

# Exercises Set 4

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## Abstract

Only the questions with a \* are compulsory (but do all of them!).

## 1 First Order Differential Equations

### Basics

**Exercise 0:** Solve the following first-order differential equation:

$$\frac{dy}{dx} = 3x^2 - 2x + 1$$

*Hint:* Integrate both sides with respect to  $x$  to find the solution.

**Exercise 1:** Solve the following first-order differential equation:

$$\frac{dy}{dx} = 5y$$

*Hint:* Calculate the derivative of  $\alpha e^{\lambda x}$  and adjust  $\alpha$  and  $\beta$ .  
Suppose that we also want  $y(0) = 2$ . Adjust further  $\alpha$  and  $\beta$ .

### Separable

**Exercise 2:** Solve the following separable differential equation:

$$\frac{dy}{dx} = \frac{x}{y}$$

*Hint:* Separate the variables  $x$  and  $y$ , and then integrate both sides to find the solution.

**Exercise 3:** Find the solution to the separable differential equation:

$$\frac{dy}{dx} = 2x^2 e^y$$

*Hint:* Separate the variables  $x$  and  $y$ , and then integrate both sides to determine the solution.

### Integrating Factor

**Exercise 4:** Solve the following linear first-order differential equation:

$$\frac{dy}{dx} + 2y = 4x$$

*Hint:* Use an integrating factor.

**Exercise 5: (\*)** Find the solution to the linear first-order differential equation:

$$\frac{dy}{dx} - \frac{1}{x}y = x^3$$

*Hint:* Use an integrating factor.

## 2 Second Order Differential Equations

### Basics

**Exercise 0:** Solve the following second-order differential equation:

$$\frac{d^2y}{dx^2} = e^x + 4\sin(2x) - 5x$$

*Hint:* Integrate twice both sides with respect to  $x$  to find the solution.

**Exercise 1:** Find one solution to the differential equation:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0$$

*Hint:* Assume a solution of the form  $y(x) = e^{rx}$  and find the values of  $r$  that satisfy the equation.

**Exercise 2:** Find the general solution to the differential equation:

$$\frac{d^2y}{dx^2} + 4y = 0$$

*Hint:* Assume a solution of the form  $y(x) = e^{rx}$  and find the values of  $r$  that satisfy the equation ( $r$  may be complex... what is exponential of a complex number?).

### Separable

**Exercise 3:** Solve the following separable differential equation:

$$y'' = (y')^2$$

*Hint:* Separate the variables.

### Non-homogeneous

**Exercise 4:** Consider the non-homogeneous linear second-order differential equation with constant coefficients:

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 6x^2 + 10x + 2$$

*Hint:* Begin by finding the general solution to the associated homogeneous equation. Then, use the method of undetermined coefficients to find a particular solution for the non-homogeneous part.

**Exercise 5:** Solve the following non-homogeneous linear second-order differential equation with constant coefficients:

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = 3e^{2t}$$

*Hint:* First, find the general solution to the homogeneous equation. Then, use the method of undetermined coefficients to find a particular solution for the non-homogeneous part.

### Boundary Conditions

**Exercise 6:** Consider the differential equation:

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 3y = 0$$

Find the particular solution of this differential equation that satisfies the boundary conditions  $y(0) = 1$  and  $y(2) = 5$ . *Hint:* First, solve the homogeneous equation, and then find a particular solution that satisfies the given boundary conditions.

**Exercise 7: (\*)** Given the differential equation:

$$\frac{d^2y}{dx^2} + 4y = 12x$$

Find the particular solution of this differential equation subject to the boundary conditions  $y(0) = 0$  and  $y'(0) = 2$ . *Hint:* Solve the homogeneous equation, find a particular solution for the non-homogeneous part, and apply the given boundary conditions.