

Mathematics Refresher Course

First Two Sessions

Paul Dubois

September 2023

Abstract

This course teaches basic mathematical methodologies for proofs. It is intended for students with a lack of mathematical background, or with a lack of confidence in mathematics. We will try to cover most of the prerequisites of the courses in the master's, i.e. basic algebra/analysis and basic applications.

1 Presentation

- Paul Dubois
- 3rd year PhD @ Centrale / TheraPanacea
- Research topic: AI applied to radiotherapy
- Email: `b00795695@essec.edu` (for any question)

Course structure

- 8*3h arranged as 1h20min lecture - 1/3h break - 1h20min lecture
- ~~No pb class planned, but lectures will have integrated live exercises~~
- ~~Interrupt if needed (do not wait for the end of the lecture)~~
- In this document, you will find the content of the first two sessions, with the small exercises we did "live".
- The remaining six sessions will be problem solving.
In case a session is spent on a topic you already, you can skip it
on the condition that you submit all compulsory exercises corresponding to that session.

- Examination

- The course is pass/fail
 - Spoiler: All of you will pass
 - ~~Home exercises, you will need 80+% to pass~~
 - ~~to complete exercises, it should take 30min to 1h~~
 - ~~2-4 exercises~~
 - ~~Hand in paper of PDF~~
 - ~~In the unlikely event of not passing, you will be able to do some extra work to pass~~
 - To pass, I will ask you, for each session, to either be in class, or submit the compulsory exercises.
 - The submission deadlines for the exercises set is exactly one week after the corresponding class.
- Submitting
 1. Solve exercises
 2. Export you work to a single PDF file (e.g. using a scanning smart-phone app)
 3. Rename your file "submission_nb_family_name.pdf" where:
 - "nb" is "2" for exercises set 2, "3" for exercise set 3, etc...
 - "family_name" is your family name in latin alphabet, capital letters

*Example: if I wanted to submit exercise set 1, the name of my file should have been "**submission_1_DUBOIS.pdf**"*
 4. Send me one new email per submission, please do not use the "reply" button, create a new email;
For the subject, you can just put the name of the file (or anything else that makes sense).

2 Sets

- sets of numbers (\mathbb{N} , \mathbb{Z} , \mathbb{R} , \mathbb{Q} , \mathbb{P})
- complex sets (with $\{\}$)
- examples (draw them):
 - $\{n \mid 4 < n < 10, n \in \mathbb{N}\}$
 - $\{2n - 1 \mid 4 < n < 10, n \in \mathbb{N}\}$
 - $\{x \mid 4 < x < 10, x \in \mathbb{R}\}$
 - $\{x \mid 4 < x^2 < 10\}$
 - $\{(x, y) \mid 0 < x < 2, 1 < y < 3, x \in \mathbb{R}, y \in \mathbb{R}\}$

- live exercises: draw set + define set from drawing
- intervals ($[a, b]$ & (a, b)); example: $[-2, 3)$
- sets unions & intersections
- examples:
 - $[0, 1) \cup (2, 3]$
 - $(0, 1) \cap [0.5, 2]$
 - $[-2, 5) \cap \mathbb{N}$
 - $[-2, 5) \cap \mathbb{Z}$
- live exercises:
 - compute and plot the intersection and union of $A = (1, 5)$ and $B = (3, 7]$.
 - compute and plot the intersection and union of $C = (-\infty, 2]$ and $D = [0, +\infty)$.
- quantifiers: \forall, \exists
- simple example: $S = \{1, 3, 5, 7, 8\}$; $\forall s \in S, s \leq 10$
- example (combined): "for any number, there is a (natural) number greater" ($\forall x \in \mathbb{R}, \exists n \in \mathbb{N} s.t. n > x$)
- live exercises:
 - $S = \{5, 6, 3, 1\}$ "all elements of S are positive"
 - $S = \{5, 6, 3, 1\}$ "there is an odd element in S "
 - $S = \{5, 6, 3, 1\}$ "there is an even element in S that is not a multiple of 4"
- implications $\implies, \impliedby, \iff$
- examples:
 - $x > 1 \implies x \text{ positive}$
 - $k \in \mathbb{Z} \impliedby k \in \mathbb{N}$
 - $k \in \mathbb{Z} \text{ and } k \geq 0 \iff k \in \mathbb{N}$
- live exercises:
 - "if x is positive, then it is the square of another number"
 - " n is pair is equivalent to $n = 2m$ for some integer m "
- extreme values (min,max vs inf,sup)
- live exercises:
 - find the extreme values of the set $A = \{x \in \mathbb{R} \mid x > 0\}$.
 - find the extreme values of the set $B = \{1 - \frac{1}{n} \mid n \in \mathbb{N}\}$.