Exercises Set 6

Paul Dubois

September 29, 2023

Abstract

As this is the last session, there will be no compulsory questions this time.

1 Lagrangian multiplier technique

1.1 Unconstrained optimization

Let $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$.
Find $(x^*, y^*) \in \mathbb{R}^2$ such that f reaches its minimum (i.e. $f(x^*, y^*) \leq f(x, y) \quad \forall (x, y) \in \mathbb{R}^2$).

1.2 Constrained optimization

Let $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$.
Suppose further that we want $3x + 5y = 2$.
Find $(x^*, y^*) \in \mathbb{R}^2$ such that $3x^* + 5y^* = 2$ and f reaches its minimum (i.e. $f(x^*, y^*) \leq f(x, y) \quad \forall (x, y) \in \mathbb{R}^2,\ 3x + 5y = 2$).

1.3 Lagrange multiplier

Let $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$.
Suppose further that we want $3x + 5y = 2$.
Let $\mathcal{L}(x, y, \lambda) = f(x, y) - \lambda(3x + 5y - 2)$.
Find the point where $\nabla f = 0$

2 Support Vector Machines

2.1 Theory

Define a line in $\mathbb{R}^2$ with parameters $\mathbf{w}$ and b defined by $\mathbf{w}.\mathbf{x} = b$ (or $\mathbf{w}.\mathbf{x} - b = 0$) for $\mathbf{x} \in \mathbb{R}^2$. This line cut the plane in 2 regions:

- $\mathbf{x} \in \mathbb{R}^2$ such that $\mathbf{w}.\mathbf{x} - b < 0$
- $\mathbf{x} \in \mathbb{R}^2$ such that $\mathbf{w}.\mathbf{x} - b > 0$

The goal is to find $\mathbf{w}$ and b such that all points of the first class are in the first region, and all points of the second class are in the second region.