Exercises Set 2

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Abstract

Only the questions with a * are compulsory (but do all of them!).

1 Systems of Linear Equations

Find the Reduced Row Echelon Form of the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 6 & 2 & 3 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

(*) Solve the following linear system using Gaussian Elimination:

$$2x + y - z = 4$$
$$3x + 2y + z = 5$$
$$x - y + 3z = 7$$

Start by writing the augmented matrix for the system and perform the necessary row operations to find the solution.

2 Vector Spaces

Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{v_3} = \begin{bmatrix} 3 \\ 5 \\ 2 \end{bmatrix}$$

Show that $\mathbf{v_1}$, $\mathbf{v_2}$, and $\mathbf{v_3}$ are linearly independent.