

# Exercises Set 6

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## Abstract

Only the questions with a \* are compulsory (but do all of them!).

## 1 Lagrangian multiplier technique



### 1.1 Unconstrained optimization

Let  $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$ .

Find  $(x^*, y^*) \in \mathbb{R}^2$  such that  $f$  reaches its minimum (i.e.  $f(x^*, y^*) \leq f(x, y) \quad \forall (x, y) \in \mathbb{R}^2$ ).

### 1.2 Constrained optimization

Let  $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$ .

Suppose further that we want  $3x + 5y = 2$ .

Find  $(x^*, y^*) \in \mathbb{R}^2$  such that  $3x^* + 5y^* = 2$  and  $f$  reaches its minimum (i.e.  $f(x^*, y^*) \leq f(x, y) \quad \forall (x, y) \in \mathbb{R}^2, 3x + 5y = 2$ ).

### 1.3 Lagrange multiplier

Let  $f(x, y) = 2x^2 - 3x + 4y^2 + 4y + 20$ .

Suppose further that we want  $3x + 5y = 2$ .

Let  $\mathcal{L}(x, y, \lambda) = f(x, y) - \lambda(3x + 5y - 2)$ .

Find the point where  $\nabla \mathcal{L} = 0$

## 2 Support Vector Machines

Define a line in  $\mathbb{R}^2$  with parameters  $\mathbf{w}$  and  $b$  defined by  $\mathbf{w} \cdot \mathbf{x} = b$  (or  $\mathbf{w} \cdot \mathbf{x} - b = 0$ ) for  $\mathbf{x} \in \mathbb{R}^2$ . This line cut the plane in 2 regions:

- $\mathbf{x} \in \mathbb{R}^2$  such that  $\mathbf{w} \cdot \mathbf{x} - b < 0$
- $\mathbf{x} \in \mathbb{R}^2$  such that  $\mathbf{w} \cdot \mathbf{x} - b > 0$

The goal is to find  $\mathbf{w}$  and  $b$  such that all points of the first class are in the first region, and all points of the second class are in the second region.