.r.t.* y *then* x ($\frac{\partial^2 h_1}{\partial y \partial x}$)
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ *w.r.t.* x *and* x ($\frac{\partial^2 h_2}{\partial x^2}$)
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ *w.r.t.* y *and* x ($\frac{\partial^2 h_2}{\partial y \partial x}$)
- $h_2(x, y, z) = 5x^3 - 18y^2 - 18x + 39z^5 + 40xy + z^2x^3y$ *w.r.t.* z *then* x *and* y ($\frac{\partial^3 h_2}{\partial x \partial y \partial z}$)

As a reminder, one step of gradient descent is done using the formula

$$x_{n+1} = x_n - \lambda * f'(x_n)$$

where x_n is the current point, λ is the 'learning rate', and f' (sometimes written $\frac{df}{dx}$) is the derivative of f, f being the function to minimize.

Question 6. *Calculate 5 steps of gradient descent with learning rate of* $\lambda = 0.8$, *starting from* $x_0 = -0.25$ *for the function* $f(x) = x^2 - x + 3$.
Conjecture what is the exact minimum of f[1]*; how far is* x_1 *from it? and* x_4*?*
What happens if the learning rate is $\lambda = 1$*? and* $\lambda = 2$*? and* $\lambda = 0.1$*? and* $\lambda = 0.01$*? (do only 3 steps)*

[1] Plotting $x_0, x_1, x_2, x_3, \ldots$ may help.