Exercises Set 3

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Abstract

Only the questions with a * are compulsory (but do all of them!).

1 Fundamental Theorem of Calculus

Statement Let f be a continuous real-valued function defined on a closed interval $[a, b]$. Let F be the function defined, for all $x \in [a, b]$, by $F(x) = \int_a^b f(t)dt$.

Then F is uniformly continuous on $[a, b]$ and differentiable on the open interval (a, b), and $F'(x) = f(x)$ for all $xin\,(a, b)$ so F is an anti-derivative of f.

Generalization / Corollary Let $f(x)$ be a continuous function on the closed interval $[a, b]$, and let $F(x)$ be an anti-derivative of $f(x)$. Prove that

$$\int_a^b f(x)\,dx = F(b) - F(a).$$

Application Evaluate the following definite integral using the Fundamental Theorem of Calculus:

$$\int_0^{\pi/2} \sin(x)\,dx$$

Evaluate the following definite integral using the Fundamental Theorem of Calculus:

$$\int_1^4 \frac{1}{x^2}\,dx$$

2 Integration techniques

Substitution **Exercise 1:** Evaluate the following integral using the method of substitution:

$$\int e^{2x}\cos(2x)\,dx$$

Hint: Let $u = e^{2x}$ and then find du to perform the substitution.

Exercise 2: Evaluate the following integral using the method of substitution:

$$\int \frac{2x}{(x^2+1)^2}\,dx$$

Hint: Let $u = x^2 + 1$ and then find du to perform the substitution.

Exercise 3: Evaluate the following integral using the method of substitution:

$$\int \frac{1}{\sqrt{1-x^2}}\,dx$$

Hint: Let $u = 1 - x^2$ and then find du to perform the substitution.

Integration by Parts **Exercise A:** Compute the following integral using integration by parts:

$$\int x\ln(x)\,dx$$

Hint: Choose $u = \ln(x)$ and $dv = x\,dx$, and then use the integration by parts formula.

Exercise B: Find the value of the integral using integration by parts:

$$\int x^2 e^x\,dx$$

Hint: Choose $u = x^2$ and $dv = e^x\,dx$, and then use the integration by parts formula.

Exercise C: Compute the following integral using integration by parts:

$$\int x\cos(x)\,dx$$

Hint: Choose $u = x$ and $dv = \cos(x)\,dx$, and then use the integration by parts formula.