

# Exercises Set 3

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## Abstract

Only the questions with a \* are compulsory (but do all of them!).

## 1 Fundamental Theorem of Calculus

**Statement** Let  $f$  be a continuous real-valued function defined on a closed interval  $[a, b]$ . Let  $F$  be the function defined, for all  $x \in [a, b]$ , by  $F(x) = \int_a^x f(t) dt$ .

Then  $F$  is uniformly continuous on  $[a, b]$  and differentiable on the open interval  $(a, b)$ , and  $F'(x) = f(x)$  for all  $x \in (a, b)$  so  $F$  is an anti-derivative of  $f$ .

**Generalization / Corollary** Let  $f(x)$  be a continuous function on the closed interval  $[a, b]$ , and let  $F(x)$  be an anti-derivative of  $f(x)$ . Prove that

$$\int_a^b f(x) dx = F(b) - F(a).$$

**Application** Evaluate the following definite integral using the Fundamental Theorem of Calculus:

$$\int_0^{\pi/2} \sin(x) dx$$

Evaluate the following definite integral using the Fundamental Theorem of Calculus:

$$\int_1^4 \frac{1}{x^2} dx$$

## 2 Integration techniques