

# Exercises Set 6

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## Abstract

Only the questions with a \* are compulsory (but do all of them!).

## 1 Lagrangian multiplier technique

### 1.1 Unconstrained optimization

Let  $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$ .

Find  $(x^*, y^*) \in \mathbb{R}^2$  such that  $f$  reaches its minimum (i.e.  $f(x^*, y^*) \leq f(x, y) \quad \forall (x, y) \in \mathbb{R}^2$ ).

### 1.2 Constrained optimization

Let  $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$ .

Suppose further that we want  $3x + 5y = 2$ .

Find  $(x^*, y^*) \in \mathbb{R}^2$  such that  $3x^* + 5y^* = 2$  and  $f$  reaches its minimum (i.e.  $f(x^*, y^*) \leq f(x, y) \quad \forall (x, y) \in \mathbb{R}^2, 3x + 5y = 2$ ).

### 1.3 Lagrange multiplier

Let  $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$ .

Suppose further that we want  $3x + 5y = 2$ .

Let  $\mathcal{L}(x, y, \lambda) = f(x, y) - \lambda(3x + 5y - 2)$ .

Find the point where  $\nabla f = 0$